II КОЛОКВИЈУМ

АНАЛИЗА ДЕФОРМАЦИЈА

1. a) $D = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 2 \\ 3 & 2 & 0 \end{bmatrix} \cdot 10^{-6}$ $\quad \vec{n} = \frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$

$n = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$ $\quad \boxed{\varepsilon_n = n^T D\, n}$

$$\varepsilon_n = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}_{1\times3} \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 2 \\ 3 & 2 & 0 \end{bmatrix}_{3\times3} \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{8}{3} & 2 & \frac{7}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} = 3{,}77 \cdot 10^{-6}$$

b) $D = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 3 \end{bmatrix} \cdot 10^{-6}$ $\quad n = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{2}\vec{j}$

$$\varepsilon_n = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{2}{\sqrt{2}} & +\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = 9{,}51 \cdot 10^{-6}$$

c)

$$D = \begin{bmatrix} 1 & 4 & 2 \\ 4 & -5 & 0 \\ 2 & 0 & 3 \end{bmatrix} \cdot 10^{-6} \qquad \vec{n} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$$

$$\varepsilon_n = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 4 & -5 & 0 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{7}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{5}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = 3{,}67 \cdot 10^{-6}$$

$$\frac{7}{3} - \frac{1}{3} + \frac{5}{3}$$

2. a)

$$D = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \\ 0 & 3 & 1 \end{bmatrix} \cdot 10^{-6} \qquad \vec{n} = \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{1}{3}\vec{k}$$

$$\vec{m} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$$

$$\frac{1}{2}\gamma_{nm} = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & \frac{5}{3} & \frac{7}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = 0{,}236 \cdot 10^{-6} \Rightarrow \boxed{\gamma_{nm} = 0{,}471}$$

b)

$$D = \begin{bmatrix} 1 & 4 & 2 \\ 4 & -1 & 3 \\ 2 & 3 & 0 \end{bmatrix} \cdot 10^{-6} \qquad \vec{n} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$$

$$\vec{m} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$$

$$\frac{1}{2}\gamma_{nm} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 4 & -1 & 3 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{6}{\sqrt{3}} & \frac{6}{\sqrt{3}} & \frac{5}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = 0{,}4082 \cdot 10^{-6}$$

3. a) $D = \begin{bmatrix} 3 & 6 & 9 \\ 6 & 12 & 18 \\ 9 & 18 & 27 \end{bmatrix}$

$$J_1 = 3 + 12 + 27 = 42$$

$$J_2 = \begin{vmatrix} 12 & 18 \\ 18 & 27 \end{vmatrix} + \begin{vmatrix} 3 & 9 \\ 9 & 27 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ 6 & 12 \end{vmatrix} = 0$$

$$J_3 = \begin{vmatrix} 3 & 6 & 9 \\ 6 & 12 & 18 \\ 9 & 18 & 27 \end{vmatrix} \begin{matrix} 3 & 6 \\ 6 & 12 \\ 9 & 18 \end{matrix} = 0$$

$$\varepsilon_k^3 - J_1 \varepsilon_k^2 + J_2 \varepsilon_k - J_3 = 0$$

$$\varepsilon^3 + 42 \varepsilon^2 = 0$$

$$\varepsilon^2 (\varepsilon - 42) = 0$$

$\boxed{\varepsilon = 0} \vee \boxed{\varepsilon = 42 \cdot 10^{-6}}$ - лин стање

б) $D = \begin{bmatrix} 3 & 6 & 9 \\ 6 & 12 & 18 \\ 9 & 18 & 3 \end{bmatrix} 10^{-6}$

$$J_1 = 15 + 3 = 18$$

$$J_2 = \begin{vmatrix} 12 & 18 \\ 18 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 9 \\ 9 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ 6 & 12 \end{vmatrix} = -360$$

$$J_3 = 0$$

$$\varepsilon_i^3 - J_1 \varepsilon^2 + J_2 \varepsilon - J_3 = 0$$

$$\varepsilon^3 - 18\varepsilon^2 - 360\varepsilon = 0$$

$$\varepsilon (\varepsilon^2 - 18\varepsilon - 360) = 0$$

$\varepsilon_1 = 0 \quad \varepsilon_2 = 30 \cdot 10^{-6} \quad \varepsilon_3 = -12 \cdot 10^{-6}$ → РАВНО СТАЊЕ НАПОНА

c) $D = \begin{bmatrix} 3 & 6 & 0 \\ 6 & 12 & 18 \\ 0 & 18 & 3 \end{bmatrix} \cdot 10^{-6}$

$J_1 = 18$

$J_2 = \begin{vmatrix} 12 & 18 \\ 18 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 0 \\ 0 & 3 \end{vmatrix} \pm \begin{vmatrix} 3 & 6 \\ 6 & 12 \end{vmatrix} = -279$

$J_3 = -972$

$\varepsilon^3 - J_1 \varepsilon^2 + J_2 \varepsilon - J_3 = 0$

$\varepsilon^3 - 18\varepsilon^2 - 279\varepsilon + 972 = 0$

d) $D = \begin{bmatrix} 3 & 2 & 9 \\ 2 & 12 & 6 \\ 9 & 6 & 3 \end{bmatrix} \cdot 10^{-6}$

$\left.\begin{matrix} J_1 = 18 \\ J_2 = -40 \\ J_3 = -768 \end{matrix}\right\} \rightarrow$ просторно стање деформ.

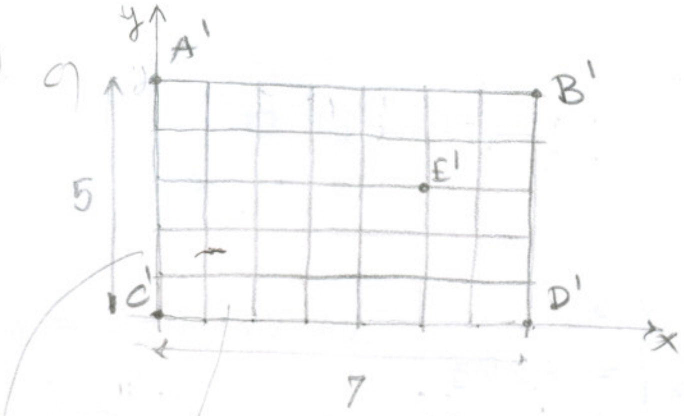
4. $x(x^\circ, t) = \begin{Bmatrix} x_1^\circ + (x_1^\circ)^2 t \\ x_2^\circ + 2(x_2^\circ x_3^\circ) t \\ x_3^\circ + 3t \end{Bmatrix}$

a) $u = \begin{Bmatrix} (x_1^\circ)^2 \\ 2(x_2^\circ x_3^\circ)^2 \\ 3 \end{Bmatrix} t$

$F_{ij} = u_{i,j} = \dfrac{\partial u_i}{\partial x_j}$

б) $F = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 2x_1^\circ & 0 & 0 \\ 0 & 4x_2^\circ (x_3^\circ)^2 & 4(x_2^\circ)^2 x_3 \\ 0 & 0 & 0 \end{bmatrix} t$

5. a)



$x_1 = x_1 + \frac{1}{6} x_4$

$x_y = x_y - \frac{1}{6} x_y$

$\vec{u} = \frac{x_1}{6}\vec{i} - \frac{x_2}{6}\vec{j}$

КОНСТИТУТИВНЕ ВЕЗЕ

1. a) $D = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot 10^{-6}$ $\quad \vec{n} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$ $\quad E = 210\,GPa$, $\nu = 0{,}25$

$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$

$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$

$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$

$\gamma_{xy} = \frac{\tau_{xy}}{G}$, $\quad G = \frac{E}{2(1+\nu)}$

$\gamma_{yz} = \frac{\tau_{yz}}{G}$

$\gamma_{zx} = \frac{\tau_{zx}}{G}$

$\sigma_x - \nu\sigma_y - \nu\sigma_z = E\varepsilon_x$

$-\nu\sigma_x + \sigma_y - \nu\sigma_z = E\varepsilon_y$

$-\nu\sigma_x - \nu\sigma_y + \sigma_z = E\varepsilon_z$

$\tau_{xy} = G\gamma_{xy} = G \cdot 0 = 0$

$\tau_{yz} = G \cdot \gamma_{yz} = G\, 2 \cdot 10^{-6} = 168000$

$\tau_{zx} = G \cdot \gamma_{zx} = G\, 2 \cdot 10^{-6} = 168000$

$\sigma_x - 0{,}25\sigma_y - 0{,}25\sigma_z = 210 \cdot 10^9 \cdot 2 \cdot 10^{-6}$

$-0{,}25\sigma_x + \sigma_y - 0{,}25\sigma_z = 210 \cdot 10^9 \cdot 2 \cdot 10^{-6}$

$-0{,}25\sigma_x - 0{,}25\sigma_y + \sigma_z = 210 \cdot 0$

$\sigma_x = 672000\ Pa$

$\sigma_y = 672000\ Pa$

$\sigma_z = 672000\ Pa$

$S = \begin{bmatrix} 672 & 0 & 168 \\ 0 & 672 & 168 \\ 168 & 168 & 672 \end{bmatrix} \cdot 10^3\ Pa$

$$f^{(n)} = \begin{bmatrix} 672 & 0 & 168 \\ 0 & 672 & 168 \\ 168 & 168 & 672 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 475{,}18 \\ 475{,}18 \\ 237{,}59 \end{bmatrix} \cdot 10^3 \ Pa$$

b)

$$D = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot 10^{-6} \qquad \vec{n} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k} \qquad E = 200\,GPa, \ \nu = 0{,}2$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}, \quad G = \frac{E}{2(1+\nu)}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] \qquad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

$$\sigma_x - \nu\sigma_y - \nu\sigma_z = E \cdot \varepsilon_x$$
$$-\nu\sigma_x + \sigma_y - \nu\sigma_z = E \cdot \varepsilon_y$$
$$-\nu\sigma_x + \nu\sigma_y + \sigma_z = E \cdot \varepsilon_z$$

$$\tau_{xy} = G\gamma_{xy} = G \cdot 4 \cdot 10^{-6} = 333333{,}33$$
$$\tau_{yz} = G\gamma_{yz} = G \cdot 2 \cdot 10^{-6} = 166666{,}67$$
$$\tau_{zx} = G\gamma_{zx} = G \cdot 0 = 0$$

$$\sigma_x - 0{,}2\sigma_y - 0{,}2\sigma_z = 200 \cdot 10^9 \cdot 2 \cdot 10^{-6}$$
$$-0{,}2\sigma_x + \sigma_y - 0{,}2\sigma_z = 200 \cdot 10^9 \cdot 0$$
$$-0{,}2\sigma_x - 0{,}2\sigma_y + \sigma_z = 0$$

$$\frac{1}{2}\gamma_{xy}$$

$$\sigma_x = 444444{,}44$$
$$\sigma_y = 111111{,}11$$
$$\sigma_z = 111111{,}11$$

$$S = \begin{bmatrix} 444 & 333 & 0 \\ 333 & 111 & 166 \\ 0 & 166 & 111 \end{bmatrix} \cdot 10^3 \ Pa$$

$$f^{(n)} = \begin{bmatrix} 444 & 333 & 0 \\ 333 & 111 & 166 \\ 0 & 166 & 111 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} 448{,}60 \\ 352{,}18 \\ 159{,}93 \end{bmatrix} \cdot 10^3 \ Pa$$

