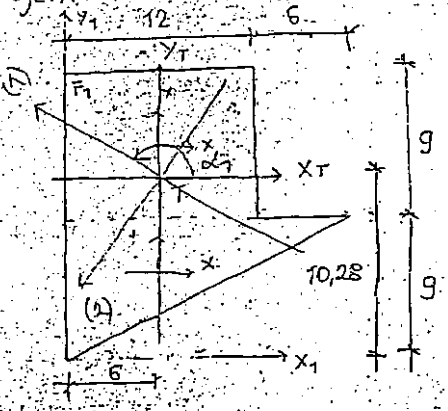
Гировић Владан 121/04

30.01.2004.

1.1 ЗА ДАТИ ПРЕСЕК ОДРЕДИТИ ПРАВАЦ ТЕЖИШНЕ ОСЕ КОЈОЈ ОДГОВАРА НАЈВЕЋИ МОМЕНАТ ИНЕРЦИЈЕ И ВРЕДНОСТИ ГЛАВНИХ ЦЕНТРАЛНИХ МОМЕНАТА ИНЕРЦИЈЕ



[cm]

$F_1 = 12 \cdot 9 = 108\ cm^2$

$F_2 = \frac{1}{2} 18 \cdot 9 = 81\ cm^2$

$F = 189\ cm^2$

$x_T = \frac{108 \cdot 6 + 81 \cdot 6}{189} = 6\ cm$

$y_T = \frac{108 \cdot 13{,}5 + 81 \cdot 6}{189} = 10{,}28\ cm$

$I_x = \frac{1}{12} 12 \cdot 9^3 + 3{,}22^2 \cdot 108 + \frac{1}{36} 18 \cdot 9^3 + 4{,}28^2 \cdot 81 = 3697{,}08\ cm^4$

$I_y = \frac{1}{12} 12^3 \cdot 9 + \frac{1}{36} \cdot 9 \cdot 18^3 = 2754\ cm^4$

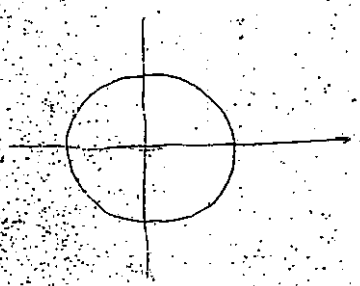
$I_{xy} = \frac{18^2 \cdot 9^2}{72} = 364{,}5\ cm^4$

$I_{1/2} = \frac{3697{,}08 + 2754}{2} \pm \sqrt{\left(\frac{3697{,}08 - 2754}{2}\right)^2 + 364{,}5^2}$

$I_1 = 3821{,}54\ cm^4 \qquad I_2 = 2629{,}54\ cm^4$

$tg\,2\alpha = \frac{-2 I_{xy}}{I_x - I_y} = \frac{-2 \cdot 364{,}5}{3697{,}08 - 2754} < 0$

$\alpha = \frac{1}{2}(360^\circ + arctg(-0{,}773)) = 161{,}15^\circ$



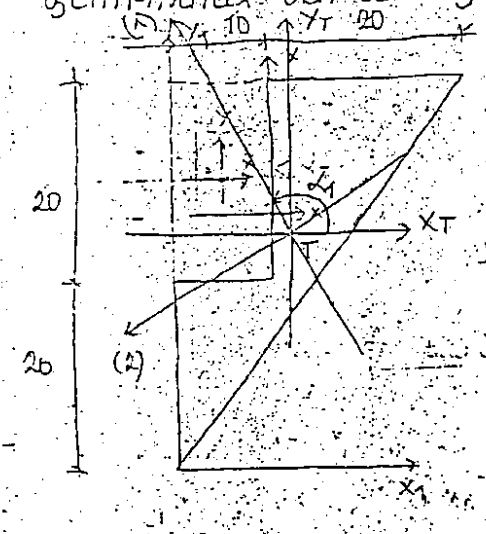
I_{yz} КОД КВАДРАТА СТАТИЧКИ МОМЕНТ ИНЕРЦИЈЕ $\neq 0$!

→ Као се тражи!

(2)

27.06.2005

w 1.1. ЗА ЗАДАТИ ПОПРЕЧНИ ПРЕСЕК ОДРЕДИТИ ПОЛОЖАЈ ГЛАВНИХ ЦЕНТРАЛНИХ ОСА ИНЕРЦИЈЕ



$F_1 = \frac{1}{2} 30 \cdot 40 = 600\ cm^2$

$F_2 = 10 \cdot 20 = 200\ cm^2$

$F = 400\ cm^2$

$x_T = \frac{600 \cdot 10 - 200 \cdot 5}{400} = 12{,}5\ cm$

$y_T = \frac{600 \cdot 26{,}67 - 200 \cdot 30}{400} = 25\ cm$

$T_1(10;\ 26{,}67)$

$T_2(5;\ 30)$

$T(12{,}5;\ 25)$

$I_x = \frac{1}{36} 30 \cdot 40^3 + 1{,}67^2 \cdot 600 - \frac{1}{12} 10 \cdot 20^3 - 5^2 \cdot 200 = 43\,340{,}01\ cm^4$

$I_y = \frac{1}{36} 30^3 \cdot 40 + 2{,}5^2 \cdot 600 - \frac{1}{12} 10^3 \cdot 20 - 7{,}5^2 \cdot 200 = 20\,833{,}33\ cm^4$

$I_{xy} = \frac{30^2 \cdot 40^2}{72} + (+1{,}67)(-2{,}5) 600 - 5 \cdot (-7{,}5) \cdot 200 = 24\,995\ cm^4$

$I_{1,2} = \frac{43\,340{,}01 + 20\,833{,}33}{2} \pm \sqrt{\left(\frac{43\,340{,}01 - 20\,833{,}33}{2}\right)^2 + 24\,995^2}$

$I_1 = 39\,995{,}12$ ~~59 498,12~~ $\qquad I_2 = 4\,675{,}22$

$tg\,2\alpha = \frac{-2 \cdot 24\,995}{43\,340{,}01 - 20\,833{,}33} < 0 \quad > 0$

$\alpha = \frac{1}{2}(360 + arctg(-2{,}56))$

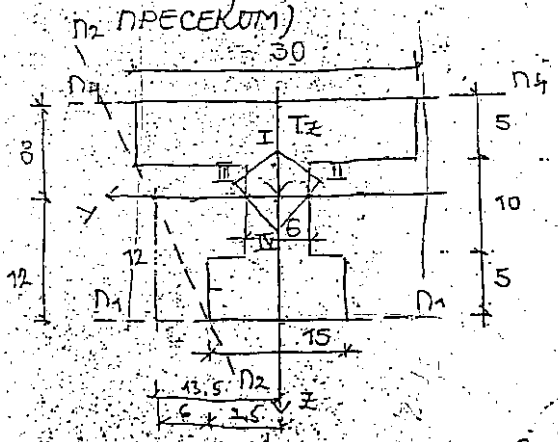
$\alpha_1 = 145{,}66°$

$\alpha_2 = 235{,}66°$

05.09.2005

1.4 а/ НАЦРТАЙ ЈЕЗГРО ПРЕСЕКА, ЗАДАТИХ ГЕОМЕТРИЈСКИХ КАРАКТЕРИСТИКА ($I_y = 12\,514{,}8$ см^4 $I_z = 12\,836{,}3$ см^4 и $F = 285$ см^2, НА СЛИЦИ ЈЕ КОТИРАН ПОЛОЖАЈ ТЕЖИШТА)

b/ НАЦРТАЈ ДИЈАГРАМ СМИЧУЋЕГ НАПОНА УСЛЕД ЗАДАТЕ ТРАНСФЕРЗАЛНЕ СИЛЕ $T_z = 10$ КН (ПРЕСЕК СМАТРАЈ ПУНИМ ПРЕСЕКОМ)



$I_y = 12514{,}8$ см^4
$I_z = 12\,836{,}3$ см^4
$F = 285$ см^2

$i_y^2 = \frac{I_y}{F} = \frac{12514{,}8}{285} = 43{,}91$ см^2

$i_z^2 = \frac{I_z}{F} = \frac{12\,836{,}3}{285} = 45{,}04$ см^2

$\frac{П_1-А_1}{(1)}$ $\rho_y = \infty$, $\rho_z = 12$

$e_y = -\frac{i_z^2}{\rho_y} = -\frac{45{,}04}{\infty} = 0$

$e_z = -\frac{i_y^2}{\rho_z} = -\frac{43{,}91}{12} = -3{,}66$

$\frac{П_2-П_2}{(2)\ (3)}$ $\rho_y = 13{,}5$, $\rho_z = 27$

$e_y = -\frac{i_z^2}{\rho_y} = -\frac{45{,}04}{13{,}5} = -3{,}34$

$e_z = -\frac{i_y^2}{\rho_z} = -\frac{43{,}91}{27} = -1{,}63$

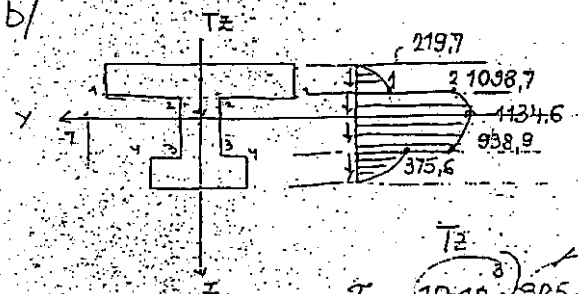
} СИМЕТРИЧНО ОА $П_3$-$П_5$

$\frac{П_4-П_4}{(4)}$ $\rho_y = \infty$, $\rho_z = -8$

$e_y = -\frac{i_z^2}{\rho_y} = -\frac{45{,}04}{\infty} = 0$

$e_z = -\frac{i_y^2}{\rho_z} = -\frac{43{,}91}{-8} = 5{,}49$

b/



$\tau_1 = 1134{,}6$ KPa

$\tau_4 = 375{,}6$ KPa

$S_y = F \cdot z_1$

$\tau = \frac{T_z \cdot S_y}{I_y \cdot b(z)}$

$S_{y_1} = 30 \cdot 5 \cdot 5{,}5 = 825$ см^3

$S_{y_2} = S_{y_1} + 6 \cdot 3 \cdot 1{,}5 = 852$ см^3

$S_{y_3} = S_{y_2} - 7 \cdot 6 \cdot 3{,}5 = 705$ см^3

$\tau_1 = \frac{10 \cdot 10^3 \cdot 825 \cdot 10^{-6}}{12514{,}8 \cdot 10^{-8} \cdot 30 \cdot 10^{-2}} = 219{,}7$ KPa

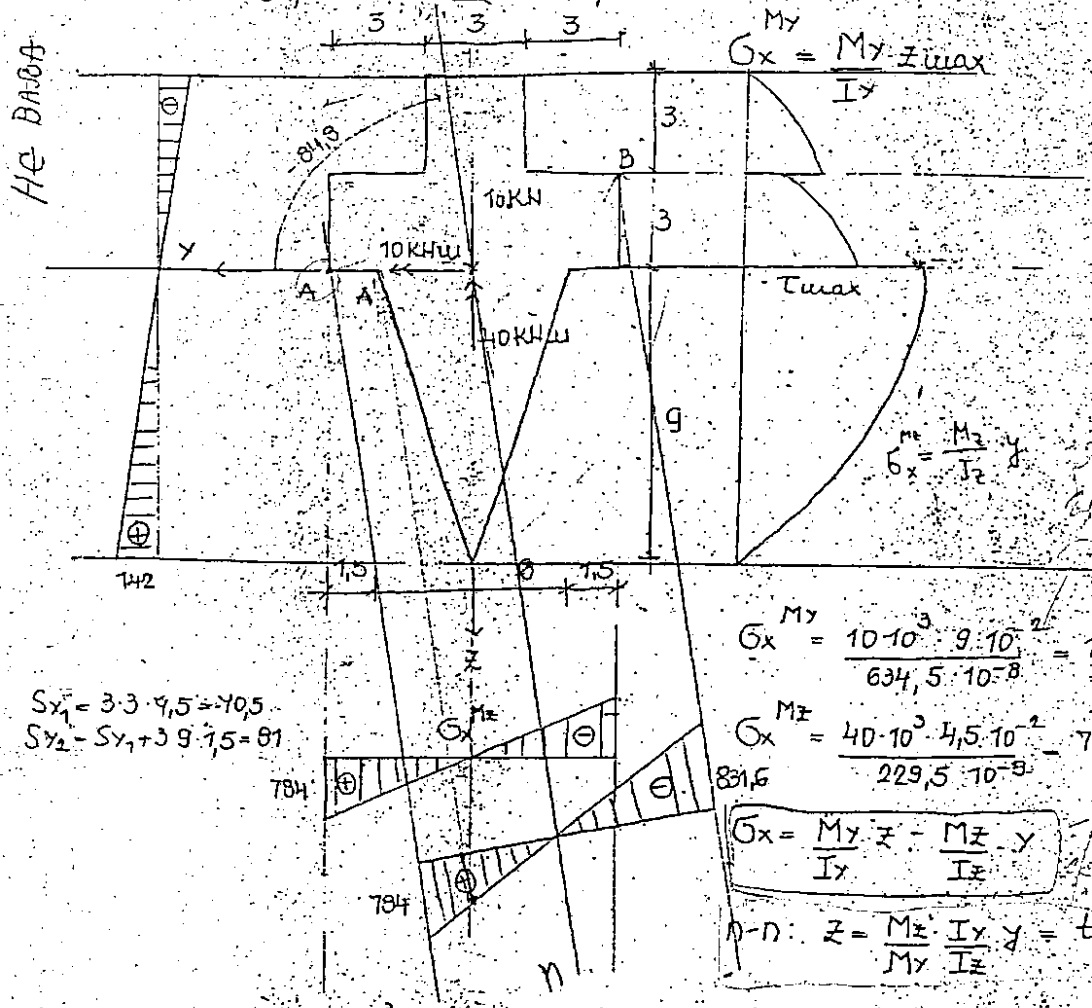
$\tau_2 = \frac{10 \cdot 10^3 \cdot 825 \cdot 10^{-6}}{12514{,}8 \cdot 10^{-8} \cdot 6 \cdot 10^{-2}} = 1098{,}7$ KPa

$\tau_3 = \frac{10 \cdot 10^3 \cdot 705 \cdot 10^{-6}}{12514{,}8 \cdot 10^{-8} \cdot 6 \cdot 10^{-2}} = 938{,}9$ KPa

02.04.2004.

1.2. НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ СИЛА У ПРЕСЕКУ. ОДРЕДИТИ ТАЧКУ У КОЈОЈ СЕ ЈАВЉА τ_x^{max} И У ЊОЈ НАПИСАТИ ТЕНЗОР НАПОНА.

$I_y = 634,5\ cm^4$ $I_z = 229,5\ cm^4$



$S_{x_1} = 3 \cdot 3 \cdot 9,5 = 10,5$

$S_{y_2} = S_{y_1} + 3 \cdot 9 \cdot 7,5 = 81$

$\sigma_x^{My} = \frac{M_y \cdot z_{max}}{I_y}$

$\sigma_x^{My} = \frac{10 \cdot 10^3 \cdot 9 \cdot 10^{-2}}{634,5 \cdot 10^{-8}} = 142\ MPa$

$\sigma_x^{Mz} = \frac{40 \cdot 10^3 \cdot 4,5 \cdot 10^{-2}}{229,5 \cdot 10^{-8}} = 784\ MPa$

$\sigma_x = \frac{M_y}{I_y} z - \frac{M_z}{I_z} y$

$n\text{-}n: \quad z = \frac{M_z}{M_y} \frac{I_y}{I_z} y = tg\alpha \cdot y$

$\tau_{max} = \frac{T_z}{I_y} \frac{S_y}{6} = \frac{10 \cdot 10^3 \cdot 81 \cdot 10^{-6}}{634,5 \cdot 10^{-8} \cdot 6 \cdot 10^{-2}} = 2,13\ MPa$

$tg\alpha = \frac{-40}{10} \frac{634,5}{229,5} = -11,1$

$\alpha = -84,83 \quad (275,17)$

$A(4,5;\ 0)$

$\sigma_x^A = \frac{M_y}{I_y} \cdot 0 - \frac{M_z}{I_z} 4,5 = \frac{(-40)}{229,5} \cdot 4,5 = -784\ MPa$

$B(-4,5;\ -3)$

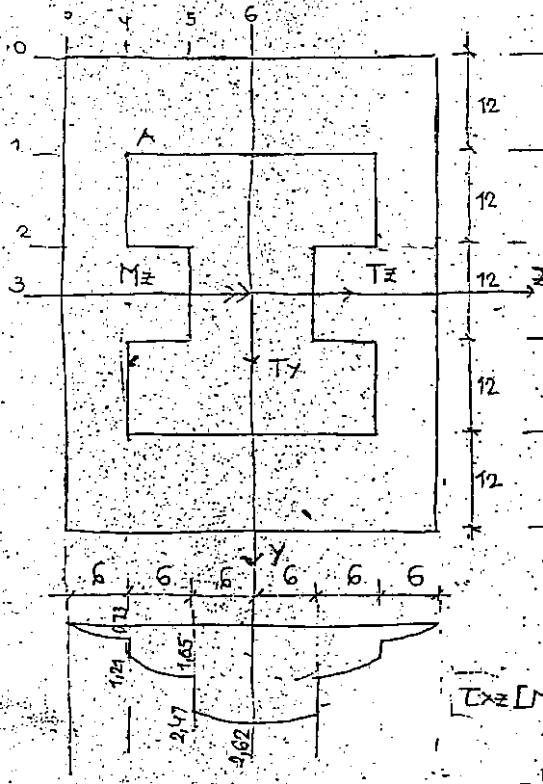
$\sigma_x^B = \frac{10 \cdot 10^3}{634,5 \cdot 10^{-8}} \cdot (-3 \cdot 10^{-2}) - \frac{(-40 \cdot 10^3)}{229,5 \cdot 10^{-8}} (-4,5 \cdot 10^{-2}) = -831,6\ MPa$

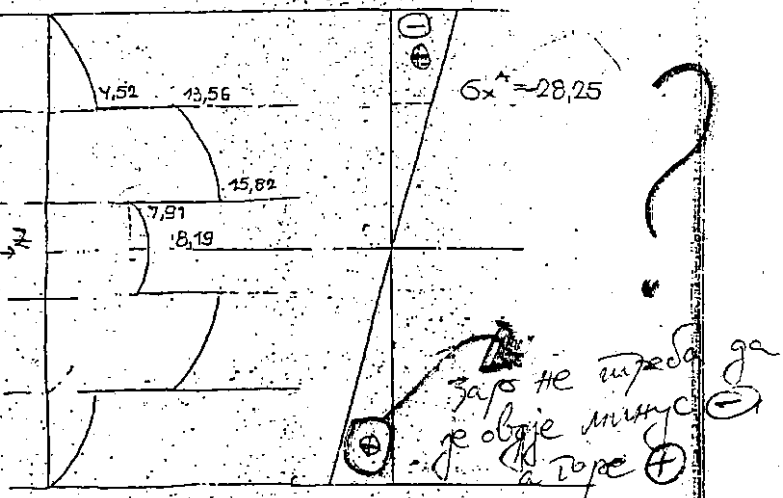
$\sigma_A' = \frac{M_y}{I_y} \cdot 0 - \frac{M_z}{I_z} 3 = -\frac{(-40 \cdot 10^3)}{229,5 \cdot 10^{-8}} \cdot 3 \cdot 10^{-2} = 523\ MPa$

$S_A' = \begin{bmatrix} 523 & 2,13 & 0 \\ 2,13 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$

30.01.2004.

(1-2) За дати попречни пресек и пресечне силе $|T_y| = 320$ kN $|T_z| = 450$ kN $|M_z| = 320$ kNcm нацртати дијаграме компонената смичућих напона и написати тензор напона у тачки A.
$I_y = 556416$ cm^4 $I_z = 203904$ cm^4





τ_{xy} [MPa] σ_x [MPa]

τ_{xz} [MPa],

$$\tau_{xz} = \frac{T_z}{I_y}\frac{S_y}{a_i} \qquad \tau_{xy} = \frac{T_y}{I_z}\frac{S_z}{b_i}$$

1: $S_z^{(1)} = 12 \cdot 36 \cdot 24 = 10368$ cm^3 — $S_{z1}^{(1)}/36 = 288$ cm^2, $S_{z1}^{(2)}/12 = 864$ cm^2

2: $S_z^{(2)} = S_z^{(1)} + 2 \cdot 6 \cdot 12 \cdot 12 = 12096$ cm^3 — $S_{z2}/12 = 1008$ cm^2, $S_{z,2}/24 = 504$ cm^2

3: $S_z^{(3)} = S_z^{(2)} + 2 \cdot 12 \cdot 6 \cdot 3 = 12528$ cm^3 — $S_{z,3}/24 = 522$ cm^2

4: $S_y^{(4)} = 6 \cdot 60 \cdot 15 = 5400$ cm^3 — $S_{y,4}/60 = 90$, $S_{y,4}/36 = 150$

5: $S_y^{(5)} = S_y^{(4)} + 3 \cdot 12 \cdot 6 \cdot 9 = 7344$ cm^3 — $S_{y,5}/36 = 204$, $S_{y,5}/24 = 306$

6: $S_y^{(6)} = S_y^{(5)} + 2 \cdot 12 \cdot 6 \cdot 3 = 7776$ cm^3 — $S_{y,6}/24 = 324$.

