# I колоквијум

1. а) $y_T = \frac{3,99 \cdot 6 + 3,49 \cdot 4,5}{10,5} = \underline{\underline{3,96 m}}$

$z_T = \frac{1,7 \cdot 6 + 3,7 \cdot 4,5}{10,5} = \underline{\underline{2,68 m}}$

б) $y_T = \frac{2,97 \cdot 8 + 4,97 \cdot 4}{12} = \underline{\underline{3,64 m}}$

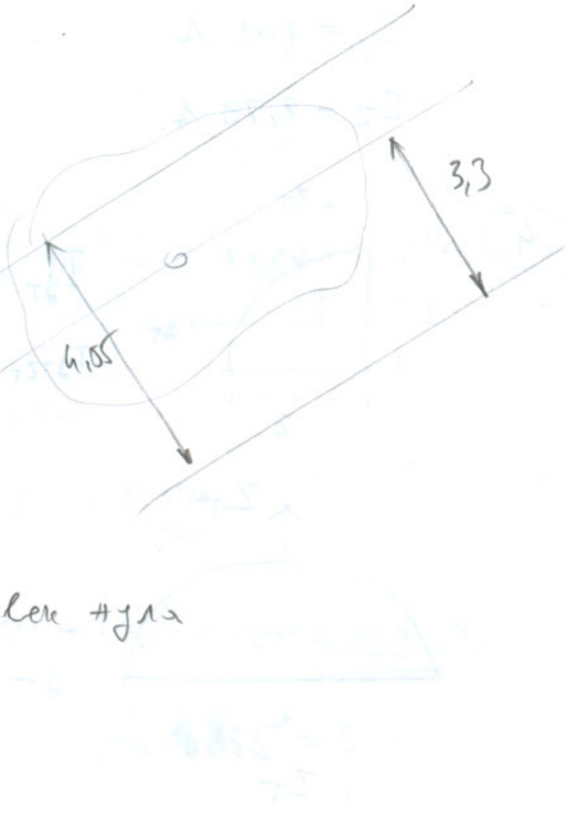
$z_T = \frac{2,73 \cdot 8 + 3,73 \cdot 4}{12} = \underline{\underline{3,06 m}}$

2. а) познато $S_{n1}$

$S_{n2} = ?$

$S_{n1} = 1,25 \cdot A \Rightarrow A = \frac{S_{n1}}{1,25}$

$S_{n2} = 3,3 \cdot A \Rightarrow \boxed{S_{n2} = \frac{3,3}{1,25} S_{n1}}$




b) $S_{n1} = 1,25 \cdot A \Rightarrow A = \frac{S_{n1}}{1,25}$

c) $S_n = 0$ → за тежишну осу је увек нула

d) $A, I_{n1} \Rightarrow$ познато $I_{n2} = ?$

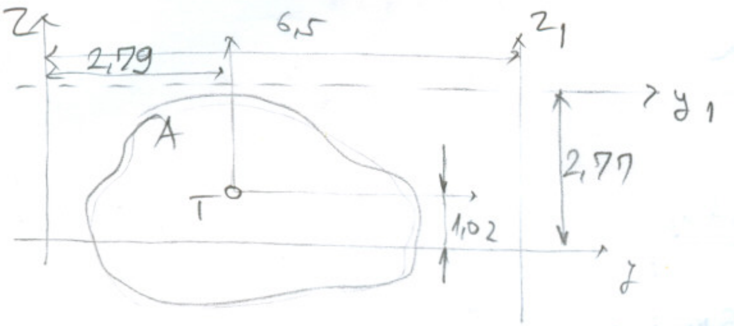
$I_{n1} = I_n + 1,25^2 A \Rightarrow I_n = I_{n1} - 1,25^2 \cdot A$

$I_{n2} = I_n + 3,3^2 \cdot A \Rightarrow I_{n2} = I_{n1} - 1,25^2 A + 3,3^2 \cdot A$

$\boxed{I_{n2} = I_{n1} + A(3,3^2 - 1,25^2)}$

e) $I_{n1} = I_n + 1,25^2 \cdot A \Rightarrow \boxed{I_n = I_{n1} - 1,25^2 \cdot A}$

3.




a) познато $A$, $I_{z1}$

$I_z = ?$

$I_{z1} = I_z + 3,71^2 \cdot A \Rightarrow I_z = I_{z1} - 3,71^2 \cdot A$

б) $I_{z1} = I_z + 3,71^2 \cdot A$

c) $I_{yz} = I_{y1z1} - 1,75 \cdot 3,71 \cdot A$

d) познато $S_y$

$S_z = ?$

$S_y = 1,02 \cdot A$

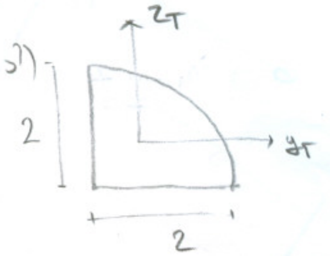
$S_z = 2,79 \cdot A$

e) познато $A, I_{y1}$ $I_y = ?$

$I_{y1} = I_y + 1,75^2 A$

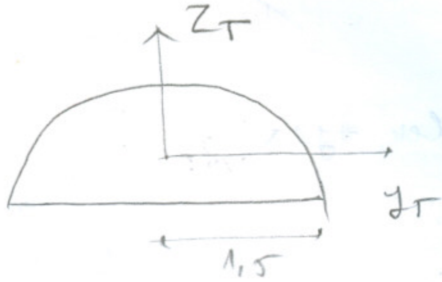
$\boxed{I_y = I_{y1} - 1,75^2 A}$

4. a)




$I_{y_T} = 0,05488 \cdot 2^4 = \underline{0,878\ m^4} = I_z$

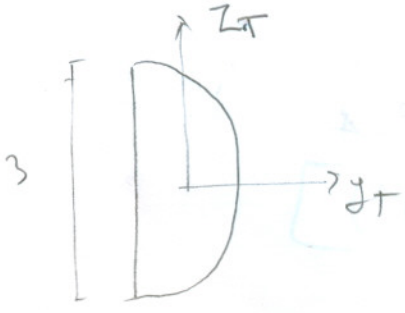
$I_{y_T z_T} = -0,01647 \cdot 2^4 = \underline{-0,264\ m^4}$




$I_{y_T} = 2 \cdot 0,05488 \cdot 1,5^4 = \underline{0,555\ m^4}$

$I_{z_T} = \frac{R^4 \pi}{8} = \underline{1,988\ m^4}$

$\underline{I_{y_T z_T} = 0}$

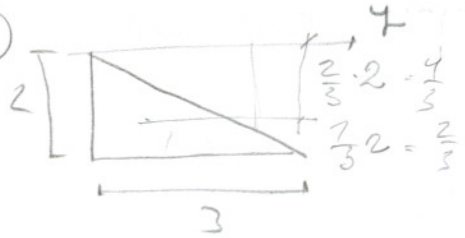



$I_{z_T} = 0,555\ m^4$

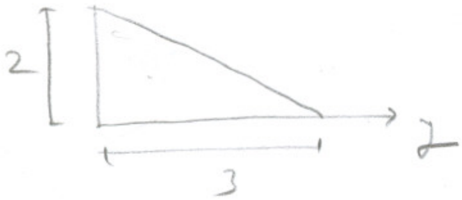
$I_{y_T} = 1,988\ m^4$

$I_{y_T z_T} = 0$

5.



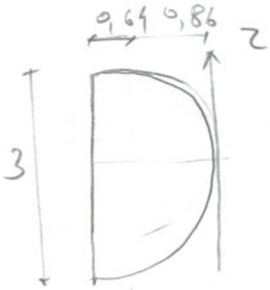

$$I_y = I_{yT} + e^2 \cdot A = \frac{1}{36}\cdot 2^3 \cdot 3 + \left(\frac{4}{3}\right)^2 \cdot 3 = 6$$



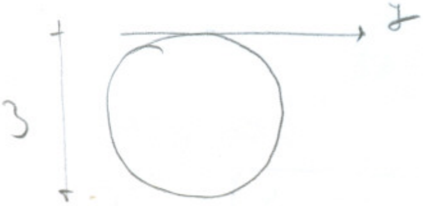

$$I_y = \frac{1}{12}\, 2^3 \cdot 3 = 2$$

$$I_y = \frac{1}{36}\, 2^3 \cdot 3 + \left(\frac{2}{3}\right)^2 \cdot 3 = 2$$

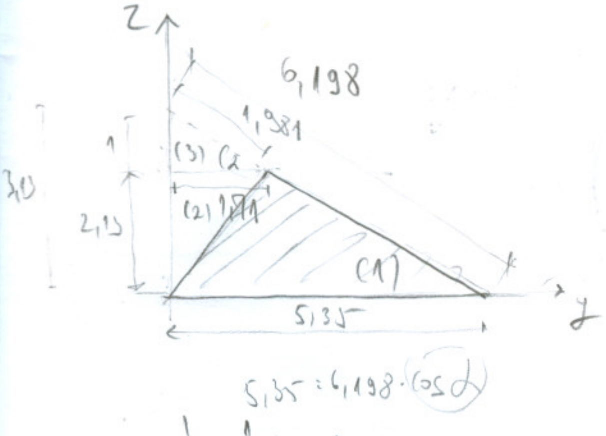

$$I_y = \frac{\overbrace{R^4\pi}^{=1{,}5^4\pi}}{8} + 1{,}5^2 \cdot \frac{1{,}5^2\pi}{2} = \underline{\underline{9{,}94\,m^4}}$$




$$I_z = 2\cdot 0{,}05488 \cdot 1{,}5^4 + 0{,}86^2 \cdot \frac{1{,}5^2\pi}{2} = \underline{\underline{3{,}17\,m^4}}$$




$$I_y = \frac{R^4\pi}{4} + 1{,}5^2 \cdot 1{,}5^2\pi = 19{,}88$$




$5{,}35 = 6{,}198 \cdot \cos\alpha$

$d_1 = \frac{1}{\sin\alpha} = 1{,}98$

$$I_{y(1)} = \frac{1}{12}\, 3{,}13^3 \cdot 5{,}35 = \underline{\underline{13{,}671}}\,m^4$$

$$I_{y(2)} = \frac{1}{36}\cdot 2{,}13^3 \cdot 1{,}71 + 1{,}42^2 \cdot 1{,}82 = \underline{\underline{4{,}129\,m^4}}$$

$$I_{y(3)} = \frac{1}{36}\cdot 1^3 \cdot 1{,}71 + 2{,}46^2 \cdot 0{,}855 = 5{,}22\,m^4$$

$$I_y = I_{y(1)} - I_{y(2)} - I_{y(3)} = \underline{\underline{4{,}322\,m^4}}$$

$$I_{z(1)} = \frac{1}{12} \, 5{,}35^3 \cdot 3{,}13 = \underline{\underline{39{,}94\,m^4}}$$

$$I_{z(2)} = \frac{1}{36} \cdot 1{,}71^3 \cdot 2{,}13 + 0{,}57^2 \cdot 1{,}82 = \underline{0{,}8872\,m^4}$$

$$I_{z(3)} = \frac{1}{36} \cdot 1{,}71^3 \cdot 1 + 0{,}57^2 \cdot 0{,}855 = \underline{\underline{0{,}4167\,m^4}}$$

$$I_z = I_{z(1)} - I_{z(2)} - I_{z(3)} = \underline{\underline{38{,}6361\,m^4}}$$

$$I_{yz(1)} = \frac{1}{24} \cdot 5{,}35^2 \cdot 3{,}13^2 = \underline{\underline{11{,}68\,m^4}}$$

$$I_{yz(2)} = -\frac{1}{72} \, 1{,}71^2 \cdot 2{,}13^2 + (-1{,}42)(-0{,}57) \cdot 1{,}82 = \underline{\underline{1{,}29\,m^4}}$$

$$I_{yz(3)} = -\frac{1}{72} \cdot 1{,}71^2 \cdot 1^2 + (-0{,}57)(-2{,}46) \cdot 0{,}855 = \underline{\underline{1{,}158\,m^4}}$$

$$I_{yz} = I_{yz(1)} - I_{yz(2)} - I_{yz(3)} = \underline{\underline{9{,}232\,m^4}}$$

6.

$$I_{y_T} = I_{y1} + (-0{,}29)^2 \cdot A_1 + I_{y2} + 0{,}38^2 \cdot A_2$$

$$I_{z_T} = I_{z1} + (-0{,}86)^2 \cdot A_1 + I_{z2} + 1{,}14^2 \cdot A_2$$

$$I_{y_T z_T} = I_{y_1 z_1} + (-0{,}29)(-0{,}86) A_1 + I_{y_2 z_2} + 1{,}14 \cdot 0{,}38 A_2$$

7. $I_y = 125\,cm^4$ $\quad I_z = 100\,cm^4$ $\quad I_{yz} = 30\,cm^4$

$$I_{1/2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y + I_z}{2}\right)^2 + I_{yz}^2} \quad \boxed{\underline{\underline{145\,cm^4}}} \Rightarrow \boxed{I_2 = 80\,cm^4}$$

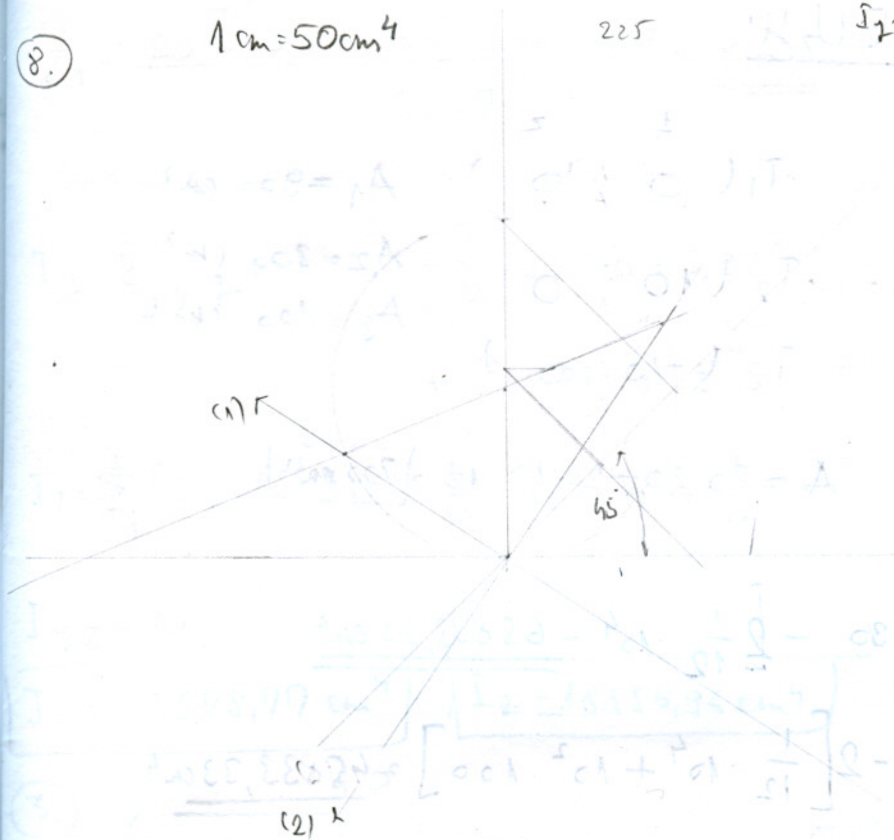
$$tg\,2\alpha = \frac{-2 I_{yz} \ominus}{I_y - I_z \oplus} = -2{,}4$$

$2\alpha = 360 - |arctg\,2{,}4|$

$2\alpha = 292{,}62°$

$$\boxed{\alpha = 146{,}31°}$$

(8.) $1\,cm = 50\,cm^4$ 225 $I_y = 125\,cm^4$ $I_z = 100\,cm^4$ $I_{yz} = 30\,cm^4$




$I_1 = 3\,cm = \frac{3 \cdot 50}{1} = 150$

$I_2 = 1{,}5\,cm = \frac{\cancel{1},5 \cdot 50}{1} = 75$

$I_{y'} = \frac{1{,}75 \cdot 50}{1} = 87{,}5$

$I_{z'} = \frac{1{,}45 \cdot 50}{1} = 72{,}5$

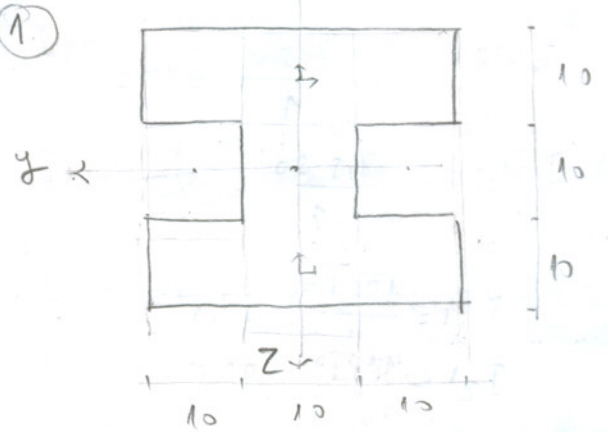
$I_{y'z'} =$

(11.) $i_1 = \sqrt{\frac{I_1}{A}} \uparrow^2$

$\frac{I_1}{A} = i_1^2 \Rightarrow \boxed{I_1 = i_1^2 \cdot A} \Rightarrow \boxed{I_2 = i_2^2 \cdot A}$

# ПРЕСЕЦИ:

1.