(2.)

a) $S = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$ MPa $\quad \vec{n} = \frac{1}{\sqrt{3}}\vec{i} + \frac{3}{5\sqrt{3}}\vec{j} + \frac{4}{5\sqrt{3}}\vec{k}$

$E = 70$ GPa

$\nu = 0{,}15$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_x = \frac{1}{E}\left[3\cdot 10^6 \cancel{-0{,}15\,(10^6 - 10^6)}^{\to 0}\right] = 4{,}28\cdot 10^{-5}$$

$\sigma_y - \nu\,(\sigma_z + \sigma_x)$

$$\varepsilon_y = \frac{1}{E}\left[10^6 - 0{,}15\,(-10^6 + 3\cdot 10^6)\right] = 10^{-5}$$

$$\varepsilon_z = \frac{1}{E}\left[-10^6 - 0{,}15\,(3\cdot 10^6 + 10^6)\right] = -2{,}28\cdot 10^{-5}$$

$$\frac{1}{E} = \frac{1}{70\cdot 10^9} \qquad G = \frac{E}{2(1+\nu)} = \frac{70\cdot 10^9}{2(1+0{,}15)}$$

$$\frac{1}{2}\gamma_{xy} = \frac{1}{2}\frac{\tau_{xy}}{G} = \frac{1}{2}\,\frac{0}{G} = 0$$

$$\frac{1}{2}\gamma_{yz} = \frac{1}{2} = \frac{1}{2}\,\frac{0}{G} = 0$$

$$\frac{1}{2}\gamma_{zx} = \frac{1}{2} = \frac{1}{\cancel{2}}\,\frac{\cancel{2}}{G} = 3{,}28\cdot 10^{-5}$$

$$D = \begin{bmatrix} 4{,}28 & 0 & 3{,}28 \\ 0 & 1 & 0 \\ 3{,}28 & 0 & -2{,}28 \end{bmatrix}\cdot 10^{-5}$$

$$\varepsilon_n = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{3}{5\sqrt{3}} & \frac{4}{5\sqrt{3}} \end{bmatrix} \begin{bmatrix} 4{,}28 & 0 & 3{,}28 \\ 0 & 1 & 0 \\ 3{,}28 & 0 & -2{,}28 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{3}{5\sqrt{3}} \\ \frac{4}{5\sqrt{3}} \end{bmatrix} =$$

$$\begin{bmatrix} 7{,}0159 & 1{,}0392 & -1{,}265 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{3}{5\sqrt{3}} \\ \frac{4}{5\sqrt{3}} \end{bmatrix} = 3{,}577\cdot 10^{-5}$$

b) $S = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ MPa $\qquad \vec{n} = \frac{3}{5\sqrt{2}}\vec{i} + \frac{4}{5\sqrt{2}}\vec{k}$

$E = 80\,GPa, \quad \nu = 0{,}25$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x + \nu(\sigma_y + \sigma_z)\right] = \frac{1}{E}\left[-2\cdot 10^6 - 0{,}25\,\cancel{(1-1)\cdot 10^6}\right] = -2{,}5\cdot 10^{-5}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] = \frac{1}{E}\left[10^6 - 0{,}25(-3)\cdot 10^6\right] = 2{,}1875\cdot 10^{-5}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = \frac{1}{E}\left[-10^6 - 0{,}25(-10^6)\right] = -3{,}375\cdot 10^{-6}$$

$$\frac{1}{2}\gamma_{xy} = \frac{1}{2}\frac{\tau_{xy}}{G} = 0 \qquad G = \frac{E}{2(1+\nu)}$$

$$\frac{1}{2}\gamma_{yz} = \frac{1}{2}\frac{\tau_{yz}}{G} = 1{,}5625\cdot 10^{-5}$$

$$\frac{1}{2}\gamma_{zx} = \frac{1}{2}\frac{\tau_{zx}}{G} = 0$$

$$D = \begin{bmatrix} -25 & 0 & 0 \\ 0 & 21{,}9 & 15{,}6 \\ 0 & 15{,}6 & -3{,}4 \end{bmatrix}\cdot 10^{-6}$$

$$\varepsilon_n = \left[\frac{3}{5\sqrt{2}} \quad 0 \quad \frac{4}{5\sqrt{2}}\right] \begin{bmatrix} -25 & 0 & 0 \\ 0 & 21{,}9 & 15{,}6 \\ 0 & 15{,}6 & -3{,}4 \end{bmatrix} \begin{bmatrix} \frac{3}{5\sqrt{2}} \\ 0 \\ \frac{4}{5\sqrt{2}} \end{bmatrix} = -45{,}048\cdot 10^{-6}$$

3. $S = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} MPa \quad \varepsilon_x = 3 \cdot 10^{-6} \quad \varepsilon_y = -1 \cdot 10^{-6}$

$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$

$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$

$\sigma_x - \nu(\sigma_y + \sigma_z) = E \cdot \varepsilon_x$

$\sigma_y - \nu(\sigma_z + \sigma_x) = E \cdot \varepsilon_y$

$3 \cdot 10^{-6} E + \nu\, 2 \cdot 10^6 = 4 \cdot 10^6$

$-10^{-6} E + \nu\, 5 \cdot 10^6 = 10^6$

$E = 1{,}05{,}9 \cdot 10^{12}\, Pa$

$\nu = 0{,}412$

$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = -1 \cdot 10^{-6}$

$\frac{1}{2}\gamma_{xy} = \frac{1}{2}\frac{\tau_{xy}}{G} = 0$

$\frac{1}{2}\gamma_{yz} = \frac{1}{2}\frac{\tau_{yz}}{G} = 0$

$\frac{1}{2}\gamma_{zx} = \frac{1}{2}\frac{\tau_{zx}}{G} = 2{,}67 \cdot 10^{-6}$

$G = \frac{E}{2(1+\nu)}$

$D = \begin{bmatrix} 3 & 0 & 2{,}67 \\ 0 & -1 & 0 \\ 2{,}67 & 0 & -1 \end{bmatrix} 10^{-6}$

4.

$\varepsilon_{n1} = 20 \cdot 10^{-6}$ $\alpha_1 = 20^\circ$

$\varepsilon_{n2} = 5 \cdot 10^{-6}$ $\alpha_2 = 65^\circ$

$\varepsilon_{n3} = -15 \cdot 10^{-6}$ $\alpha_3 = 110^\circ$

$$\varepsilon_{n1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cdot \cos(2 \cdot 20) + \frac{1}{2} \gamma_{xy} \sin(2 \cdot 20) \quad / \cdot 2$$

$$\varepsilon_{n2} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cdot \cos(2 \cdot 65^\circ) + \frac{1}{2} \gamma_{xy} \sin(2 \cdot 65) \quad / \cdot 2$$

$$\varepsilon_{n3} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos(2 \cdot 110^\circ) + \frac{1}{2} \gamma_{xy} \sin(2 \cdot 110^\circ) \quad / \cdot 2$$

$$2\varepsilon_{n1} = \varepsilon_x + \varepsilon_y + (\varepsilon_x - \varepsilon_y) \cos 40 + \gamma_{xy} \sin 40$$

$$2\varepsilon_{n2} = \varepsilon_x + \varepsilon_y + (\varepsilon_x - \varepsilon_y) \cos 130 + \gamma_{xy} \sin 130$$

$$2\varepsilon_{n3} = \varepsilon_x + \varepsilon_y + (\varepsilon_x - \varepsilon_y) \cos 220^\circ + \gamma_{xy} \sin 220^\circ$$

$$\varepsilon_x (1 + \cos 40) + \varepsilon_y (1 - \cos 40) + \gamma_{xy} \sin 40 = 2\varepsilon_{n1}$$

$$\varepsilon_x \cdot (1 + \cos 130) + \varepsilon_y (1 - \cos 130) + \gamma_{xy} \sin 130 = 2\varepsilon_{n2}$$

$$\varepsilon_x (1 + \cos 220) + \varepsilon_y (1 - \cos 220) + \gamma_{xy} \sin 220 = 2\varepsilon_{n3}$$

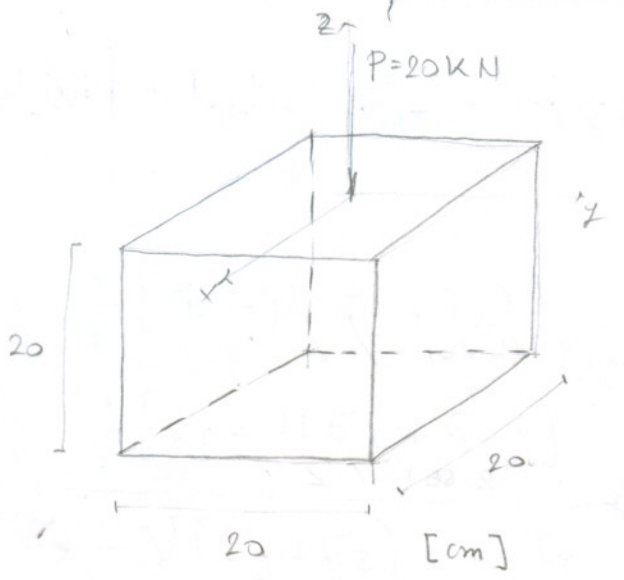
$\varepsilon_x = 14{,}30 \cdot 10^{-6}$

$\varepsilon_y = -9{,}30 \cdot 10^{-6}$

$\gamma_{xy} = 26{,}33 \cdot 10^{-6}$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{1}{2}\gamma_{xy}\right)^2} \Rightarrow \begin{array}{l} \varepsilon_1 = 20{,}18 \cdot 10^{-6} \\ \varepsilon_2 = -15{,}18 \cdot 10^{-6} \end{array}$$

5.