$\tau_{xy}^{(1)} = 4,52$ MPa
$\tau_{xy}^{(2)} = 13,56$ MPa
$\tau_{xz}^{(2)} = 15,82$
$\tau_{xy}^{(2)} = 7,91$
$\tau_{xy}^{(2)} = 8,19$
$\tau_{xz}^{(4)} = 0,73$
$\tau_{xz}^{(5)} = 1,21$
$\tau_{xz} = 1,65$
$\tau_{xz} = 2,47$
$\tau_{xz} = 2,62$

$$K_z = \frac{T_z}{I_y} = \frac{450 \cdot 10^3}{556416 \cdot 10^{-8}} \qquad K_y = \frac{T_y}{I_z} = \frac{320 \cdot 10^3}{203904 \cdot 10^{-8}}$$

$$\sigma_x^A = \frac{M_z}{I_z} y_A = \frac{320 \cdot 10^3 (18 \cdot 10^{-2})}{203904 \cdot 10^{-8}} = -28,25 \text{ MPa}$$

$$S = \begin{matrix} \\ A \\ 1 \\ 2 \end{matrix} \begin{bmatrix} x & y & z \\ -28,25 & 13,56 & 1,21 \\ 13,56 & 0 & 0 \\ 1,21 & 0 & 0 \end{bmatrix} \text{MPa}$$

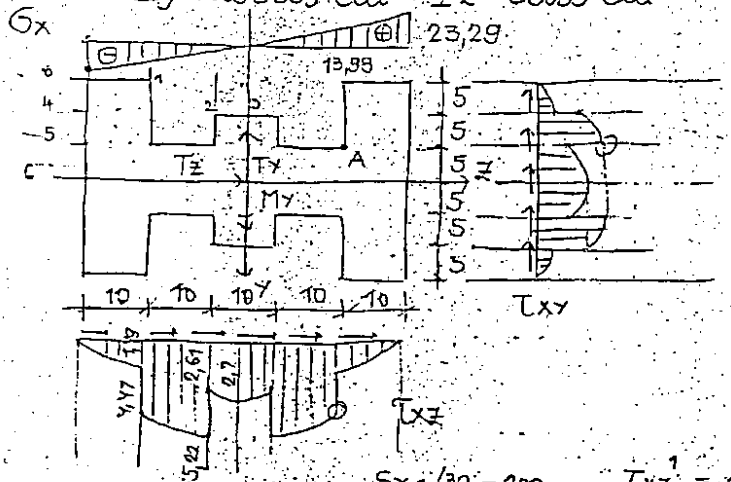
Зашто се узимају ~~Све~~ Све једнакости. Како знам шта узимам

(6)

30.01.2004.

1.2 За дати попречни пресек и пресечне силе $|T_y| = 280$ kN, $|T_z| = 200$ kN, $|M_y| = 250$ kNm нацртати дијаграме компонеталних смичућих напона и написати тензор напона за тачку A.

$I_y = 268333\ cm^4$ $I_z = 53333\ cm^4$



T_y $\tau_{xy} = \frac{T_y \cdot S_z^*}{I_z \cdot t}$

$S_{y,1} = 10 \cdot 30 \cdot 20 = 6000$ — $S_{y,1}/30 = 200$, $S_{y,1}/10 = 600$

$S_{y,2} = S_{y,1} + 10 \cdot 10 \cdot 10 = 7000$ — $S_{y,2}/10 = 700$, $S_{y,2}/20 = 350$

$S_{y,3} = S_{y,2} + 5 \cdot 20 \cdot 2,5 = 7250$ — $S_{y,3}/20 = 362,5$

$S_{z,4} = 2 \cdot 5 \cdot 10 - 12,5 = 1250$ — $S_{y,4}/20 = 62,5$, $S_{y,4}/30 = 41,7$

$S_{z,5} = S_{z,4} + 5 \cdot 30 \cdot 7,5 = 2375$ — $S_{y,5}/30 = 79,2$, $S_{y,5}/50 = 47,5$

$S_{z,6} = S_{z,5} + 5 \cdot 50 \cdot 2,5 = 3000$ → $S_{y,6}/50 = 60$

$\tau_{xz}^1 = 1,49$ MPa

$\tau_{xz}^1 = 4,47$

$\tau_{xz}^2 = 5,22$

$\tau_{xz}^2 = 2,61$

$\tau_{xz}^3 = 2,7$

$\tau_{xy}^4 = 3,28$

$\tau_{xy}^4 = 2,19$

$\tau_{xy}^5 = 4,16$

$\tau_{xy}^5 = 2,49$

$\tau_{xy}^6 = 3,15$

$\tau_{xy} = \underbrace{\frac{T_y}{I_z}} \cdot \frac{S_z^*}{b(y)}$ $\tau_{xz} = \underbrace{\frac{T_z}{I_y}} \cdot \frac{S_y^*}{b(z)}$

$K_y = \frac{-280 \cdot 10^3}{53333 \cdot 10^{-8}}$ $K_z = \frac{200 \cdot 10^3}{268333 \cdot 10^{-8}}$

$\sigma_x = \frac{M_y}{I_y} \cdot z = \frac{250 \cdot 10^3 \cdot 25 \cdot 10^{-2}}{268333 \cdot 10^{-8}} = 23,29$ MPa

$\sigma_x^4 = \frac{250 \cdot 10^3 \cdot 15 \cdot 10^{-2}}{268333 \cdot 10^{-8}} = 13,98$ MPa

$$S_A = \begin{bmatrix} 13,98 & 4,16 & 4,47 \\ 4,16 & 0 & 0 \\ 4,47 & 0 & 0 \end{bmatrix}$$

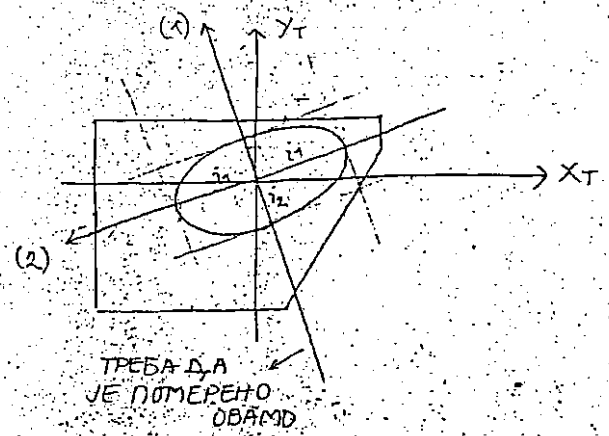
$$i_1 = \sqrt{\frac{I_1}{F}} = \sqrt{\frac{2062,78}{130}} = 4 \text{ cm}$$

$$i_2 = \sqrt{\frac{I_2}{F}} = \sqrt{\frac{971,19}{130}} = 2,73 \text{ cm}$$

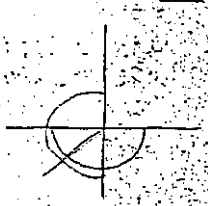
$$tg\,2\alpha = -\frac{2I_{xy}}{I_x - I_y} = -\frac{2 \cdot 291,95}{1053,61 - 2000,36} = 0,616$$

$2\alpha < 0$

$$\alpha = \frac{1}{2}(180 + arctg\,0,616) = 105,85°$$



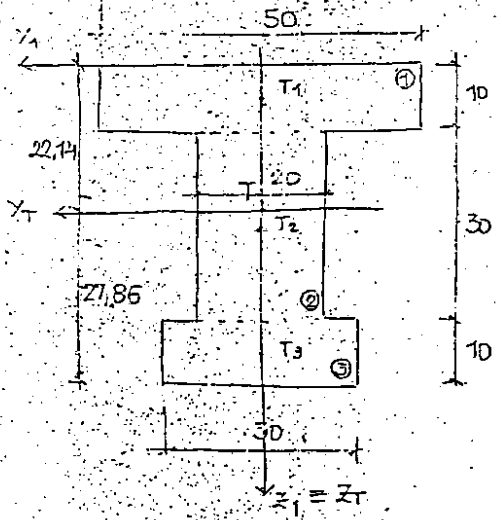
ТРЕБА ДА ЈЕ ПОМЕРЕНО ОВАМО



ГЕОМЕТРИЈСКЕ КАРАКТЕРИСТИКЕ ПОПРЕЧНОГ ПРЕСЕКА ГРЕДЕ

W 05.09.2003

1.1. ОДРЕДИТИ ВРЕДНОСТИ ГЛАВНИХ ЦЕНТРАЛНИХ МОМЕНАТА ИНЕРЦИЈЕ ЗА ПОПРЕЧНИ ПРЕСЕК НА СЛИЦИ



$F_1 = 10 \cdot 50 = 500\ cm^2$
$F_2 = 20 \cdot 30 = 600\ cm^2$
$F_3 = 10 \cdot 30 = 300\ cm^2$
$F = 1400\ cm^2$

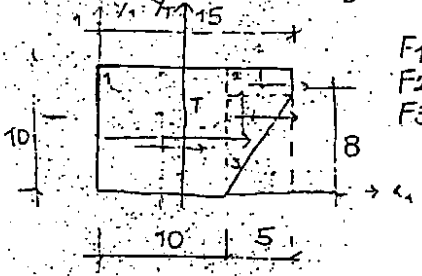
$z_T = \frac{500 \cdot 5 + 600 \cdot 25 + 300 \cdot 45}{1400} = 22,14\ cm$

$I_y = \frac{1}{12} 50 \cdot 10^3 + 500 \cdot 17,14^2 + \frac{1}{12} 20 \cdot 30^3 + 600 \cdot 2,86^2 + \frac{1}{12} 30 \cdot 10^3 + 300 \cdot 22,86^2 = 360\,238,11\ cm^4$

$I_z = \frac{1}{12} 50^3 \cdot 10 + \frac{1}{12} 30 \cdot 20^3 + \frac{1}{12} 10 \cdot 30^3 = 146\,666,67\ cm^4$

$I_{yz} = 0$

1.2. ОДРЕДИТИ ПОЛУПРЕЧНИК ЕЛИПСЕ ИНЕРЦИЈЕ И НАЦРТАТИ ЕЛИПСУ ИНЕРЦИЈЕ ЗА ПОПРЕЧНИ ПРЕСЕК НА СЛИЦИ



$F_1 = 10 \cdot 10 = 100\ cm^2$
$F_2 = 5 \cdot 2 = 10\ cm^2$
$F_3 = 1/2\ 5 \cdot 8 = 20\ cm^2$
$F = 130\ cm^2$

$x_T = \frac{100 \cdot 5 + 10 \cdot 12,5 + 20 \left(\frac{5}{3} + 10\right)}{130}$

$x_T = 6,6\ cm$

$y_T = \frac{100 \cdot 5 + 10 \cdot 9 + 20 \frac{2 \cdot 8}{3}}{130} = 5,36\ cm$

$I_x = \frac{1}{12} 15 \cdot 10^3 + 0,36^2 \cdot 10 \cdot 15 - \left[\frac{1}{36} 5\ 8^3 + 2,69^2 \frac{1}{2} 8 \cdot 5\right] = 1053,61\ cm^4$

$I_y = \frac{1}{12} 15^3 \cdot 10 + 0,9^2 \cdot 10 \cdot 15 - \left[\frac{1}{36} \cdot 5^3 \cdot 8 + 6,73^2 \frac{8 \cdot 5}{2}\right] = 2000,36\ cm^4$

$I_{yz} = 0 + 0,9(-0,36)150 - \left[\frac{8^2 \cdot 5^2}{72} + 6,73 \cdot (-2,69) \cdot 20\right] = 291,25\ cm^4$

$I_{1,2} = \frac{1053,61 + 2000,36}{2} \pm \sqrt{\left(\frac{1053,61 - 2000,36}{2}\right)^2 + 291,25^2}$

$I_1 = 2082,78\ cm^4$

$I_2 = 971,19\ cm^4$

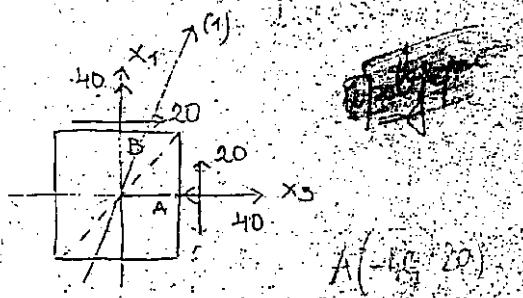
24.09.2004. АНАЛИЗА НАПОНА W

1.2. СТАЊЕ НАПОНА У НЕКОЈ ТАЧКИ ДАТО ЈЕ ТЕНЗОРОМ НАПОНА S. НАЦРТАТИ Мохр-ОВ КРУГ НАПОНА ЗА ДЕВИЈАТОРСКИ ДЕО ТЕНЗОРА НАПОНА.

$$S = \begin{bmatrix} 20 & 0 & 20 \\ 0 & -20 & 0 \\ 20 & 0 & -60 \end{bmatrix} \text{MPa}$$

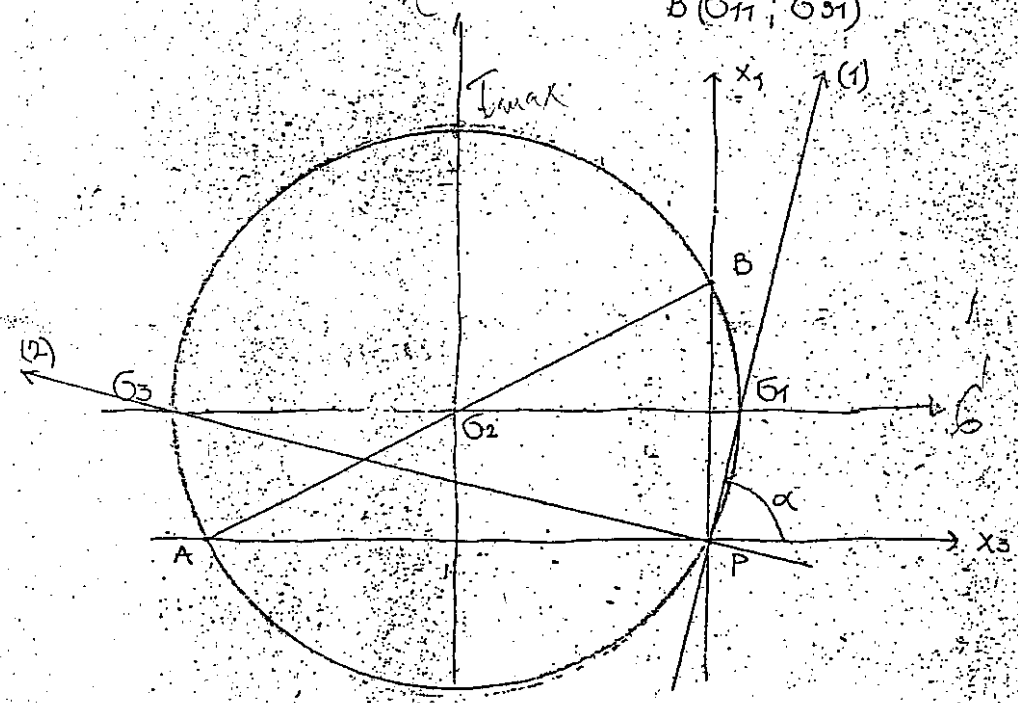
$$\bar{\sigma} = \frac{1}{3}(20 - 20 - 60) = -20 \text{ MPa}$$

$$S_{dev} = \begin{bmatrix} 40 & 0 & 20 \\ 0 & 0 & 0 \\ 20 & 0 & -40 \end{bmatrix} \text{MPa}$$



A(-40; 20)
B(40; 20)

A(σ_{33}; $-\sigma_{31}$)
B(σ_{11}; σ_{31})



(10)

02.04.2004. IV

1.4. ОДРЕДИТИ σ_y ТАКО ДА МАТРИЦА НАПОНА ОДГОВАРА РАВНОМ СТАЊУ НАПОНА. ЗА ТАКО ОДРЕЂЕНО σ_y ОДРЕДИТИ ВРЕДНОСТИ ГЛАВНИХ НАПОНА И ДИЛАТАЦИЈА

$$S = \begin{bmatrix} 5 & 7 & -3 \\ 7 & \sigma_y & -6 \\ -3 & -6 & 10 \end{bmatrix} \text{ MPa} \qquad E = 210 \text{ GPa} \quad \nu = 1/3$$

$$\begin{vmatrix} 5 & 7 & -3 \\ 7 & \sigma_y & -6 \\ -3 & -6 & 10 \end{vmatrix} \begin{matrix} 5 & 7 \\ 7 & \sigma_y \\ -3 & -6 \end{matrix} = 50\sigma_y + 126 + 126 - 490 - 180 - 9\sigma_y = 0$$

$$41\sigma_y = 418 \qquad \sigma_y = 10{,}2 \text{ MPa}$$

$$I_1 = 5 + 10{,}2 + 10 = 25{,}2$$

$$I_2 = \begin{vmatrix} 10{,}2 & -6 \\ -6 & 10 \end{vmatrix} + \begin{vmatrix} 5 & -3 \\ -3 & 10 \end{vmatrix} + \begin{vmatrix} 5 & 7 \\ 7 & 10{,}2 \end{vmatrix} = 66 + 41 + 2 = 109$$

$$I_3 = 0$$

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

$$\sigma^3 - 25{,}2\sigma^2 + 109\sigma = 0 \qquad \sigma(\sigma^2 - 25{,}2\sigma + 109) = 0 \qquad \sigma_1 = 0$$

$$\sigma_{1,2} = \frac{25{,}2 \pm \sqrt{635{,}04 - 436}}{2} \begin{matrix} \nearrow 19{,}65 \\ \searrow 5{,}54 \end{matrix}$$

$\sigma_1 = 19{,}65$ MPa
$\sigma_2 = 5{,}54$ MPa
$\sigma_3 = 0$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = -\frac{1{,}73}{E} \qquad G = \frac{E}{2(1+\nu)} = \frac{E}{2{,}67}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] = \frac{5{,}2}{E}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = \frac{4{,}93}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2{,}67}{E} \cdot 7 = 18{,}69/E \qquad D = \frac{1}{E}\begin{bmatrix} -1{,}73 & 9{,}34 & -4 \\ 9{,}34 & 5{,}2 & -8{,}01 \\ -4 & -8{,}01 & 4{,}93 \end{bmatrix}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} = \frac{2{,}67}{E}(-6) = -16{,}02/E$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} = \frac{2{,}67}{G} \cdot (-3) = -8{,}01/E$$

$$\begin{vmatrix} -1{,}73 - E & 9{,}34 & -4 \\ 9{,}34 & 5{,}2 - E & -8{,}01 \\ -4 & -8{,}01 & 4{,}93 - E \end{vmatrix} = 0 \Rightarrow \varepsilon_1, \varepsilon_2, \varepsilon_3$$

Zabranili alg

(8) Дато је следеће поље напона:

$\sigma_x = 50x^3 + 2y \qquad \tau_{xy} = 100z + 8y^2$

$\sigma_y = 40x^3 + 500 \qquad \tau_{yz} = 0$

$\sigma_z = 60y^3 + 30z^3 \qquad \tau_{zx} = xz^3 + 4x^2y$

а) Одредити расподелу запреминских сила из услова равнотеже и компоненте запреминских сила у тачки (2,2,1)

б) Ако се усвоји да је $E = 30 \cdot 10^6$ Pa и $\nu = 0{,}25$

одредити тензор деформација у тој тачки

$$\begin{bmatrix} 50x^3+2y & 100z+8y^2 & xz^3+4x^2y \\ 100z+8y^2 & 40x^3+500 & 0 \\ xz^3+4x^2y & 0 & 60y^3+30z^3 \end{bmatrix} \xrightarrow{A(2,2,1)} \begin{bmatrix} 404 & 420 & 34 \\ 420 & 820 & 0 \\ 34 & 0 & 510 \end{bmatrix}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_x = 0$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + F_y = 0$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + F_z = 0$$

$150x^2 + 16y + 3xz^2 + F_x = 0 \qquad F_x = -(150x^2 + 16y + 3xz^2)$

$0 + 0 + 0 + F_y = 0 \qquad F_y = 0$

$z^3 + 8xy + 0 + 90z^2 + F_z = 0 \qquad F_z = -(91z^3 + 8xy)$

A(2,2,1) $\qquad F_x = -(150 \cdot 4 + 16 \cdot 2 + 3 \cdot 2 \cdot 1) = -926$

$F_y = 0$

$F_z = -(91 \cdot 1^3 + 8 \cdot 2 \cdot 2) = -123$

$E = 30 \cdot 10^6$ Pa $\qquad \nu = 0{,}25 \qquad \frac{1+\nu}{E} = \frac{1+0{,}25}{30 \cdot 10^6} = 0{,}0417 \cdot 10^{-6}$ 1/Pa

$$S = \begin{bmatrix} 404 & 420 & 34 \\ 420 & 820 & 0 \\ 34 & 0 & 510 \end{bmatrix}$$

$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\delta_{ij}\sigma_{kk}$

$\varepsilon_{12} = \frac{1+\nu}{E}\sigma_{12} = 17{,}51 \cdot 10^{-6}$

$\varepsilon_{11} = \frac{1}{E}[\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})] = 2{,}38 \cdot 10^{-6}$

$\varepsilon_{22} = \frac{1}{E}[\sigma_{22} - \nu(\sigma_{11} + \sigma_{33})] = 19{,}72 \cdot 10^{-6}$

$\varepsilon_{23} = \frac{1+\nu}{E}\sigma_{23} = 0$

$\varepsilon_{33} = \frac{1}{E}[\sigma_{33} - \nu(\sigma_{22} + \sigma_{11})] = 6{,}8 \cdot 10^{-6}$

$\varepsilon_{31} = \frac{1+\nu}{E}\sigma_{31} = 1{,}42 \cdot 10^{-6}$

(12)

① ДАТО ЈЕ ПОЉЕ ПОМЕРАЊА

$W\ U_1 = 3x_1x_2^2 \quad U_2 = 2x_3x_1 \quad U_3 = x_3^2 - x_1x_2$

ОДРЕДИТИ ТЕНЗОР ДЕФОРМАЦИЈЕ И ПРОВЕРИТИ ДА ЛИ СУ ЗАДОВОЉЕНИ УСЛОВИ КОМПАТИБИЛНОСТИ ДЕФОРМАЦИЈА

$$G = [U_{ij}]^T = \left[\frac{\partial U}{\partial x_j}\right] = \begin{bmatrix} \frac{\partial U_1}{\partial x_1} & \frac{\partial U_1}{\partial x_2} & \frac{\partial U_1}{\partial x_3} \\ \frac{\partial U_2}{\partial x_1} & \frac{\partial U_2}{\partial x_2} & \frac{\partial U_2}{\partial x_3} \\ \frac{\partial U_3}{\partial x_1} & \frac{\partial U_3}{\partial x_2} & \frac{\partial U_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 3x_2^2 & 6x_1x_2 & 0 \\ 2x_3 & 0 & 2x_1 \\ -x_2 & -x_1 & 2x_3 \end{bmatrix}$$

$$D = \frac{1}{2}(U_{i,j} + U_{j,i}) = \begin{bmatrix} 3x_2^2 & \frac{1}{2}(2x_3 + 6x_1x_2) & -\frac{x_2}{2} \\ \frac{1}{2}(2x_3 + 6x_1x_2) & 0 & \frac{1}{2}(2x_1 - x_1) \\ -\frac{x_2}{2} & \frac{1}{2}(2x_1 - x_1) & 2x_3 \end{bmatrix}$$

$$D = \begin{bmatrix} 3x_2^2 & x_3 + 3x_1x_2 & -x_2/2 \\ x_3 + 3x_1x_2 & 0 & x_1/2 \\ -x_2/2 & x_1/2 & 2x_3 \end{bmatrix}$$