$T_1(0\ ;\ 0)$ $\quad A_1 = 900\ cm^2$

$T_2(10\ ;\ 0)$ $\quad A_2 = 100\ cm^2$

$T_3(-10\ ;\ 0)$ $\quad A_3 = 100\ cm^2$

$$A = 30\cdot 30 - 2\cdot 10\cdot 10 = \boxed{700\ cm^2}$$

$$I_y = \sum_{i=1}^{3}(I_{yi} + c_i^2 A_i) = \frac{1}{12}\cdot 30^3\cdot 30 - 2\frac{1}{12}\cdot 10^4 = \underline{\underline{65833{,}33\ cm^4}}$$

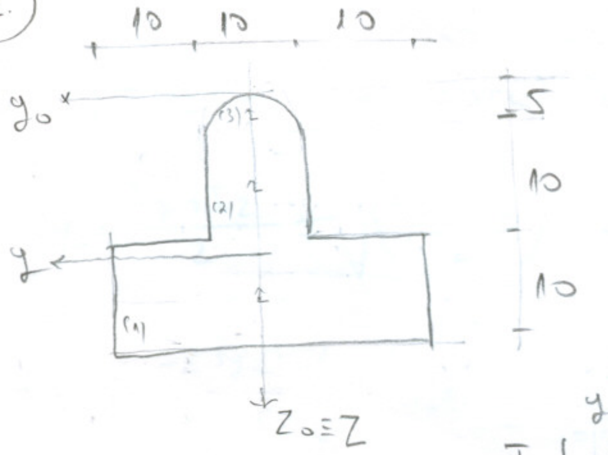
$$I_z = \sum_{i=1}^{3}(I_{zi} + b_i^2 A_i) = \frac{1}{12}30^4 - 2\left[\frac{1}{12}\cdot 10^4 + 10^2\cdot 100\right] = \underline{\underline{45833{,}33\ cm^4}}$$

$$\boxed{I_{yz} = 0\ cm^4}$$

$$I_{12} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} \Rightarrow \boxed{\begin{array}{l} I_1 = 65833{,}33\ cm^4 \\ I_2 = 45833{,}33\ cm^4 \end{array}}$$

$$\boxed{\alpha = 0^\circ}$$

2.




$A_1 = 300\ cm^2$

$A_2 = 100\ cm^2$

$A_3 = 39{,}27\ cm^2$

$\boxed{A = 439{,}27\ cm^2}$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | 20 | $T_1$ | 0 | 3,97 |
| $T_2$ | 0 | 10 | $T_2$ | 0 | -6,03 |
| $T_3$ | 0 | 2,88 | $T_3$ | 0 | -13,15 |

$$Z_T = \frac{20 \cdot 300 + 10 \cdot 100 + 2{,}88 \cdot 39{,}27}{439{,}27} = \underline{\underline{16{,}03 \text{ cm}}}$$

$$y_T = 0$$

$$I_y = \sum_{i=1}^{3} (I_y + c_i^2 A_i) = \frac{1}{12} 10^3 \cdot 30 + 3{,}97^2 \cdot 300 + \frac{1}{12} \cdot 10^3 \cdot 10 + (-6{,}03)^2 \cdot 100 +$$

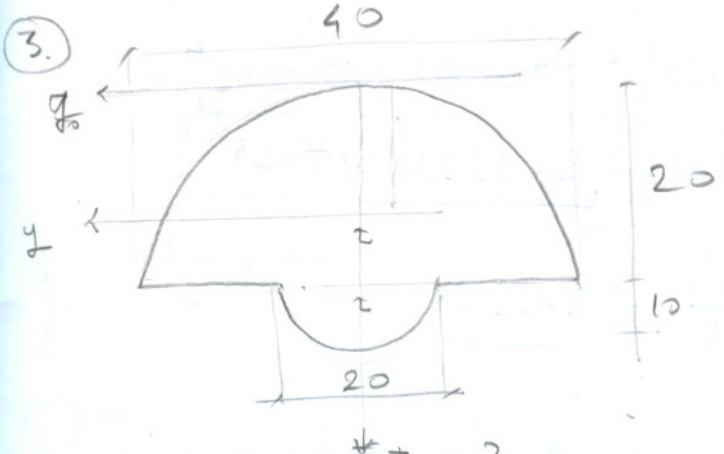
$$+ 2 \cdot 0{,}05488 \cdot 5^4 + (-13{,}15)^2 \cdot 39{,}27 = \boxed{18556{,}96 \text{ cm}^4}$$

$$I_z = \sum_{i=1}^{3} (I_z + b_i^2 A_i) = \frac{1}{12} \cdot 30^3 \cdot 10 + \frac{1}{12} \cdot 10^4 + \frac{5^4 \pi}{8} = \boxed{23578{,}77 \text{ cm}^4}$$

$$I_{yz} = 0$$

$$\boxed{I_1 = 23578{,}77 \text{ cm}^4} \quad \boxed{I_2 = 18556{,}96 \text{ cm}^4}$$

3.




$$A_1 = 200\pi \text{ cm}^2$$
$$A_2 = 50\pi \text{ cm}^2$$
$$A_3 = 250\pi \text{ cm}^2$$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | 11,51 | $T_1$ | 0 | -2,54 |
| $T_2$ | 0 | 24,24 | $T_2$ | 0 | 10,19 |

$$z_0 = 2$$

$$Z_T = \frac{11{,}51 \cdot 200\pi + 24{,}24 \cdot 50\pi}{250\pi} = \boxed{14{,}05 \text{ cm}}$$

$$I_y = \sum_{i=1}^{2} (I_y + c_i^2 A_i) = 2 \cdot 0{,}05488 \cdot 20^4 + (-2{,}54)^2 \cdot 200\pi + 2 \cdot 0{,}05488 \cdot 10^4 +$$

$$10{,}19^2 \cdot 50\pi = \boxed{39023{,}40 \text{ cm}^4}$$

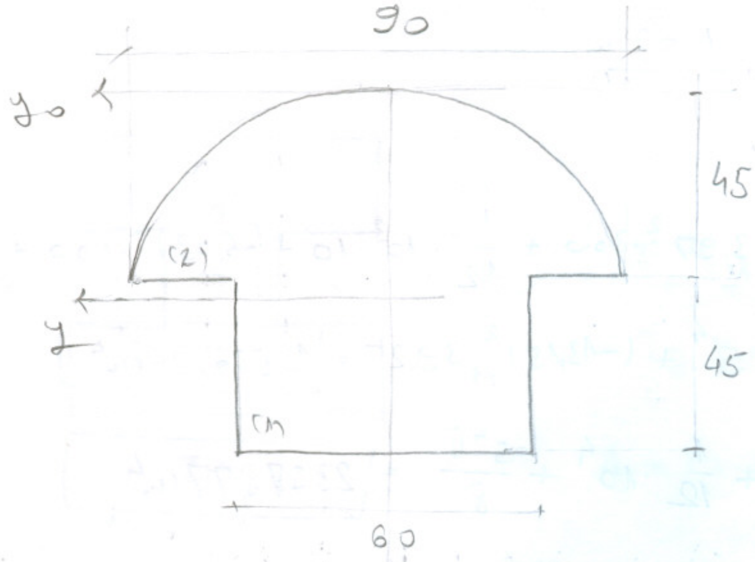
$$I_z = \sum_{i=1}^{2} (I_z + b_i^2 A_i) = \frac{20^4 \pi}{8} + \frac{10^4 \pi}{8} = \boxed{66758{,}84 \text{ cm}^4}$$

3432811,6719
2420311,6719

$$\boxed{I_{yz} = 0 \text{ cm}^4}$$

$$\boxed{\begin{aligned} I_1 &= 66758{,}84 \text{ cm}^4 \\ I_2 &= 39024{,}40 \text{ cm}^4 \end{aligned}} \quad \boxed{\alpha = 0}$$

4.




$z_0 = z$

$A_1 = 2700 \text{ cm}^2$

$A_2 = 3180{,}86 \text{ cm}^2$

$A = 5880{,}86 \text{ cm}^2$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | 25,90 | $T_1$ | 0 | -22,5 |
| $T_2$ | 0 | 67,5 | $T_2$ | 0 | 19,1 |

$$Z_T = \frac{25{,}90 \cdot 2700 + 67{,}5 \cdot 3180{,}86}{5880{,}86} = \boxed{48{,}40 \text{ cm}}$$

$$I_y = \sum_{i=1}^{2} (I_{yi} + c_i^2 A_i) = \frac{1}{12} \cdot 45^3 \cdot 60 + (-22{,}5)^2 \cdot 2700 + 2 \cdot 0{,}05488 \cdot 45^4 + 19{,}1^2 \cdot 3180{,}86 = \boxed{3432338{,}0317 \text{ cm}^4}$$

$$I_z = \sum_{i=1}^{2} (I_{zi} + b_i^2 A_i) = \frac{1}{12} \cdot 60^3 \cdot 45 + \frac{45^4 \pi}{8} = \boxed{2420311{,}672 \text{ cm}^4}$$

$\boxed{I_{yz} = 0 \text{ cm}^4}$ $\boxed{\alpha = 0}$

$\boxed{I_1 = I_y \Rightarrow I_2 = I_z}$

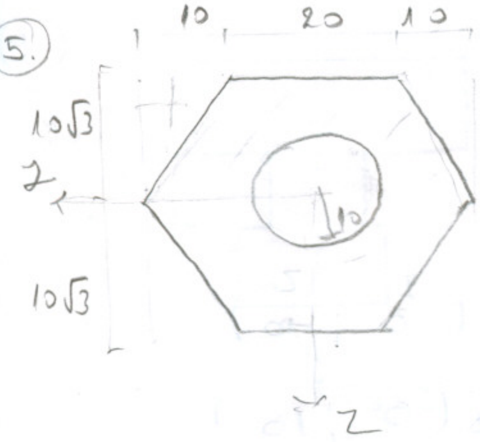
– у односу на задате осе

| | $y$ | $z$ |
|---|---|---|
| $T_1$ | 0 | 45 |
| $T_2$ | 0 | |

$$I_y = \frac{1}{12} 45^3 \cdot 60 + 22{,}5^2 \cdot 2700 + 2 \cdot 0{,}05488 \cdot 45^4 + 19{,}039^2 \cdot 3180{,}86 = \boxed{3432872{,}631 \text{ cm}^4}$$

$$I_z = \frac{1}{12} \cdot 60^3 \cdot 45 + \frac{45^4 \pi}{8} = \boxed{2420311{,}672 \text{ cm}^4}$$

5.



86,60

$A_1 = 40 \cdot 20\sqrt{3} = 1385{,}64\ cm^2$

$A_2 = 4 \cdot \frac{10 \cdot 10\sqrt{3}}{2} = 346{,}41\ cm^2$

$A_3 = 10^2 \pi = 100\pi\ cm^2$

$A = A_1 - A_2 - A_3 = \boxed{725{,}07\ cm^2}$

$$I_y = \frac{1}{12}(20\sqrt{3})^3 \cdot 40 + 4\left[\frac{1}{36}(10\sqrt{3})^3 \cdot 10 + 11{,}547^2 \cdot 86{,}60\right] - \frac{10^4\pi}{4} =$$

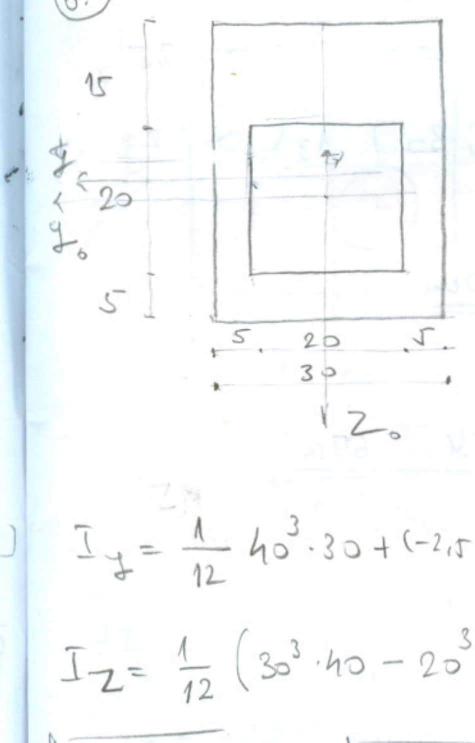
$$\Rightarrow \boxed{I_y = 78749{,}96\ cm^4}$$

$$I_z = \frac{1}{12} \cdot 40^3 \cdot 20\sqrt{3} - 4\left[\frac{1}{36} \cdot 10^3 \cdot 10\sqrt{3} + 16{,}67^2 \cdot 86{,}60\right] - \frac{10^4\pi}{4} =$$

$$\Rightarrow \boxed{I_z = 78712{,}89\ cm^4}$$

$\boxed{I_{yz} = 0\ cm^4}$

6.