$\Delta l_z = 3\,mm$

$\Delta l_x = \Delta l_y = 1\,mm$

$\sigma_z = \dfrac{P}{A_0} = \dfrac{20\,kN}{400\,cm^2} \cdot 10 = 0,5\ MPa$

$\varepsilon_z = \dfrac{\Delta l}{l_0} = \dfrac{-3}{200} = -0,015$

$\varepsilon_y = \varepsilon_x = \dfrac{1}{200} = 5 \cdot 10^{-3}$

$\varepsilon_y = -\nu \varepsilon_z$

$\nu = -\dfrac{\varepsilon_y}{\varepsilon_z} = -\dfrac{5 \cdot 10^{-3}}{-0,015} = \dfrac{1}{3}$

$E = \dfrac{\sigma_z}{\varepsilon_z} = 33,33\ MPa$

$G = \dfrac{E}{2(1+\nu)} = 12,50\ MPa$

$K = \dfrac{E}{3(1-2\nu)} = \underline{\underline{33,33\ MPa}}$

6.

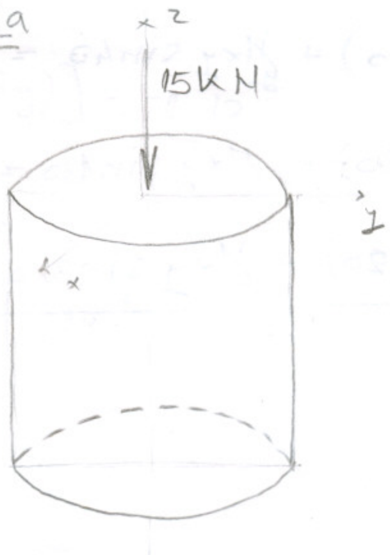
$d = 20\,cm$

$H = 20\,cm$

$P = 15\,kN$

$\Delta l_z = 0,2\,mm$

$\Delta l_y = 0,07\,mm$



$\sigma_z = \dfrac{P}{A_0} = \dfrac{15\,kN}{100\pi} \cdot 10 = \underline{\underline{0,477\,MPa}}$

$\varepsilon_z = \dfrac{-0,2}{200} = -10^{-3}$

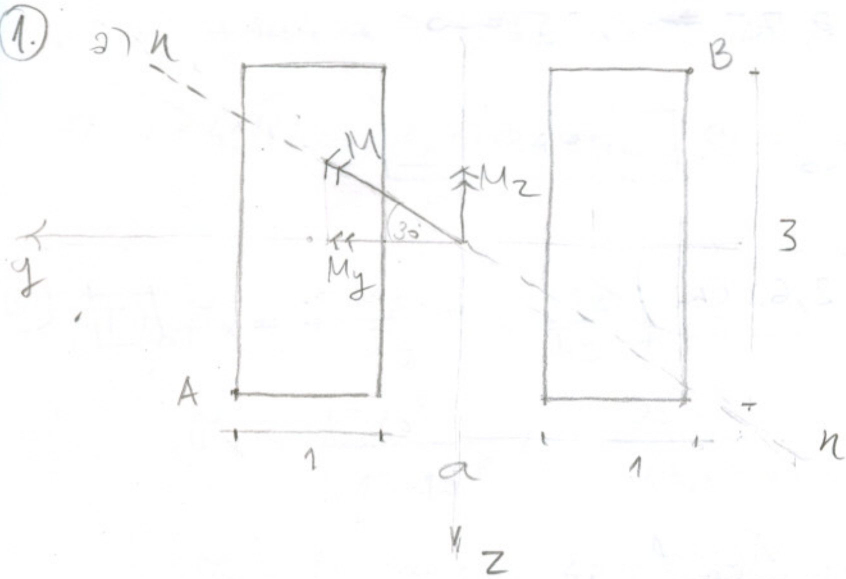
$\varepsilon_y = \dfrac{0,07}{200} = 3,5 \cdot 10^{-4}$

$E = \dfrac{\sigma_z}{\varepsilon_z} = \dfrac{0,477}{10^{-3}} = \underline{\underline{477\,MPa}}$

$\nu = -\dfrac{\varepsilon_y}{\varepsilon_z} = \dfrac{-3,5 \cdot 10^{-4}}{-10^{-3}} = \underline{\underline{0,35}}$

7. ЧИСТО ПРАВО САВИЈАЊЕ – ЧИСТО КОСО САВИЈАЊЕ

1. a)



$$\sigma_x = +\frac{M_y}{I_y} z + \frac{M_z}{I_z} y$$

$$\sigma_x = 0 \Rightarrow \frac{M_y}{I_y} z = -\frac{M_z}{I_z} y$$

$$z = -\frac{M_z}{I_z}\frac{I_y}{M_y} y = -\frac{M_z}{M_y}\frac{I_y}{I_z} y$$

$$z = -\frac{\cancel{M}\sin\varphi}{\cancel{M}\cos\varphi}\frac{I_y}{I_z} y$$

$$\boxed{z = -\operatorname{tg}\varphi \frac{I_y}{I_z} y}$$

$$-\operatorname{tg}\varphi \frac{I_y}{I_z} = -\frac{\sqrt{3}}{3}$$

$$\cancel{-\frac{\sqrt{3}}{3}} \frac{I_y}{I_z} = \cancel{-\frac{\sqrt{3}}{3}} \Rightarrow \boxed{I_y = I_z}$$

$$I_y = 2\,\frac{1}{12}\,1\cdot 3^3 = \underline{\underline{4{,}5\ cm^4}}$$

$$I_z = 2\left(\frac{1}{12}\cdot 1^3\cdot 3 + \left(\frac{a}{2} + 0{,}5\right)^2\cdot 3\right) = 4{,}5$$

$$0{,}25 \quad + \left(\frac{a^2}{4} + 2\frac{a}{2}\cdot 0{,}5 + 0{,}5^2\right)3 = 2{,}25$$

$$0{,}25+\left(\frac{a^2}{4}+\cancel{2}\frac{a}{\cancel{2}}0{,}5+0{,}5^2\right)\cdot 3=2{,}25$$

$$0{,}25+\frac{3}{4}a^2+\frac{3}{2}a+0{,}75-2{,}25=0$$

$$\frac{3}{4}a^2+\frac{3}{2}a-\frac{5}{4}=0$$

$$\boxed{a=0{,}63\text{ cm}} \quad \vee \quad \boxed{a=-2{,}63\text{ cm}}$$

b) $$\sigma_x=+\frac{M_y}{I_y}z+\frac{M_z}{I_z}y$$

$$\sigma_x=0 \Rightarrow \frac{M\cos\varphi}{I_y}z=-\frac{M\sin\varphi}{I_z}y$$

$$z=-\frac{\sin\varphi}{I_z}y\,\frac{I_y}{\cos\varphi}$$

$$z=-\operatorname{tg}\varphi\,\frac{I_y}{I_z}\,y$$

$$-\operatorname{tg}\varphi\,\frac{I_y}{I_z}=-1$$

$$\cancel{-1}\,\frac{I_y}{I_z}=\cancel{-1} \Rightarrow \boxed{I_y=I_z}$$

$$I_y=\frac{1}{12}\left((3+2a)^3\cdot 4-4a^3\cdot 1\right)$$

$$I_z=\frac{1}{12}\left(4^3(3+2a)-4a\right)$$

$$(3+2a)^3\cdot 4-4a^3=4^3(3+2a)-4a$$

$$(3+2a)(3+2a)^2\cdot 4-4a^3=192+128a-4a$$

$$4(3+2a)(9+12a+4a^2)-4a^3=192+128a-4a$$

$$4(27+36a+12a^2+18a+24a^2+8a^3)-4a^3-192-128a+4a$$

$108 + 144a + 48a^2 + 72a + 36a^2 + 32a^3 - 4a^3 - 192 - 128a + 4a = 0$

$28a^3 + 144a^2 + 32a - 84 = 0$

$a_1 = \cancel{-4{,}186\,cm}$ $\boxed{a_2 = 0{,}494\,cm}$ $a_3 = \cancel{-4{,}45\,cm}$

2. $\boxed{a)}$ $\sigma_x = +\frac{M_y}{I_y} z + \frac{M_z}{I_z} y$

$\sigma_x = \frac{15 \cdot 10^3}{12 \cdot 10^{-8}} z - \frac{20 \cdot 10^3}{46{,}833 \cdot 10^{-8}} y$