УСЛОВИ КОМПАТИБИЛНОСТИ (Saint-Venant)

$$\frac{\partial^2 \varepsilon_{11}}{\partial x_2 \partial x_3} = \frac{\partial}{\partial x_1}\left(\frac{\partial \varepsilon_{12}}{\partial x_3}^{1} - \frac{\partial \varepsilon_{23}}{\partial x_1}^{1/2} + \frac{\partial \varepsilon_{31}}{\partial x_2}^{-1/2}\right) \quad 0 = 0$$

$$\frac{\partial^2 \varepsilon_{22}}{\partial x_3 \partial x_1} = \frac{\partial}{\partial x_2}\left(\frac{\partial \varepsilon_{12}}{\partial x_3}^{1} + \frac{\partial \varepsilon_{23}}{\partial x_1}^{1/2} - \frac{\partial \varepsilon_{31}}{\partial x_2}^{-1/2}\right) \quad 0 = 0$$

$$\frac{\partial^2 \varepsilon_{33}}{\partial x_1 \partial x_2} = \frac{\partial}{\partial x_3}\left(-\frac{\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2}\right) \quad 0 = 0$$

- писмо за знаци!

$$2 \cdot \frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} = \frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} \quad 2 \cdot 3 = 6 + 0 \quad 6 = 6$$

$$2 \cdot \frac{\partial^2 \varepsilon_{23}}{\partial x_2 \partial x_3} = \frac{\partial^2 \varepsilon_{11}}{\partial x_3^2} + \frac{\partial^2 \varepsilon_{33}}{\partial x_2^2} \quad 0 - 0 + 0 \quad 0 = 0$$

$$2 \cdot \frac{\partial^2 \varepsilon_{31}}{\partial x_3 \partial x_1} = \frac{\partial^2 \varepsilon_{33}}{\partial x_1^2} + \frac{\partial^2 \varepsilon_{11}}{\partial x_3^2} \quad 0 = 0 + 0 \quad 0 = 0$$

КОМПОНЕНТЕ ДЕФОРМАЦИЈА ИСПУЊАВАЈУ УСЛОВЕ КОМПАТИБИЛНОСТИ

⑥ СТАЊЕ ДЕФОРМАЦИЈА У ТАЧКИ ЈЕ ЗАДАТО КОМПОНЕНТАМА ДЕФОРМАЦИЈЕ:

$$\varepsilon_{22} = 100\cdot10^{-6} \quad \varepsilon_{33} = -200\cdot10^{-6} \quad \gamma_{23} = 300\cdot10^{-6}$$

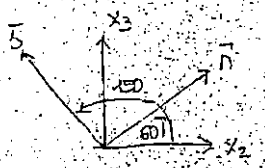
а/ ОДРЕДИТИ ДИЛАТАЦИЈУ ПРАВЦА $\vec{n}$ КОЈИ СА ОСОМ x_2 ГРАДИ УГАО ОД 60°, КАО И КЛИЗАЊЕ ИЗМЕЂУ $\vec{n}$ И ЊЕМУ УПРАВНОГ ПРАВЦА $\vec{s}$

b/ ОДРЕДИТИ ГЛАВНЕ ДИЛАТАЦИЈЕ И ЊИХОВЕ ПРАВЦЕ

c/ ПРОВЕРИТИ ДОБИЈЕНЕ РЕЗУЛТАТЕ НА Mohr-ОВОМ КРУГУ

а/

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 100 & 150 \\ 0 & 150 & -200 \end{bmatrix}\cdot10^{-6} \qquad \varphi = 60°$$



$$\varepsilon_n = \frac{\varepsilon_{22}+\varepsilon_{33}}{2} + \frac{\varepsilon_{22}-\varepsilon_{33}}{2}\cos 2\varphi + \varepsilon_{23}\sin 2\varphi$$

$$\varepsilon_n = \frac{100-200}{2} + \frac{100+200}{2}\cos 120° + 150\sin 120° = 4{,}904\cdot10^{-6}$$

$$\frac{1}{2}\gamma_u = \frac{100+200}{2}\sin 120° - 150\cos 120° = 204{,}9\cdot10^{-6} \qquad \boxed{\gamma_u = 409{,}8\cdot10^{-6}}$$

b/

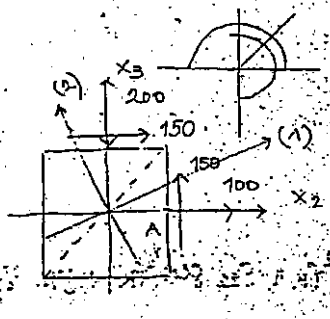
$$\varepsilon_{1,2} = \frac{\varepsilon_{22}+\varepsilon_{33}}{2} \pm \sqrt{\left(\frac{\varepsilon_{22}-\varepsilon_{33}}{2}\right)^2 + \varepsilon_{23}^2} = \frac{100-200}{2} \pm \sqrt{\left(\frac{100+200}{2}\right)^2 + 150^2}$$

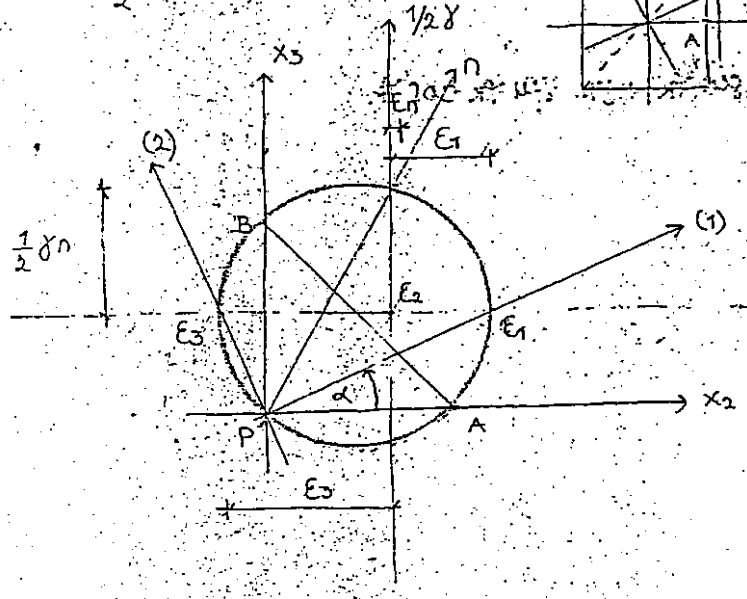
$$\varepsilon_1 = 162{,}13\cdot10^{-6} \qquad \varepsilon_2 = -262{,}13\cdot10^{-6}$$

$$\Rightarrow \quad \varepsilon_1 = 162{,}13\cdot10^{-6} \qquad \varepsilon_2 = 0 \qquad \varepsilon_3 = -262{,}13\cdot10^{-6}$$

$$\operatorname{tg} 2\alpha = \frac{2\varepsilon_{23}}{\varepsilon_{22}-\varepsilon_{33}} = \frac{300}{100+200} = 1 \qquad (>0,\ >0)$$

$$\alpha = \frac{1}{2}\operatorname{arctg} 1 = 22{,}5°$$





(19)

c/ $\vec{n} = \vec{n}_1 \qquad \vec{n}_1 = \{1/2, -1/2, 1/\sqrt{2}\}$

$\vec{b} = \vec{n}_2 \qquad \vec{n}_2 = \{-1/2, 1/2, 1/\sqrt{2}\}$

$$\vec{n}_3 = \vec{n}_1 \times \vec{b} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{vmatrix} = \vec{e}_1\left(-\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right) - \vec{e}_2\left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) + \vec{e}_3\left(\frac{1}{4} - \frac{1}{4}\right)$$

$$\vec{n}_3 = \left\{-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\} = -\frac{1}{\sqrt{2}}\{1, 1, 0\}$$

d/ $\bar{\varepsilon} = \frac{1}{3} I_1 = \frac{1}{3} \cdot 6 \cdot 10^{-4} = 2 \cdot 10^{-4}$

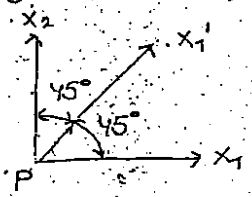
$$D_{dev} = \begin{bmatrix} 1-2 & -3 & \sqrt{2} \\ -3 & 1-2 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4-2 \end{bmatrix} \cdot 10^{-4} = \begin{bmatrix} -1 & -3 & \sqrt{2} \\ -3 & -1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 2 \end{bmatrix} \cdot 10^{-4}$$

W

⑤ ПОМОЋУ МЕРНИХ ТРАКА ИЗМЕРЕНЕ СУ ДИЛАТАЦИЈЕ У ТАЧКИ P ЗА ТРИ ПРАВЦА X_1, X_1', X_2 КОЈИ МЕЂУСОБНО ГРАДЕ УГЛОВЕ ОД 45°. ОДРЕДИТИ КЛИЗАЊЕ АКО ЈЕ:

$\varepsilon_{11} = 5 \cdot 10^{-4}$ $\varepsilon_{11}' = 4 \cdot 10^{-4}$ $\varepsilon_{22} = 7 \cdot 10^{-4}$

$\gamma_{12} = ?$



$$\varepsilon_n = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} + \frac{\varepsilon_{11} - \varepsilon_{22}}{2} \cos 2\alpha + \varepsilon_{12} \sin 2\alpha$$

$$\varepsilon_{x_1'} = \frac{5+7}{2} + \frac{5-7}{2} \cos 90^\circ + \varepsilon_{12} \sin 90^\circ = 4$$

$\varepsilon_{12} = 4 - 6$ $\qquad \varepsilon_{12} = -2 \cdot 10^{-4}$ $\qquad \gamma_{12} = 2 \cdot \varepsilon_{12}$ $\qquad \boxed{\gamma_{12} = -4 \cdot 10^{-4}}$

$$\varepsilon^{(k)} = \varepsilon_{11} \cos^2 \varphi_k + \varepsilon_{22} \sin^2 \varphi_k + \gamma_{12} \sin \varphi_k \cos \varphi_k$$

$$4 \cdot 10^{-4} = 5 \cdot 10^{-4} \frac{1}{2} + 7 \cdot 10^{-4} \frac{1}{2} + \gamma_{12} \frac{1}{2}$$

$$8 \cdot 10^{-4} = 5 \cdot 10^{-4} + 7 \cdot 10^{-4} + \gamma_{12}$$

$$\boxed{-4 \cdot 10^{-4} = \gamma_{12}}$$

(1) СТАЊЕ ДЕФОРМАЦИЈА У НЕКОЈ ТАЧКИ ТЕЛА ЈЕ ЗАДАТО ТЕНЗОРОМ D.
w a/ ОДРЕДИТИ ДИЛАТАЦИЈУ ПРАВЦА $\vec{n}=\{1/2, -1/2, 1/\sqrt{2}\}$ И КЛИЗАЊЕ ИЗМЕЂУ $\vec{n}$ И $\vec{s}=\{-1/2, 1/2, 1/\sqrt{2}\}$.

b/ ОДРЕДИТИ ГЛАВНЕ ДИЛАТАЦИЈЕ
c/ ПОКАЗАТИ ДА СУ $\vec{n}$ И $\vec{s}$ ГЛАВНИ ПРАВЦИ ДИЛАТАЦИЈА
d/ ОДРЕДИТИ ДЕВИЈАТОРСКИ ДЕО ТЕНЗОРА ДЕФОРМАЦИЈА

$$D=\begin{bmatrix}1 & -3 & \sqrt{2}\\ -3 & 1 & -\sqrt{2}\\ \sqrt{2} & -\sqrt{2} & 4\end{bmatrix}\cdot 10^{-4}$$

a/ $\vec{n}=\{1/2, -1/2, 1/\sqrt{2}\}$

$$\varepsilon_n = n^T D n = \begin{bmatrix}1/2 & -1/2 & 1/\sqrt{2}\end{bmatrix}\begin{bmatrix}1 & -3 & \sqrt{2}\\ -3 & 1 & -\sqrt{2}\\ \sqrt{2} & -\sqrt{2} & 4\end{bmatrix}\begin{bmatrix}1/2\\ -1/2\\ 1/\sqrt{2}\end{bmatrix}\cdot 10^{-4} =$$

$$= \begin{bmatrix}3 & -3 & 3\sqrt{2}\end{bmatrix}\begin{bmatrix}1/2\\ -1/2\\ 1/\sqrt{2}\end{bmatrix}\cdot 10^{-4} = \left(\frac{3}{2}+\frac{3}{2}+3\right)\cdot 10^{-4} = 6\cdot 10^{-4}$$

$5\cdot 10^{-4}$

$\vec{s}=\{-1/2, 1/2, 1/\sqrt{2}\}$

$$\varepsilon_{ns} = n^T D s = \begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}1 & -3 & \sqrt{2}\\ -3 & 1 & -\sqrt{2}\\ \sqrt{2} & -\sqrt{2} & 4\end{bmatrix}\begin{bmatrix}-1/2\\ 1/2\\ 1/\sqrt{2}\end{bmatrix}\cdot 10^{-4} =$$

$$= \begin{bmatrix}3 & -3 & 3\sqrt{2}\end{bmatrix}\begin{bmatrix}-1/2\\ 1/2\\ 1/\sqrt{2}\end{bmatrix}\cdot 10^{-4} = \left(-\frac{3}{2}-\frac{3}{2}+3\right)\cdot 10^{-4} = 0$$

$$\gamma_{ns} = 2\varepsilon_{ns} = 0$$

b/ $I_1 = 1+1+4 = 6\cdot 10^{-4}$

$$I_2 = \begin{vmatrix}1 & -\sqrt{2}\\ -\sqrt{2} & 4\end{vmatrix} + \begin{vmatrix}1 & \sqrt{2}\\ \sqrt{2} & 4\end{vmatrix} + \begin{vmatrix}1 & -3\\ -3 & 1\end{vmatrix} = -2+2-8 = -4\cdot 10^{-8}$$

$$I_3 = \begin{vmatrix}1 & -3 & \sqrt{2}\\ -3 & 1 & -\sqrt{2}\\ \sqrt{2} & -\sqrt{2} & 4\end{vmatrix}\begin{matrix}1 & -3\\ -3 & 1\\ \sqrt{2} & -\sqrt{2}\end{matrix} = 4+6+6-2-2-36 = -24\cdot 10^{-12}$$

$$\varepsilon^3 - I_1\varepsilon^2 + I_2\varepsilon - I_3 = 0$$

$$\varepsilon^3 - 6\varepsilon^2 - 4\varepsilon + 24 = 0$$

$$(\varepsilon^2 - 4\varepsilon - 12)(\varepsilon - 2) = 0 \quad 1^\circ\ \varepsilon_1 = 2$$

$$\varepsilon_{1,2} = \frac{4 \pm \sqrt{16+48}}{2} \qquad \varepsilon_1 = 6 \quad \varepsilon_2 = -2$$

$$\varepsilon_1 = 6 \quad \varepsilon_2 = 2 \quad \varepsilon_3 = -2$$

$$(\varepsilon^3 - 6\varepsilon^2 - 4\varepsilon + 24) : (\varepsilon - 2) = \varepsilon^2 - 4\varepsilon - 12$$
$$-\varepsilon^3 + 2\varepsilon^2$$
$$-4\varepsilon^2 - 4\varepsilon$$
$$+4\varepsilon^2 \mp 8\varepsilon$$
$$-12\varepsilon + 24$$
$$\mp 12\varepsilon \pm 24$$
$$=$$

(16)

(3) Стање деформације у некој тачки тела је задато тензором D. Одредити тензор деформације D' ако је позната матрица A трансформације D.

$$D = \begin{bmatrix} 2 & -2 & 0 \\ -2 & -\sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{bmatrix} 10^{-6} \qquad A = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & 1/2 & -1/2 \end{bmatrix}$$

$$D' = A \cdot D \cdot A^T$$

$$D' = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/2 & -1/2 \\ -1/\sqrt{2} & 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 \\ -2 & \sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & 1/2 \\ 1/\sqrt{2} & -1/2 & -1/2 \end{bmatrix} 10^{-6} =$$

$$= \begin{bmatrix} -2/\sqrt{2} & 1 & -1 \\ \frac{2-\sqrt{2}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-2-\sqrt{2}}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & 1/2 \\ 1/\sqrt{2} & -1/2 & -1/2 \end{bmatrix} \cdot 10^{-6} =$$

$$= \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1-\sqrt{2} & -1 \\ 2 & -1 & 1+\sqrt{2} \end{bmatrix} 10^{-6}$$

② Стање деформације у некој тачки тела је задато тензором. Одредити главне дилатације и правце главних дилатација.

$$D=\begin{bmatrix}3 & 1 & 1\\ 1 & 0 & 2\\ 1 & 2 & 0\end{bmatrix}\cdot 10^{-6}$$

$$I_1 = 3+0+0 = 3\cdot 10^{-6}$$

$$I_2 = \begin{vmatrix}0 & 2\\ 2 & 0\end{vmatrix}+\begin{vmatrix}3 & 1\\ 1 & 0\end{vmatrix}+\begin{vmatrix}3 & 1\\ 1 & 0\end{vmatrix} = -4-1-1 = -6\cdot 10^{-12}$$

$$I_3 = \begin{vmatrix}3 & 1 & 1\\ 1 & 0 & 2\\ 1 & 2 & 0\end{vmatrix}\begin{matrix}3 & 1\\ 1 & 0\\ 1 & 2\end{matrix} = 2+2-12 = -8\cdot 10^{-18}$$

$$\varepsilon^3 - I_1\varepsilon^2 + I_2\varepsilon + I_3 = 0$$

$$\varepsilon^3 - 3\varepsilon^2 - 6\varepsilon + 8 = 0$$

$$(\varepsilon^3-3\varepsilon^2-6\varepsilon+8):(\varepsilon-1)=\varepsilon^2-2\varepsilon-8$$
$$-\varepsilon^3+\varepsilon^2$$
$$-2\varepsilon^2-6\varepsilon$$
$$+2\varepsilon^2+2\varepsilon$$
$$-8\varepsilon+8$$
$$+8\varepsilon+8$$
$$0$$

$$(\varepsilon^2-2\varepsilon-8)(\varepsilon-1)=0$$

$$\varepsilon_1 = 1 \qquad \varepsilon_{1,2} = \frac{2\pm\sqrt{4+32}}{2}$$

$$\varepsilon_1 = 4 \quad \varepsilon_2 = -2$$

$$\boxed{\varepsilon_1 = 4 \quad \varepsilon_2 = -1 \quad \varepsilon_3 = -2}$$

$\varepsilon_1 = 4$

$$D=\begin{bmatrix}-1 & 1 & 1\\ 1 & -4 & 2\\ 1 & 2 & -4\end{bmatrix}$$

$$C_1^{(1)} = \begin{vmatrix}1 & 1\\ -4 & 2\end{vmatrix} = 6 \qquad C_2^{(1)} = -\begin{vmatrix}-1 & 1\\ 1 & 2\end{vmatrix} = -3$$

$$C_3^{(1)} = \begin{vmatrix}-1 & 1\\ 1 & -4\end{vmatrix} = 3 \qquad \sqrt{36+9+9} = \sqrt{54} = 3\sqrt{6}$$

$$\vec{n}_1 = \left\{2/\sqrt{6}\,;\ 1/\sqrt{6}\,;\ 1/\sqrt{6}\right\}$$

$\varepsilon_2 = 1$

$$D=\begin{bmatrix}2 & 1 & 1\\ 1 & -1 & 2\\ 1 & 2 & -1\end{bmatrix}$$

$$C_1^{(2)} = \begin{vmatrix}1 & 1\\ -1 & 2\end{vmatrix} = 3 \qquad C_2^{(2)} = -\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix} = -3$$

$$C_3^{(2)} = \begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix} = -3 \qquad \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

$$\vec{n}_2 = \left\{1/\sqrt{3}\,;\ -1/\sqrt{3}\,;\ -1/\sqrt{3}\right\}$$

$\varepsilon_3 = -2$

$$D=\begin{bmatrix}5 & 1 & 1\\ 1 & 2 & 2\\ 1 & 2 & 2\end{bmatrix}$$

$$C_1^{(3)} = \begin{vmatrix}1 & 1\\ 2 & 2\end{vmatrix} = 0 \qquad C_2^{(3)} = -\begin{vmatrix}5 & 1\\ 1 & 2\end{vmatrix} = -9$$

$$C_3^{(3)} = \begin{vmatrix}5 & 1\\ 1 & 2\end{vmatrix} = 9 \qquad \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$$

$$\vec{n}_3 = \left\{0\,;\ -1/\sqrt{2}\,;\ 1/\sqrt{2}\right\}$$

АНАЛИЗА ДЕФОРМАЦИЈА

W

① СТАЊЕ ДЕФОРМАЦИЈА У НЕКОЈ ТАЧКИ ТЕЛА ЈЕ ЗАДАТО ТЕНЗОРОМ ДЕФОРМАЦИЈА D.