$A_1 = 1200\ cm^2$

$A_2 = 400\ cm^2$

$A = A_1 - A_2 = 800\ cm^2$

$I_y = 131666{,}67$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | -5 | $T_1$ | 0 | -2,5 |
| $T_2$ | 0 | 0 | $T_2$ | 0 | 2,5 |

$z_T = \frac{-5 \cdot 400}{800} = -2{,}5\ cm \quad y_T = 0$

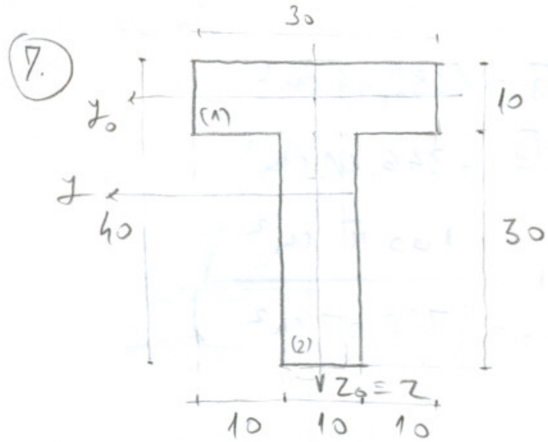
$$I_y = \frac{1}{12} 40^3 \cdot 30 + (-2{,}5)^2 \cdot 1200 - \left[\frac{1}{12} 20^3 \cdot 20 + 2{,}5^2 \cdot 400\right] = 151666{,}67\ cm^4$$

$$I_z = \frac{1}{12}\left(30^3 \cdot 40 - 20^3 \cdot 20\right) = 76666{,}67\ cm^4$$

$\boxed{I_{yz} = 0} \Rightarrow \boxed{\alpha_1 = 0}$

$I_1 = 151666{,}67\ cm^4$
$I_2 = 76666{,}67\ cm^4$

7.




$A_1 = 300\,cm^2$

$A_2 = 300\,cm^2$

$\boxed{A = 600\,cm^2}$

$T_1(\overset{y_0}{0}\,;\,\overset{z_0}{0})$ $\quad T_1(\overset{y}{0}\,;\,\overset{z}{-10})$

$T_2(0\,;\,20)$ $\quad T_2(0\,;\,10)$

$$I_y = \overbrace{\frac{1}{12}10^3\cdot 30}^{+(-10)^2\cdot 300} + \frac{1}{12}30^3\cdot 10 + 10^2\cdot 300$$

$$z_T = \frac{20\cdot 300}{600} = \underline{10cm}$$

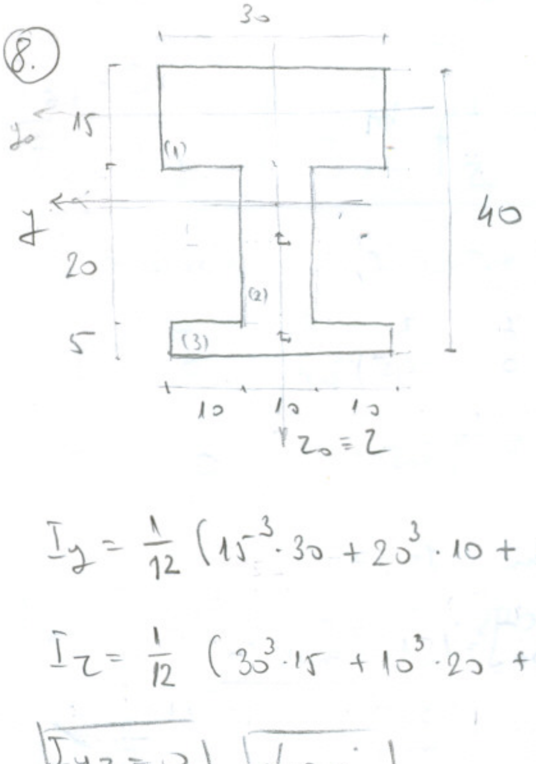
$$\Rightarrow \boxed{I_y = 85000\,cm^4}$$

$$I_z = \frac{1}{12}\cdot 30^3\cdot 10 + \frac{1}{12}10^3\cdot 30 = \boxed{25000\,cm^4}$$

$\boxed{I_{yz} = 0} \Rightarrow \boxed{\alpha_1 = 0^\circ}$

$$\boxed{\begin{matrix} I_1 = 85000\,cm^4 \\ I_2 = 25000\,cm^4 \end{matrix}}$$

8.




$A_1 = 450\,cm^2$

$A_2 = 200\,cm^2$

$A_3 = 150\,cm^2$

$\boxed{A = 800\,cm^2}$

$T_1(\overset{y_0}{0}\,;\,\overset{z_0}{0})$ $\quad T_1(\overset{y}{0}\,;\,\overset{z}{-10})$

$T_2(0\,;\,17{,}5)$ $\quad T_2(0\,;\,7{,}5)$

$T_3(0\,;\,30)$ $\quad T_3(0\,;\,20)$

$$z_T = \frac{17{,}5\cdot 200 + 30\cdot 150}{800} = \underline{10cm}$$

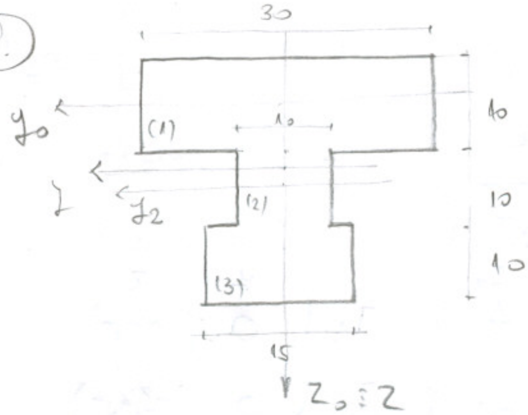
$$I_y = \overbrace{\frac{1}{12}(15^3\cdot 30 + 20^3\cdot 10 + 5^3\cdot 30)}^{+(-10)^2\cdot 450} + 7{,}5^2\cdot 200 + 20^2\cdot 150 = \underline{131666{,}67\,cm^4}$$

$$I_z = \frac{1}{12}(30^3\cdot 15 + 10^3\cdot 20 + 30^3\cdot 5) = \underline{46666{,}67\,cm^4}$$

$\boxed{I_{yz} = 0}$ $\quad \boxed{\alpha_1 = 0^\circ}$

$$\boxed{\begin{matrix} I_1 = 131666{,}67\,cm^4 \\ I_2 = 46666{,}67\,cm^4 \end{matrix}}$$

9.




$A_1 = 300 \text{ cm}^2$

$A_2 = 100 \text{ cm}^2$

$A_3 = 150 \text{ cm}^2$

$\boxed{A = 550 \text{ cm}^2}$

$I_y = 45428,2215$

$I_z = 26105,42$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | 0 | $T_1$ | 0 | $-7,27$ |
| $T_2$ | 0 | 10 | $T_2$ | 0 | 2,73 |
| $T_3$ | 0 | 20 | $T_3$ | 0 | 12,73 |

$$z_T = \frac{10 \cdot 100 + 20 \cdot 150}{550} = \underline{\underline{7,27 \text{ cm}}}$$

$$I_y = \underbrace{\frac{1}{12}\left(10^3 \cdot 30 + 10^3 \cdot 10 + 10^3 \cdot 15\right)}_{A} + \overbrace{2,73^2 \cdot 100 + 12,73^2 \cdot 150}^{(-7,27)^2 \cdot 300} = \underline{\underline{45492,43 \text{ cm}^4}}$$

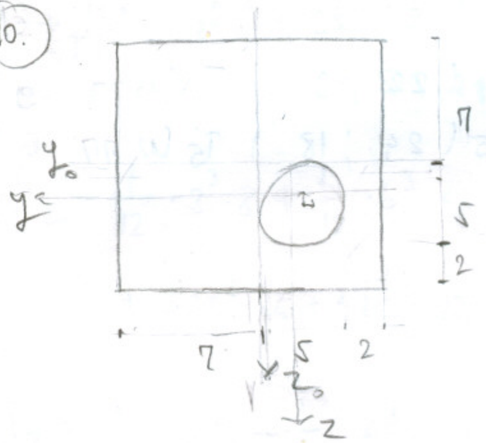
$$I_z = \frac{1}{12}\left(30^3 \cdot 10 + 10^4 + 15^3 \cdot 10\right) = \underline{\underline{26145,83 \text{ cm}^4}}$$

$\boxed{I_{yz} = 0} \Rightarrow \boxed{\alpha_1 = 0°}$

$\boxed{I_1 = I_y}$ $\boxed{I_2 = I_z}$

$$I_{y2} = A + 10^2 \cdot 100 + 10^2 \cdot 150 = \underline{\underline{29583,33 \text{ cm}^4}}$$

10.




$A_1 = 196 \text{ cm}^2$

$A_2 = 19,63 \text{ cm}^2$

$\boxed{A = 176,37 \text{ cm}^2}$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | 0 | $T_1$ | 0,278 | $-0,278$ |
| $T_2$ | $-2,5$ | 2,5 | $T_2$ | $-2,22$ | 2,22 |

$I_y =$

$$y_T = \frac{-2,5 \cdot 19,63}{176,37} = \underline{\underline{-0,278 \text{ cm}}}$$

$$z_T = \underline{\underline{0,278 \text{ cm}}}$$

$$I_y = \frac{1}{12} 14^4 + \frac{2,5^4 \pi}{4} + 2,22^2 \cdot 19,63 + (-0,278)^2 \cdot 196 = \boxed{3343,90 \text{ cm}^4} \quad \operatorname{tg} 2\alpha_1 = \frac{-2 I_{yz}}{I_y - I_z} = \infty$$

$$I_z = \frac{1}{12} 14^4 + \frac{2,5^4 \pi}{4} + 2,22^2 \cdot 19,63 + (+0,278)^2 \cdot 196 = \boxed{3343,90 \text{ cm}^4} \quad 2\alpha_1 = 90° \Rightarrow \boxed{\alpha_1 = 45°}$$

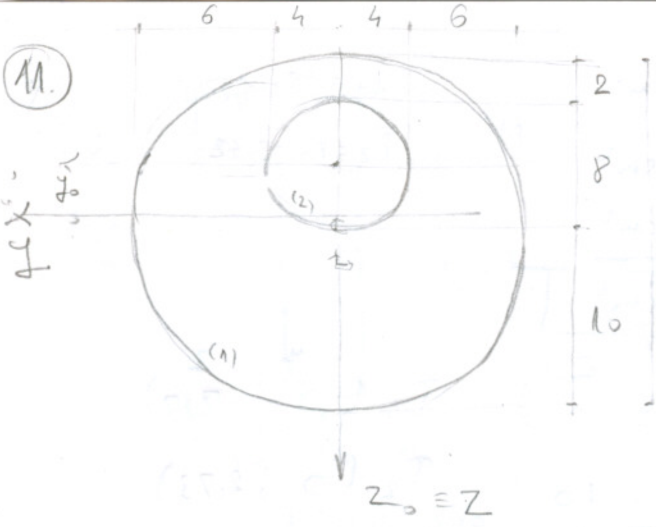
$$I_{yz} = -2,22^2 \cdot 19,63 - 0,278^2 \cdot 196 = \boxed{-111,89 \text{ cm}^4}$$

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} \Rightarrow \boxed{\begin{array}{l} I_1 = 3455,79 \text{ cm}^4 \\ I_2 = 3232,01 \text{ cm}^4 \end{array}}$$

$\boxed{\begin{array}{l} I_y = 3034,27 \\ I_z = 3034,27 \\ I_{yz} = 136,38 \end{array}}$

$\begin{array}{l} I_1 = 3170,65 \\ I_2 = 2897,89 \end{array}$

11.




$A_1 = 100\pi\ cm^2$

$A_2 = 16\pi\ cm^2$

$\boxed{A = 116\pi\ cm^2}$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | 4 | $T_1$ | 0 | 0,55 |
| $T_2$ | 0 | 0 | $T_2$ | 0 | −3,45 |

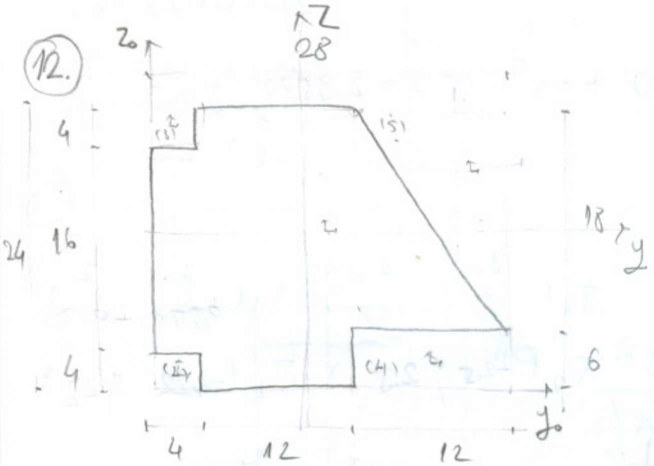
$$z_T = \frac{4 \cdot 100\pi}{116\pi} = \underline{\underline{3,45\ cm}}$$

$$I_y = \frac{10^4\pi}{4} + 0,55^2 \cdot 100\pi - \left[\frac{4^4\pi}{4} + (-3,45)^2 \cdot 16\pi\right] = \underline{\underline{8748,36\ cm^4}}$$

$$I_z = \frac{10^4\pi}{4} - \frac{4^4\pi}{4} = \boxed{7652,91\ cm^4}$$

$\boxed{I_{yz} = 0}$ $\boxed{\alpha = 0}$ $\boxed{I_1 = I_y\ ;\ I_2 = I_z}$

12.




| | | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|---|
| $A_1 = 672\ cm^2$ | $T_1$ | 14 | 12 | $T_1$ | 2,77 | 0 |
| $A_2 = 16\ cm^2$ | $T_2$ | 2 | 2 | $T_2$ | −9,23 | −10 |
| $A_3 = 16\ cm^2$ | $T_3$ | 2 | 22 | $T_3$ | −9,23 | 10 |
| $A_4 = 72\ cm^2$ | $T_4$ | 22 | 3 | $T_4$ | 10,77 | −9 |
| $A_5 = 108\ cm^2$ | $T_5$ | 24 | 18 | $T_5$ | 12,77 | 6 |

$\boxed{A = 460\ cm^2}$

$$y_T = \frac{14 \cdot 672 - 2 \cdot 16 - 2 \cdot 16 - 22 \cdot 72 - 24 \cdot 108}{460} = \underline{\underline{11,23\ cm}}$$

$$z_T = \frac{12 \cdot 672 - 2 \cdot 16 - 22 \cdot 16 - 3 \cdot 72 - 18 \cdot 108}{460} = \underline{\underline{12\ cm}}$$

$$I_y = \frac{1}{12} 24^3 \cdot 28 + 0 - 2\left[\frac{1}{12} 4^4 + 10^2 \cdot 16\right] - \left[\frac{1}{36} 18^3 \cdot 12 + (-9)^2 \cdot 72\right] - \left[\frac{1}{12} 6^3 \cdot 12 + 6^2\, 108\right]$$

$$\Rightarrow \boxed{I_y = 17133,33\ cm^4}$$

$$I_z = \frac{1}{12} 28^3 \cdot 24 + 2,77^2 \cdot 672 - \left[2\left(\frac{1}{12} \cdot 4^4 + (-9,23)^2 \cdot 16\right) + \frac{1}{36}\, 12^3 \cdot 18 + 10,77^2 \cdot 72 + \right.$$

$$\left. + \frac{1}{12} \cdot 12^3 \cdot 6 + 12,77^2 \cdot 108\right]$$

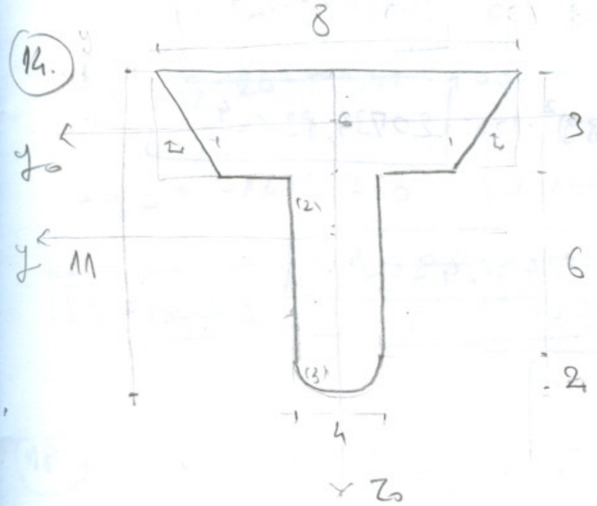
$$\Rightarrow \boxed{I_z = 18599,99\ cm^4}$$

$$I_{yz} = -\left[-9{,}23 \cdot (-10) \cdot 16 \cdot 2 + \frac{1}{72} 18^2 \cdot 12^2 + (-9) \cdot 10{,}77 \cdot 72 + 12{,}77 \cdot 6 \cdot 108\right)$$

$$\Rightarrow I_{yz} =$$

$I_{yz} = -648{,}00$

14.