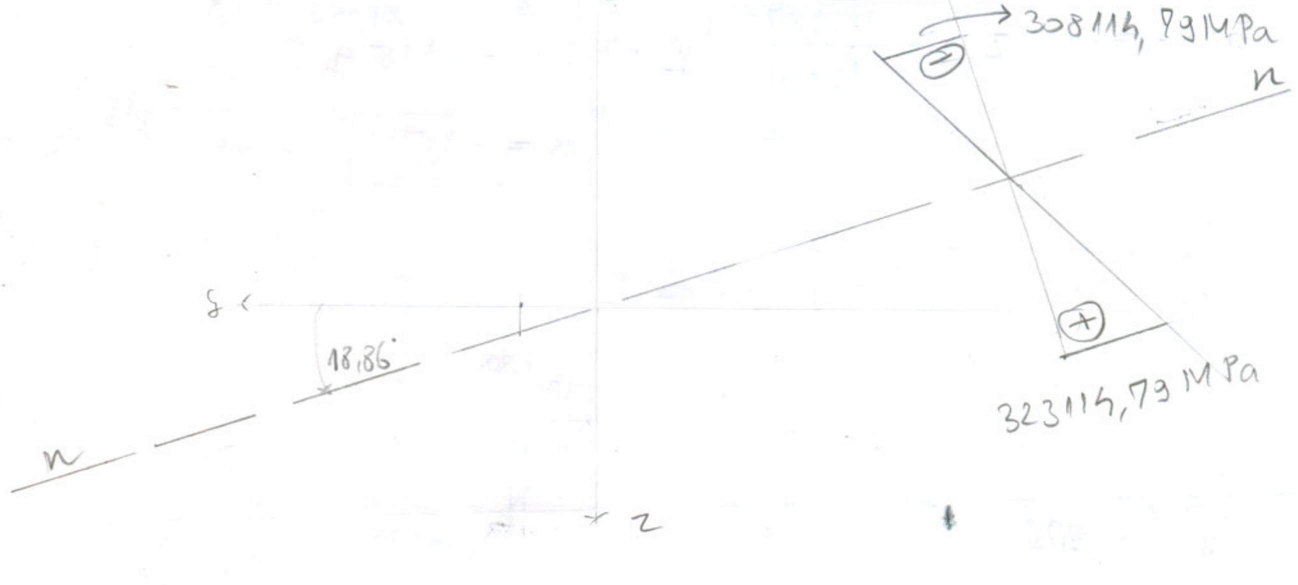
$\sigma_x = (125000z - 42704{,}93y)$ MPa $\quad (1{,}25 \cdot 10^{11} z - 4{,}27 \cdot 10^{10} y)$ Pa

b) $W_y = \frac{I_y}{z_{max}} = \frac{12}{1{,}56} = 7{,}69\,cm^3$

$W_z = \frac{I_z}{y_{max}} = \frac{46{,}833}{3} = 15{,}611\,cm^3$

c) $\sigma_x = 0 \Rightarrow 125000z - 42704{,}93y = 0$

$z = 0{,}342y \Rightarrow \alpha = 18{,}86$



d) $\sigma_{x,A} = 125000(-1{,}44) - 42707{,}93(-3) = -51876{,}21$ MPa

e) $\sigma_{x,B} = (125000\, z_B - 42704{,}93\, y_B)$ MPa

$\sigma_{x,B} = 125000 \cdot 1{,}56 - 42704{,}93 \cdot (+3) = \underline{\underline{323114{,}79 \text{ MPa}}}$

$\tau_{max} = \frac{1}{2} \sigma_{x,B} = \underline{\underline{161557{,}395 \text{ MPa}}}$

$\sigma_{x,C} = 125000 \cdot (-1{,}44) - 42704{,}93 \cdot 3 = -308114{,}79$ MPa

$\boxed{b1)}$ $\Rightarrow \sigma_x = + \frac{M_y}{I_y} z + \frac{M_z}{I_z} y$

$\sigma_x = \frac{20 \cdot 10^3}{10{,}0481 \cdot 10^{-8}} z + \frac{5 \cdot 10^3}{38{,}4996 \cdot 10^{-8}} y$

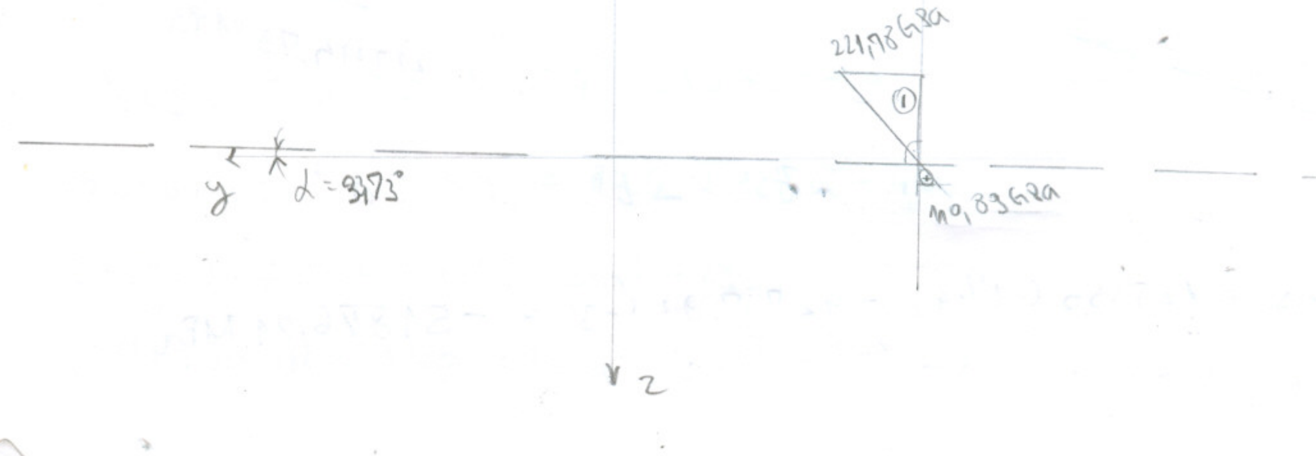
$\boxed{\sigma_x = (199{,}042\, z + 12{,}987\, y) \text{ GPa}}$

b) $W_y = \frac{I_y}{z_{max}} = \frac{10{,}0481}{1{,}69} = \underline{\underline{5{,}946 \text{ cm}^3}}$

$W_z = \frac{I_z}{y_{max}} = \frac{38{,}4996}{3} = \underline{\underline{12{,}833 \text{ cm}^3}}$

c) $\sigma_x = 0 \Rightarrow \quad z = -\frac{12{,}987}{199{,}042} y \Rightarrow z = -0{,}065\, y$

$\boxed{\alpha = -3{,}73^\circ}$



d) A (-3 ; 0,69)

$\sigma_{x,A} = 199,042 \cdot 0,59 + 12,987\,(-3) = \underline{\underline{98,38\ GPa}}$

e) B (3 , -1,31)

$\sigma_{x,B} = 199,042 \cdot (-1,31) + 12,987 \cdot 3 = \underline{\underline{-221,78\ GPa}}$

$\sigma_{x,max} = \underline{\underline{221,78\ GPa}}$

$\tau_{max} = \frac{1}{2}\sigma_{x\,max} = \underline{\underline{110,89\ GPa}}$ $\qquad \sigma_x = -\frac{M_y}{I_y} z + \frac{M_z}{I_z} y$

$\boxed{3)}$ a) $\sigma_x = -\frac{M_y}{I_y} z + \frac{M_z}{I_z} y$

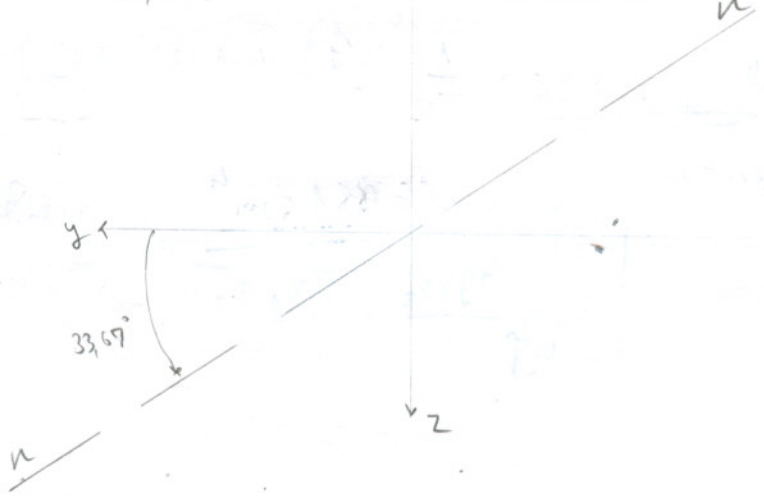
$\sigma_x = -\frac{10 \cdot 10^3}{10,2130 \cdot 10^{-8}} z + \frac{15 \cdot 10^3}{23 \cdot 10^{-8}} y$

$\boxed{\sigma_x = (-97,914\,z + 65,22\,y)\ GPa}$

b) $W_y = \frac{I_y}{z_{max}} = \frac{10,2130}{1,69} = \underline{\underline{6,043\ cm^3}}$

$W_z = \frac{I_z}{y_{max}} = \frac{23}{3} = \underline{\underline{7,67\ cm^3}}$

c) $z = \frac{65,22}{97,914} y \Rightarrow \boxed{z = 0,666\,y} \Rightarrow \boxed{\alpha = 33,67^\circ}$



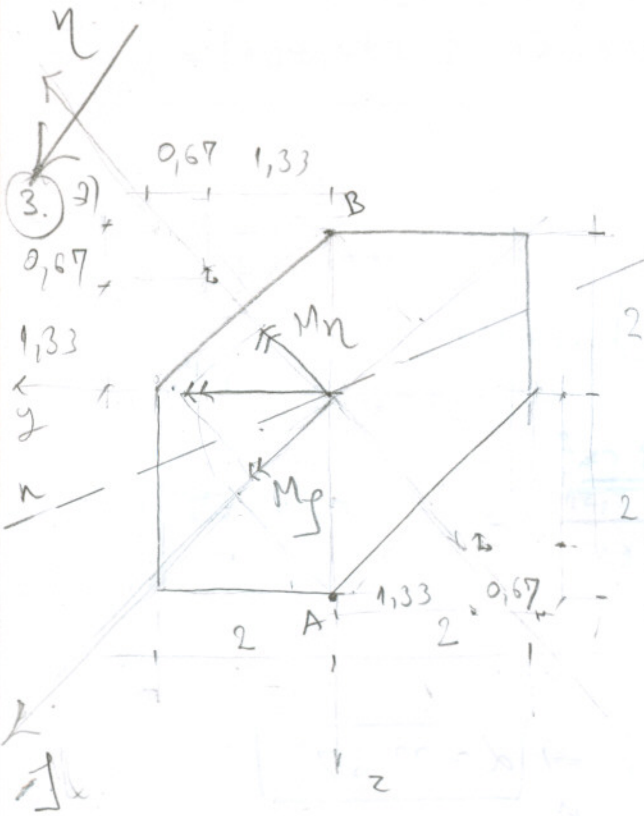
d) $\sigma_x = (-97{,}914\,z + 65{,}22\,y)$ GPa

$$A\begin{pmatrix} y & & z \\ 3 & ; & -1{,}31 \end{pmatrix}$$

$\sigma_{x,A} = -97{,}914\,(-1{,}31) + 65{,}22 \cdot 3 = \underline{\underline{323{,}327 \text{ GPa}}}$

e) $B\begin{pmatrix} y & & z \\ -3 & ; & 1{,}69 \end{pmatrix}$

3. a)