а/ ОДРЕДИТИ ДИЛАТАЦИЈУ ПРАВЦА $\vec{n} = \{1/3; 2/3; -2/3\}$

б/ ОДРЕДИТИ КЛИЗАЊЕ ИЗМЕЂУ ПРАВЦА $\vec{n}$ И ЊЕМУ УПРАВНОГ ПРАВЦА $\vec{s}$ КОЈИ ЛЕЖИ У РАВНИ x_1x_2

$$D = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 5 & 0 \\ -1 & 0 & 6 \end{bmatrix} \cdot 10^{-6}$$

а/ $\varepsilon_n = n^T D n = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 5 & 0 \\ -1 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} \cdot 10^{-6} =$

$$= \begin{bmatrix} 1 & 3 & -\frac{13}{3} \end{bmatrix} \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} \cdot 10^{-6} = \left(\frac{1}{3} + 2 + \frac{13}{3}\frac{2}{3}\right) \cdot 10^{-6} = \frac{47}{9} \cdot 10^{-6} = 5{,}22 \cdot 10^{-6}$$

б/ $\vec{s} \perp \vec{n}$, $\vec{s} \in x_1 O x_2 \Rightarrow s_3 = 0$ $\qquad |\vec{s}| = 1$

$\vec{s} = s_1 \vec{e}_1 + s_2 \vec{e}_2 + 0 \cdot \vec{e}_3$ $\qquad \sqrt{s_1^2 + s_2^2} = 1$

$\vec{s} \cdot \vec{n} = 0 \quad \frac{1}{3} s_1 + s_2 \cdot \frac{2}{3} = 0$ $\qquad (-2s_2)^2 + s_2^2 = 1$

$s_1 = -2s_2$ $\qquad 4s_2^2 + s_2^2 = 1$

$\boxed{s_1 = \mp \frac{2}{\sqrt{5}}}$ $\qquad 5s_2^2 = 1$

$\boxed{s_2 = \pm \frac{1}{\sqrt{5}}}$

1° $\vec{s} = \{-2/\sqrt{5}; 1/\sqrt{5}; 0\}$

2° $\vec{s} = \{2/\sqrt{5}; -1/\sqrt{5}; 0\}$

1° $\varepsilon_{ns} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 5 & 0 \\ -1 & 0 & 6 \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix} =$

$$= \begin{bmatrix} 1 & 3 & -\frac{13}{3} \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix} \cdot 10^{-6} = \left(-\frac{2}{\sqrt{5}} + \frac{3}{\sqrt{5}}\right) \cdot 10^{-6} = \frac{1}{\sqrt{5}} \cdot 10^{-6}$$

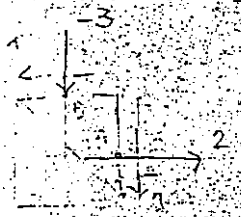
$\gamma_{ns} = 2\varepsilon_{ns} \Rightarrow \boxed{\gamma_{ns} = 2 \cdot \frac{1}{\sqrt{5}} \cdot 10^{-6}}$

2° ИСТА ВРЕДНОСТ КЛИЗАЊА, САМО СУПРОТНОГ ЗНАКА

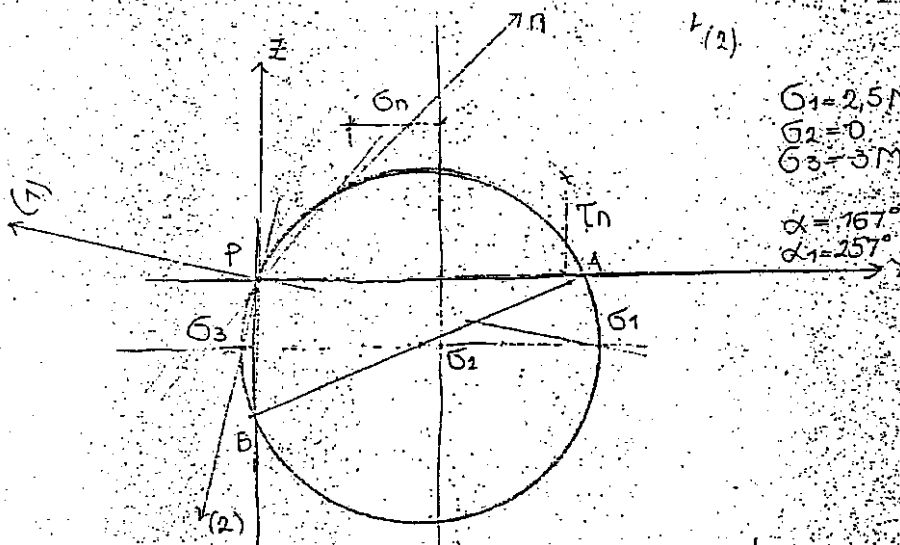
28.01.2005. W

1.2. Дат је тензор напона S. Одредити помоћу Моровог круга
a/ вредности главних напона, њихове правце и вредност максималног смичућег напона
b/ компоненталне напоне за раван чија нормала полови угао између оса Y и Z.
c/ аналитички проверити правац главног напона σ_2

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & -3 \end{bmatrix} MPa$$



A(σ_{22}; $-\sigma_{23}$)
B(σ_{33}; σ_{32})



$\sigma_1 = 2{,}5\ MPa$
$\sigma_2 = 0$
$\sigma_3 = -3\ MPa$

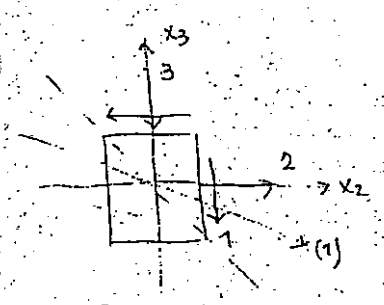
$\alpha = 167°$
$\alpha_1 = 257° \Rightarrow$ ПРАВАЦ σ_2

$\sigma_n = -1{,}8\ MPa$
$\tau_n = 1{,}5\ MPa$

$$tg\,2\alpha = \frac{2\sigma_{23}}{\sigma_{22}-\sigma_{33}} = \frac{2(-1)}{2+3} = -\frac{2}{5} \quad \begin{matrix} <0 \\ >0 \end{matrix}$$

$$\alpha = \frac{1}{2}\left(360 + arctg\left(-\tfrac{2}{5}\right)\right) = 169{,}7°$$

$x_1 = 259{,}7°$,

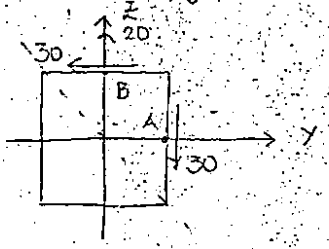


(20)

27.05.2005. IV

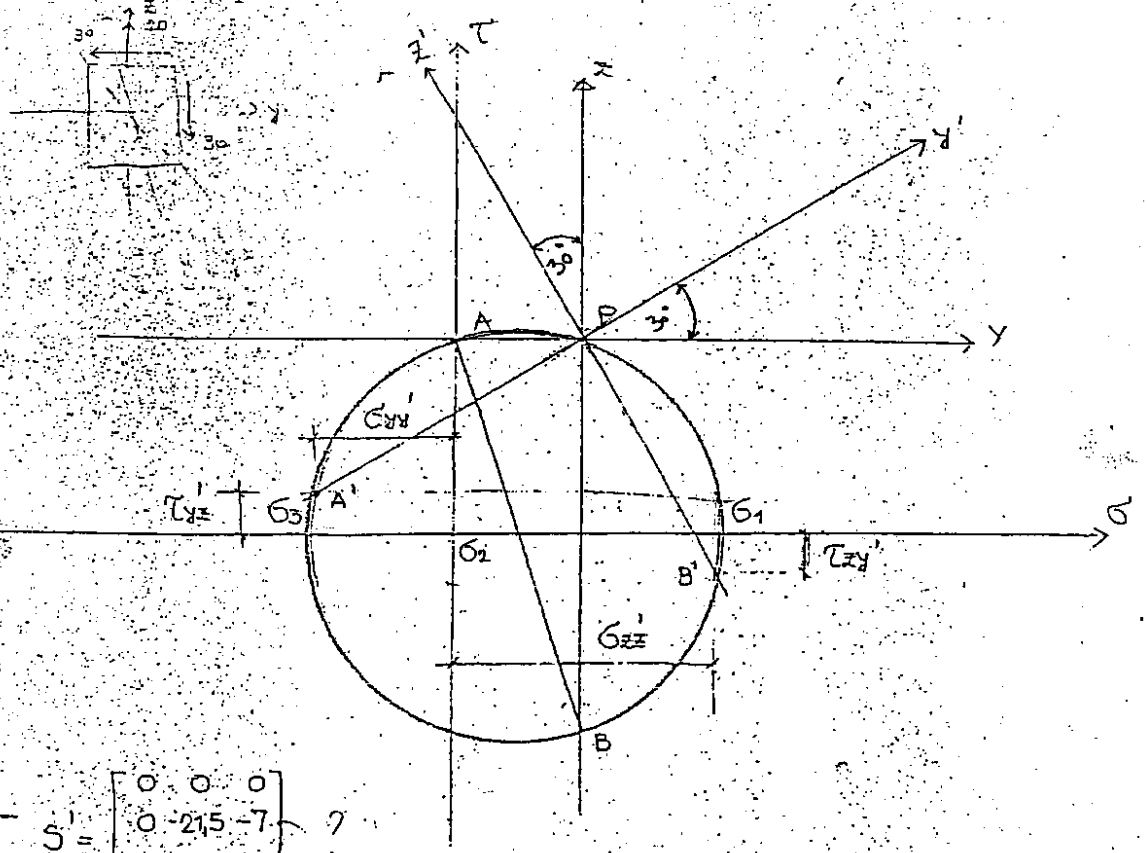
1.2. ДАТ ЈЕ ТЕНЗОР НАПОНА S У СИСТЕМУ КООРДИНАТА x, y, z. ПОМОЋУ МОРОВОГ КРУГА ОДРЕДИТИ ТЕНЗОР НАПОНА S' У РОТИРАНОМ КООРДИНАТНОМ СИСТЕМУ x, y', z' ОКО ОСЕ x ЗА $+30°$

$$S_{xyz} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -30 \\ 0 & -30 & 20 \end{bmatrix} \text{MPa}$$



$A(\sigma_{22}, -\sigma_{23}) = (0; 30)$

$B(\sigma_{33}, \sigma_{32}) = (20; -30)$



$$S' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -21{,}5 & -7 \\ 0 & -7 & 41 \end{bmatrix}$$

05.03.2005.

1.3. ОДРЕДИТИ ИНТЕНЗИТЕТ ТОТАЛНОГ НАПОНА ЗА РАВАН СА НОРМАЛОМ $\vec{n}$, АКО ЈЕ ЗАДАТ ТЕНЗОР НАПОНА И ПРАВАЦ НОРМАЛЕ

$$\vec{n} = \frac{1}{\sqrt{2}}\vec{e}_1 - \frac{1}{\sqrt{2}}\vec{e}_2 \qquad S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \text{MPa}$$

$$p_i^{(n)} = \sigma_{ij} n_j$$

$$p_1^{(n)} = \sigma_{11} n_1 + \sigma_{12} n_2 + \sigma_{13} n_3 = 1 \cdot \frac{1}{\sqrt{2}} - 2\frac{1}{\sqrt{2}} = -1/\sqrt{2}$$

$$p_2^{(n)} = 2 \cdot \frac{1}{\sqrt{2}} - 4\frac{1}{\sqrt{2}} = -2/\sqrt{2}$$

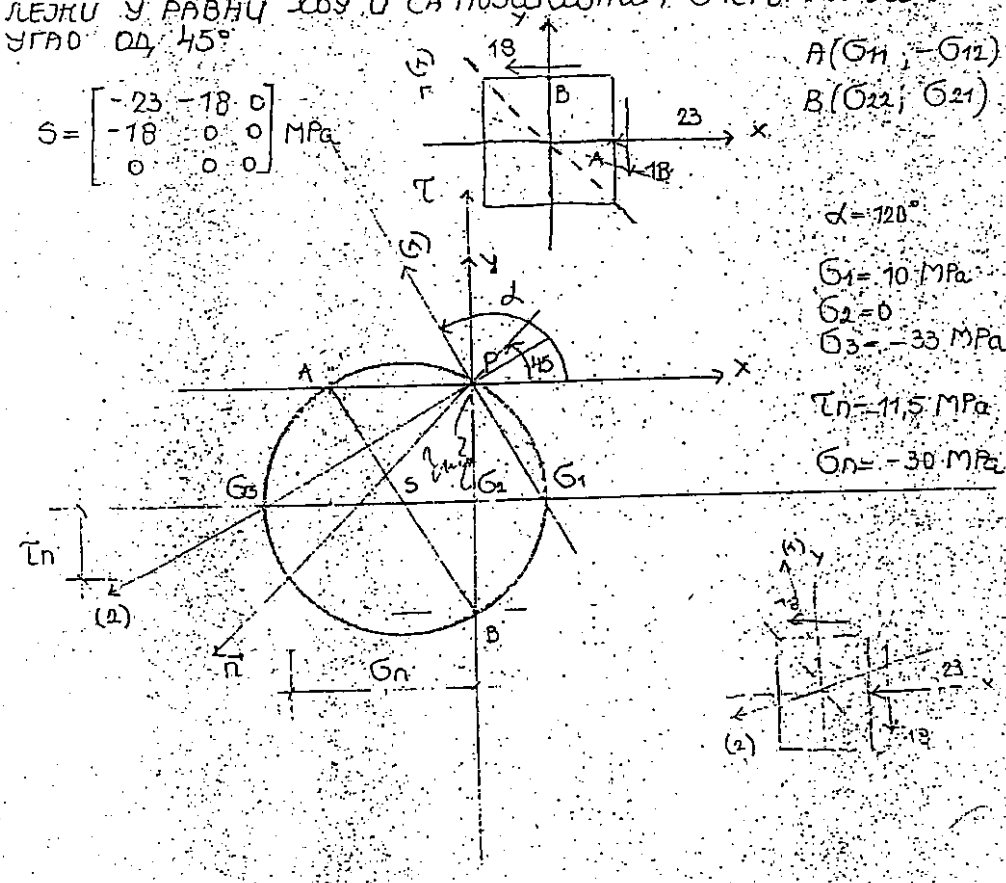
$$p_3^{(n)} = 3\frac{1}{\sqrt{2}} - 6\frac{1}{\sqrt{2}} = -3/\sqrt{2}$$

$$|\vec{p}^{(n)}| = \sqrt{\frac{1}{2} + \frac{4}{2} + \frac{9}{2}} = \sqrt{7}$$

31.01.2004. W

1.3. ДАТ ЈЕ ТЕНЗОР НАПОНА S. КОНСТРУИСАТИ Mohr-ов КРУГ НАПОНА И ОДРЕДИТИ ПОМОЋУ ЊЕГА ВРЕДНОСТИ ГЛАВНИХ НАПОНА И ЊИХОВЕ ПРАВЦЕ КАО И КОМПОНЕНТАЛНЕ НАПОНЕ ЗА РАВАН ЧИЈА НОРМАЛА ЛЕЖИ У РАВНИ xOy И СА ПОЗИТИВНИМ СМЕРОМ x-ОСЕ ГРАДИ УГАО ОД 45°

$$S = \begin{bmatrix} -23 & -18 & 0 \\ -18 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$



$A(\sigma_{11}; -\sigma_{12})$

$B(\sigma_{22}; \sigma_{21})$

$\alpha = 120°$

$\sigma_1 = 10$ MPa

$\sigma_2 = 0$

$\sigma_3 = -33$ MPa

$\tau_n = -11{,}5$ MPa

$\sigma_n = -30$ MPa

(22)

c) $I_1 = 1+4+\frac{1}{4} = \frac{21}{4}$ $\quad I_2 = 0 \quad I_3 = 0$

$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$ $\qquad 1^\circ\ \sigma_{1,2} = 0$

$\sigma^3 - \frac{21}{4}\sigma^2 = 0 \qquad \sigma^2(\sigma - \frac{21}{4}) = 0 \qquad 2^\circ\ \sigma = \frac{21}{4}$

$\sigma_1 = \frac{21}{4} \quad \sigma_2 = 0 \quad \sigma_3 = 0$

$\tau_{max} = \frac{1}{2}|\sigma_1 - \sigma_2| = \frac{21}{8} = 2{,}625$ MPa

d)

$$S' = \begin{bmatrix} 5{,}25 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

✓

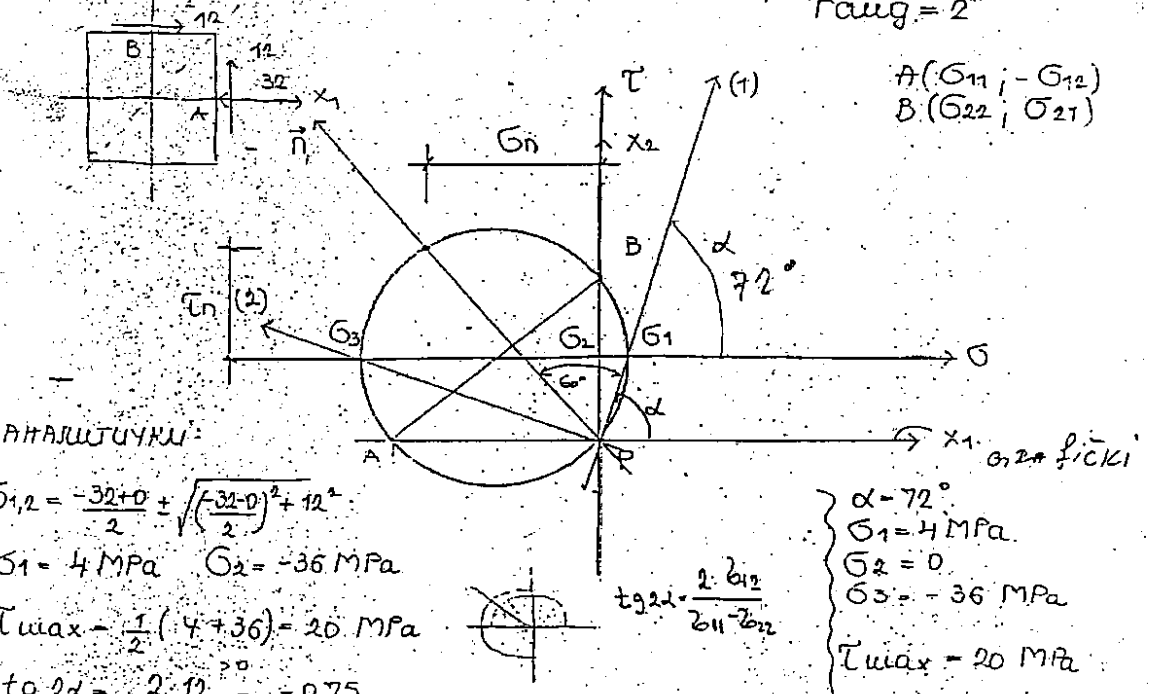
⑧ СТАЊЕ НАПОНА У НЕКОЈ ТАЧКИ НАПРЕГНУТОГ ТЕЛА ЈЕ ЗАДАТО ТЕНЗОРОМ S. ПОМОЋУ Mohr-ОВОГ КРУГА ОДРЕДИТИ ВРЕДНОСТ КОМПОНЕНТАЛНИХ НАПОНА ЗА РАВАН ЧИЈА НОРМАЛА ЛЕЖИ У РАВНИ x_0y И СА ГЛАВНОМ ОСОМ (1) ГРАДИ УГАО ОД 60°. ДОБИЈЕНО РЕШЕЊЕ ПРОВЕРИТИ АНАЛИТИЧКИ

$$S = \begin{bmatrix} -32 & 12 & 0 \\ 12 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

$\det[S] = 0$

$\text{rang}[S] = 2$

РАВАНСКО СТАЊЕ НАПОНА:
$D = 0$
$D_i \neq 0$
$\text{rang} = 2$



$A(\sigma_{11}; -\sigma_{12})$
$B(\sigma_{22}; \sigma_{21})$

АНАЛИТИЧКИ:

$\sigma_{1,2} = \frac{-32+0}{2} \pm \sqrt{\left(\frac{-32-0}{2}\right)^2 + 12^2}$

$\sigma_1 = 4$ MPa $\quad \sigma_2 = -36$ MPa

$\tau_{max} = \frac{1}{2}(4+36) = 20$ MPa

$tg\,2\alpha = \frac{2 \cdot 12}{-32-0} = -0{,}75$ $\quad (12 > 0,\ -32-0 < 0)$

$2\alpha + 60° = \varphi$

$tg\,2\alpha = \frac{2 b_{12}}{b_{11} - b_{22}}$

$\alpha = \frac{1}{2}(180 + \text{arctg}(-0{,}75))$

$\alpha = 71{,}56° \quad \varphi = 131{,}56°$

$\sigma_n = \frac{-32+0}{2} + \frac{-32-0}{2}\cos 263{,}12 + 12 \cdot \sin 263{,}12 = -26$ MPa

$\tau_n = \frac{-32-0}{2}\sin 263{,}12 - 12\cos 263{,}12 = 17{,}3$ MPa

$\alpha = 72°$
$\sigma_1 = 4$ MPa
$\sigma_2 = 0$
$\sigma_3 = -36$ MPa
$\tau_{max} = 20$ MPa
$\sigma_n = -26$ MPa
$\tau_n = 17$ MPa

⑦ W Показати да је стање напона у некој тачки које је задато тензором напона S линеарно, а затим одредити

a/ косинусе праваца са којима су напони паралелни

b/ величину тоталног напона за раван управну на ту праву

c/ величину максималног смичућег напона

d/ написати тензор напона у координатном систему чија је једна оса паралелна са правом под a/

$$S = \begin{bmatrix} 1 & 2 & 1/2 \\ 2 & 4 & 1 \\ 1/2 & 1 & 1/4 \end{bmatrix} \text{ MPa}$$

Линеарно стање напона: $D=0$ $D_1=0$ $\Gamma=1$

! потребан и довољан услов: $\vec{p}^{(2)} = \lambda_2 \vec{p}^{(1)}$ $\vec{p}^{(3)} = \lambda_3 \vec{p}^{(1)}$

$\Rightarrow$ вектори $\vec{p}^{(1)}, \vec{p}^{(2)}, \vec{p}^{(3)}$ колинеарни

$$\vec{p}^{(1)} = \vec{e}_1 + 2\vec{e}_2 + \frac{1}{2}\vec{e}_3$$

$$\vec{p}^{(2)} = 2\vec{e}_1 + 4\vec{e}_2 + \vec{e}_3 = 2\vec{p}^{(1)}$$

$$\vec{p}^{(3)} = \frac{1}{2}\vec{e}_1 + \vec{e}_2 + \frac{1}{4}\vec{e}_3 = \frac{1}{2}\vec{p}^{(1)}$$

a) $|\vec{n}| = 1 \quad \sqrt{n_1^2 + n_2^2 + n_3^2} = 1$

$$|\vec{p}^{(1)}| = \sqrt{\sigma_{11}^2 + \sigma_{12}^2 + \sigma_{13}^2} = \sqrt{1+4+\frac{1}{4}} = \frac{\sqrt{21}}{2}$$

$$n_1 = \cos\sphericalangle(\vec{p}^{(1)}, x_1) = \frac{2\sigma_{11}}{\sqrt{21}} = \frac{2}{\sqrt{21}}$$

$$n_2 = \cos\sphericalangle(\vec{p}^{(1)}, x_2) = \frac{2\sigma_{12}}{\sqrt{21}} = \frac{4}{\sqrt{21}}$$

$$n_3 = \cos\sphericalangle(\vec{p}^{(1)}, x_3) = \frac{2\sigma_{13}}{\sqrt{21}} = \frac{1}{\sqrt{21}}$$

$$\vec{n} = \begin{Bmatrix} 2/\sqrt{21} \\ 4/\sqrt{21} \\ 1/\sqrt{21} \end{Bmatrix} = \frac{1}{\sqrt{21}} \begin{Bmatrix} 2 \\ 4 \\ 1 \end{Bmatrix}$$

b/ $\vec{n} = \left[2/\sqrt{21} ; 4/\sqrt{21} ; 1/\sqrt{21} \right]$

$p^{(n)} = \sigma_{ij} n_j$, $4+16+1$

$$p^{(1)} = 1\cdot\frac{2}{\sqrt{21}} + 2\cdot\frac{4}{\sqrt{21}} + \frac{1}{2}\cdot\frac{1}{\sqrt{21}} = \frac{21}{2\sqrt{21}}$$

$$p^{(2)} = 2\cdot\frac{2}{\sqrt{21}} + 4\cdot\frac{4}{\sqrt{21}} + 1\cdot\frac{1}{\sqrt{21}} = \frac{21}{\sqrt{21}}$$

$$p^{(3)} = \frac{1}{2}\cdot\frac{2}{\sqrt{21}} + 1\cdot\frac{4}{\sqrt{21}} + \frac{1}{4}\cdot\frac{1}{\sqrt{21}} = \frac{21}{4\sqrt{21}}$$

$$p^{(n)} = \begin{Bmatrix} \frac{21}{2\sqrt{21}} \\ \frac{21}{\sqrt{21}} \\ \frac{21}{4\sqrt{21}} \end{Bmatrix} = \frac{21}{\sqrt{21}} \begin{bmatrix} 1/2 \\ 1 \\ 1/4 \end{bmatrix}$$