|  | $y_0$ | $z_0$ |  | $y$ | $z$ |
|---|---|---|---|---|---|
| $A_1 = 24\,cm^2$ | $T_1($ 0 ; | 0 $)$ | $T_1($ 0 ; | $-3{,}25)$ |  |
| $A_2 = 24\,cm^2$ | $T_2($ 0 ; | $4{,}5)$ | $T_2($ 0 ; | $1{,}25)$ |  |
| $A_3 = 2\pi\,cm^2$ | $T_3($ 0 ; | $8{,}35)$ | $T_3($ 0 ; | $5{,}1)$ |  |
| $A_4 = 1{,}5\,cm^2$ | $T_4($ 0 ; | $2)$ | $T_4($ 0 ; | $-1{,}25)$ |  |
| $A_5 = 1{,}5\,cm^2$ | $T_5($ 0 ; | $2)$ | $T_5($ 0 ; | $-1{,}25)$ |  |

$$\boxed{A = 51{,}28\,cm^2}$$

$A = A_1 + A_2 + A_3 - A_4 - A_5$

$$\boxed{\begin{array}{l} I_y = 520{,}7590 \\ I_z = 125{,}7832 \\ I_{yz} = 0 \end{array}}$$

$$z_T = \frac{4{,}5 \cdot 24 + 8{,}35 \cdot 2\pi + 2 \cdot 1{,}5 + 2 \cdot 1{,}5}{51{,}28} = \underline{\underline{3{,}25\,cm}}$$

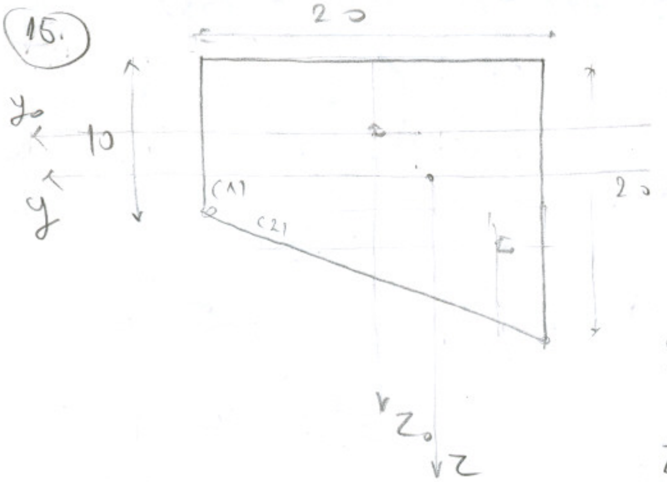
$$I_y = \frac{1}{12} \cdot 3^3 \cdot 8 + \frac{1}{12} \cdot 6^3 \cdot 4 + 2 \cdot 0{,}05488 \cdot 2^4 - 2\left(\frac{1}{36} \cdot 3^3 \cdot 1 + 1{,}25^2 \cdot 1{,}5\right) +$$

$$+ 3{,}25^2 \cdot 24 + 1{,}25^2 \cdot 24 + 5{,}1^2 \cdot 2\pi = \boxed{539{,}99\,cm^4}$$

$$I_z = \frac{1}{12} \cdot 8^3 \cdot 3 + \frac{1}{12} \cdot 4^3 \cdot 6 + \frac{2^4\pi}{8} - 2\left[\frac{1}{36} \cdot 1^3 \cdot 3\right] = \boxed{166{,}11\,cm^4}$$

$\boxed{I_{yz} = 0}$ $\boxed{\alpha = 0}$ $\boxed{I_1 = I_y \,;\, I_2 = I_z}$

(15.)




$A_1 = 200\,cm^2 \quad T_1(0\ ;\ 0) \quad T_1(4{,}44\ ;\ -4{,}44)$

$A_2 = 100\,cm^2 \quad T_2(-13{,}33\ ;\ 13{,}33) \quad T_2(-8{,}89\ ;\ 8{,}89)$

$\boxed{A = 300\,cm^2}$

$y_T = \dfrac{-13{,}33 \cdot 100}{300} = \underline{\underline{-4{,}44\,cm}}$

$z_T = \dfrac{13{,}33 \cdot 100}{300} = \underline{\underline{4{,}44\,cm}}$

$\boxed{\begin{array}{l} I_y = 6851{,}8519 \\ I_z = 9629{,}6296 \\ I_{yz} = -2407{,}4074 \\ I_1 = 11020{,}0613 \\ I_2 = 5461{,}4202 \end{array}}$

$I_y = \frac{1}{12}10^3 \cdot 20 + (-4{,}44)^2 \cdot 200 + \frac{1}{36} \cdot 10^3 \cdot 20 + 8{,}89^2 \cdot 100 = \boxed{14068{,}15\,cm^4}$

$I_z = \frac{1}{12}20^3 \cdot 10 + 4{,}44^2 \cdot 200 + \frac{1}{36} \cdot 20^3 \cdot 10 + (8{,}89)^2 \cdot 100 = \boxed{20734{,}82\,cm^4}$

$I_{yz} = -4{,}44^2 \cdot 200 - \frac{1}{72}10^2 \cdot 20^2 - 8{,}89^2 \cdot 100 = \boxed{-12401{,}49\,cm^4}$

$I_{1,2} = \dfrac{I_y + I_z}{2} \pm \sqrt{\left(\dfrac{I_y - I_z}{2}\right)^2 + I_{yz}^2} \Rightarrow \boxed{\begin{array}{l} I_1 = 30243{,}13\,cm^4 \\ I_2 = 4559{,}84\,cm^4 \end{array}}$

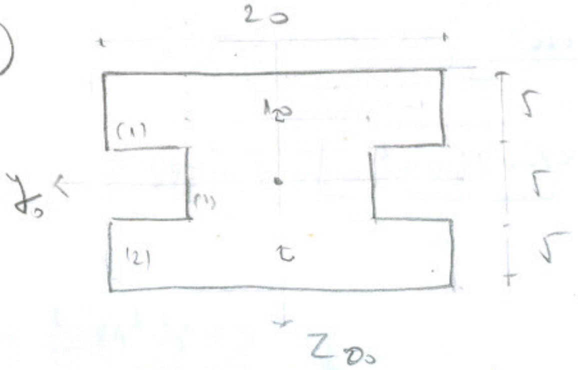
$tg\,2\alpha_1 = \dfrac{-2I_{yz}}{I_y - I_z} \begin{array}{c} (+) \\ (-) \end{array} = -3{,}72$

$2\alpha_1 = 180° - |arctg(-3{,}72)|$

$2\alpha_1 = 105{,}04°$

$\boxed{\alpha_1 = 52{,}52°}$

(16.)




$A_1 = 100\,cm^2 \quad T_1(0\ ;\ -7{,}5)$

$A_2 = 100\,cm^2 \quad T_2(0\ ;\ 0)$

$A_3 = 50\,cm^2 \quad T_3(0\ ;\ 7{,}5)$

$A = 250\,cm^2$

$I_y = 5520{,}83$

$I_y = 2\left(\frac{1}{12} \cdot 5^3 \cdot 20 + (-7{,}5)^2 \cdot 100\right) + \frac{1}{12} \cdot 5^3 \cdot 10 = \boxed{5520{,}83\,cm^4}$

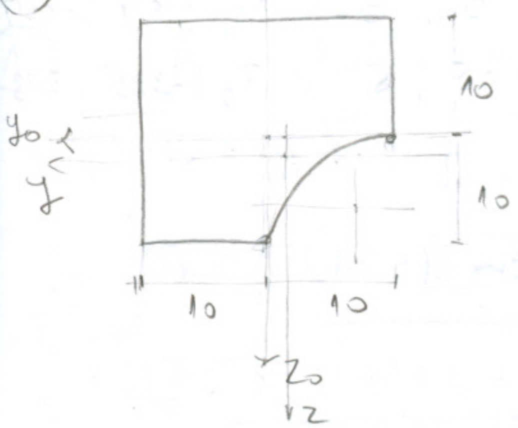
$I_z = 2\,\frac{1}{12} \cdot 20^3 \cdot 5 + \frac{1}{12}\,10^3 \cdot 5 = \boxed{7083{,}33\,cm^4}$

$\boxed{I_{yz} = 0\,cm^4}$

$\boxed{\alpha = 0°}$

$\boxed{I_1 = I_y \quad I_2 = I_z}$

19.




$A_1 = 400\,cm^2$

$A_2 = 78{,}54\,cm^2$

$\boxed{A = 321{,}46\,cm^2}$

$y_T = \dfrac{-5{,}76 \cdot 78{,}54}{321{,}46} = -1{,}41\,cm$

$z_T = 1{,}41\,cm$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | 0 | $T_1$ | 1,41 | -1,41 |
| $T_2$ | -5,76 | 5,76 | $T_2$ | -4,35 | 4,35 |

$\boxed{I_y = I_z = 9546{,}7307 \quad I_{yz} = 3073{,}0476}$

$\boxed{I_1 = 6473{,}7435 \quad I_2 = 12619{,}8873}$

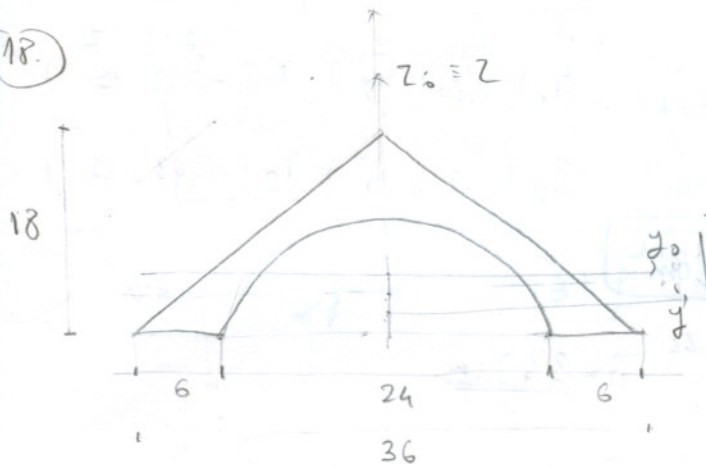
$$I_y = \frac{1}{12}\cdot 20^4 + (-1{,}41)^2 \cdot 400 - \left(0{,}05488\cdot 10^4 + 4{,}35^2 \cdot 78{,}54\right) = \boxed{12093{,}60\,cm^4}$$

$$I_z = \frac{1}{12}\cdot 20^4 + 1{,}41^2 \cdot 400 - \left(0{,}05488\cdot 10^4 + (-4{,}35)^2 \cdot 78{,}54\right) = \boxed{12093{,}60\,cm^4}$$

$$I_{yz} = -1{,}41^2 \cdot 400 - \left(0{,}01647\cdot 10^4 - 4{,}35^2 \cdot 78{,}54\right) = 526{,}23\,cm^4$$

$\boxed{I_1 = 13032{,}9\,cm^4}$ $\boxed{I_2 = 11980{,}44\,cm^4}$ $2\alpha = 90$ $\boxed{\alpha = 45^\circ}$

18.




$A_1 = 324\,cm^2$

$A_2 = 72\pi\,cm^2$

$\boxed{A = 97{,}80\,cm^2}$

$z_T = \dfrac{-0{,}91 \cdot 72\pi}{97{,}80} = -2{,}10\,cm$

| | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|
| $T_1$ | 0 | 0 | $T_1$ | 0 | 2,10 |
| $T_2$ | 0 | -0,91 | $T_2$ | 0 | 1,19 |

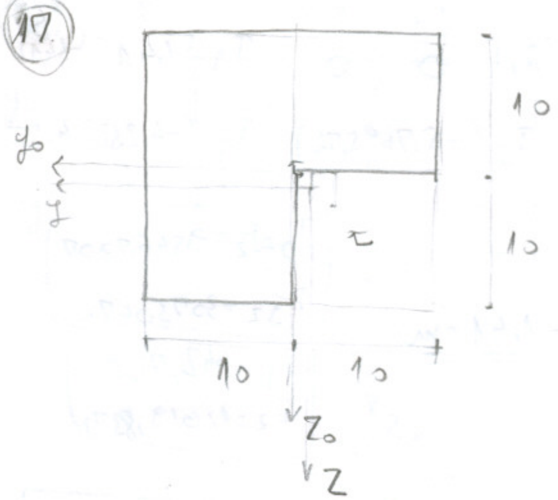
$$I_y = \frac{1}{36}\cdot 18^3 \cdot 36 + 2{,}1^2 \cdot 324 - \left(2\cdot 0{,}05488\cdot 12^4 + 1{,}19^2 \cdot 72\pi\right) = \boxed{4664{,}54\,cm^4}$$

$$I_z = \frac{1}{48}\, 36^3 \cdot 18 - \frac{12^4\pi}{8} = \boxed{9352{,}99\,cm^4}$$

$\boxed{I_{yz} = 0\,cm^4}$ $\boxed{\alpha = 0^\circ}$

$\boxed{I_1 = I_y \;;\; I_2 = I_z}$

17.




| | | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|---|
| $A_1 = 400\,cm^2$ | $T_1$ | 0 | 0 | $T_1$ | 1,67 | -1,67 |
| $A_2 = 100\,cm^2$ | $T_2$ | -5 | 5 | $T_2$ | -3,33 | 3,33 |

$\boxed{A = 300\,cm^2}$

$y_T = \frac{-500}{300} = \boxed{-1,67\,cm}$

$\boxed{z_T = 1,67\,cm}$

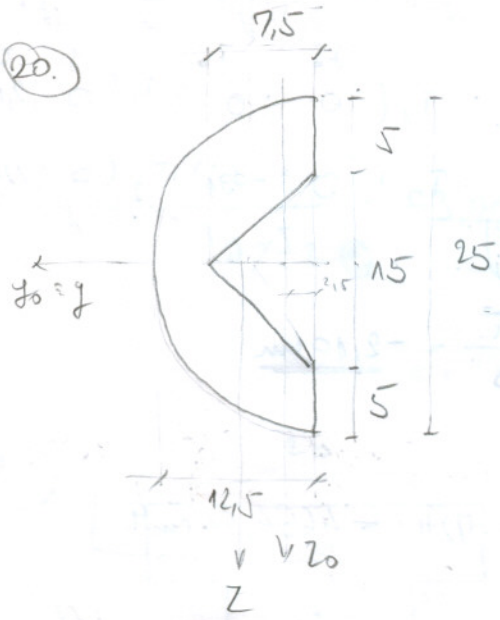
$I_y = \frac{1}{12}20^4 + (-1,67)^2 \cdot 400 - \left(\frac{1}{12}10^4 + 3,33^2 \cdot 100\right) = \boxed{12506,67\,cm^4}$