$I_y = \frac{1}{12}4^4 - 2\left(\frac{1}{36}2^3 \cdot 2 + 1{,}33^2 \cdot 2\right) =$

$= \underline{\underline{13{,}369 \text{ cm}^4}}$

$I_z = \frac{1}{12}4^4 - 2\left(\frac{1}{36}\cdot 2^4 + 1{,}33^2\, 2\right)$

$= \underline{\underline{13{,}369 \text{ cm}^4}}$

$I_{yz} = \frac{1}{72}\cdot 2^4 + (1{,}33)\cdot(-1{,}33)\cdot 2 +$

$\frac{1}{72}\cdot 2^4 + (-1{,}33)(1{,}33)\cdot 2 \Big) =$

$= \underline{\underline{+6{,}631 \text{ cm}^4}}$ 6,6867

x: 560,5697

y: 626,2255

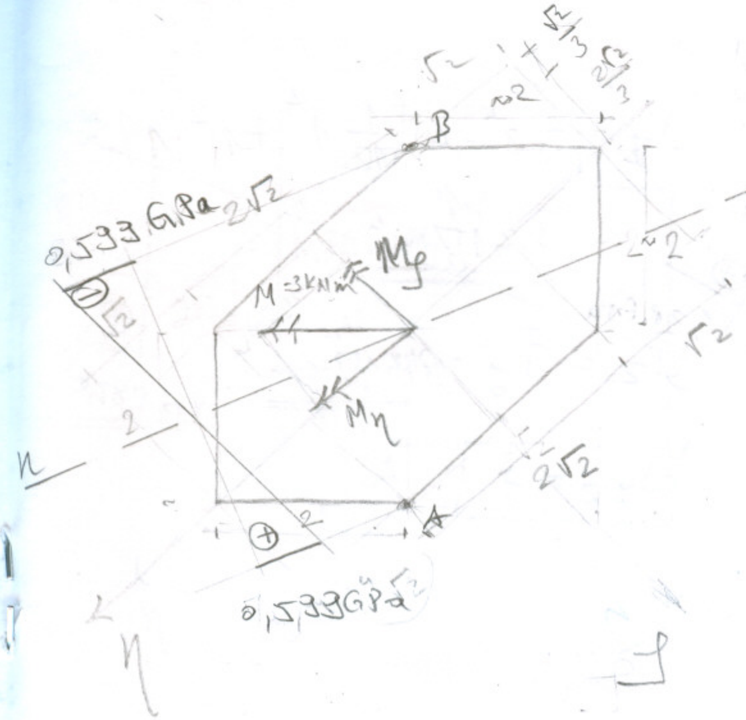
$\boxed{\begin{array}{l} I_\eta = 20 \text{ cm}^4 \\ I_\zeta = 6{,}738 \text{ cm}^4 \end{array}}$

$2\alpha = 90$

$\alpha = 45$

$tg\, 2\alpha_1 = \frac{\mp 2 I_{yz}}{I_y - I_z} = -\infty$

$\boxed{I_{\eta,\zeta} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}}$



$$I_\eta = \frac{1}{12}(2\sqrt{2})^3 \cdot 4\sqrt{2} - 4\left(\frac{1}{36}(\sqrt{2})^4 + \left(\frac{2\sqrt{2}}{3}\right)^2 \cdot 1\right) = 6{,}67\,cm^4$$

$$I_\zeta = \frac{1}{12}(4\sqrt{2})^3 \cdot 2\sqrt{2} - 4\left(\frac{1}{36}(\sqrt{2})^4 + \left(\frac{2\sqrt{2}}{3}\right)^2 \cdot 1\right) = 20\,cm^4$$

$$I_{\eta\zeta} = 0$$

x: 598,0858

y: 395,4986

$$2a^2 = 4$$

$$a = \sqrt{\frac{4}{2}} = \sqrt{2}$$

$$M_\eta = 3 \cdot \frac{\sqrt{2}}{2} = M_\zeta = 2{,}12\,kNm$$

$$\sigma_x = +\frac{M_\eta}{I_\eta}\zeta + \frac{M_\zeta}{I_\zeta}\eta$$

$$\sigma_x = 0 \Rightarrow \frac{M_\eta}{I_\eta}\zeta = -\frac{M_\zeta}{I_\zeta}\eta$$

$$\sigma_x = +\frac{M_\eta}{I_\eta}\zeta - \frac{M_\zeta}{I_\zeta}\eta$$

$$\zeta = \frac{M_\zeta}{I_\zeta}\frac{I_\eta}{M_\eta}\eta = \frac{I_\eta}{I_\zeta}\frac{M \cdot 0{,}7071}{M \cdot 0{,}7071}\eta$$

$$\zeta = \frac{20}{6{,}738}\eta \Rightarrow \zeta = 2{,}968\,\eta \Rightarrow \alpha = 71{,}38°$$

$A(-\sqrt{2}; \sqrt{2})$

$B(\sqrt{2}; -\sqrt{2})$

$$\sigma_{x,A} = \frac{2{,}12}{20}\sqrt{2} - \frac{2{,}12}{6{,}738}(-\sqrt{2}) = 0{,}595\,GPa$$

$$\sigma_{x,B} = \frac{2{,}12}{20}(-\sqrt{2}) - \frac{2{,}12}{6{,}738}\sqrt{2} = -0{,}595\,GPa$$

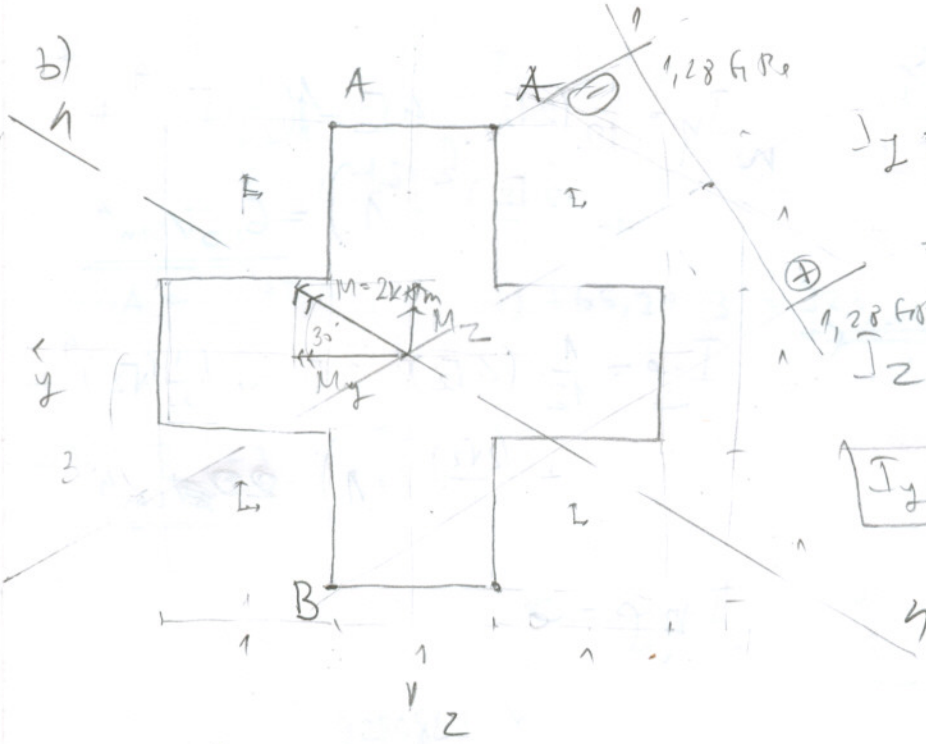
$$\zeta = -\frac{M_\zeta}{M_\eta}\frac{I_\eta}{I_\zeta}\eta = -\frac{2{,}12}{2{,}12}\frac{6{,}67}{20}\eta \Rightarrow \boxed{\zeta = -0{,}3335\,\eta}$$

$$\Rightarrow \boxed{\alpha = -18{,}44°}$$

$$\sigma_{x,A} = \frac{2{,}12 \cdot 10^3}{6{,}67 \cdot 10^{-8}}(\sqrt{2}) \cdot 10^{-2} + \frac{2{,}12 \cdot 10^3}{20 \cdot 10^{-8}}\sqrt{2} \cdot 10^{-2} = 0{,}599\,GPa$$

$$\sigma_{x,B} = \frac{2{,}12 \cdot 10^3}{6{,}67 \cdot 10^{-8}}(-\sqrt{2}) \cdot 10^{-2} + \frac{2{,}12}{20 \cdot 10^{-8}}(-\sqrt{2}) \cdot 10^{-2} = -0{,}599\,GPa$$

b)



$$J_y = \frac{1}{12}3^4 - 4\left(\frac{1}{12}1^4 + 1^2 \cdot 1\right) =$$

$$= \underline{2{,}417\ cm^4}$$

$$J_z = J_y = \underline{2{,}417\ cm^4}$$

$$\boxed{J_{yz} = 0}$$

$$\sigma_x = +\frac{M_y}{I_y} z + \frac{M_z}{I_z} y \qquad M_y = 2 \cdot \cos 30 = \sqrt{3}$$

$$M_z = 2 \cdot \sin 30 = 1$$

$$\sigma_x = 0 \Rightarrow \frac{M_y}{I_y} z = -\frac{M_z}{I_z} \cdot y$$

$$z = -\frac{M_z}{I_z} \cdot \frac{I_y}{M_y} \cdot y = -\underbrace{\frac{I_y}{I_z}}_{1} \underbrace{\frac{M \sin\theta}{M \cos\theta}}_{tg30} \cdot y$$

$$A\left(\overset{y}{-0{,}5}, \overset{z}{-1{,}5}\right) \Rightarrow \boxed{z = -\frac{\sqrt{3}}{3} y}$$

$$B(+0{,}5, 1{,}5)$$

$$\sigma_{x,A} = \frac{M_y}{I_y} z_A + \frac{M_z}{I_z} y_A$$

$$\sigma_{x,A} = \frac{\sqrt{3}}{2{,}417}(-1{,}5) + \frac{1}{2{,}417} \cdot 0{,}5 = \underline{-1{,}28\ GPa}$$