(29)

$$\sigma_n = \frac{\sigma_{33}+\sigma_{11}}{2} + \frac{\sigma_{33}-\sigma_{11}}{2}\cos 2\gamma \pm \sigma_{31}\sin 2\gamma$$

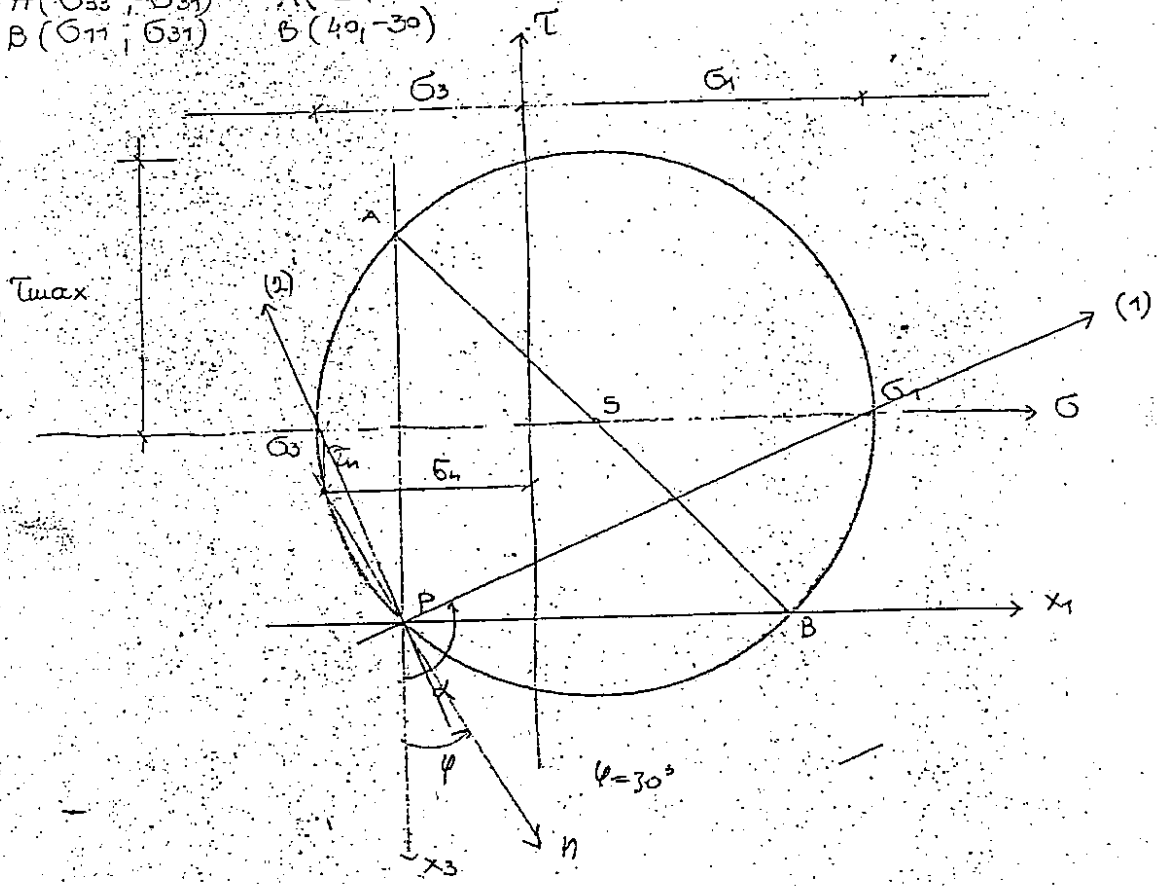
$$\tau_n = \frac{\sigma_{33}-\sigma_{11}}{2}\sin 2\gamma \mp \sigma_{31}\cos 2\gamma \qquad \gamma = 30°$$

$$\sigma_n = \frac{-20+40}{2} + \frac{-20-40}{2}\cos 60° - 30\sin 60° = -30{,}98\ MPa$$

$$\tau_n = \frac{-20-40}{2}\sin 60° + 30\cos 60° = -10{,}98\ MPa$$

$A(\sigma_{33}; -\sigma_{31})$ $\quad A(-20; 30)$

$B(\sigma_{11}; \sigma_{31})$ $\quad B(40; -30)$



$$\tau_{max} = \frac{1}{2}|\sigma_1 - \sigma_3| = \frac{1}{2}|\sqrt{65} + \sqrt{65}| = \sqrt{65} = 8{,}06\ MPa$$

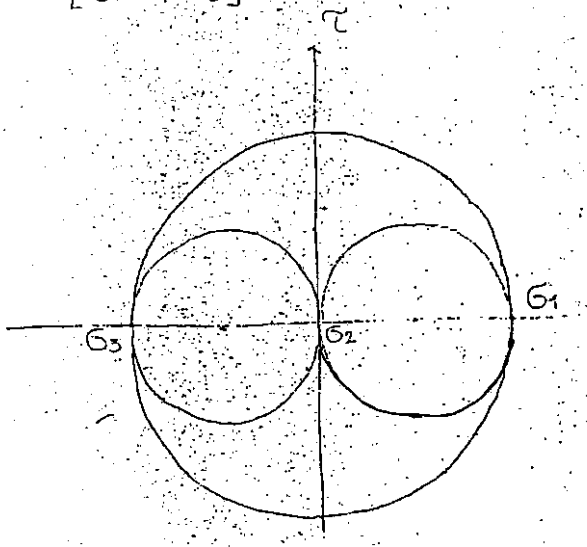
$S = S_{SFERNI} + S_{DEVIJATORSKI}$

$$S_{SFERNI} = \begin{bmatrix} \overline{\sigma} & 0 & 0 \\ 0 & \overline{\sigma} & 0 \\ 0 & 0 & \overline{\sigma} \end{bmatrix} \quad S_{DEVIJAT.} = \begin{bmatrix} \sigma_{11}-\overline{\sigma} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\overline{\sigma} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\overline{\sigma} \end{bmatrix} \quad \overline{\sigma} = \frac{1}{3}(\sigma_{11}+\sigma_{22}+\sigma_{33})$$

$$\overline{\sigma} = \frac{1}{3}(\sigma_{11}+\sigma_{22}+\sigma_{33}) = \frac{1}{3}(0+0+0) = 0$$

$$S_{DEVIJAT.} = \begin{bmatrix} 0 & 7 & 0 \\ 7 & 0 & -4 \\ 0 & -4 & 0 \end{bmatrix} \qquad I_1^{dev} = 0$$

d/



W

(6) Стање напона у некој тачки напрегнутог тела је задато тензором напона S. Одредити главне напоне и правце главних напона аналитичким и графичким путем (Мохр-ов круг). Одредити компоненталне напоне за раван чија нормала лежи у равни zox и са осом z гради угао $\gamma = 30°$

$$S = \begin{bmatrix} 40 & 0 & -30 \\ 0 & 0 & 0 \\ -30 & 0 & -20 \end{bmatrix} MPa$$

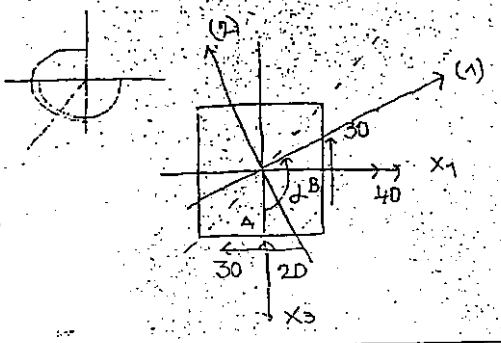
$$\sigma_{1,2}^{+} = \frac{\sigma_{33}+\sigma_{11}}{2} \pm \sqrt{\left(\frac{\sigma_{33}-\sigma_{11}}{2}\right)^2 + \sigma_{31}^2} = \frac{-20+40}{2} \pm \sqrt{\left(\frac{-20-40}{2}\right)^2 + (-30)^2}$$

$\sigma_1 = 52{,}43\ MPa \qquad \sigma_3 = -32{,}43\ MPa$

$$tg\,2\alpha = \frac{\oplus 2\sigma_{31}}{\sigma_{33}-\sigma_{11}} = \frac{2(-30)}{-20-40} < 0 = 1$$

$$\alpha = \frac{1}{2}(180 + arctg\ 1) = 112°30'00''$$

$$\tau_{max} = \frac{1}{2}|\sigma_1 - \sigma_3| = 42{,}43\ MPa$$



(5) СТАЊЕ НАПОНА У НЕКОЈ ТАЧКИ НАПРЕГНУТОГ ТЕЛА ЈЕ ЗАДАТО ТЕНЗОРОМ σ НАПОНА S, ГДЕ ЈЕ C ПРОИЗВОЉНА КОНСТАНТА.

a/ ПОКАЗАТИ, АКО СУ ЗАНЕМАРЕНЕ ЗАПРЕМИНСКЕ СИЛЕ, ДА СУ УСЛОВИ РАВНОТЕЖЕ ЗАДОВОЉЕНИ

b/ ОДРЕДИТИ ТОТАЛНИ НАПОН У ТАЧКИ $A(4,-4,7)$ ЗА РАВАН $2x+2y-z=-7$

c/ ОДРЕДИТИ ГЛАВНЕ НАПОНЕ, МАКСИМАЛНИ СМИЧУЋИ НАПОН И ПРВУ ИНВАРИЈАНТУ ДЕВИЈАТОРСКОГ ДЕЛА ТЕНЗОРА НАПОНА ЗА $C=1$

d/ НАЦРТАТИ Mohr-ОВ КРУГ НАПОНА ЗА ТАЧКУ A.

$$S=\begin{bmatrix} 0 & Cz & -Cy \\ Cz & 0 & -Cx \\ -Cy & -Cx & 0 \end{bmatrix}$$

a/ $x_1=x \quad x_2=y \quad x_3=z \quad \sigma_{ij,j}+F_i=0 \quad F_i=0$

$$F_1=-\left(\frac{\partial\sigma_{11}}{\partial x_1}+\frac{\partial\sigma_{12}}{\partial x_2}+\frac{\partial\sigma_{13}}{\partial x_3}\right)$$

$$F_1=-\left(\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y}+\frac{\partial\sigma_{13}}{\partial z}\right)=0 \qquad F_1=-(0+0+0)=0$$

$$F_2=-\left(\frac{\partial\sigma_{21}}{\partial x}+\frac{\partial\sigma_{22}}{\partial y}+\frac{\partial\sigma_{23}}{\partial z}\right)=0 \qquad F_2=-(0+0+0)=0$$

$$F_3=-\left(\frac{\partial\sigma_{31}}{\partial x}+\frac{\partial\sigma_{32}}{\partial y}+\frac{\partial\sigma_{33}}{\partial z}\right)=0 \qquad F_3=-(0+0+0)=0$$

b/ $A(\overset{x}{4},\overset{y}{-4},\overset{z}{7})$

$$S=\begin{bmatrix} 0 & 7C & 0 \\ 7C & 0 & -4C \\ 0 & -4C & 0 \end{bmatrix}$$

$2x+2y-z=-7$

$$|\vec{n}|=\sqrt{2^2+2^2+(-1)^2}=\sqrt{4+4+1}=3 \qquad \vec{n}=\left\{\frac{2}{3};\frac{2}{3};-\frac{1}{3}\right\}$$

$$\rho_1^{(n)}=0\cdot\frac{2}{3}+7C\cdot\frac{2}{3}-0\cdot\frac{1}{3}=\frac{14}{3}C$$

$$\rho_2^{(n)}=7C\cdot\frac{2}{3}+0\cdot\frac{2}{3}+4C\cdot\frac{1}{3}=6C$$

$$\rho_3^{(n)}=0\cdot\frac{2}{3}-4C\cdot\frac{2}{3}-0\cdot\frac{1}{3}=-\frac{8}{3}C$$

$$\vec{\rho}^{(n)}=C\begin{Bmatrix} 14/3 \\ 6 \\ -8/3 \end{Bmatrix}$$

c/ $C=1$

$$S=\begin{bmatrix} 0 & z & 0 \\ z & 0 & -x \\ 0 & -x & 0 \end{bmatrix}$$

$$I_1=0$$

$$I_2=\begin{vmatrix} 0 & -x \\ -x & 0 \end{vmatrix}+\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}+\begin{vmatrix} 0 & z \\ z & 0 \end{vmatrix}=-x^2-z^2$$

$$I_3=\begin{vmatrix} 0 & z & 0 \\ z & 0 & -x \\ 0 & -x & 0 \end{vmatrix}\begin{matrix} 0 & z \\ z & 0 \\ 0 & -x \end{matrix}=0$$

$$\sigma^3-I_1\sigma^2+I_2\sigma-I_3=0$$

$$\sigma^3-(x^2+z^2)\sigma=0 \qquad 1^\circ\ \sigma=0$$

$$\sigma\left(\sigma^2-(x^2+z^2)\right)=0 \qquad 2^\circ\ \sigma^2-(x^2+z^2)=0 \quad \sigma^2=x^2+z^2$$

$$\sigma=\pm\sqrt{x^2+z^2}$$

$$\left.\begin{aligned} \sigma_1&=\sqrt{x^2+z^2} \\ \sigma_2&=0 \\ \sigma_3&=-\sqrt{x^2+z^2} \end{aligned}\right\} \quad \sigma_1=\sqrt{65} \quad \sigma_2=0 \quad \sigma_3=-\sqrt{65}$$

(4) W

СТАЊЕ НАПОНА У НЕКОЈ ТАЧКИ НАПРЕГНУТОГ ТЕЛА ЗАДАТО ЈЕ ТЕНЗОРОМ НАПОНА S.

a/ ПОКАЗАТИ ДА ТЕНЗОР КАРАКТЕРИШЕ РАВНО СТАЊЕ НАПОНА

b/ ОДРЕДИТИ ЈЕДНАЧИНУ НЕНАПРЕГНУТЕ РАВНИ И БЕЛИЧУШНУ ГЛАВНИХ НАПОНА

c/ ОДРЕДИТИ τ_{max}

d/ НАПИСАТИ ТЕНЗОР НАПОНА У СИСТЕМУ ГЛАВНИХ ОСА

$$S = \begin{bmatrix} 100 & 50 & -40 \\ 50 & 0 & 0 \\ -40 & 0 & 0 \end{bmatrix} MPa$$

a/ ДА БИ СТАЊЕ НАПОНА БИЛО РАВНО, ПОТРЕБНО ЈЕ И ДОВОЉНО ДА

$$\det[S] = 0$$

$$\begin{vmatrix} 100 & 50 & -40 \\ 50 & 0 & 0 \\ -40 & 0 & 0 \end{vmatrix} \begin{matrix} 100 & 50 \\ 50 & 0 \\ -40 & 0 \end{matrix} = 0+0+0-0-0-0 = 0 \Rightarrow \text{РАВНО СТАЊЕ}$$

b/ $\vec{\rho}^{(x)}(\vec{\rho}^{(y)} \times \vec{\rho}^{(z)}) = 0$ ↙ БИЛО КОЈА ДВА

$\vec{r} = (x, y, z)$ $\quad \vec{r} \cdot (\vec{\rho}^{(x)} \times \vec{\rho}^{(y)}) = 0$

$$\begin{vmatrix} 100 & 50 & -40 \\ x & y & z \\ -40 & 0 & 0 \end{vmatrix} \begin{matrix} 100 & 50 \\ x & y \\ -40 & 0 \end{matrix} = -2000z - 1600y = 0 \qquad \boxed{5z + 4y = 0}$$

$I_1 = 100$ MPa

$$I_2 = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 100 & -40 \\ -40 & 0 \end{vmatrix} + \begin{vmatrix} 100 & 50 \\ 50 & 0 \end{vmatrix} = 0 - 1600 - 2500 = -4100 \text{ MPa}$$

$I_3 = 0$ $\quad \therefore \sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$

$\sigma^3 - 100\sigma^2 - 4100\sigma = 0$

$\sigma(\sigma^2 - 100\sigma - 4100) = 0$

1° $\sigma = 0$

2° $\sigma_{1,2} = \dfrac{100 \pm \sqrt{100^2 + 16400}}{2}$

$\sigma = 131{,}24 \quad \sigma = -31{,}24$ MPa

$\sigma_1 = 131{,}24$ MPa $\quad \sigma_2 = 0 \quad \sigma_3 = -31{,}24$ MPa

c/ $\tau_{max} = \frac{1}{2}|\sigma_1 - \sigma_3| = \frac{1}{2}(131{,}24 + 31{,}24) = 81{,}24$ MPa

d/

$$S = \begin{bmatrix} 131{,}24 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -31{,}24 \end{bmatrix} MPa$$

(28)

$\sigma_1 = 4 \cdot 10^2$ MPa $\sigma_2 = 1 \cdot 10^2$ MPa $\sigma_3 = -2 \cdot 10^2$ MPa

$$\tau_{max} = \frac{1}{2}|\sigma_1 - \sigma_3|$$

$$\tau_{max} = \frac{1}{2}|4 \cdot 10^2 + 2 \cdot 10^2| = 3\,10^2 \text{ MPa}$$

$\sigma_1 = 4$

$$S = \begin{bmatrix} 3-4 & 1 & 1 \\ 1 & 0-4 & 2 \\ 1 & 2 & 0-4 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -4 & 2 \\ 1 & 2 & -4 \end{bmatrix} 10^2$$

$$C_1^{(1)} = \begin{vmatrix} 1 & 1 \\ -4 & 2 \end{vmatrix} = 2+4 = 6$$

$$C_2^{(1)} = \ominus \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} = -(-2-1) = 3$$

$$C_3^{(1)} = \begin{vmatrix} -1 & 1 \\ 1 & -4 \end{vmatrix} = 4-1 = 3$$

$$n_1^{(1)} = \frac{C_1^{(1)}}{\sqrt{(C_1^{(1)})^2 + (C_2^{(1)})^2 + (C_3^{(1)})^2}}$$

$\sqrt{36+9+9} = 7{,}35$

$n_1^{(1)} = \frac{6}{7{,}35} = 0{,}816$ $n_2^{(1)} = \frac{3}{7{,}35} = 0{,}408$ $n_3^{(1)} = 0{,}408$

$\sigma_2 = 1$

$$S = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix} 10^2$$

$$C_1^{(2)} = \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = 2+1 = 3$$

$$C_2^{(2)} = \ominus \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -(4-1) = -3$$

$$C_3^{(2)} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2-1 = -3$$

$\sqrt{9+9+9} = 5{,}20$

$n_1^{(2)} = \frac{3}{5{,}2} = 0{,}577$ $n_2^{(2)} = -0{,}577$ $n_3^{(2)} = -0{,}577$

$\sigma_3 = -2$

$$S = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix} \cdot 10^2$$

$$C_1^{(3)} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2-2 = 0$$

$$C_2^{(3)} = \ominus \begin{vmatrix} 5 & 1 \\ 1 & 2 \end{vmatrix} = -(10-1) = -9$$

$$C_3^{(3)} = \begin{vmatrix} 5 & 1 \\ 1 & 2 \end{vmatrix} = 10-1 = 9$$

$\sqrt{0+81+81} = 12{,}73$

$n_1^{(3)} = \frac{0}{12{,}73} = 0$ $n_2^{(3)} = \frac{-9}{12{,}73} = -0{,}707$ $n_3^{(3)} = 0{,}707$

$$C = \begin{bmatrix} 0{,}816 & 0{,}408 & 0{,}408 \\ 0{,}577 & -0{,}577 & -0{,}577 \\ 0 & -0{,}707 & 0{,}707 \end{bmatrix}$$

② СТАЊЕ НАПОНА У НЕКОЈ ТАЧКИ НАПРЕГНУТОГ ТЕЛА ЈЕ ЗАДАТО ТЕНЗОРОМ НАПОНА S, ГДЕ СУ a, b, c НЕОДРЕЂЕНЕ КОНСТАНТЕ, А σ_0 ЗАДАТИ НАПОН. ОДРЕДИТИ ОВЕ КОНСТАНТЕ ТАКО ДА ЈЕ ТОТАЛНИ НАПОН У ОКТАЕДАРСКОЈ РАВНИ ЈЕДНАК 0.

$$S = \sigma_0 \begin{bmatrix} 1 & 1 & a \\ 1 & b & c \\ a & c & 2b \end{bmatrix} \qquad \vec{n} = \left\{ \pm\frac{1}{\sqrt{3}} ; \pm\frac{1}{\sqrt{3}} ; \pm\frac{1}{\sqrt{3}} \right\}$$

$\vec{p}^{(n)} = 0$ $\quad p_1^{(n)} = 0 \quad \sigma_0\left(1\cdot\frac{1}{\sqrt{3}} + 1\cdot\frac{1}{\sqrt{3}} + a\cdot\frac{1}{\sqrt{3}}\right) = 0$

$p_i^{(n)} = \sigma_{ij} n_j$ $\quad \frac{2}{\sqrt{3}} = -\frac{a}{\sqrt{3}} \quad \boxed{a = -2}$

$p_2^{(n)} = 0 \quad \sigma_0\left(1\cdot\frac{1}{\sqrt{3}} + b\cdot\frac{1}{\sqrt{3}} + c\cdot\frac{1}{\sqrt{3}}\right) = 0$

$\frac{1}{\sqrt{3}} + \frac{b}{\sqrt{3}} + \frac{c}{\sqrt{3}} = 0 \quad b + c = -1$

$c = -1 - b$

$p_3^{(n)} = 0 \quad \sigma_0\left(a\cdot\frac{1}{\sqrt{3}} + c\cdot\frac{1}{\sqrt{3}} + 2b\cdot\frac{1}{\sqrt{3}}\right) = 0$

$a + c + 2b = 0 \quad \boxed{c = -4}$

$-2 - 1 - b + 2b = 0 \quad \boxed{b = 3}$

③ СТАЊЕ НАПОНА У НЕКОЈ ТАЧКИ НАПРЕГНУТОГ ТЕЛА ЈЕ ЗАДАТО ТЕНЗОРОМ НАПОНА S. ОДРЕДИТИ ВЕЛИЧИНЕ И ПРАВЦЕ ГЛАВНИХ НАПОНА КАО И ВЕЛИЧИНУ МАКСИМАЛНОГ СМИЧУЋЕГ НАПОНА

$$S = \begin{bmatrix} 300 & 100 & 200 \\ 100 & 0 & 200 \\ 100 & 200 & 0 \end{bmatrix} MPa = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \cdot 10^2 MPa$$