$I_z = \frac{1}{12}20^4 + 1,67^2 \cdot 400 - \left(\frac{1}{12} \cdot 10^4 + (3,33)^2 \cdot 100\right) = \boxed{12506,67\,cm^4}$

$I_{yz} = -1,67^2 \cdot 400 - \left(-3,33^2 \cdot 100\right) = \boxed{-6,67\,cm^4}$

$\boxed{I_1 = 12513,34\,cm^4 \quad I_2 = 12500\,cm^4}$ $\boxed{2\alpha = 90 \quad \alpha = 45}$

20.




| | | $y_0$ | $z_0$ | | $y$ | $z$ |
|---|---|---|---|---|---|---|
| $A_1 = 245,44\,cm^2$ | $T_1$ | 2,80 | 0 | $T_1$ | -0,83 | 0 |
| $A_2 = 56,25\,cm^2$ | $T_2$ | 0 | 0 | $T_2$ | -3,63 | 0 |

$\boxed{A = 189,19\,cm^2}$

$y_T = \frac{2,80 \cdot 245,44}{189,19} = \underline{3,63\,cm}$

$I_y = \frac{12,5^4 \pi}{8} - \frac{1}{48}15^3 \cdot 7,5 = \boxed{9060,04\,cm^4}$

$I_z = 2 \cdot 0,05488 \cdot 12,5^4 + (-0,83)^2 \cdot 245,44 - \left(\frac{1}{36} \cdot 7,5^3 \cdot 15 + (-3,63)^2 \cdot 56,25\right)$

$\Rightarrow \boxed{I_z = 1931,79\,cm^4}$

$\boxed{I_{yz} = 0\,cm^4}$ $\boxed{\alpha = 0°}$ $\boxed{I_1 = I_y;\ I_2 = I_z}$

# АНАЛИЗА НАПОНА

1. a) $\vec{f}^{(n)} = (1{,}5\vec{i} + 3\vec{j} + 5\vec{k})\ \text{MPa}$

$\vec{n} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\cdot\vec{j} + \frac{1}{\sqrt{3}}\cdot\vec{k}$

$\sigma_n = \frac{1}{\sqrt{3}}(1{,}5 + 3 + 5) = \boxed{5{,}485\ \text{MPa}}$

$|\vec{f}^{(n)}| = \sqrt{1{,}5^2 + 3^2 + 5^2} = \boxed{6{,}021\ \text{MPa}}$

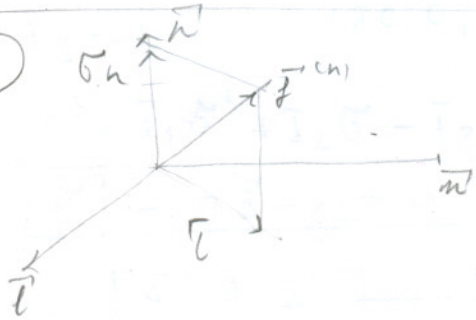
$\tau = \sqrt{6{,}021^2 - 5{,}485^2} = \boxed{2{,}483\ \text{MPa}}$

b) $\boxed{\begin{array}{l}\sigma_n = 1{,}061\ \text{MPa}\\ |\vec{f}^{(n)}| = 6{,}021\ \text{MPa}\end{array}} \Rightarrow \boxed{\tau = 5{,}927\ \text{MPa}}$

c) $\boxed{\begin{array}{l}\sigma_n = 9{,}4\ \text{MPa}\\ |\vec{f}^{(n)}| = 10{,}677\ \text{MPa}\end{array}} \Rightarrow \boxed{\tau = 5{,}064\ \text{MPa}}$

d) $\boxed{\begin{array}{l}\sigma_n = 0{,}267\ \text{MPa}\\ |\vec{f}^{(n)}| = 2{,}449\ \text{MPa}\end{array}} \Rightarrow \boxed{\tau = 2{,}434\ \text{MPa}}$

2.




a) $\vec{f}^{(n)} = \sigma_n \vec{n} + \vec{\tau}$

$\vec{\tau} = \vec{f}^{(n)} - \sigma_n \cdot \vec{n}$

$\vec{\tau} = (1{,}5;\ 3;\ 5) - \left(\frac{5{,}485}{\sqrt{3}},\ \frac{5{,}485}{\sqrt{3}},\ \frac{5{,}485}{\sqrt{3}}\right)$

$\boxed{\vec{\tau} = -1{,}67\vec{i} - 0{,}16\vec{j} + 1{,}83\vec{k}}$

b) $\vec{f}^{(n)} = \sigma_n\cdot\vec{n} + \vec{\tau} \Rightarrow \vec{\tau} = \vec{f}^{(n)} - \sigma_n\cdot\vec{n} =$

$= (-1{,}5;\ 3;\ 5) - \left(\frac{1{,}061}{\sqrt{2}};\ \frac{1{,}061}{\sqrt{2}};\ 0\right)$

$\vec{\tau} = -2{,}25\vec{i} + 2{,}250\vec{j} + 5\vec{k}$

c) $\vec{\tau} = (-5, 8, 5) - (0;\ 0{,}8\cdot 9{,}4,\ 0{,}6\cdot 9{,}4) = \underline{-5\vec{i} + 0{,}48\vec{j} - 0{,}64\vec{k}}$

(3.) $S = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 3 & 4 \\ 0 & 4 & 0 \end{bmatrix}$ MPa $\quad \vec{n} = \frac{4}{\sqrt{17}}\vec{i} + \frac{1}{\sqrt{17}}\vec{j}$

$$\vec{f}^{(n)} = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 3 & 4 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} \frac{4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \\ 0 \end{bmatrix} = \begin{bmatrix} 5{,}336 \\ 2{,}668 \\ 0{,}97 \end{bmatrix} \text{ MPa}$$

$$\sigma = 5{,}336 \cdot \frac{4}{\sqrt{17}} + 2{,}668 \cdot \frac{1}{\sqrt{17}} + 0 = \boxed{5{,}824 \text{ MPa}} \quad 5{,}824$$

$$\boxed{5{,}353 \text{ MPa}}$$

(5.) a) $\vec{f}^{(m)} = 2\vec{i} + 3\vec{j} + \vec{k} \quad \vec{m} = 0{,}8\vec{i} + 0{,}6\vec{k} \quad \vec{n}_1 = 0{,}8\vec{i} + 0{,}6\vec{j}$

$$\vec{f}^{(m)} \circ \vec{n}_1 = \vec{f}^{(n_1)} \circ \vec{m}$$

$$\vec{f}^{(n_1)} = \sigma_1 \circ \vec{n}_1$$

$$\vec{f}^{(m)} \circ \vec{n}_1 = \sigma_1 \cdot \vec{n}_1 \circ \vec{m}$$

$$\boxed{\sigma_1 = \frac{\vec{f}^{(m)} \circ \vec{n}_1}{\vec{n}_1 \circ \vec{m}}} = \frac{(2,3,1)\cdot(0{,}8;0{,}6;0)}{(0{,}8;0{,}6;0)\,(0{,}8;0;0{,}6)} =$$

$$= \frac{3{,}4}{0{,}64} = \underline{5{,}3125 \text{ MPa}}$$

b) $$\sigma_1 = \frac{\vec{f}^{(m)} \circ \vec{n}_1}{\vec{n}_1 \circ \vec{m}} = \frac{(1,3,1)\cdot(0{,}8;0;0{,}6)}{(0{,}8;0;0{,}6)\left(\frac{4}{5\sqrt{2}};\frac{1}{\sqrt{2}};\frac{3}{5\sqrt{2}}\right)} =$$

$$= \frac{1{,}4}{\frac{1}{\sqrt{2}}\left(\frac{4}{5}\cdot 0{,}8 + \frac{3}{5}\cdot 0{,}6\right)} = \boxed{1{,}98 \text{ MPa}}$$

$$\frac{1{,}4}{0{,}707}$$

6.

$$S=\begin{bmatrix}5&2&0\\2&3&4\\0&4&0\end{bmatrix} MPa$$

$I_1=5+3+0=8$

$$I_2=\begin{vmatrix}3&4\\4&0\end{vmatrix}+\begin{vmatrix}5&0\\0&0\end{vmatrix}+\begin{vmatrix}5&2\\2&3\end{vmatrix}=-16+15-4=\underline{\underline{-5}}$$

$I_3=-80$

$\sigma^3-I_1\sigma^2+I_2\sigma-I_3=0$

$$\boxed{\sigma^3-8\sigma^2-5\sigma+80=0}$$

$$S=\begin{bmatrix}0&2&0\\2&3&2\\0&2&-3\end{bmatrix}$$

$I_1=0$

$$I_2=\begin{vmatrix}3&2\\2&-3\end{vmatrix}+\begin{vmatrix}0&0\\0&-3\end{vmatrix}+\begin{vmatrix}0&2\\2&3\end{vmatrix}=-13-4=-17$$

$I_3=-12$

$\sigma^3-I_1\sigma^2+I_2\sigma-I_3=0$

$$\boxed{\sigma^3-17\sigma-12=0}$$

$$S=\begin{bmatrix}5&1&2\\1&0&3\\2&3&-5\end{bmatrix}$$

$I_1=0$

$$I_2=\begin{vmatrix}0&3\\3&-5\end{vmatrix}+\begin{vmatrix}5&2\\2&-5\end{vmatrix}+\begin{vmatrix}5&1\\1&0\end{vmatrix}=-9-29-1=-39$$

$I_3=-28$

$\sigma^3-I_1\sigma^2+I_2\sigma-I_3=0$

$$\boxed{\sigma^3-39\sigma+28=0}$$

7.

$$S=\begin{bmatrix}5&2&0\\2&3&4\\0&4&0\end{bmatrix}$$

$$\bar{\sigma}=\frac{1}{3}(5+3)=2{,}67$$

$$S=\begin{bmatrix}2{,}67&0&0\\0&2{,}67&0\\0&0&2{,}67\end{bmatrix}+\begin{bmatrix}2{,}33&2&0\\2&0{,}33&4\\0&4&-2{,}67\end{bmatrix}$$

$$I_2=\begin{vmatrix}0{,}33&4\\4&-2{,}67\end{vmatrix}+\begin{vmatrix}2{,}33&0\\0&-2{,}67\end{vmatrix}+\begin{vmatrix}2{,}33&2\\2&0{,}33\end{vmatrix}=\underline{\underline{-26{,}33}}$$

$$=-0{,}33\cdot 2{,}67-16-2{,}33\cdot 2{,}67+2{,}33\cdot 0{,}33-4=$$

$$S=\begin{bmatrix}0&2&0\\2&3&2\\0&2&-3\end{bmatrix}\qquad \bar{\sigma}=\frac{1}{3}\cdot 0=0$$

$$I_2=\begin{vmatrix}3&2\\2&-3\end{vmatrix}+\begin{vmatrix}0&0\\0&-3\end{vmatrix}+\begin{vmatrix}0&2\\2&3\end{vmatrix}=-9-4-4=\boxed{-17}$$

$$S=\begin{bmatrix}5&1&2\\1&0&3\\2&3&-5\end{bmatrix}\Rightarrow I_2=\begin{vmatrix}0&3\\3&-5\end{vmatrix}+\begin{vmatrix}5&2\\2&-5\end{vmatrix}+\begin{vmatrix}5&1\\1&0\end{vmatrix}=\underline{\underline{-39}}$$

8.

$$\vec{f}^{(m)}\circ\vec{n}^{[k]}=\vec{f}^{[n](k)}\circ\vec{m}$$

$$\vec{f}^{(m)}\circ\vec{n}^{[k]}=\sigma^{(k)}\cdot\vec{n}^{[k]}\circ\vec{m}$$

$$\sigma^{(k)}=\frac{\vec{f}^{(m)}\circ\vec{n}^{[k]}}{\vec{n}^{[k]}\circ\vec{m}}$$

$$\underline{k=1}\Rightarrow\sigma_1=\frac{(1,2,-2)\,(0,0,1)}{(0,0,1)\cdot\left(\frac{2}{\sqrt{5}},0,-\frac{1}{\sqrt{5}}\right)}=\frac{-2}{-\frac{1}{\sqrt{5}}}=\underline{\underline{4{,}72\ \mathrm{MPa}}}$$

$$\underline{k=2} \Rightarrow \sigma_2 = \frac{(1,2,-2)\cdot\left(\frac{1}{\sqrt2},\frac{1}{\sqrt2},0\right)}{\left(\frac{1}{\sqrt2},\frac{1}{\sqrt2},0\right)\left(\frac{2}{\sqrt5},0,-\frac{1}{\sqrt5}\right)} = \frac{\frac{3}{\sqrt2}}{\frac{2}{\sqrt{10}}} = \underline{\underline{1{,}34\ MPa}}$$

$$\underline{k=3} \Rightarrow \sigma_3 = \frac{(1,2,-2)\cdot\left(\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)}{\left(\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)\left(\frac{2}{\sqrt5},0,-\frac{1}{\sqrt5}\right)} = -13{,}46$$

5) $n_1 = \left\{\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right\}$ $\quad m = \left\{\frac{1}{\sqrt2}, \frac{4}{5\sqrt2}, -\frac{3}{5\sqrt2}\right\}$

$n_2 = \left\{-\frac{4}{3\sqrt2}, -\frac{1}{3\sqrt2}, \frac{1}{3\sqrt2}\right\}$ $\quad \vec f^{(m)} = \{2, 1, -2\}$

$n_3 = \left\{-\frac{1}{\sqrt2}, 0, -\frac{1}{\sqrt2}\right\}$

$$\sigma_1 = \frac{\vec f^{(m)}\circ\vec n_1}{\vec n_1\circ\vec m} = \frac{(2,1,-2)\cdot\left(\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)}{\left(\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)\left(\frac{1}{\sqrt2},\frac{4}{5\sqrt2},-\frac{3}{5\sqrt2}\right)} = 3{,}14\ MPa$$

$$\sigma_2 = \frac{(2,1,-2)\left(-\frac{4}{3\sqrt2},-\frac{1}{3\sqrt2},\frac{1}{3\sqrt2}\right)}{\left(-\frac{4}{3\sqrt2},-\frac{1}{3\sqrt2},\frac{1}{3\sqrt2}\right)\cdot\left(\frac{1}{\sqrt2},\frac{4}{5\sqrt2},-\frac{3}{5\sqrt2}\right)} = 2{,}35\ MPa$$

$$\sigma_3 = \frac{(2,1,-2)\left(-\frac{1}{\sqrt2},0,-\frac{1}{\sqrt2}\right)}{\left(-\frac{1}{\sqrt2},0,-\frac{1}{\sqrt2}\right)\left(\frac{1}{\sqrt2},\frac{4}{5\sqrt2},-\frac{3}{5\sqrt2}\right)} = 3{,}53\ MPa$$