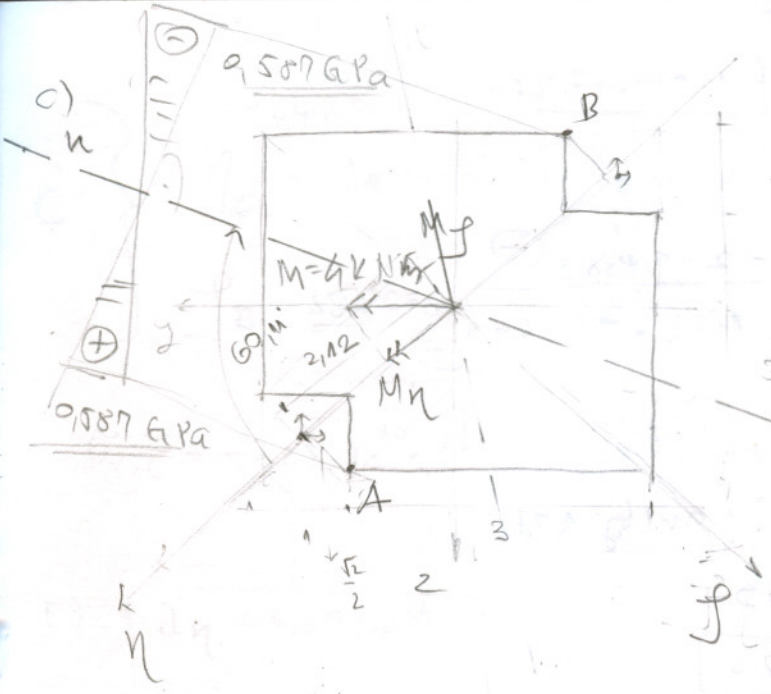
$$\sigma_{x,B} = \frac{\sqrt{3}}{2{,}417} 1{,}5 + \frac{1}{2{,}417}(-0{,}5) = \underline{1{,}28\ GPa}$$

$A(0{,}5; -1{,}5)$

$$\sigma_{x,A} = -\frac{M_y}{I_y} z_A + \frac{M_z}{I_z} y_A = -\frac{\sqrt{3}}{2{,}417}(-1{,}5) + \frac{1}{2{,}417} \cdot 0{,}5 = 1{,}28$$

$$\sigma_{x,B} = +\frac{M_y}{I_y} z_B + \frac{M_z}{I_z} y_B =$$

c)



$I_y = \frac{1}{12}4^4 - 2\left(\frac{1}{12}\cdot 1^4 + 1{,}5^2\cdot 1\right) =$

$= \underline{16{,}67\,cm^4}$

$I_z = \frac{1}{12}4^4 - 2\left(\frac{1}{12}1^4 + 1{,}5^2\cdot 1\right) =$

$= \underline{16{,}67\,cm^4}$

$I_{yz} = 0 - (1{,}5(+1{,}5)\cdot 1 + (1{,}5)(+1{,}5)\cdot 1)$

$\boxed{I_{yz} = -4{,}5\,cm^4}$

$I_{\eta,\xi} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} \Rightarrow I_\eta = \underline{21{,}17\,cm^4}$

$I_\xi = \underline{12{,}17\,cm^4}$

x: 457,1651
y: 716,6310

3>pinepata:

$\boxed{\begin{array}{l} I_x = 21{,}1667 \\ I_y = 12{,}1667 \end{array}}$

$tg\,2\alpha_1 = \frac{-2I_{yz}}{I_y - I_z} = \frac{9}{0} = +\infty \Rightarrow 2\alpha_1 = 90° \Rightarrow \boxed{\alpha_1 = 45°}$

$\sigma_x = + \frac{M_\eta}{I_\eta}\xi + \frac{M_\xi}{I_\xi}\eta$

$\sigma_x = 0 \Rightarrow \xi = -\frac{M_\xi}{I_\xi}\cdot\frac{I_\eta}{M_\eta}\eta = \frac{I_\eta}{I_\xi}\frac{M\cdot\frac{\sqrt2}{2}}{M\frac{\sqrt2}{2}}\eta \Rightarrow \xi = -1{,}740\,\eta$

$\boxed{\alpha = -60{,}11°}$

$\eta \quad \xi$

A(2,12 ; 0,707)

B(-2,12 ; -0,707)

$\sigma_{x,A} = \frac{2\sqrt2\cdot 1}{21{,}17}\cdot 0{,}707 + \frac{2\sqrt2}{12{,}17}\cdot 2{,}12 = \underline{0{,}587\,GPa}$ 0,189

$\sigma_{x,B} = \frac{2\sqrt2}{21{,}17}(-0{,}707) + \frac{2\sqrt2}{12{,}17}\cdot(-2{,}12) = \underline{-0{,}587\,GPa}$

4\. a) $I_{\eta,\zeta} = \frac{1,5396 + 2,5660}{2} \pm \sqrt{\left(\frac{1,5396 - 2,5660}{2}\right)^2 + 0,18889^2}$

$I_\eta = 3,079\ cm^4$

$I_\zeta = 1,026\ cm^4$

$tg\, 2\alpha_1 = \frac{-2 \cdot 0,18889}{1,5396 - 2,5660} = 1,732$

$2\alpha_1 = 180° + arctg\, 1,732$

$2\alpha_1 = 240°$

$\boxed{\alpha_1 = 120°}$

$\theta = 60° = 180° - 120°$

$\sigma_x = -\frac{M_\eta}{I_\eta}\zeta + \frac{M_\zeta}{I_\zeta}\eta = \left(-\frac{10^3}{3,079 \cdot 10^{-8}}\zeta + \frac{\sqrt{3} \cdot 10^3}{1,026 \cdot 10^{-8}}\eta\right) = \left(-3,25 \cdot 10^{10}\zeta + 1,69 \cdot 10^{11}\eta\right) Pa$

$\sigma_x = 0 \Rightarrow \zeta = \frac{M_\zeta}{I_\zeta}\frac{I_\eta}{M_\eta}\eta = \frac{I_\eta}{I_\zeta} \cdot \frac{M \sin\theta}{M \cos\theta}\eta$

$\zeta = tg\,60 \cdot \frac{3,079}{1,026}\eta \Rightarrow \zeta = 5,198\,\eta \Rightarrow \boxed{\varphi = 79,11°}$

$M_\eta = M \cdot \cos 60 = 1\ kNm$

$M_\zeta = M \cdot \sin 60 = \sqrt{3}\ kNm$

$y' = y\cos\alpha + z\sin\alpha$

$z' = z\cos\alpha - y\sin\alpha$

A $\begin{cases} \eta_A = y_A \cos\alpha_1 + z_A \sin\alpha_1 = -1,73 \cdot \cos 120 + \sin 120 = -1,025 \cdot 10^{-3}\ cm \\ \zeta_A = z_A \cos\alpha_1 - y_A \sin\alpha_1 = \cos 120 + 1,73 \sin 120 = 1,988\ cm \end{cases}$

$\sigma_{x,A} = -3,25 \cdot 10^{10}\, 1,988 \cdot 10^{-2} + 1,69 \cdot 10^{11} \cdot (-1,025 \cdot 10^{-3}) = -0,648\ GPa$

$C(0{,}58;\ -1)$ (y = 0,58; z = -1)

B, C

$$\eta_C = y_C\cos\alpha_1 + z_C\sin\alpha_1 = 0{,}58\cdot\cos 120 - \sin 120 = \underline{-1{,}156\ cm}$$

$$\zeta_C = z_C\cos\alpha_1 - y_C\sin\alpha_1 = -\cos 120 - 0{,}58\sin 120 = \underline{-2{,}295\cdot 10^{-3}\ cm}$$

$$\sigma_{x,C} = -3{,}25\cdot 10^{10}\cdot(-2{,}295\cdot 10^{-5}) + 1{,}69\cdot 10^{11}\cdot(-1{,}156)\cdot 10^{-2} = \underline{-1{,}9507\ GPa}$$

Г) $\boxed{\begin{array}{l} I_\eta = 6{,}67\ cm^4 \\ I_\zeta = 4{,}42\ cm^4 \end{array}}$

$\operatorname{tg} 2\alpha_1 = +\infty \Rightarrow \boxed{\alpha_1 = 45°}$

$$\sigma_x = -\frac{M_\eta}{I_\eta}\zeta - \frac{M_\zeta}{I_\zeta}\eta \Rightarrow \boxed{\sigma_x = (-2{,}12\cdot 10^{10}\zeta - 3{,}20\cdot 10^{10}\eta)\ Pa}$$

$$\zeta = -\frac{M_\zeta}{I_\zeta}\cdot\frac{I_\eta}{M_\eta}\eta = -\frac{M\cdot\frac{\sqrt2}{2}}{M\cdot\frac{\sqrt2}{2}}\frac{I_\eta}{I_\zeta}\eta \Rightarrow \boxed{\zeta = -1{,}51\eta}$$

$A(0,\ -1{,}37)$ $\boxed{\varphi = -56{,}47°}$

A

$$\eta_A = y_A\cos\alpha_1 + z_A\sin\alpha_1 = -1{,}37\frac{\sqrt2}{2} = \underline{-0{,}969\ cm}$$

$$\zeta_A = z_A\cos\alpha_1 - y_A\sin\alpha_1 = -1{,}37\frac{\sqrt2}{2} = \underline{-0{,}969\ cm}$$

$$\boxed{\sigma_{x,A} = 0{,}516\ GPa}$$

B
$\eta_B =$
$\zeta_B = 0$

$y' = y\cos\varphi + z\sin\varphi$

$z' = z\cos\varphi - y\sin\varphi$

ЕКСЦЕНТРИЧНО НАПРЕЗАЊЕ

1. а) $\boxed{\sigma_x = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z}$

$M_z = e_y N = 3N$

$M_y = e_z N = -1{,}31 N$

$e_y = 3 \text{ cm}$

$e_z = -1{,}31 \text{ cm}$

$i_y^2 = \frac{I_y}{A} = \frac{10{,}21}{12} = 0{,}851 \text{ cm}^2$

$i_z^2 = \frac{I_z}{A} = \frac{23}{12} = 1{,}917 \text{ cm}^2$

$p_y = -\frac{i_z^2}{e_y} = -\frac{1{,}917}{3} = -0{,}639 \text{ cm}$

$p_z = -\frac{i_y^2}{e_z} = -\frac{0{,}851}{-1{,}31} = 0{,}650 \text{ cm}$

$\sigma_x^c = -\frac{P}{A}\left(1 - \frac{y}{p_y} - \frac{z}{p_z}\right) = -\frac{P}{12 \cdot 10^{-4}}\left(1 - \frac{3}{-0{,}639} - \frac{-1{,}31}{0{,}650}\right) = -6425{,}18 P \, [Pa]$