$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33} = 3\cdot 10^2 MPa$

$$I_2 = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = -4 - 1 - 1 = -6\cdot 10^4 MPa$$

$$I_3 = \det|\sigma_{ij}| = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} \begin{matrix} 3 & 1 \\ 1 & 0 \\ 1 & 2 \end{matrix} = 0 + 2 + 2 - 0 - 12 = -8\cdot 10^6 MPa$$

СЕКУЛАРНА ЈЕДНАЧИНА: $\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$

$\sigma^3 - 3\sigma^2 - 6\sigma + 8 = 0 \qquad (\sigma^3 - 3\sigma^2 - 6\sigma + 8) : (\sigma + 2) = \sigma^2 - 5\sigma + 4$

$-\sigma^3 \mp 2\sigma^2$

$-5\sigma^2 - 6\sigma$

$\mp 5\sigma^2 \mp 10\sigma$

$4\sigma + 8$

$-4\sigma \mp 8$

0

$(\sigma^2 - 5\sigma + 4)(\sigma + 2) = 0$

$\sigma_{1,2} = \frac{5 \pm \sqrt{25 - 16}}{2} = \begin{cases} 4\cdot 10^2 \\ 1\cdot 10^2 \end{cases}$

$\sigma + 2 = 0 \qquad \sigma = -2\cdot 10^2$

АНАЛИЗА НАПОНА

① СТАЊЕ НАПОНА У НЕКОЈ ТАЧКИ НАПРЕГНУТОГ ТЕЛА ЗАДАТО ЈЕ ТЕНЗОРОМ НАПОНА S. ЗА РАВАН СА НОРМАЛОМ $\vec{n}$ ОДРЕДИТИ ИНТЕНЗИТЕТ И ПРАВАЦ ВЕКТОРА ТОТАЛНОГ НАПОНА $\vec{p}^{(n)}$, НОРМАЛНИ НАПОН σ И ТОТАЛНИ СМИЧУЋИ НАПОН τ.

$$S = \begin{bmatrix} 4 & -3 & 5 \\ -3 & 2 & 0 \\ 5 & 0 & 6 \end{bmatrix} MPa \qquad \vec{n} = \left\{ \frac{1}{3} ; \frac{2}{3} ; -\frac{2}{3} \right\}$$

$$\boxed{p_i^{(n)} = \sigma_{ij} n_j}$$

$$p_1^{(n)} = \sigma_{11} n_1 + \sigma_{12} n_2 + \sigma_{13} n_3$$

$$p_1^{(n)} = 4 \cdot \frac{1}{3} - 3 \cdot \frac{2}{3} - 5 \cdot \frac{2}{3} = -4$$

$$p_2^{(n)} = -3 \cdot \frac{1}{3} + 2 \cdot \frac{2}{3} - 0 \cdot \frac{2}{3} = 1/3$$

$$p_3^{(n)} = 5 \cdot \frac{1}{3} + 0 \cdot 2/3 - 6 \cdot \frac{2}{3} = -7/3$$

$$\vec{p}^{(n)} = -4\vec{e}_1 + 1/3\,\vec{e}_2 - 7/3\,\vec{e}_3$$

$$p^{(n)} = \begin{Bmatrix} -4 \\ 1/3 \\ -7/3 \end{Bmatrix}$$

ИНТЕНЗИТЕТ:

$$|p^{(n)}| = \sqrt{(-4)^2 + \left(\frac{1}{3}\right)^2 + \left(-\frac{7}{3}\right)^2} = 4{,}64 \left[MPa\right]$$

$$\boxed{\sigma_n = p_i^{(n)} n_i} = -4 \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{7}{3} = 4/9 \left[MPa\right]$$

$$|p^{(n)}|^2 = \sigma_n^2 + \tau_n^2$$

$$\tau_n = \sqrt{|p^{(n)}|^2 - \sigma_n^2} \qquad \tau_n = \sqrt{4{,}64^2 - \left(\frac{4}{9}\right)^2} = 4{,}62 \left[MPa\right]$$

ПРАВАЦ:

$$|\vec{m}| = 1 \;\wedge\; \vec{m} \parallel \vec{p}^{(n)}$$

$$m_1 = \frac{p_1^{(n)}}{|p^{(n)}|} = \frac{-4}{4{,}64} = -0{,}86$$

$$m_2 = \frac{p_2^{(n)}}{|p^{(n)}|} = \frac{1/3}{4{,}64} = 0{,}07$$

$$m_3 = \frac{p_3^{(n)}}{|p^{(n)}|} = \frac{-7/3}{4{,}64} = -0{,}5$$

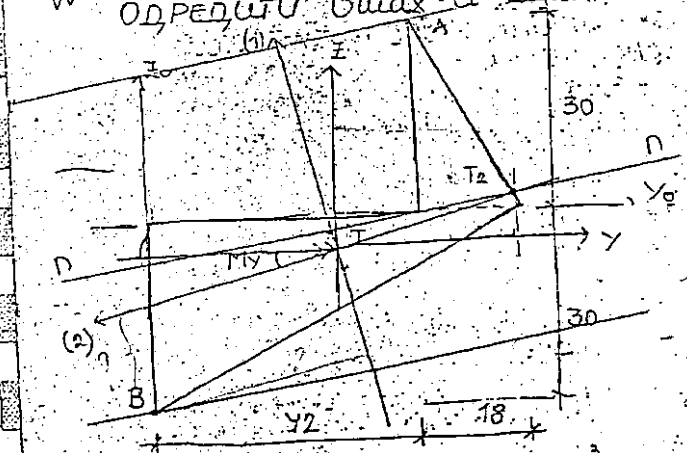
$$\vec{m} = \{ -0{,}86 ; 0{,}07 ; -0{,}5 \}$$

ЈОВИЧИЋ ЖЕЛИМИР 335/04

ОТПОРНОСТ МАТЕРИЈАЛА

28.01.2005.

1.1. У ПОПРЕЧНОМ ПРЕСЕКУ ДЕЛУЈУ ПРЕСЕЧНЕ СИЛЕ: $M_y = 100$ KNm
ПРЕМА СЛИЦИ И НОРМАЛНА СИЛА $N = -400$ KN.
ОДРЕДИТИ σ_{max} И τ_{max}.



$T_1(20; -10)$ $F_1 = 900\ cm^2$
$T_2(-8; 10)$ $F_2 = 270\ cm^2$
$F = 1170\ cm^2$

$y_T = \frac{900 \cdot 20 + 270 \cdot (-8)}{1170} = 26{,}46\ cm$

$z_T = \frac{900 \cdot (-10) + 270 \cdot 10}{1170} = -5{,}38\ cm$

$T_1'(-6{,}46; -4{,}62)$

$T_2'(21{,}54; 15{,}38)$

$I_y = \frac{1}{36} 60 \cdot 30^3 + 4{,}62^2 \cdot 900 + \frac{1}{36} 18 \cdot 30^3 + 15{,}38^2 \cdot 270 = 141576{,}95\ cm^4$

$I_z = \frac{1}{36} 60^3 30 + 6{,}46^2 \cdot 900 + \frac{1}{36} 18^3 30 + 21{,}54^2 \cdot 270 = 347690{,}77\ cm^4$

$I_{yz} = \frac{1}{72} 60^2 30^2 + 6{,}46 \cdot 4{,}62 \cdot 900 - \left[\frac{1}{72} 18^2 30^2 + 21{,}54 \cdot 15{,}38 \cdot 270\right] = 157\,257{,}68\ cm^4$

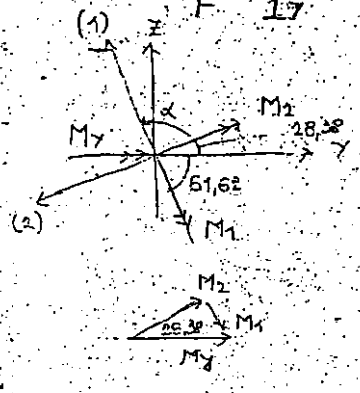
$I_{1,2} = \frac{141576{,}95 + 347690{,}77}{2} \pm \sqrt{\left(\frac{141576{,}95 - 347690{,}77}{2}\right)^2 + 157257{,}68^2}$

$I_1 = 432651{,}69 \qquad I_2 = 56\,616{,}03$

$tg\,2\alpha = \frac{-2 \cdot 157257{,}68}{141576{,}95 - 347690{,}77} < 0 \qquad \alpha = \frac{1}{2}(180 + arctg\ 1{,}526) = 118{,}38°$

$\sigma_x = \frac{N}{F} + \frac{M_y}{I_y} z - \frac{M_z}{I_z} y$ — $\sigma_x = \frac{N}{F} + \frac{M_1}{I_1} \cdot "2" - \frac{M_2}{I_2} \cdot "1"$

ОВО ЈЕ ОБЛИК ЗА ЕКСЦЕНТРИЧНО НАПРЕЗАЊЕ



$M_1 = M_y \cdot \sin 28{,}38 = 47{,}53$

$M_2 = M_y \cdot \cos 28{,}38 = 87{,}98$

НЕУТРАЛНА ОСА: $\sigma_x = 0$

$0 = \frac{N}{F} + \frac{M_1}{I_1} \cdot "2" - \frac{M_2}{I_2} \cdot "1"$

$0 = \frac{-400 \cdot 10^3}{1170 \cdot 10^{-4}} + \frac{47{,}53 \cdot 10^3}{432651{,}69 \cdot 10^{-8}} \cdot "2" + \frac{87{,}98 \cdot 10^3}{56\,616{,}03 \cdot 10^{-8}} \cdot "1"$

$0 = -0{,}34188 - 1{,}09857 \cdot "2" + 15{,}53977 \cdot "1"$

ЗА $"1" = 0 \Rightarrow "2" = -0{,}3112\ w = -31{,}12\ cm$ w

$"2" = 0 \Rightarrow "1" = 0{,}022\ w = +2{,}2\ cm$ w

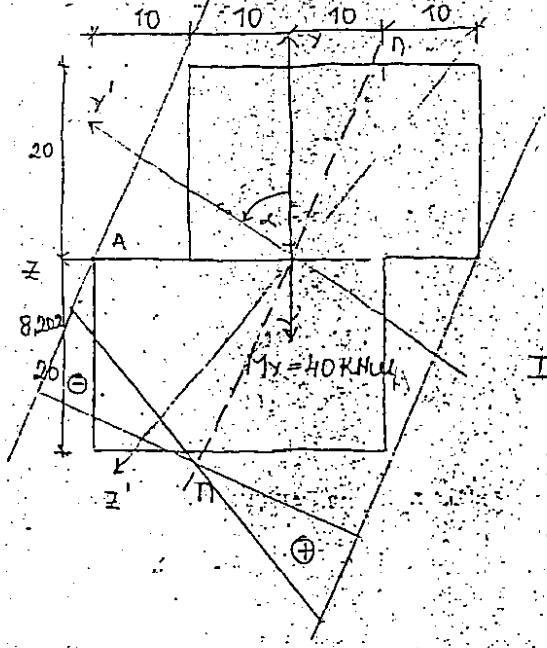
КОЈА ОСА ЈЕ "2" = $\frac{M_2}{M_1} \frac{J_x}{J_2} \cdot "1"$ — ЧИЈА ЈЕ $\frac{N}{F}$?

(32) 27.06.2003.

1.3. ЗА ДАТ ПОПРЕЧНИ ПРЕСЕК ПОЗНАТИ СУ МОМЕНТИ ИНЕРЦИЈЕ ЗА ТЕЖИШНЕ ОСЕ

$I_y = 120\,000\ cm^4$ $I_z = 160\,000\ cm^4$ $I_{yz} = -60\,000\ cm^4$

УСЛЕД ЗАДАТОГ МОМЕНТА $M_y = 40\ kNm$ ОДРЕДИТИ ПОЛОЖАЈ ТАЧАКА У КОЈИМА ЈЕ КОМПОНЕНТАЛНИ НАПОН ЈЕДНАК НУЛИ, ПОЛОЖАЈ ТАЧКЕ У КОЈОЈ СЕ ЈАВЉА НАЈВЕЋИ НЕГАТИВАН НОРМАЛНИ КОМПОНЕНТАЛНИ НАПОН КАО И ВРЕДНОСТ ТОГ НАПОНА.



$I_y = 120\,000\ cm^4$
$I_z = 160\,000\ cm^4$
$I_{yz} = -60\,000\ cm^4$

$$tg\,2\alpha = \frac{-2\cdot(-60\,000)}{120\,000-160\,000} = -3 \quad (>0,\ <0)$$

$$\alpha = \frac{1}{2}\left(180 + arctg(-3)\right) = 54{,}22^\circ$$

$$I_{1,2} = \frac{120\,000+160\,000}{2} \pm \sqrt{\left(\frac{120\,000-160\,000}{2}\right)^2 + (-60\,000)^2}$$

$I_1 = 203\,246\ cm^4$ $I_2 = 76\,754\ cm^4$

$\sin\alpha = 0{,}811$ $\cos\alpha = 0{,}585$

$M_y' = M_y \cdot \cos\alpha = 40 \cdot 0{,}585 = 23{,}4\ kNm$

$M_z' = M_y \cdot \sin\alpha = 40 \cdot 0{,}811 = 32{,}44\ kNm$

$$\sigma_x = -\frac{23{,}4\cdot 10^{-3}}{203\,246\cdot 10^{-8}}\cdot z' - \frac{32{,}44\cdot 10^{-3}}{76754\cdot 10^{-8}}\cdot y'$$

$$\sigma_x = 0 \Rightarrow y' = -\frac{23{,}4\cdot 76754}{32{,}44\cdot 203246}\cdot z'$$

$$y' = -0{,}272\, z'$$

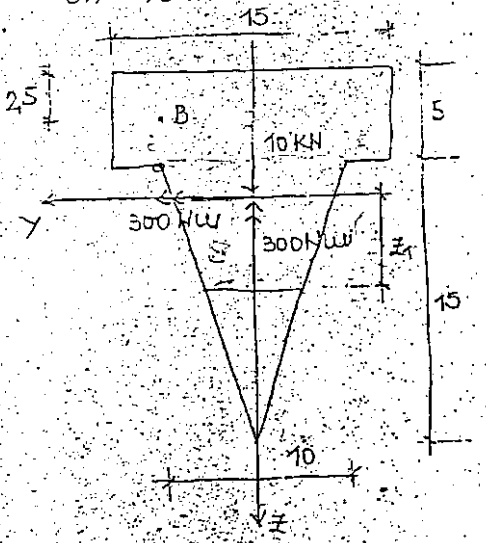
$A(0,20)$ $y' = z\cdot\sin\alpha + y\cdot\cos\alpha = 16{,}22\ cm$

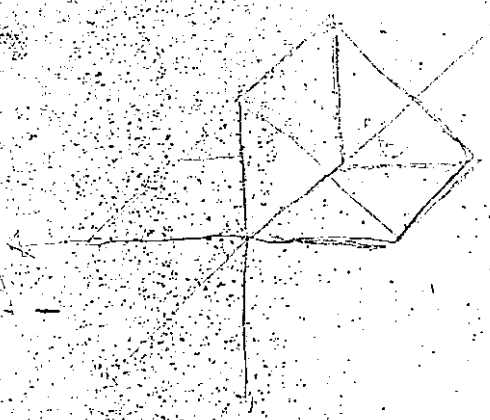
$z' = z\cdot\cos\alpha - y\cdot\sin\alpha = 11{,}7\ cm$

$$\sigma_x^A = \frac{-23{,}4\cdot 10^{-3}}{203\,246\cdot 10^{-8}}\cdot 11{,}7\cdot 10^{-2} - \frac{32{,}44\cdot 10^{-3}}{76754\cdot 10^{-8}}\cdot 16{,}22\cdot 10^{-2} \cdot 10^{9} = -8{,}202\ MPa$$

* ЗАДАТИ ПРЕСЕК:
a/ ОДРЕДИТИ КОМПОНЕНТУ τ_i ВЛАКНА У КОМЕ СЕ ЈАВЉА τ_{max} СМИЧУЋИ НАПОН τ_x^{max}
b/ ОДРЕДИТИ ВРЕДНОСТ И ПРАВАЦ τ_x^{max}
c/ НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА
d/ ОДРЕДИТИ НОРМАЛНИ И ТОТАЛНИ СМИЧУЋИ НАПОН У ТАЧКИ B ЗА РАВАН ЧИЈА НОРМАЛА ПОЛОВИ УГАО УЗМЕЂУ ОСА X И Z

КОНСУЛТАЦИЈЕ!?

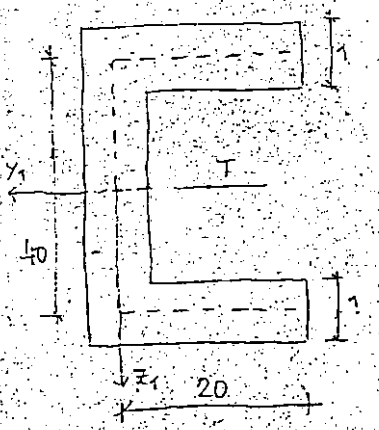


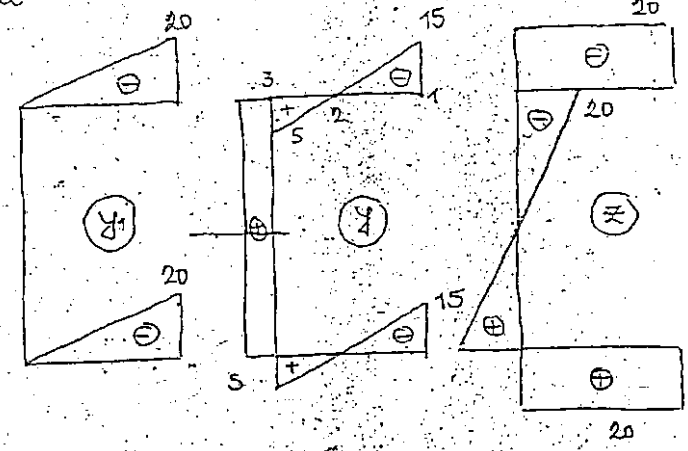


(39)

ZADANIE

W 1) $I_y, I_z, I_\omega, S_y^*, S_z^*, S_\omega^*$



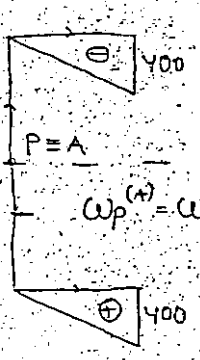


$$y_{1T} = \frac{S_{z1}}{F} = \frac{\int t y_1 ds}{F} = \frac{t\int y_1 ds}{F} \qquad F = 80\ cm^2 \Rightarrow y_{1T} = -\frac{400}{80} = -5$$

$$S_{z1} = 1 \cdot \frac{(-20)\cdot 20}{2} \cdot 2 = -400$$

$$I_y = t\int z^2 ds = 1\left(20\cdot 20\cdot 20 + \frac{20}{3}\cdot 20\cdot 20\right)\cdot 2 = 21\,333{,}3\ cm^4$$

$$I_z = t\int y^2 ds = 1\left[\left(\frac{15}{3}15^2 + \frac{5}{3}5^2\right)\cdot 2 + 40\cdot 5\cdot 5\right] = 3\,333{,}3\ cm^4$$

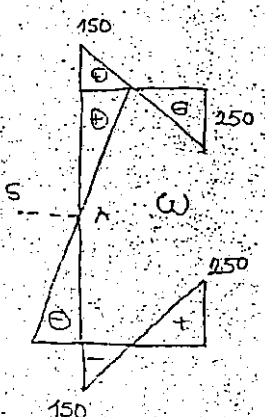


$$I_{z\omega_P} = t\int z\,\omega_P\, ds$$

$$= 1\left(\frac{20\cdot 400}{2}\cdot 20\right)\cdot 2 = 160\,000\ cm^5$$

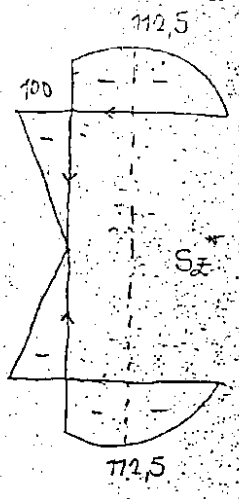
$I_{y\omega_P} = 0$ – JEP JE Y OSA – OSA SIMETRIJE

$$y_S = y_{POLA} + \frac{I_{z\omega_P}}{I_y} = y_P + \frac{160\,000}{21333} = 5 + 7{,}5$$



$$I_\omega = t\int \omega^2 ds = 1\left[\frac{20}{3}\left(-150\right)^2 + (-150)\cdot 150 + 150^2\right) + \frac{20}{3}\cdot 150^2\right]\cdot 2$$

$$I_\omega = 933\,333{,}3\ cm^4$$

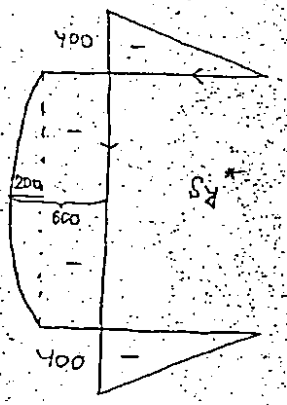


$$S_z^* = t\int y\,ds$$

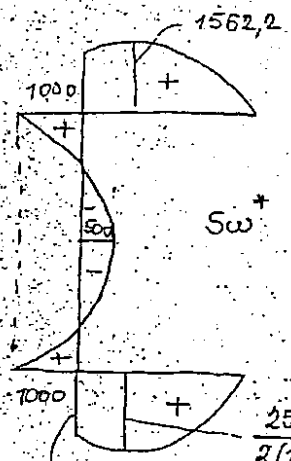
$$S_z^*(2) = t\cdot F_y^{1\to 2} = 1\cdot\frac{(-15)\cdot 15}{2} = -112{,}5$$

$$S_z^*(3) = S_z^*(2) + t\cdot F_y^{2\to 3} = -100$$

$$S_z^*(4) = S_z^*(3) + t\,F_y^{3\to 4} = -100\cdot 1\cdot 50\cdot 20 = 0$$



$$S_y^* = t\int z\,ds$$



$$S\omega^* = t\int \omega\,ds$$

$$\frac{250^2\cdot 20}{2(150+250)} = 1562{,}2$$

$$1\cdot\frac{250-150}{2}\cdot 20 = 1000$$

$$\tau = \frac{T_z\cdot S_y^*}{J_y\cdot t}$$

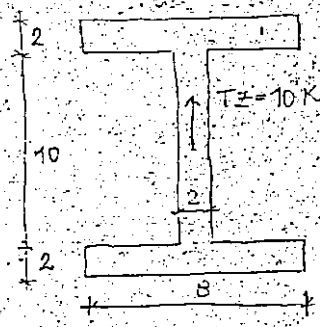
ВЕЖБЕ

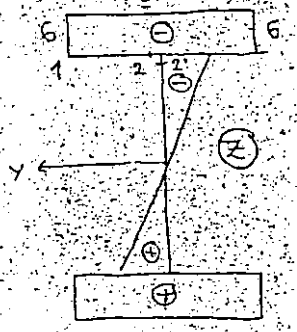
W ЗА ДАТУ ПРЕСЕК НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА

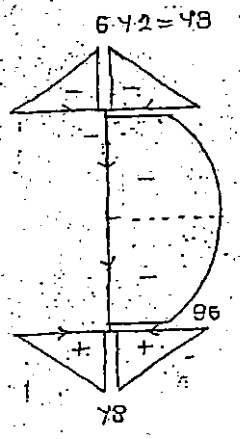
$I_y = \frac{1}{12}(8 \cdot 14^3 - 6 \cdot 10^3) = 1329{,}3 \text{ cm}^4$ ←

ŠTO NE?