$\vec n_1 = \frac{1}{3}\{1, -2, 2\}$ $\quad \vec m = \frac{1}{\sqrt2}\left\{1, \frac{4}{5}, -\frac{3}{5}\right\}$

$\vec n_2 = \frac{1}{3\sqrt2}\{-4, -1, 1\}$ $\quad \vec f^{(m)} = \{2, 1, -2\}$

$\vec n_3 = \frac{1}{\sqrt2}\{0, -1, -1\}$

$$\frac{\frac{1}{\sqrt2}(-1+2)}{\frac{1}{2}(} = \frac{\frac{1}{\sqrt2}}{}$$

$$\sigma_1 = \frac{(2,1,-2)\cdot\frac{1}{3}(1,-2,2)}{\frac{1}{3}(2,1,-2)\ \frac{1}{\sqrt2}\left(1,\frac{4}{5},-\frac{3}{5}\right)} = \frac{-\frac{4}{3}}{\frac{1}{3\sqrt2}\cdot 2{,}4} = -3{,}14$$

$\sigma_2 = 2{,}88$

$\sigma_3 =$

8.

$$S = \begin{bmatrix} 3x+3y & 2x & x+z^2 \\ 2x & 7 & 2+xy \\ x+z^2 & 2+yz & x+y \end{bmatrix}$$

$3 + 0 + 2z + f_x = 0 \Rightarrow f_x = -3-2z$

$2 + 0 + 0 + f_y = 0 \Rightarrow f_y = -2$

$1 + z + 0 + f_z = 0 \Rightarrow f_z = -(1+z)$

$$\boxed{\vec{F} = (-3,-2z)\vec{i} + 2\vec{j} - (1+z)\vec{k}}$$

$$S = \begin{bmatrix} 3x+2y & 2z & x+z^2 \\ 2z & 0 & y \\ x+z^2 & y & x+y \end{bmatrix} \cdot 10^{-3}\,Pa$$

$3 + 0 + 2z + f_x = 0 \Rightarrow \boxed{f_x = -3-2x}$

$0 + 0 + 0 + f_y = 0 \quad \boxed{f_y = 0}$

$1 + 1 + 0 + f_z = 0 \Rightarrow \boxed{f_z = -2}$

$$\boxed{\vec{F} = (-3-2x)\vec{i} - 2\vec{k}}$$

$$S = \begin{bmatrix} xz & 2x & z^2 \\ 2x & 7+x & x\cdot y \\ z^2 & x\cdot y & 4y \end{bmatrix} \cdot 10^{-3}\,Pa$$

$z + 0 + 2z + f_x = 0 \Rightarrow f_x = -3z$

$2 + 0 + 0 + f_y = 0 \Rightarrow f_y = -2$

$0 + x + 0 + f_z = 0 \Rightarrow f_z = -x$

$$\boxed{\vec{F} = -3z\vec{i} - 2\vec{j} - x\vec{k}}$$

9.

$$S=\left|\begin{array}{ccc|cc} 4 & 2 & 0 & 4 & 2 \\ 2 & A & 1 & 2 & A \\ 0 & 1 & -2 & 0 & 1 \end{array}\right| = -8A - 4 + 8 = 0$$

$$8A = 4 \Rightarrow \boxed{A = \frac{1}{2}}$$

$$S=\left|\begin{array}{ccc|cc} 4 & 1 & 0 & 4 & 1 \\ 1 & A & 5 & 1 & A \\ 0 & 5 & -1 & 0 & 5 \end{array}\right| = -4A - 100 + 1 \Rightarrow 4A = -99$$

$$\boxed{A = -\frac{99}{4}}$$

$$S=\left|\begin{array}{ccc|cc} 4 & 1 & 0 & 4 & 1 \\ 1 & 2 & A & 1 & 2 \\ 0 & A & -1 & 0 & A \end{array}\right| = -8 - 4A^2 + 1 = 0$$

$$4A^2 = -7 \Rightarrow A^2 = \sqrt{-\frac{7}{4}}$$

$$\boxed{A = \pm \frac{i\sqrt{7}}{2}}$$

10.

$$S=\begin{bmatrix} 1 & 0 & -1 \\ 0 & 3 & 1 \\ -1 & 1 & 4/3 \end{bmatrix}$$

$$\left|\begin{array}{ccc|cc} x & y & z & x & y \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 3 & 1 & 0 & 3 \end{array}\right| = 3z + 3x - y = 0$$

$$\boxed{3x - y + 3z = 0}$$

$$\left|\begin{array}{ccc|cc} x & y & z & x & y \\ 2 & 1 & 2 & 2 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{array}\right| = x + 2y - z - 2y = 0$$

$$\boxed{x - z = 0}$$

$$\left|\begin{array}{ccc|cc} x & y & z & x & y \\ 4 & \sqrt{6} & -1 & 4 & \sqrt{6} \\ \sqrt{6} & 2 & 0 & \sqrt{6} & 2 \end{array}\right| = -\sqrt{6}y + 8z - 6z + 2x = 0$$

$$\boxed{2x - \sqrt{6}y + 2z = 0}$$

(11.) $S = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$ $S = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ MPa $\sigma_x = 5$ $\sigma_y = 3$ $\tau_{xy} = 2$

11.1 $\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \Rightarrow \boxed{\begin{matrix}\sigma_1 = 6{,}236\ \text{MPa} \\ \sigma_2 = 1{,}764\ \text{MPa}\end{matrix}}$

$tg\, 2\alpha = \frac{2\tau_{xy} \oplus}{\sigma_x - \sigma_y \oplus} = 2 \Rightarrow 2\alpha = 63{,}43°$

$\boxed{\alpha_1 = 31{,}72°}$

$\boxed{\alpha_2 = 121{,}72°}$

11.2 $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}(6{,}236 - 1{,}764) = \boxed{2{,}236\ \text{MPa}}$

11.3 $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\varphi + \tau_{xy}\sin 2\varphi = \boxed{6{,}23\ \text{MPa}}$

$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\varphi - \tau_{xy}\cos 2\varphi = \boxed{-0{,}13\ \text{MPa}}$

$\frac{5-3}{2}\sin 60 - 2 \cdot 2\cos 60 =$

$|f^{(n)}| = \sqrt{\sigma_n^2 + \tau_n^2} = 6{,}23\ \text{MPa}$

$f^{(n)} = \tau_{max} = 2{,}236\ \text{MPa}$

$\varphi = 45$

$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\varphi + \tau_{xy}\sin 2\varphi = 4 + \cos 90 + 2 \cdot \sin 90 = 6$

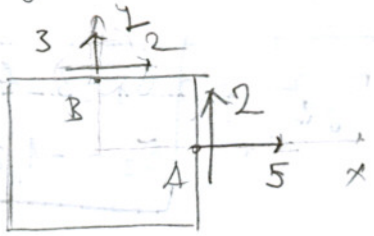
$\tau_n = \frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\varphi + \tau_{xy}\cos 2\varphi = \sin 90 - 2\cos 90 = 1$

$|f^{(n)}| = \sqrt{36+1} = 6{,}08$

$$S = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\sigma_x = 5$ $\tau_{xy} = 2$

$\sigma_y = 3$




$\sigma$ $\tau$

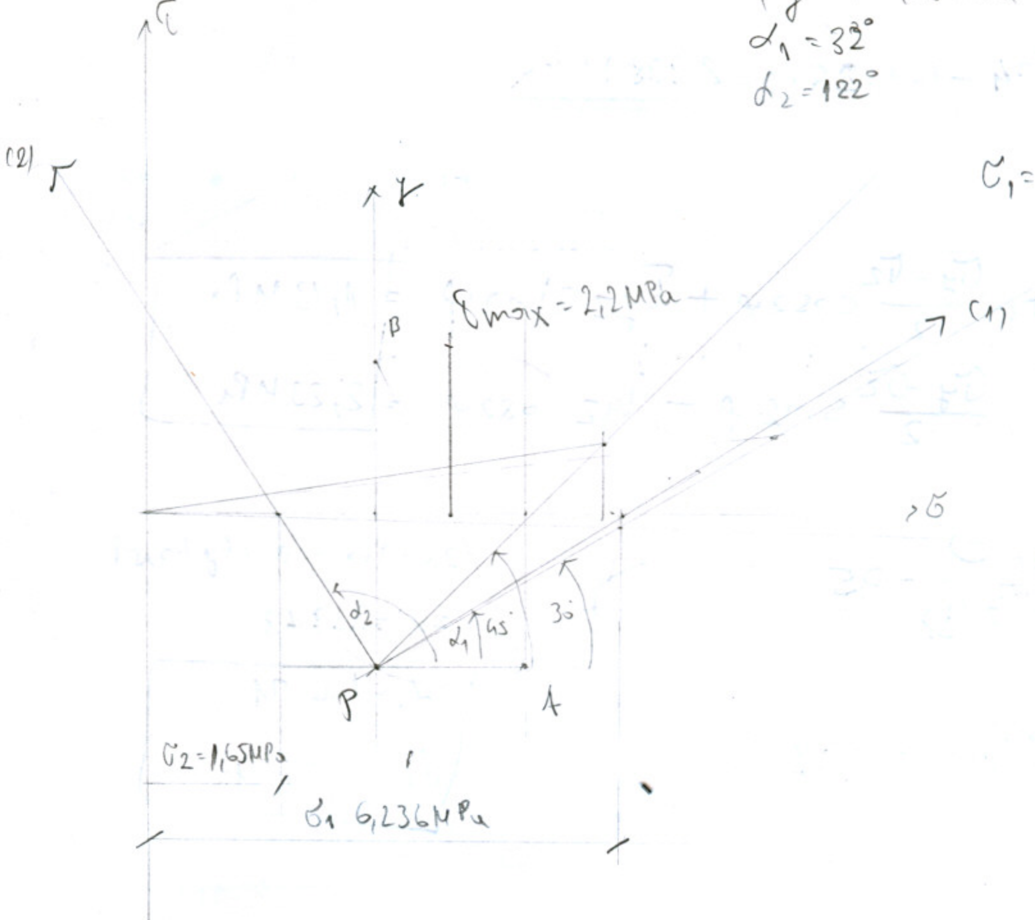
$A\ (5, -2)$

$B\ (3, 2)$

| $\varphi = 30°$ | $\varphi = 45°$ |
|---|---|
| $\sigma_n = 6{,}23\ MPa$ | $\sigma_n = 6\ MPa$ |
| $\tau = -0{,}15\ MPa$ | $\tau = 0{,}95\ MPa$ |
| $\lvert f^{(n)} \rvert = 6{,}25\ MPa$ | $\lvert f^{(n)} \rvert = 6{,}05\ MPa$ |

$\alpha_1 = 32°$

$\alpha_2 = 122°$



(11.)

$$S=\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_y & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_z\end{bmatrix} \qquad S=\begin{bmatrix}5 & 2 & 0\\ 2 & 3 & 0\\ 0 & 0 & 0\end{bmatrix}\text{MPa} \qquad \begin{matrix}\sigma_x=5\\ \sigma_y=3\\ \tau_{xy}=2\end{matrix}$$

11.1 $\sigma_{1,2}=\frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} \Rightarrow \boxed{\begin{matrix}\sigma_1=6{,}236\text{ MPa}\\ \sigma_2=1{,}764\text{ MPa}\end{matrix}}$

$tg\,2\alpha=\frac{2\tau_{xy}\,\oplus}{\sigma_x-\sigma_y\,\oplus}=2 \Rightarrow 2\alpha=63{,}43^\circ$

$\boxed{\alpha_1=31{,}72^\circ}$

$\boxed{\alpha_2=121{,}72^\circ}$

11.2 $\tau_{max}=\frac{1}{2}(\sigma_1-\sigma_2)=\frac{1}{2}(6{,}236-1{,}764)=\boxed{2{,}236\text{ MPa}}$

11.3 $\sigma_n=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos2\varphi+\tau_{xy}\sin2\varphi\approx\boxed{6{,}23\text{ MPa}}$

$\tau=\frac{\sigma_x-\sigma_y}{2}\sin2\varphi-\tau_{xy}\cos2\varphi=\boxed{-0{,}13\text{ MPa}}$

$\frac{5-3}{2}\sin60-2\cdot2\cos60$

$|f^{(n)}|=\sqrt{\sigma_n^2+\tau_n^2}\approx6{,}23\text{ MPa}$

$f^{(n)}=\tau_{max}=2{,}236\text{ MPa}$

$\varphi=45^\circ$

$\sigma_n=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos2\varphi+\tau_{xy}\sin2\varphi=4+\cos90+2\cdot\sin90=6$

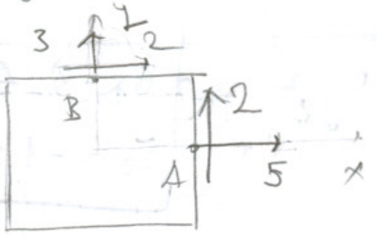
$\tau_n=\frac{\sigma_x-\sigma_y}{2}\cdot\sin2\varphi+\tau_{xy}\cos2\varphi=\sin90-2\cos90=1$

$|f^{(n)}|=\sqrt{36+1}=6{,}08$

$$S = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\sigma_x = 5 \quad \tau_{xy} = 2$

$\sigma_y = 3$




$\sigma \quad \tau$

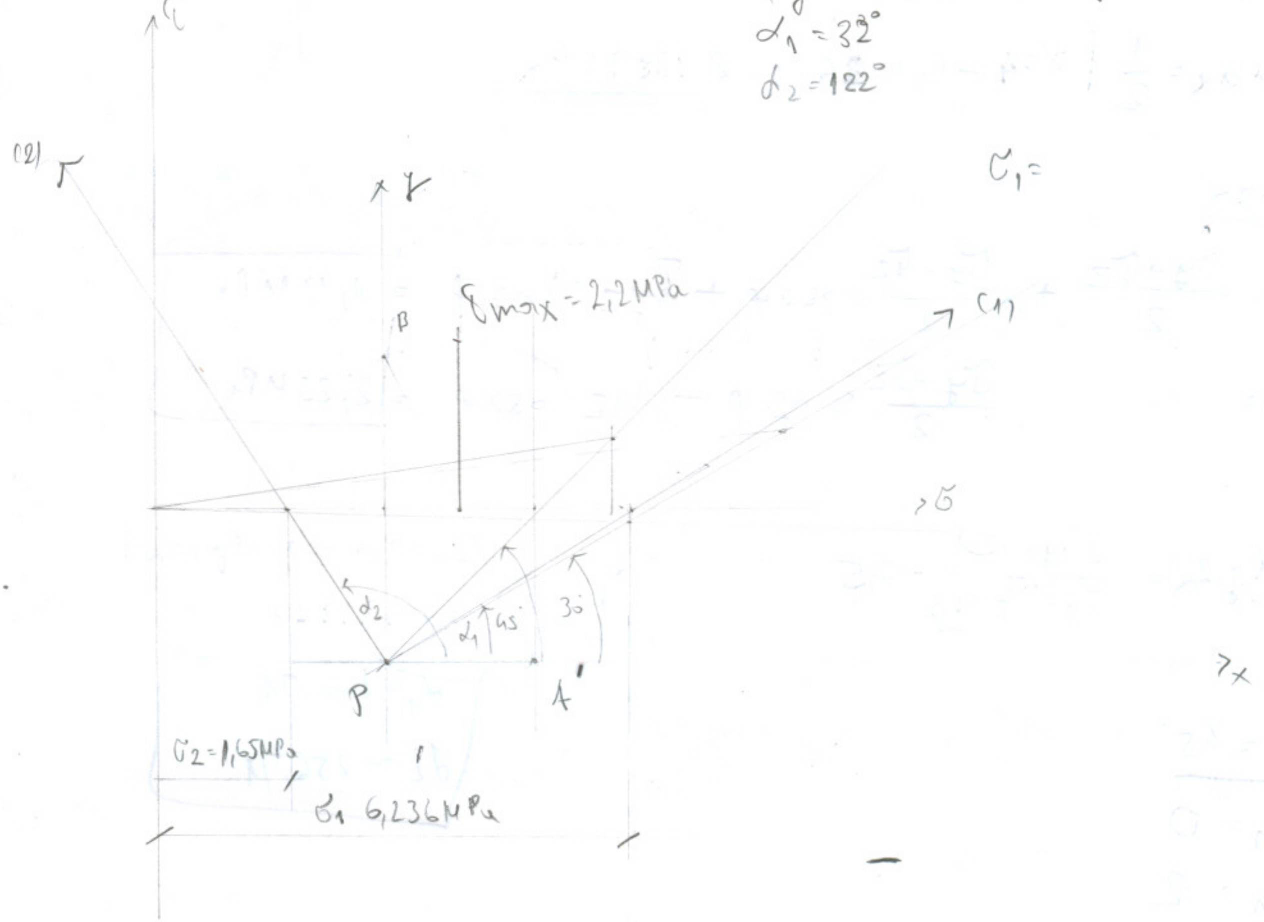
$A\,(5, -2)$

$B\,(3, 2)$

| $\varphi = 30^\circ$ | $\varphi = 45^\circ$ |
|---|---|
| $\sigma_n = 6{,}23\,MPa$ | $\sigma_n = 6\,MPa$ |
| $\tau = -0{,}15\,MPa$ | $\tau = 0{,}95\,MPa$ |
| $\lvert \vec{f}^{(n)} \rvert = 6{,}25\,MPa$ | $\lvert \vec{f}^{(n)} \rvert = 6{,}05\,MPa$ |