$\tau_{max} = \frac{1}{2}|\sigma_x^c| = -3212{,}59 \, P \, [Pa]$

б) $\boxed{\sigma_x = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z}$

$C\left(\overset{y}{-3} ; \overset{z}{0{,}69}\right)$

$\boxed{\begin{aligned} e_y &= -3 \text{ cm} \\ e_z &= 0{,}69 \text{ cm} \end{aligned}}$

$i_y^2 = \frac{I_y}{A} = \frac{10{,}05}{15} = \boxed{0{,}67 \text{ cm}^2}$

$i_z^2 = \frac{I_z}{A} = \frac{38{,}50}{15} = \boxed{2{,}57 \text{ cm}^2}$

$p_y = -\frac{i_z^2}{e_y} = -\frac{2{,}57}{-3} = 0{,}857 \text{ cm}$

$p_z = -\frac{i_y^2}{e_z} = -\frac{0{,}67}{0{,}69} = -0{,}971 \text{ cm}$

$\sigma_x^c = -\frac{P}{A}\left(1 - \frac{y_c}{p_y} - \frac{z_c}{p_z}\right) = -\frac{P}{15 \cdot 10^{-4}}\left(1 - \frac{-3}{0{,}857} - \frac{0{,}69}{-0{,}971}\right) = -3474{,}13 P \, [Pa]$

$\sigma_{max} = |\sigma_x^c| = 3474{,}13 \, P \, [Pa]$

$\tau_{max} = \frac{1}{2} \sigma_{max} = 1737{,}065 \, P \, [Pa]$

B) $\boxed{\sigma_x = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z}$

$$\boxed{\begin{aligned} e_y &= 1{,}88 \text{ cm} \\ e_z &= -2 \text{ cm} \end{aligned}}$$

$C(\overset{y}{1{,}88};\overset{z}{-2})$

$i_y^2 = \frac{I_y}{A} = \frac{22}{12} = \underline{\underline{1{,}83 \text{ cm}^2}}$

$i_z^2 = \frac{I_z}{A} = \frac{20{,}81}{12} = \underline{\underline{1{,}73 \text{ cm}^2}}$

$P_y = -\frac{i_z^2}{e_y} = -\frac{1{,}73}{1{,}88} = \underline{\underline{-0{,}92 \text{ cm}}}$

$P_z = -\frac{i_y^2}{e_z} = -\frac{1{,}83}{-2} = \underline{\underline{0{,}915 \text{ cm}}}$

$$\sigma_x^c = -\frac{P}{A}\left(1 - \frac{y}{P_y} - \frac{z}{P_z}\right) = -\frac{P}{12 \cdot 10^{-4}}\left(1 - \frac{1{,}88}{-0{,}92} - \frac{-2}{0{,}915}\right) =$$

$$= \underline{-4357{,}72 P \,[\text{Pa}]}$$

$\sigma_{max} = |\sigma_x^c| = 4357{,}72 P \,[\text{Pa}]$

$\tau_{max} = \frac{1}{2}\sigma_{max} = \underline{\underline{2178{,}86 P \,[\text{Pa}]}}$

H) $\boxed{\sigma_x = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z}$

$C(\overset{y}{0};\overset{z}{-1{,}36})$

$$\boxed{\begin{aligned} e_y &= \infty \\ e_z &= -1{,}36 \text{ cm} \end{aligned}}$$

$$\boxed{\begin{aligned} I_\eta &= 6{,}197 \text{ cm}^4 \\ I_\zeta &= 3{,}255 \text{ cm}^4 \end{aligned}}$$

$A = 7 \text{ cm}^2$

$tg\, 2\alpha_1 = -1{,}80 \Rightarrow \boxed{\alpha_1 = 149{,}53°}$

$\frac{\ominus}{\oplus}$

c) $\begin{cases} \eta_c = e_\eta = \cancel{y_c \cos\alpha_1} + z_c \sin\alpha_1 = -1{,}36 \cdot \sin\alpha_1 = \underline{-0{,}69 \text{ cm}} \\ \zeta_c = e_\zeta = z_c \cos\alpha_1 - \cancel{y_c \sin\alpha_1} = -1{,}36 \cdot \cos\alpha_1 = \underline{\underline{1{,}17 \text{ cm}}} \end{cases}$

$i_\eta^2 = \frac{I_\eta}{A} = \underline{\underline{0{,}885 \text{ cm}^2}}$

$i_\zeta^2 = \frac{I_\zeta}{A} = \underline{0{,}465 \text{ cm}^2}$

$$\sigma_x^{\,c} = -\frac{P}{A}\left(1 - \frac{\eta_c}{p_\eta} - \frac{\zeta_c}{p_\zeta}\right)$$

$$p_\eta = -\frac{i_\zeta^2}{e_\eta} = -\frac{0{,}465}{-0{,}69} = \underline{\underline{0{,}67\ cm}}$$

$$p_\zeta = -\frac{i_\eta^2}{e_\zeta} = -\frac{0{,}885}{1{,}17} = \underline{\underline{-0{,}76\ cm}}$$

$$A\left(\overset{y}{1{,}21}\ ;\ \overset{z}{-1{,}36}\right)$$

$$\eta_A = y_A\cos\alpha_1 + z_A\sin\alpha_1 = 1{,}21\cdot\cos\alpha_1 + 1{,}36\sin\alpha_1 = \underline{\underline{-1{,}73\,cm}}$$

$$\zeta_A = z_A\cos\alpha_1 - y_A\sin\alpha_1 = -1{,}36\cos\alpha_1 - 1{,}21\sin\alpha_1 = \underline{\underline{0{,}56\,cm}}$$

$$\sigma_{x,A} = -\frac{P}{A}\left(1 + \frac{e_\eta}{i_\zeta^2}\eta_A + \frac{e_\zeta}{i_\eta^2}\zeta_A\right) =$$

$$= -\frac{P}{7\cdot10^{-4}}\left(1 + \frac{-0{,}69}{0{,}465}(-1{,}73) + \frac{1{,}17}{0{,}885}\cdot 0{,}56\right) = \underline{\underline{-6153{,}48\,P\ [Pa]}}$$

P [N]

сила

$$\sigma_{x,A} = -\frac{P}{A}\left(1 - \frac{\eta_A}{p_\eta} - \frac{\zeta_A}{p_\zeta}\right) =$$

$$= -\frac{P}{7\cdot10^{-4}}\left(1 - \frac{-1{,}73}{0{,}67} - \frac{0{,}56}{-0{,}76}\right) = \underline{\underline{-6169{,}90\,P\ [Pa]}}$$

e) $\boxed{\sigma_x = \frac{N}{A} + \frac{M_z}{I_z} y + \frac{M_y}{I_y} z}$ $\qquad A = (2{,}13 + 2{,}94) \cdot 1 + 3 = 8{,}07\ cm^2$

$I_1 = I_\eta = 15{,}432\ cm^4 \qquad tg\, 2\alpha_1 = 3{,}797$

$I_2 = I_\zeta = 7{,}676\ cm^4$

$2\alpha_1 = 180 + arctg\, 3{,}797$

$\boxed{\alpha_1 = 127{,}62^\circ}$

$\eta_c = e_\eta = \cancel{y_c \cos\alpha_1}^{\,0} + z_c \sin\alpha_1 = -0{,}99\ cm$

$\zeta_c = e_\zeta = z_c \cos\alpha_1 - \cancel{y_c \sin\alpha_1}^{\,0} = 0{,}76\ cm$

$C(\overset{y}{0}\ ; \overset{z}{-1{,}25})$

$i_\eta^2 = \frac{I_\eta}{A} = 1{,}91\ cm^2 \qquad p_\eta = -\frac{i_\zeta^2}{e_\eta} = -\frac{0{,}95}{-0{,}99} = 0{,}96\ cm$

$i_\zeta^2 = \frac{I_\zeta}{A} = 0{,}95\ cm^2 \qquad p_\zeta = -\frac{i_\eta^2}{e_\zeta} = -\frac{1{,}91}{0{,}76} = -2{,}51\ cm$

$A(\overset{y}{2{,}13}\ ; \overset{z}{-1{,}25})$

$\boxed{\begin{aligned} \eta_A &= -2{,}29\ cm \\ \zeta_A &= -0{,}92\ cm \end{aligned}}$

$$\sigma_x^A = -\frac{P}{A}\left(1 + \frac{e_\eta}{i_\zeta^2}\eta_A + \frac{e_\zeta}{i_\eta^2}\zeta_A\right) =$$

$$= -\frac{P}{8{,}07 \cdot 10^{-4}}\left(1 + \frac{-0{,}99}{0{,}95}(-2{,}29) + \frac{0{,}76}{1{,}91}(-0{,}92)\right) = -3742{,}69\ P\ [Pa]$$

$$\sigma_x^A = -\frac{P}{A}\left(1 - \frac{\eta_A}{p_\eta} - \frac{\zeta_A}{p_\zeta}\right) =$$

$$= -\frac{P}{8{,}07 \cdot 10^{-4}}\left(1 - \frac{-2{,}29}{0{,}96} - \frac{0{,}92}{-2{,}51}\right) = -4649{,}26\ P\ [Pa]$$