$I_y = t\int z^2 dS = 1440 \text{ cm}^4$







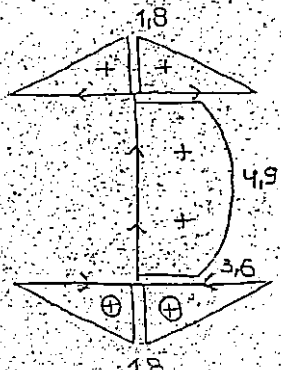
$$\tau_{xs} = -\frac{T_z S_y^*}{I_y t}$$

$\tau_{xs}(1) = 0$

$$\tau_{xs}(2) = -\frac{(-10 \cdot 10^3)(-48 \cdot 10^{-6})}{1329{,}3 \cdot 10^{-8} \cdot 2 \cdot 10^{-2}} = -1{,}8 \text{ MPa}$$

$$\tau_{xs}(2') = -\frac{(-10 \cdot 10^3)(-96 \cdot 10^{-6})}{1329{,}3 \cdot 10^{-8} \cdot 2 \cdot 10^{-2}} = -3{,}6 \text{ MPa}$$

$$\tau_{xs}(3) = -\frac{(-10 \cdot 10^3)(-132 \cdot 10^{-6})}{1329{,}3 \cdot 10^{-8} \cdot 2 \cdot 10^{-2}} = -4{,}96 \text{ MPa}$$



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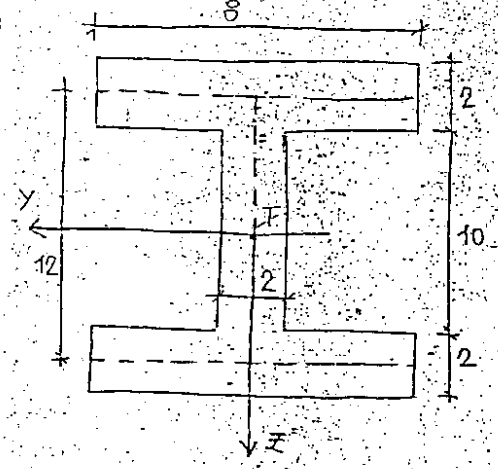
ВЕЖБЕ > W

ЗА ТАКОЗВАНИ ПРЕСЕК ОДРЕДИТИ $I_y, I_z, I_\omega, S_y^*, S_z^*, S_\omega$



ПРЕСЕК ИМА ДВЕ ОСЕ СИМЕТРИЈЕ

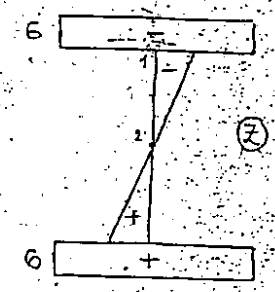
$T \equiv A \equiv P \equiv S$

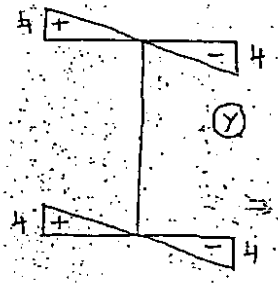
$\omega_P^{(A)} = \omega_S^{(A)} = \bar{\omega}$

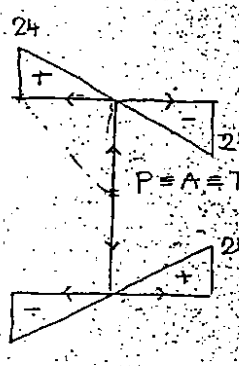
$S\omega^{(A)} = 0$

$\triangle \times \triangle = \frac{1}{3} ab$

$2 \cdot \frac{1}{2} \cdot 4 \cdot 6$



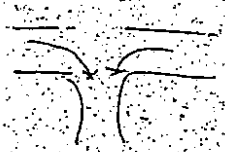




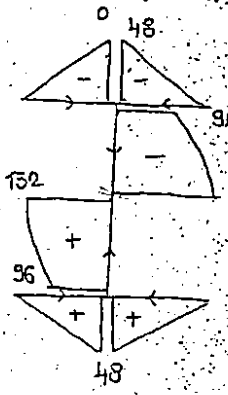
$$I_y = t\int z^2 dS = 2\left[2 \cdot 6 \cdot 8 \cdot 6 + 2 \cdot \frac{6}{3} \cdot 6 \cdot 6\right] = 1440 \text{ cm}^4$$

$$I_z = t\int y^2 dS = 2 \cdot 4 \cdot \frac{4}{3} \cdot 4 \cdot 4 = 170{,}67 \text{ cm}^4$$

$$I_\omega = t\int \omega^2 dS = 2 \cdot 4 \cdot \frac{4}{3} \cdot 24 \cdot 24 = 6144 \text{ cm}^6$$

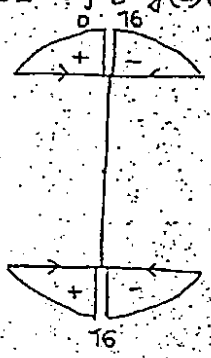


$$S_y^* = \int_0^{s^*} t \cdot z(s)\, ds \qquad S_{y,\text{лево}} = (-6) \cdot 4 \cdot 2$$



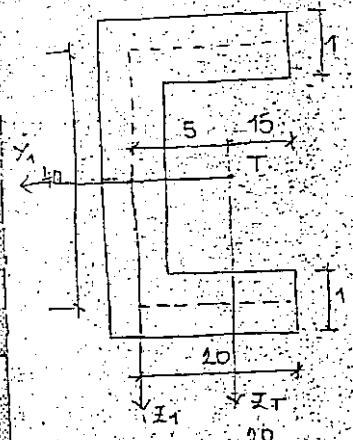
$$S_\omega^* = t\int \omega\, ds$$

$$S_z^* = \int_0^{s^*} t \cdot y(s)\, ds$$



ВЕЖБЕ 4

Одредити $I_y, I_z, I_\omega, S_y^*, S_z^*, S_\omega^*$



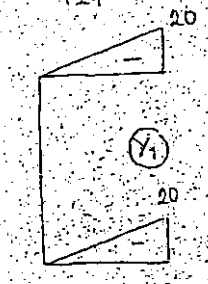
пресек има једну осу симетрије

T, A ≡ P, S леже на оси симетрије

$\omega_P^{(A)} \neq \omega_S^{(A)}$

$S_\omega^{(A)} = 0 \Rightarrow \omega_S^{(A)} = \omega$

$$a \square \times b \triangle = \frac{1}{2} ab$$

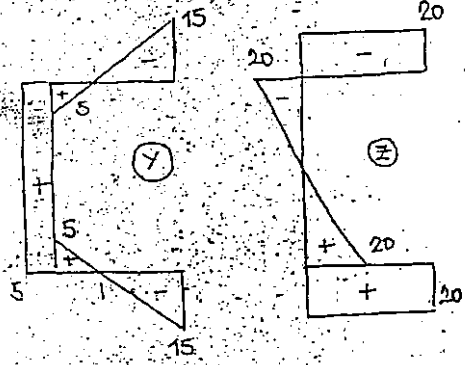


$$y_T = \frac{S_{z1}}{F} = \frac{t\int y_1 ds}{F}$$

$$F = 2 \cdot 20 \cdot 1 + 40 \cdot 1 = 80 \text{ cm}^2$$

$$S_{z1} = 2 \cdot 1 \cdot \frac{(-20) \cdot 20}{2} = -400$$

$$y_T = \frac{-400}{80} = -5$$

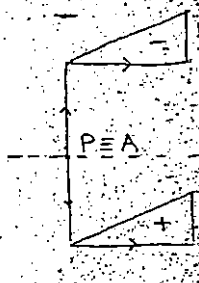


$$I_y = t\int z^2 ds = 1\left[2 \cdot 20 \cdot 20 \cdot 20 + 2 \cdot \frac{20}{3} 20 \cdot 20\right]$$

$$I_y = 21\,333{,}3 \text{ cm}^4$$

$$I_z = t\int y^2 ds = 1\left[2 \cdot \frac{15}{3} 15^2 + 2 \cdot \frac{5}{3} 5^2 + 40 \cdot 5 \cdot 5\right]$$

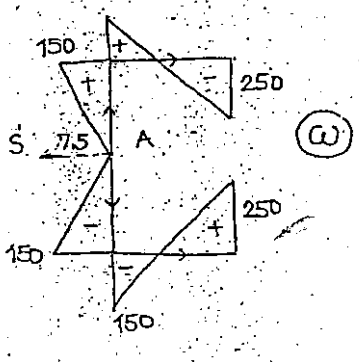
$$I_z = 3\,333{,}3 \text{ cm}^4$$



$$I_{z\omega_P} = t\int z\omega_P ds = 1\left[\frac{20}{2} 20 \cdot 400 \cdot 2\right] = 160\,000 \text{ cm}^4$$

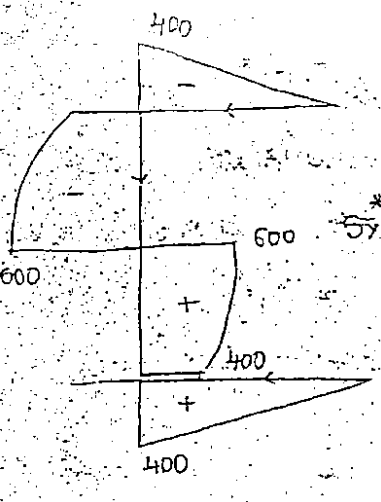
$\omega_P^{(A)} = \omega_P$. $I_{y\omega_P} = 0$ јер је y оса симетрије

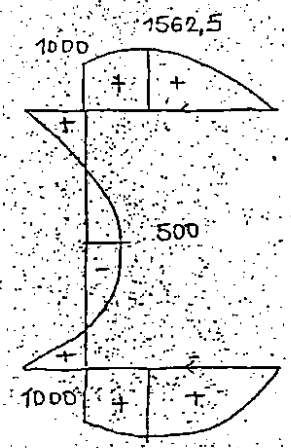
$$y_S = y_P + \frac{I_{z\omega_P}}{I_y} = 5 + \frac{160\,000}{21\,333{,}3} = 12{,}5 \text{ cm}$$



$$I_\omega = t\int \omega^2 ds = 1\left[2 \cdot \frac{20}{3}(150^2 - 150 \cdot 250 + 250^2) + 2 \cdot \frac{20}{3} 150^2\right] = 933\,333{,}3 \text{ cm}^6$$

40





znaki ?

$$\frac{(0{,}75a^2)^2 \cdot a}{2\cdot(0{,}75a^2+0{,}25a^2)} \cdot 0{,}1a$$

ВЕЖБЕ W

$\omega, I_\omega, S_\omega^*, I_1, I_2$

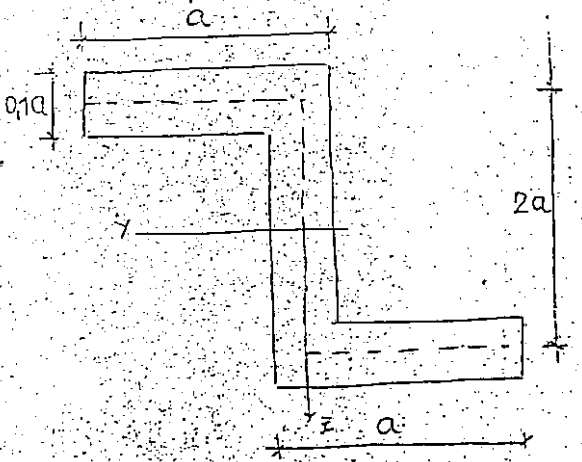
УЧЕНТРАЛНО СИМЕТРИЧАН ПРЕСЕК

$T \equiv A \equiv P \equiv S$

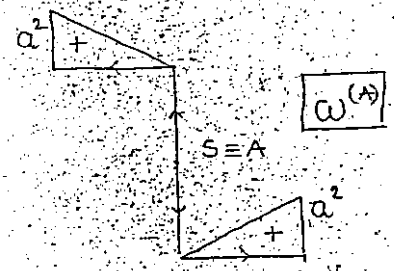
$\omega_P^{(A)} = \omega_S^{(A)} = \omega^{(A)}$

$S\omega^{(A)} \neq 0 \quad \omega^{(A)} \neq \omega$

$\omega = \omega^{(A)} + \omega_0$



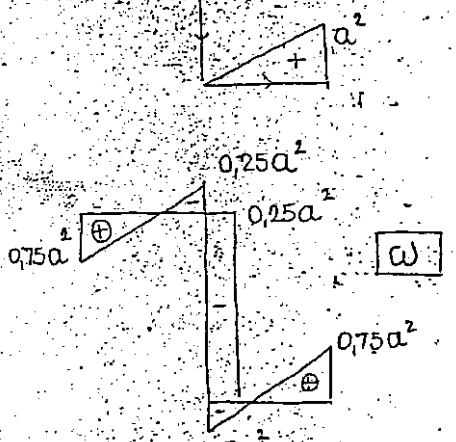
$F = 2 \cdot 0{,}1a \cdot a + 2a \cdot 0{,}1a - 0{,}4a^2$



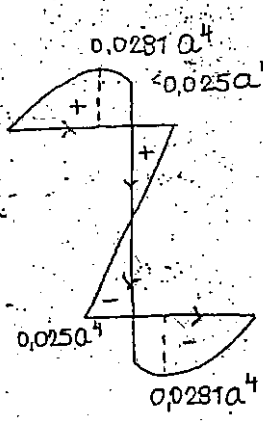
$\omega^{(A)}$

$S\omega^{(A)} = 2 \cdot 0{,}1a \cdot \frac{1}{2} a^2 a = 0{,}1a^4$

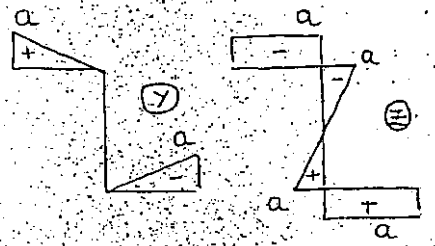
$\omega_0 = -\frac{S\omega^{(A)}}{F} = -\frac{0{,}1a^4}{0{,}4a^2} = -0{,}25a^2$



ω



$S\omega^*$



$\oplus$

$I_z = 0{,}1a\left[2 \cdot \frac{a}{3} \cdot a a\right] = 0{,}067a^4$

$I_Y = 0{,}1a\left[2 \cdot a^3 + 2 \cdot \frac{a}{3} a^2\right] = 0{,}267a^4$

$I_{Yz} = 0{,}1a\left[-2 \cdot \frac{a}{2} a \cdot a\right] = -0{,}1a^4$

$I_{1,2} = \frac{0{,}267a^4 + 0{,}067a^4}{2} \pm \sqrt{\left(\frac{0{,}267a^4 - 0{,}067a^4}{2}\right)^2 + (-0{,}1a^4)^2}$

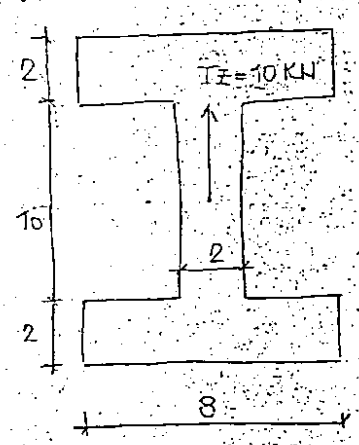
$I_1 = 0{,}308a^4 \qquad I_2 = 0{,}026a^4$

$tg\,2\alpha = \frac{2 \cdot 0{,}1a^4}{0{,}267a^4 - 0{,}067a^4} = 1 \qquad \alpha = 22{,}5^\circ$

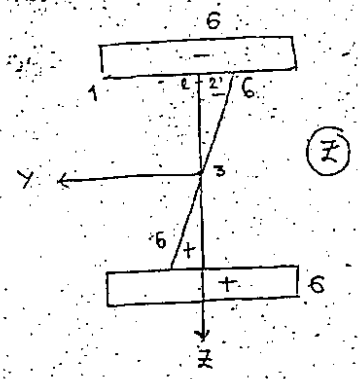
(42)

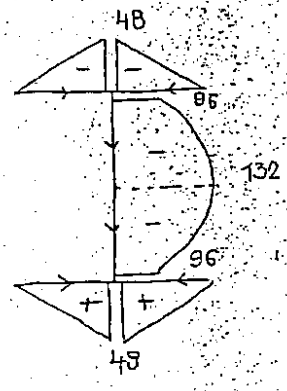
ВЕЖБЕ

√ ЗА ДАТИ ПРЕСЕК НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА



$$I_y = \frac{1}{12}(8\cdot 14^3 - 6\cdot 10^3) = 1329{,}3 \text{ cm}^4$$





$$\tau_{xs} = -\frac{T_z S_y^*}{I_y t}$$

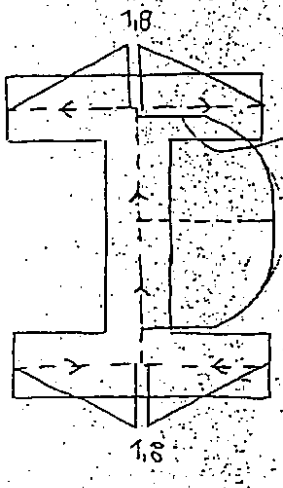
S_y^*

$\tau_{xs}(1) = 0$

$$\tau_{xs}(2) = -\frac{(-10\cdot 10^3)\cdot(-48\cdot 10^{-6})}{1329{,}3\cdot 10^{-8}\cdot 2\cdot 10^{-2}} = -1{,}8 \text{ MPa}$$

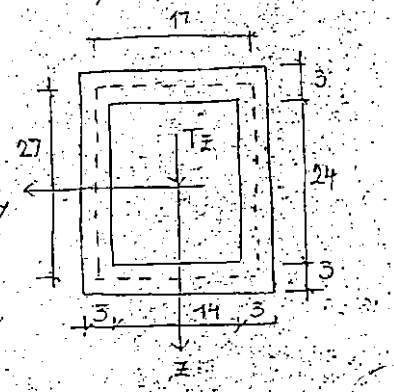
$$\tau_{xs}(2') = -\frac{(-10\cdot 10^3)\cdot(-96\cdot 10^{-6})}{1329{,}3\cdot 10^{-8}\cdot 2\cdot 10^{-2}} = -3{,}6 \text{ MPa}$$

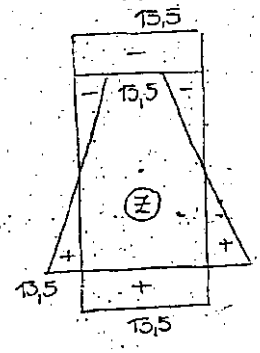
$$\tau_{xs}(3) = -\frac{(-10\cdot 10^3)\cdot(-132\cdot 10^{-6})}{1329{,}3\cdot 10^{-8}\cdot 2\cdot 10^{-2}} = -4{,}96 \text{ MPa}$$



ВЕЖБЕ W

Одредити дијаграме смичућег напона $T_z = 10$ kN

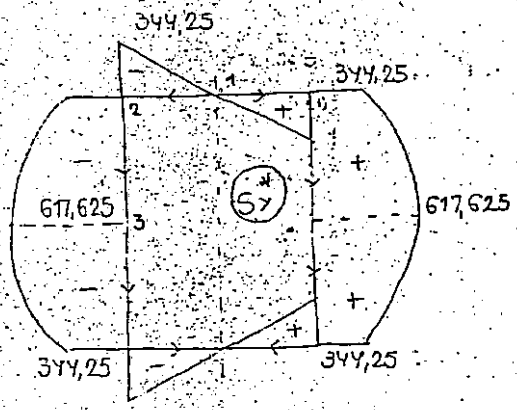




$I_y = \frac{1}{12}[20 \cdot 30^3 - 14 \cdot 24^3] = 28872\ cm^4$

$I_y = t\int z^2 dS =$

$= 28431\ cm^4$



$S_y^*(1) = 0$

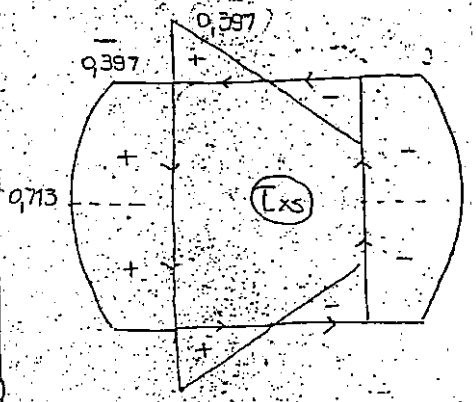
$S_y^*(2) = -13,5 \cdot \frac{27}{2} \cdot 3 = -344,25$

$S_y^*(3) = S_y^*(2) + \frac{(-13,5) \cdot 27/2}{2} \cdot 3 = -617,625$

$\tau_{xs}(1) = 0$

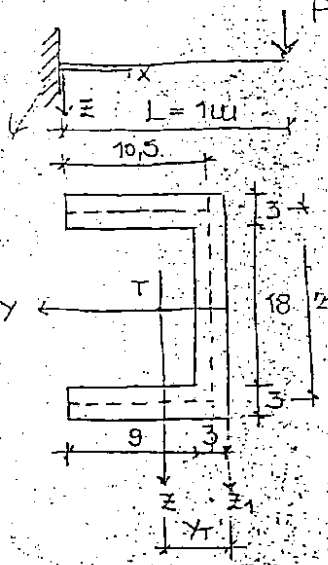
$\tau_{xs}(2) = -\frac{10 \cdot 10^3 \cdot (-344,25 \cdot 10^{-6})}{28872 \cdot 10^{-8} \cdot 3 \cdot 10^{-2}} = 0,397\ MPa$