$\alpha_1 = 32^\circ$

$\alpha_2 = 122^\circ$



$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & -1 \end{bmatrix} MPa \qquad \sigma_y = 3 \quad \tau_{yz} = -1 \quad \sigma_z = -1$$

1. $$\sigma_{1,2} = \frac{\sigma_y + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_y + \sigma_z}{2}\right)^2 + \tau_{yz}^2} \Rightarrow \boxed{\begin{array}{l}\sigma_1 = 3{,}24\ MPa \\ \sigma_2 = -1{,}236\ MPa\end{array}}$$

2. $$\tau_{max} = \frac{1}{2}\left|3{,}24 - (-1{,}236)\right| = \underline{\underline{2{,}238\ MPa}}$$

3. $\varphi = 30°$

$$\sigma_n = \frac{\sigma_y + \sigma_z}{2} + \frac{\sigma_y - \sigma_z}{2}\cos 2\varphi + \tau_{yz}\sin 2\varphi = \boxed{1{,}13\ MPa}$$

$$\tau_n = \frac{\sigma_y - \sigma_z}{2}\sin 2\varphi - \tau_{yz}\cos 2\varphi = \boxed{2{,}23\ MPa}$$

1' $$tg\,2\alpha = \frac{2\tau_{yz}}{\sigma_y - \sigma_z} = -0{,}5$$

$2\alpha = 360° - arctg\,0{,}51$

$2\alpha = 333{,}43°$

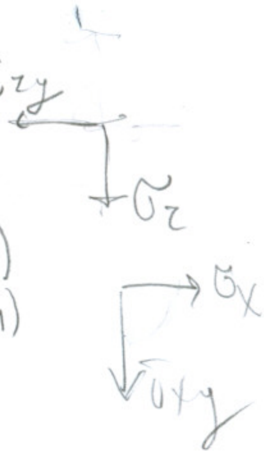
$$\boxed{\begin{array}{l}\alpha_1 = 166{,}71 \\ \alpha_2 = 256{,}71\end{array}}$$

$\varphi = 45°$

$\sigma_n = 0$

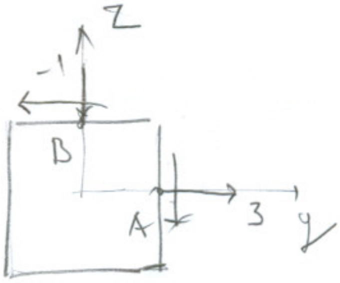
$\tau_n = 2$

4. $$\left|\vec{f}^{(n)}\right|_{30°} = \sqrt{1{,}13^2 + 2{,}23^2} = 2{,}5\ MPa$$




45°

$\left|\vec{f}^{(n)}\right| = 2\ MPa$




A(3, 1)

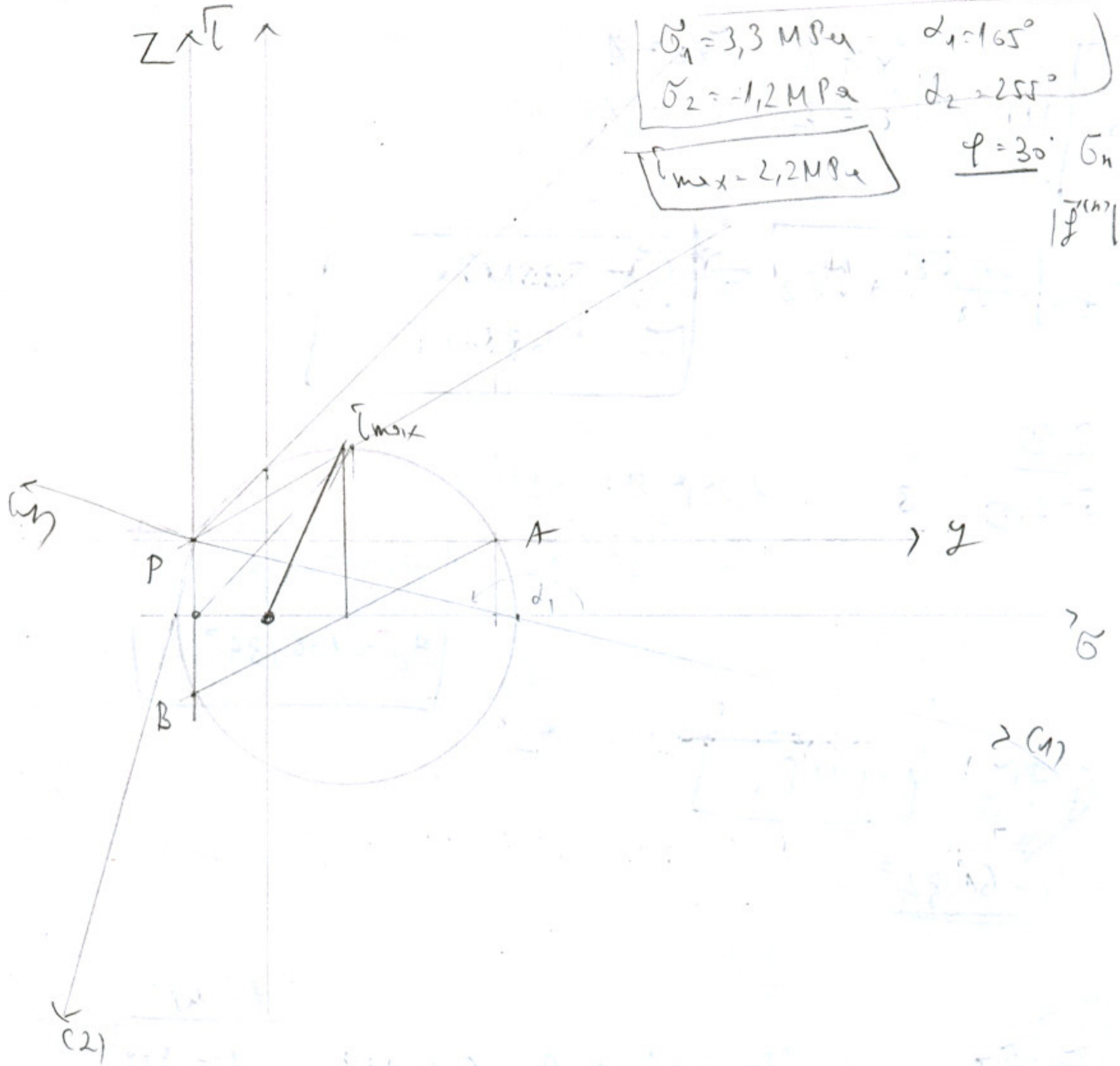
B(-1, -1)

$\boxed{\begin{array}{ll} \sigma_1 = 3{,}3\ \mathrm{MPa} & \alpha_1 = 165^\circ \\ \sigma_2 = -1{,}2\ \mathrm{MPa} & \alpha_2 = 255^\circ \end{array}}$

$\boxed{\tau_{max} = 2{,}2\ \mathrm{MPa}}$

$\underline{\varphi = 30^\circ}$ $\sigma_n = 1{,}1\ \mathrm{MPa}$

$|\bar{f}^{(n)}| = 2{,}5\ \mathrm{MPa}$



$$S = \begin{bmatrix} 5 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix} MPa \quad \sigma_x = 5 \quad \sigma_{xz} = 1 \quad \sigma_z = 2$$

1. $\sigma_1 = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \sigma_{xz}^2} \Rightarrow \boxed{\begin{aligned}\sigma_1 &= 5{,}30\,MPa \\ \sigma_2 &= 1{,}68\,MPa\end{aligned}}$

$$tg\,2\alpha = \frac{2 \cdot 1}{5-2 \cdot 1} = \frac{2}{3} \qquad 2\alpha = arctg\left(\frac{2}{3}\right)$$

$$2\alpha = 33{,}69 \rightarrow \boxed{\begin{aligned}\alpha_1 &= 16{,}84^\circ \\ \alpha_2 &= 106{,}84^\circ\end{aligned}}$$

2. $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \boxed{1{,}81\,MPa}$

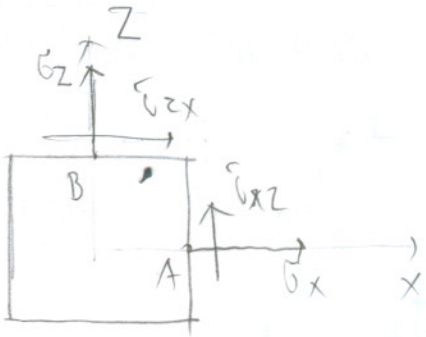
$$n^{(\tau)} = 45 + \alpha_1 = \underline{61{,}84^\circ}$$

3. $\underline{\varphi = 30^\circ}$ $\qquad\qquad$ $\underline{\varphi = 45^\circ}$

$$\sigma_n = \frac{\sigma_x + \sigma_z}{2} + \frac{\sigma_x - \sigma_z}{2} \cos 2\varphi + \tau_{xz} \sin 2\varphi = \underline{5{,}11\,MPa} \qquad 4{,}5\,MPa$$

$$\tau = \frac{\sigma_x - \sigma_z}{2} \sin 2\varphi - \tau_{xz} \cos 2\varphi = \underline{\underline{0{,}80\,MPa}} \qquad 1{,}5\,MPa$$

$$|\vec{f}^{(n)}| = \sqrt{5{,}11^2 + 0{,}80^2} = \underline{5{,}17\,MPa} \qquad \frac{45^\circ}{\underline{4{,}74\,MPa}}$$




A (5, −1)
B (2, 1)

$\varphi = 30°$ $\varphi = 45°$

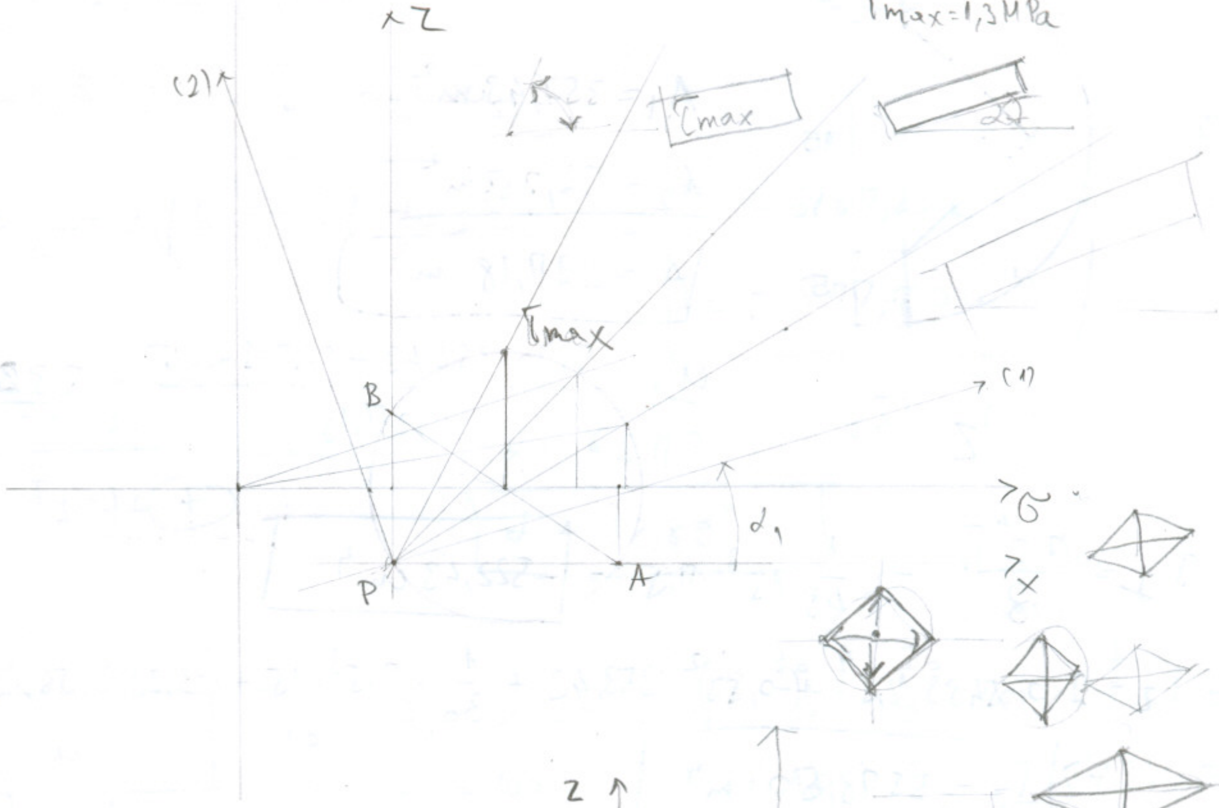
$\sigma_n = 5{,}4\,MPa$

$\tau_n = 0{,}15\,MPa$

$\sigma_1 = 5{,}3\,MPa$ $\alpha_1 = 12°$

$\sigma_2 = 1{,}7\,MPa$ $\alpha_2 = 102°$

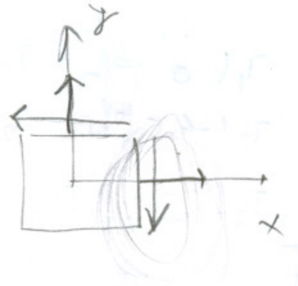
$\tau_{max} = 1{,}3\,MPa$





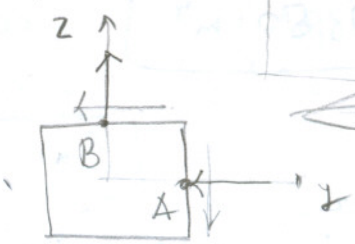
$\sigma_x = 5$ $\tau_{xy} = -2$

$\sigma_y = 3$



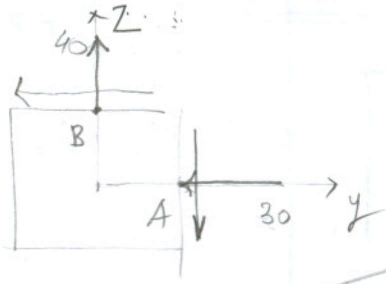

A(-30,20)