2. а) $p_z = 1,69\ cm$

$p_y = 1,69\ cm$

$i_y^2 = \frac{I_y}{A} = \frac{10,21}{12} = 0,85\ cm^2$

$i_z^2 = \frac{I_z}{A} = \frac{23}{12} = 1,92\ cm^2$

$e_y = -\frac{i_z^2}{p_y} = -\frac{1,92}{1,69} = -1,14\ cm$

$e_z = -\frac{i_y^2}{p_z} = -\frac{0,85}{1,69} = -0,50\ cm$

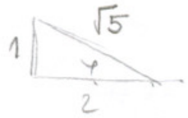
$A(-1,14;\ -0,5)$

$n_1 - n_1 \Rightarrow p_{y_1} = \infty \Rightarrow e_{y_1} = -\frac{i_z^2}{p_{y_1}} = -\frac{1,92}{\infty} = 0$

$p_{z_1} = 1,69 \Rightarrow e_{z_1} = -\frac{i_y^2}{p_{z_1}} = -\frac{0,85}{1,69} = -0,50$

$A_1(0;\ -0,5)$

г) $p_z = -1,47\ cm$



$\cos\varphi = \frac{2}{\sqrt{5}} \Rightarrow \varphi = \arccos\left(\frac{2}{\sqrt{5}}\right) = 26,56°$

$A = 4 \cdot 3 - \frac{2 \cdot 1}{2} = 11\ cm^2$

$$\boxed{\begin{array}{l} i_y^2 = \frac{I_y}{A} = \frac{8,49}{11} = 0,77\ cm^2 \\ i_z^2 = \frac{I_z}{A} = \frac{10,33}{11} = 0,94\ cm^2 \end{array}}$$

$z = tg(-\varphi)\, y - 1,47$

$z = -0,50\, y - 1,47$

$z = 0 \Rightarrow y = -\frac{1,47}{0,50} = -2,94\ cm$

$p_y = -2,94\ cm$

$e_y = -\frac{i_z^2}{p_y} = -\frac{0,94}{-2,94} = 0,32\ cm$

$e_z = -\frac{i_y^2}{p_z} = -\frac{0,77}{-1,47} = 0,52\ cm$

$A(0,32;\ 0,52)$

$n_1 - n_1 \Rightarrow p_{y_1} = \infty \Rightarrow e_{y_1} = 0$

$p_{z_1} = 1,53 \Rightarrow e_{z_1} = -\frac{i_y^2}{p_{z_1}} = -\frac{0,77}{1,53} = -0,50\ cm$

$n_2 - n_2 \Rightarrow p_{y_2} = -2 \Rightarrow e_{y_2} = -\frac{i_z^2}{p_{y_2}} = -\frac{0,94}{-2} = 0,47\ cm$

$p_{z_2} = \cancel{1,5} \Rightarrow e_{z_2} = -0,50\ cm$

$n_3 - n_3 \Rightarrow P_{y3} = \infty \quad \Rightarrow e_{y3} = 0$

$P_{z3} = -1{,}47 \quad \Rightarrow e_{z3} = -\dfrac{0{,}77}{-1{,}47} = \underline{\underline{0{,}52 cm}}$

$n_4 - n_4 \Rightarrow P_{y4} = 2 \quad \Rightarrow e_{y4} = -\dfrac{0{,}94}{2} = \underline{\underline{-0{,}47}} cm$

$P_{z4} = \infty \quad \Rightarrow \underline{\underline{e_{z4} = 0}}$

B) $\boxed{\begin{array}{l} i_y^2 = \dfrac{I_y}{A} = \dfrac{22}{12} = 1{,}83 cm^2 \\ i_z^2 = \dfrac{20{,}81}{12} = 1{,}73 cm^2 \end{array}}$

$P_y = 1{,}88 \quad \Rightarrow e_y = -\dfrac{i_z^2}{P_y} = -\dfrac{1{,}73}{1{,}88} = \underline{\underline{-0{,}92 cm}}$

$P_z = 2 \quad \Rightarrow e_z = -\dfrac{i_y^2}{P_z} = -\dfrac{1{,}83}{2} = \underline{\underline{-0{,}915 cm}}$

$n_1 - n_1 \Rightarrow P_{y1} = \infty \quad \Rightarrow e_{y1} = 0$

$P_{z1} = 2 \Rightarrow e_{z1} = -\dfrac{1{,}83}{2} = \underline{\underline{-0{,}915}} cm \qquad \boxed{A_1 (0; -0{,}915)}$

$n_2 - n_2 \Rightarrow P_{y2} = -2{,}62 cm \quad \Rightarrow e_{y2} = -\dfrac{1{,}73}{-2{,}62} = \underline{\underline{0{,}66 cm}}$

$P_{z2} = \infty \quad \Rightarrow e_{z2} = 0 \qquad \boxed{A_2 (0{,}66; 0)}$

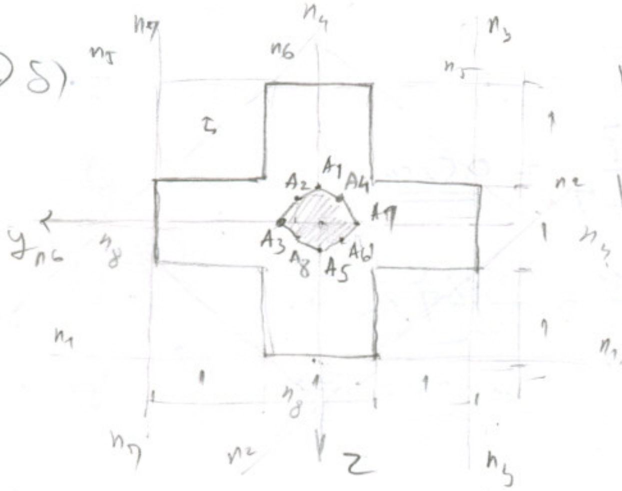
$n_3 - n_3 \Rightarrow P_{y3} = \infty \quad \Rightarrow e_{y3} = 0$

$P_{z3} = -2 \quad \Rightarrow \underline{\underline{e_{z3} = -\dfrac{1{,}83}{-2} = 0{,}915 cm}}$

$n_4 - n_4 \Rightarrow P_{y4} = 1{,}88 \quad \Rightarrow e_{y4} = -\dfrac{1{,}73}{1{,}88} = -\underline{\underline{0{,}92}}$

$P_{z4} = \infty \quad \Rightarrow e_{z4} = 0$

3. 5).



$\boxed{I_y = \frac{1}{12}3^4 - 4\left(\frac{1}{12}+1\right) = \underline{\underline{2,417\,cm^4}}}$

$\boxed{I_z = I_y = 2,417\,cm^4}$ $\boxed{A = 5\,cm^2}$

$i_y^2 = \frac{2,417}{5} = 0,48\,cm^2$

$i_z^2 = \frac{2,417}{5} = 0,48\,cm^2$

$n_1 - n_1 \Rightarrow p_{y1} = \infty \Rightarrow e_{y1} = 0$

$p_{z1} = 1,5 \Rightarrow e_{z1} = -\frac{0,48}{1,5} = -\underline{\underline{0,32\,cm}}$ $\boxed{A_1(0, -0,32)}$

$n_2 - n_2 \Rightarrow p_{y2} = -2 \Rightarrow e_{y2} = -\frac{0,48}{-2} = 0,24\,cm$ $\boxed{A_2(0,24; -0,24)}$

$p_{z2} = 2 \Rightarrow e_{z2} = -\frac{0,48}{2} = -0,24\,cm$

$n_3 - n_3 \Rightarrow p_{y3} = -1,5 \Rightarrow e_{y3} = -\frac{0,48}{-1,5} = \underline{\underline{0,32\,cm}}$

$p_{z3} = \infty \Rightarrow e_{z3} = 0$

$\boxed{A_3(0,32; 0)}$

$n_4 - n_4 \Rightarrow p_{y4} = -2 \Rightarrow e_{y4} = -0,24\,cm$

$p_{z4} = -2 \Rightarrow e_{y4} = -0,24\,cm$ $\boxed{A_4(-0,24; -0,24)}$

$n_5 - n_5 \Rightarrow p_{y5} = \infty \Rightarrow e_{y5} = 0$

$\boxed{A_5(0; 0,32)}$

$p_{z5} = -1,5 \Rightarrow e_{z5} = -\frac{0,48}{-1,5} = \underline{\underline{0,32\,cm}}$

$n_6 - n_6 \Rightarrow p_{y6} = 2 \Rightarrow e_{y6} = -\frac{0,48}{2} = -0,24\,cm$

$\boxed{A_6(-0,24; 0,24)}$

$p_{z6} = -2 \Rightarrow e_{z6} = -\frac{0,48}{-2} = 0,24\,cm$

$n_7 - n_7 \Rightarrow p_{y7} = 1,5 \Rightarrow e_{y7} = -\frac{0,48}{1,5} = -0,32$ $\boxed{A_7(-0,32; 0)}$

$p_{z7} = \infty \Rightarrow e_{z7} = 0$

$n_8 - n_8 \Rightarrow \underline{p_{y8} = -2} \Rightarrow e_{y8} = -\frac{0,48}{-2} = \underline{\underline{0,24\ cm}}$

$\underline{p_{z8} = 2} \Rightarrow e_{z8} = -\frac{0,48}{2} = \underline{\underline{-0,24\ cm}}$ $\boxed{A_8 (0,24 ; -0,24)}$

B) $e_y = 0,37$ $p_y = -\frac{i_z^2}{e_y} = -\frac{1,73}{0,37} = \underline{\underline{-4,68\ cm}}$

$e_z = 1$

$\boxed{\begin{matrix} i_y^2 = 1,83\ cm^2 \\ i_z^2 = 1,73\ cm^2 \end{matrix}}$ $p_z = -\frac{i_y^2}{e_z} = -\frac{1,83}{1} = \underline{\underline{-1,83\ cm}}$

g) $e_y = -0,91\ cm$ $p_y = -\frac{i_z^2}{e_y} = -\frac{2,75}{-0,91} = \underline{\underline{3,02\ cm}}$

$e_z = 0,56\ cm$ $p_z = -\frac{i_y^2}{e_z} = -\frac{0,70}{0,56} = \underline{\underline{-1,25\ cm}}$

$i_y^2 = \frac{12}{17} = \underline{\underline{0,70\ cm^2}}$

$i_z^2 = \frac{46,83}{17} = \underline{\underline{2,75\ cm^2}}$