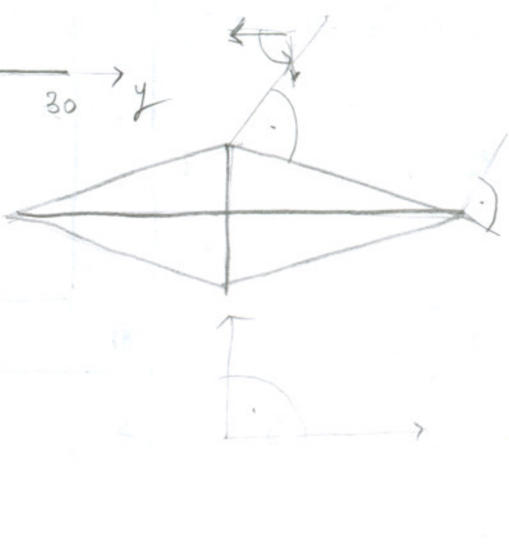
B(



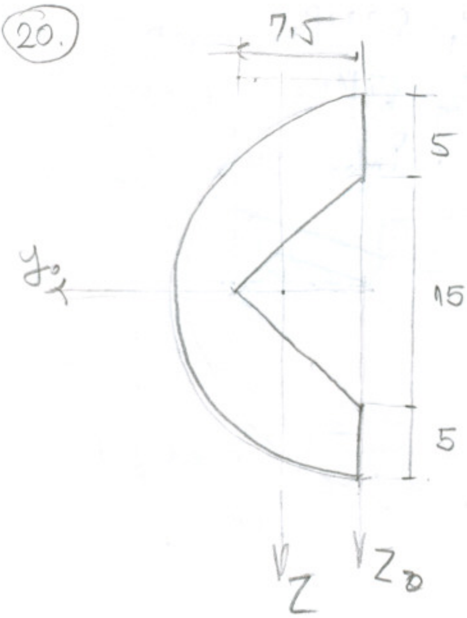

$\sigma_y = -30$ $\tau_{yz} = -20$

$\sigma_z = 40$



20.




$T_1(5{,}30 ; 0)$ $T_1(-0{,}53 ; 0)$

$T_2(2{,}5 ; 0)$ $T_2(-3{,}33 ; 0)$

$A_1 = 353{,}43 cm^2$

$A_2 = 56{,}25 cm^2$

$\boxed{A = 297{,}18 cm^2}$

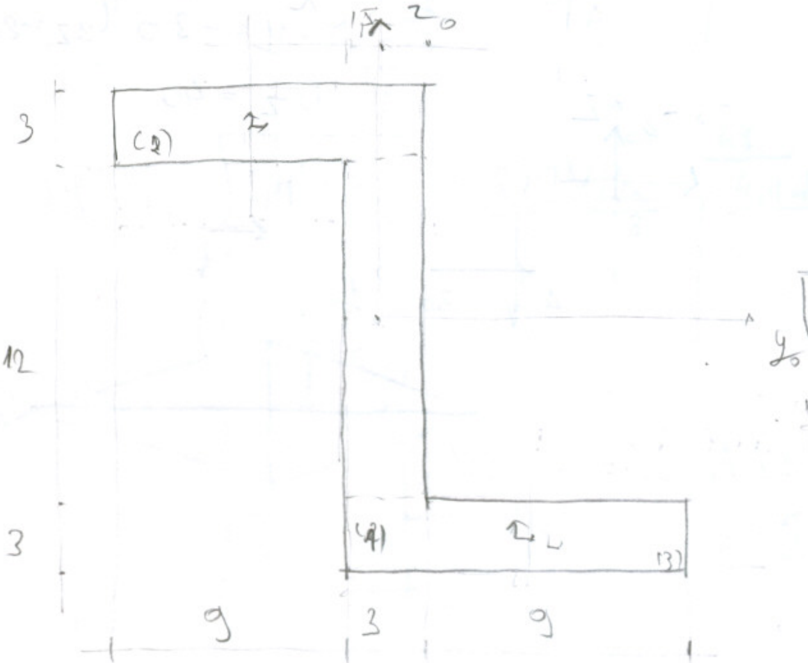
$$y_T = \frac{5{,}30 \cdot 353{,}43 - 2{,}5 \cdot 56{,}25}{297{,}18} = \underline{5{,}83 cm}$$

$$I_y = \frac{12{,}5^4 \pi}{8} - \frac{1}{48} 15^3 \cdot 7{,}5 = \boxed{-522{,}43 cm^4}$$

$$I_z = 2 \cdot 0{,}05488 \cdot 12{,}5^4 + (-0{,}53)^2 \cdot 353{,}43 + \frac{1}{36} \cdot 7{,}5^3 \cdot 15 + (-3{,}33)^2 \, 56{,}25 =$$

$$\Rightarrow \boxed{I_z = 3578{,}50 cm^4}$$

$I_{yz} = 0$




$A_1 = 36 cm^2$ $T_1(0 ; 0)$ $T_1( ; )$

$A_2 = 36 cm^2$ $T_2(-4{,}5 ; 7{,}5)$ $T_2( ; )$

$A_3 = 36 cm^2$ $T_3(4{,}5 ; -7{,}5)$ $T_3( ; )$

$\boxed{A = 108 cm^2}$

$$I_y = \frac{1}{12}\cdot 12^3\cdot 3 + 2\cdot\left[\frac{1}{12}\cdot 3^3\cdot 12 + 7,5^2\cdot 36\right] = \underline{4536\ cm^4}$$

x $$I_z = \frac{1}{12}\cdot 3^3\cdot 12 + 2\cdot\left[\frac{1}{12}\, 12^3\cdot 3 + 4,5^2\cdot 36\right] = \underline{2349\ cm^4}$$

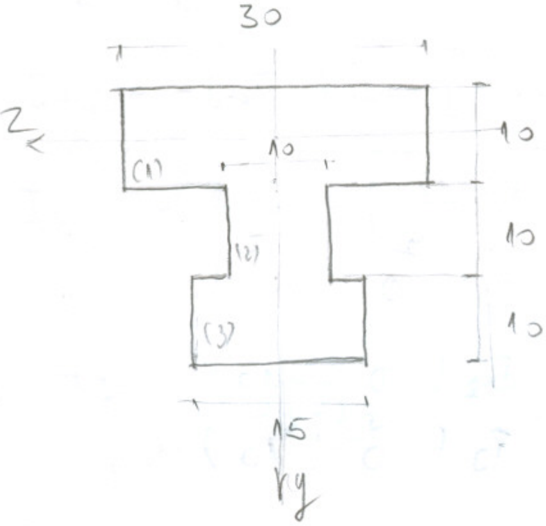
$$I_{yz} = -4,5\cdot 7,5\cdot 36\cdot 2 = \underline{-2430\ cm^4}$$

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} \Rightarrow \boxed{\begin{array}{l} I_1 = 6107,20\ cm^4 \\ I_2 = 777,80\ cm^4 \end{array}}$$

$$tg\, 2\alpha = \frac{-2I_{yz}\ (+)}{I_y - I_z\ (+)} = 2,22 \Rightarrow 2\alpha = 65,77$$

$$\boxed{\alpha = 32,89^\circ}$$

x: 465,2311
y: 665,2404




| | $y_0$ | $z_0$ | $y$ | $z$ |
|---|---|---|---|---|
| $A_1 = 300\ cm^2$ | $T_1($ 0 | 0 $)$ | $T_1($ -8,64 | 0 $)$ |
| $A_2 = 100\ cm^2$ | $T_2($ 10 | 0 $)$ | $T_2($ 1,36 | 0 $)$ |
| $A_3 = 150\ cm^2$ | $T_3($ 25 | 0 $)$ | $T_3($ 16,36 | 0 $)$ |

$$\boxed{A = 550\ cm^2}$$

$$y_T = \frac{10\cdot 100 + 25\cdot 150}{550} = \underline{8,64\ cm}$$

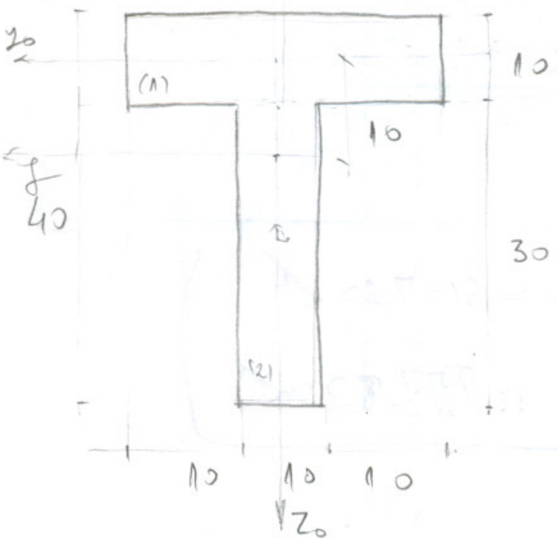
$$I_y = \frac{1}{12}\cdot 30^3\cdot 10 + (-8,64)^2\cdot 300 + \frac{1}{12}\cdot 10^3\cdot 10 + 1,36^2\cdot 100 + \frac{1}{12}\cdot 15^3\cdot 10 + 16,36^2\cdot 150$$

$$\Rightarrow \boxed{I_y = 88873,11\ cm^4}$$

$$I_z = \frac{1}{12}\left(10^3\cdot 30 + 10^4 + 10^3\cdot 15\right) \boxed{4583,33\ cm^4}$$

$$\boxed{I_{yz} = 0\ cm^4}$$

$$\boxed{I_1 = I_y \quad I_2 = I_z}$$



$A_1 = 300\,cm^2 \quad T_1(0\,;\,0) \quad T_1(0\,;\,-10)$

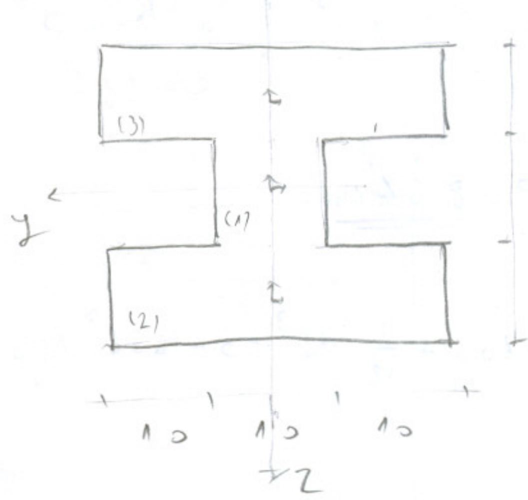
$A_2 = 300\,cm^2 \quad T_2(0\,;\,20) \quad T_2(0\,;\,10)$

$A = 600\,cm^2$

$$y_T = \frac{0 + 20\cdot 300}{600} = \boxed{10\,cm}$$

$$I_y = \frac{1}{12}10^3\cdot 30 + (-10)^2\cdot 300 + \frac{1}{12}30^3\cdot 10 + 10^2\cdot 300 = \boxed{85000\,cm^4}$$

$$I_z = \frac{1}{12}\left(30^3\cdot 10 + 10^3\cdot 30\right) = \boxed{25000\,cm^4}$$



$A_1 = 100\,cm^2 \quad T_1(0\,;\,0)$

$A_2 = 300\,cm^2 \quad T_2(0\,;\,-10)$

$A_3 = 300\,cm^2 \quad T_3(0\,;\,10)$

$\boxed{A = 700\,cm^2}$

$$I_y = \frac{1}{12}\cdot 10^4 + 2\left(\frac{1}{12}\cdot 10^3\cdot 30 + 10^2\cdot 300\right) = \boxed{65833{,}33\,cm^4}$$

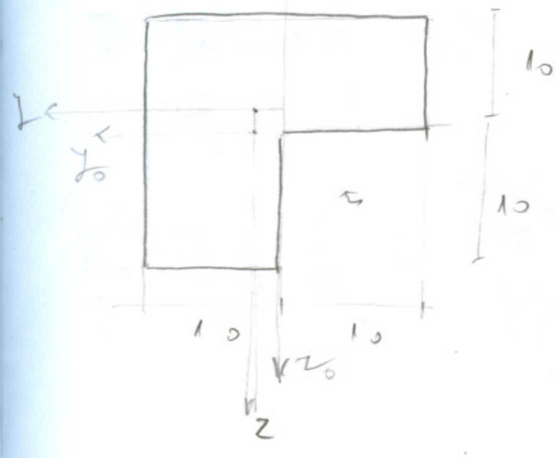
$$I_z = \frac{1}{12}\cdot 10^4 + 2\left(\frac{1}{12}\cdot 30^3\cdot 10\right) = \boxed{45833{,}33\,cm^4}$$

$\boxed{I_{yz} = 0\,cm^4}$

$\boxed{\alpha = 0}$

$\boxed{I_1 = 65833{,}33\,cm^4}$

$\boxed{I_2 = 45833{,}33\,cm^4}$




$A_1 = 400\,cm^2 \quad T_1(\overset{y_0}{0}\,;\,\overset{z_0}{0}) \quad T_1(\overset{y}{-1{,}67}\,;\,\overset{z}{\underline{1{,}67}})$

$A_2 = 100\,cm^2 \quad T_2(-5\,;\,5) \quad T_2(-6{,}67\,;\,6{,}67)$

$\boxed{A = 300\,cm^2}$

$$y_T = \frac{0 + 5\cdot 100}{300} = \underline{\underline{1{,}67\,cm}}$$

$$z_T = \frac{0 - 5\cdot 100}{300} = \underline{\underline{-1{,}67\,cm}}$$

$$I_y = \frac{1}{12}\cdot 20^4 + 1{,}67^2\cdot 400 - \left[\frac{1}{12}\cdot 10^4 + 6{,}67^2\cdot 100\right] = \boxed{9166{,}67\,cm^4}$$

$$I_z = \frac{1}{12}\cdot 20^4 + (-1{,}67)^2\cdot 400 - \left[\frac{1}{12}\cdot 10^4 + (-6{,}67)^2\cdot 100\right] = \boxed{9166{,}67\,cm^4}$$

$$I_{yz} = -1{,}67^2\cdot 400 - \left[-6{,}67^2\cdot 100\right] = \boxed{3333{,}33\,cm^4}$$

$$I_{1;2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} \Rightarrow \boxed{I_1 = 12500\,cm^4}$$

$$\boxed{I_2 = 5833{,}34\,cm^4}$$

$$tg\,2\alpha = \frac{-2I_{yz}}{I_y - I_z} = \infty \qquad 2\alpha = 90 \qquad \boxed{\alpha = 45}$$