$\tau_{xs}(3) = -\frac{10 \cdot 10^3 (-617,625 \cdot 10^{-6})}{28872 \cdot 10^{-8} \cdot 3 \cdot 10^{-2}} = 0,713\ MPa$

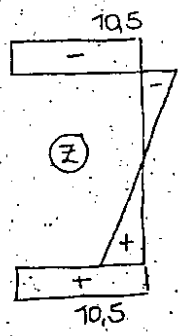


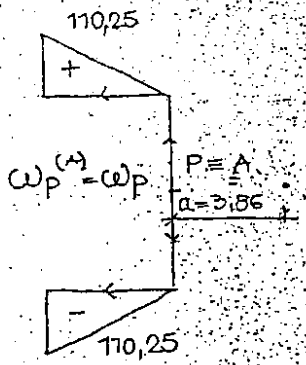
ВЕЖБЕ

ЗА ЗАДАТУ КОНЗОЛНУ НОСАЧ НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА ЗА ПРЕСЕК У УКЉЕШТЕЊУ



$$y_T = \frac{2 \cdot 3 \cdot 12 \cdot 6 + 18 \cdot 3 \cdot 1{,}5}{2 \cdot 3 \cdot 12 + 18 \cdot 3} = 4{,}1 \text{ cm}$$



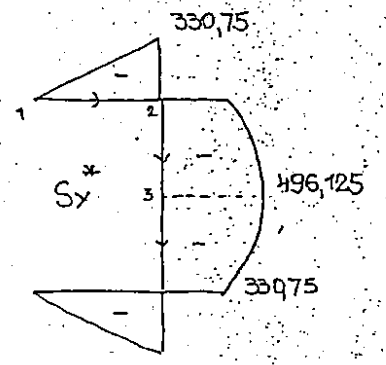


$$I_y \omega_P = 0$$

$$I_z \omega_P = 3 \cdot \left[2 \cdot \left(-\frac{10{,}5}{2} \cdot 10{,}5 \cdot 110{,}25 \right) \right] = -36\,465{,}19 \text{ cm}^5$$

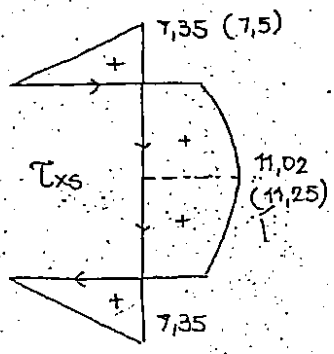
$$I_y = \frac{1}{12}\left(12 \cdot 24^3 - 9 \cdot 18^3\right) = 9450 \text{ cm}^4$$

$$y_S = y_P + \frac{I_z \omega_P}{I_y} = y_P + \frac{-36465{,}19}{9450} = y_P - 3{,}86$$



$$\tau_{xs}(1) = 0$$

$$\tau_{xs}(2) = -\frac{63 \cdot 10^3 \cdot (-330{,}75 \cdot 10^{-6})}{9450 \cdot 10^{-8} \cdot 3 \cdot 10^{-2}} = 7{,}35$$



$$\tau_{xs}(3) = -\frac{63 \cdot 10^3 \cdot (-496{,}125 \cdot 10^{-6})}{9450 \cdot 10^{-8} \cdot 3 \cdot 10^{-2}}$$

$$\tau_{xs}(3) = 11{,}02 \text{ MPa}$$

ЗАДАТАК

P_1 = ПОВРШИНА ДИЈ. × ДЕБЉИНА

$P_1 = \frac{7,5 \cdot 10^{-6} \cdot 10,5 \cdot 10^{-2}}{2} \cdot 3 \cdot 10^{-2} = 11812,5\ N = 11,812\ KN$ (11,825)

P_2 = ▯ × ↓↓ = (7,35 · 21 + $\frac{2}{3}$ · СТРЕЛА · ДУЖИНА) · 3 $(10^6 \cdot 10^{-2} \cdot 10^{-2})$

= $(7,35 \cdot 21 + \frac{2}{3} \cdot 3,67 \cdot 21) \cdot 3 (10^6 \cdot 10^{-2} \cdot 10^{-2}) = 62$ KN (65)

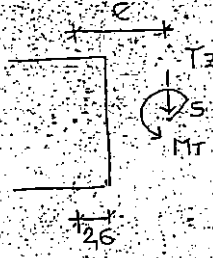
P_3 = ◺ × ↓↓ = P_1 = 11,812 KN (11,825)

a : $P_1 \frac{h}{2} - P_2 a + P_3 \frac{h}{2} = 0$

$(P_1 + P_3)\frac{h}{2} - P_2 a = 0$ $\quad a = (11,812 \cdot 2 \cdot \frac{21}{2}) \frac{1}{62} = 4$ (3,94)

$\sum M_{spoj} = 0$

$a \cdot M_t$



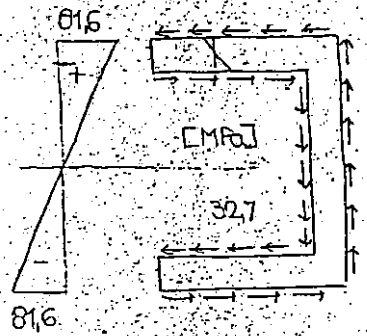
$M_t = T_z \cdot e$

$e = a + 2,6 = 3,94 + 2,6 - 6,54$

$M_t = 63$ KN 6,54 cm = 63 0,0654 = 4,12 KNm

$t = const. \Rightarrow W = \frac{\Sigma b_i t^3}{3}$ $\quad W = (10,5 + 21 + 10,5) 3^2 = 126\ cm^3$

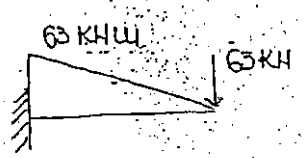
$\tau^{M_t} = \frac{M_t}{W_t} = \frac{4,12 \cdot 10^3}{126 \cdot 10^{-6}} = 32,7$ MPa → НА БОЧНОЈ ИВИЦИ ВИДА



УКУПАН ДИЈ НАПОНА БИ БИО ЗБИР ОВОГ И ПРЕДХОДНО НАЦРТАНОГ τ_{xs}.

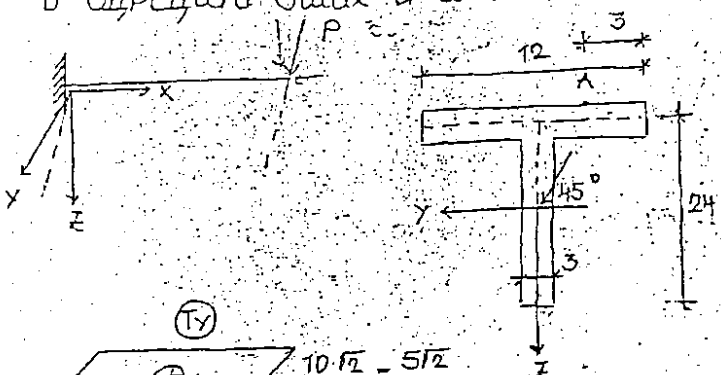
$\sigma_x^{M_y} = \frac{M_y}{I_y} z_{max} = \frac{63 \cdot 10^3 \cdot 12 \cdot 10^6}{9450} = 81,6$ MPa

M_y - МОМЕНТ САВИЈАЊА



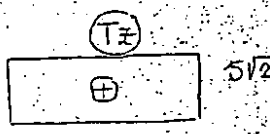
(46) ЗАДАТАК

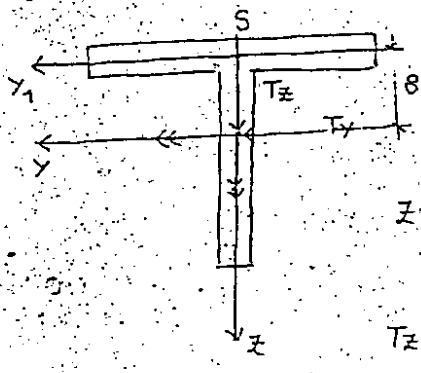
Нацртати дијаграме пресечних сила, компонената напона и одредити σ_{max} и τ_{max} за пресек у укљештењу



$P = 10$ kN

✓ Зракасти пресек (сива се где је S)

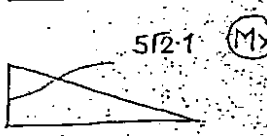


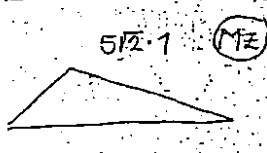


Редукован у односу на S, а не на T

$F = 3(12+24) = 108$ cm^2

$z_T = \frac{24 \cdot 3 \cdot 12}{108} = 8$ cm

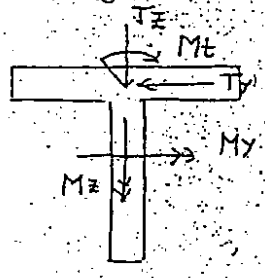




T_z пролази кроз S → не даје M_t

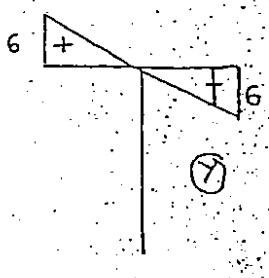
$M_t = T_y \cdot 8 = 5\sqrt{2} \cdot 0{,}08 = 0{,}4\sqrt{2}$

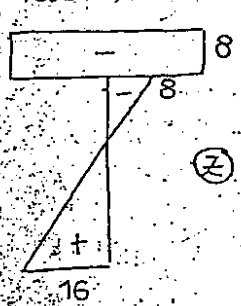
* Пребацивање силе у S



$M_t = 565$ Ncm
$M_y = 5\sqrt{2}$ kNcm
$M_z = -5\sqrt{2}$ kNcm

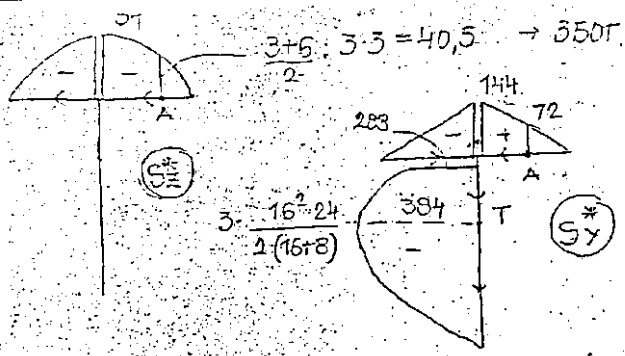
* Дијаграми y и z координате





$I_y = 3\left[8 \cdot 12 \cdot 8 + \frac{8}{3} 8^2 + \frac{16}{3} 16^2\right] = 6912$ cm^4

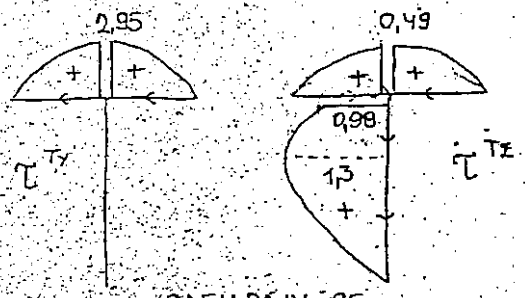
$I_z = 3\left[2 \cdot \frac{6}{3} 6^2\right] = 432$ cm^4



* ДИЈАГРАМИ КОМПОНЕНТАЛНИХ НАПОНА

$$\tau^{T_y} = -\frac{T_y S_z^*}{I_z \cdot t}$$

$$\tau_{T_y}^{(1)} = -\frac{5\sqrt{2}\,10^3\,(-54\cdot 10^{-6})}{432\cdot 10^{-8}\cdot 3\cdot 10^{-2}} = 2{,}95\ MPa$$

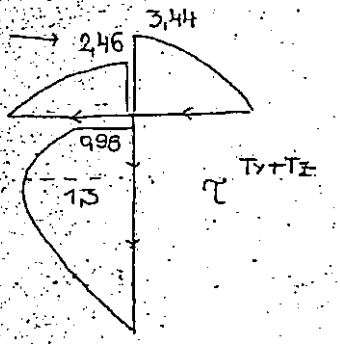


$$\tau^{T_z} = -\frac{T_z S_y^*}{I_y \cdot t}$$

САБИРАЈУ СЕ

$$\tau^{M_t}$$

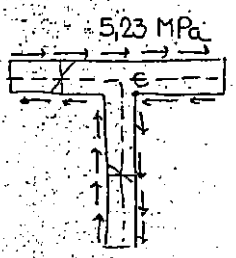
СУПРОТНЕ СТРЕЛИЦЕ →



$t = const. \Rightarrow W_t = \frac{1}{3}\sum b_i t_i^2$

$W_t = \frac{1}{3}(12+24)\cdot 3^2 = 108\ cm^3$

$$\tau^{M_t} = \frac{M_t}{W_t} = \frac{565\cdot 10^{-2}\ Ncm}{108\cdot 10^{-6}} = 5{,}23\ MPa$$

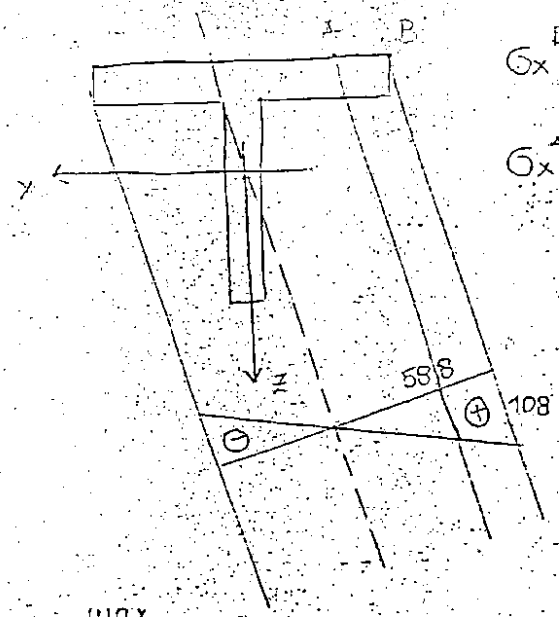


$$\sigma_x = -\frac{M_y}{I_y} z - \frac{M_z}{I_z} y$$

$\sigma_x = 0 \qquad y = -\frac{M_y}{M_z}\frac{I_z}{I_y} z$

$$y = -\frac{5\sqrt{2}\cdot 10^5 \cdot 432}{5\sqrt{2}\cdot 10^5 \cdot 6912} z \qquad \boxed{z = -16y}$$

48



$$\sigma_x^B = 5\sqrt{2}\cdot 10^3\left(\frac{9,5}{6912}+\frac{6}{432}\right)10^6 = 108\ MPa$$

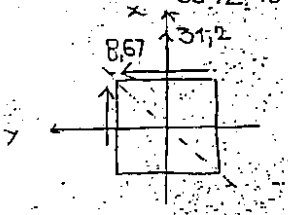
$$\sigma_x^A = 5\sqrt{2}\cdot 10^3\left(\frac{9,5}{6912}+\frac{3}{432}\right)10^6 = 58,8\ MPa$$

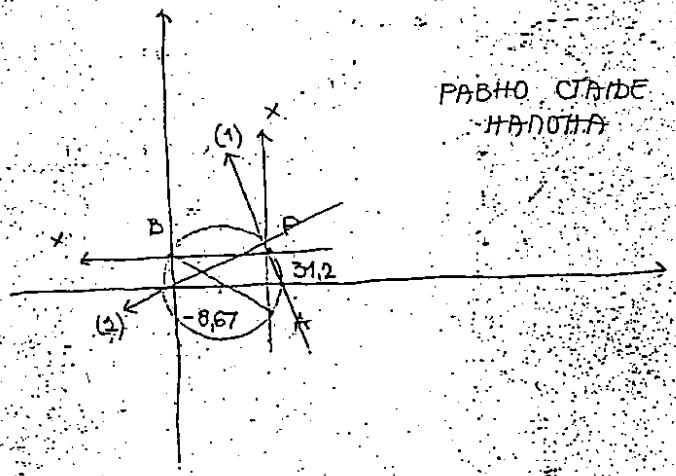
τ_x^{max} СЕ ЈАВЉА У С

$\tau_x^{max} = \tau_x^C = 3,44+5,23 = 8,67\ MPa$

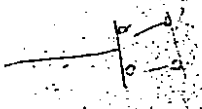
σ_x^C → ПОТРЕБНЕ СУ КООРДИНАТЕ $C(-1,5;-6,5)$

$$\sigma_x^C = +\frac{5\sqrt{2}\cdot 10^5\cdot 6,5}{6912\cdot 10^6} + \frac{5\sqrt{2}\cdot 10^5\cdot 1,5}{432\cdot 10^6} = 31,2\ MPa$$





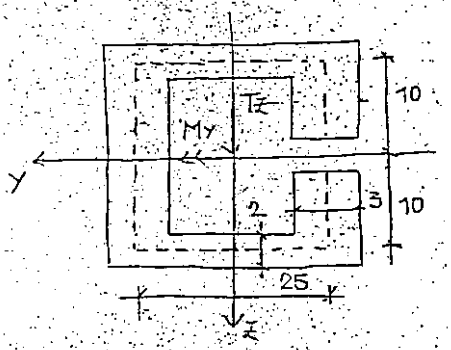
РАВНО СТАЊЕ НАПОНА

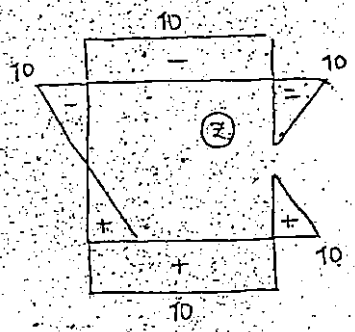


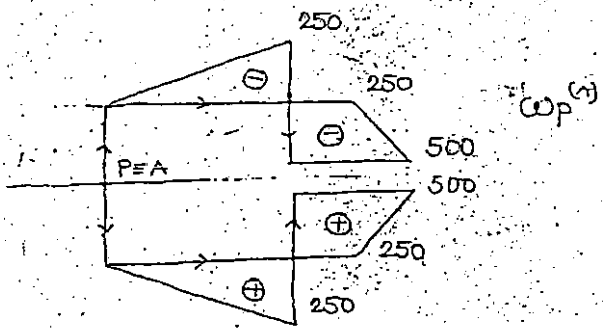
05.09.2003. W

1.5. НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ ПРЕСЕЧНИХ СИЛА КОЈЕ ДЕЛУЈУ У ТЕЖИШТУ И ОДРЕДИТИ τ_{max}, КАО И ТАЧКУ У КОЈОЈ СЕ ЈАВЉА

$T_z = 5$ KN
$M_y = 10$ KNm







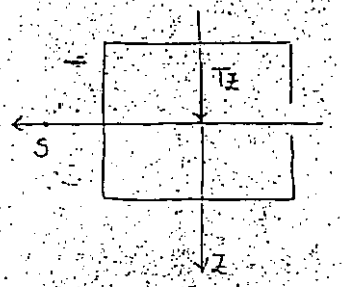
$$I_y = 3 \cdot 4 \cdot \frac{10}{3} 10^2 + 2 \cdot 2 \cdot 25 \cdot 10^2 = 14\,000 \text{ cm}^4$$

$$I_{z\omega p} = 2 \cdot 2 \cdot \frac{25}{2} \cdot 10 \cdot 250 + 2 \cdot 3 \cdot \frac{10}{6} \left[250 (2 \cdot 10 + 0) + 500 (2 \cdot 0 + 10) \right] = 225\,000 \text{ cm}^6$$

$$y_S = y_p + \frac{I_{z\omega p}}{I_y} = y_p + \frac{225\,000}{14\,000} = y_p + 16{,}07 = 28{,}57 \text{ cm}$$

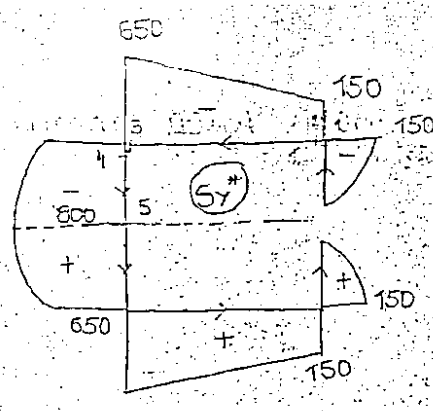
$$M_t = T_z \cdot y_S = 5 \text{ KN} \cdot 0{,}2857 \text{ m} = 1{,}428 \text{ KNm}$$

$$I_t = \frac{1}{3} \sum b_i t_i^3 = \frac{1}{3} (40 \cdot 3^3 + 50 \cdot 2^3) = 493{,}33 \text{ cm}^4$$



$$\tau^{Mt} = \frac{M_t}{I_t} \cdot t \quad \begin{cases} \tau_1^{Mt} = \dfrac{1{,}428 \cdot 10^3 \cdot 3 \cdot 10^{-2}}{493{,}33 \cdot 10^{-8}} = 8{,}68 \text{ MPa} \\ \tau_2^{Mt} = \dfrac{1{,}428 \cdot 10^3 \cdot 2 \cdot 10^{-2}}{493{,}33 \cdot 10^{-8}} = 5{,}79 \text{ MPa} \end{cases}$$

$$\tau^{T_z} = -\frac{T_z S_y^*}{I_x \cdot t}$$



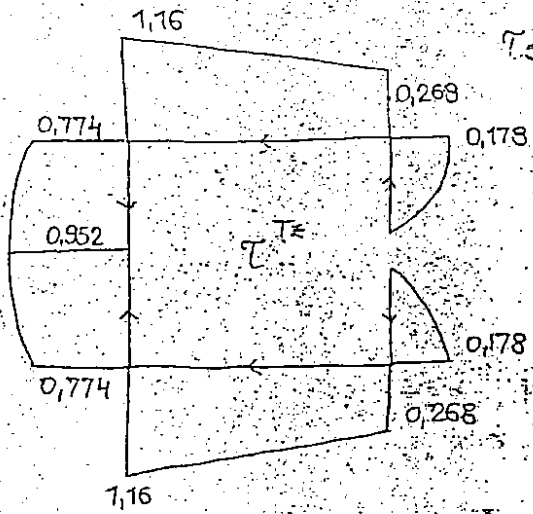
$$\tau_1^{Tz} = -\frac{5\cdot10^3(-150\cdot10^{-6})}{14000\cdot10^{-8}\cdot3\cdot10^{-2}} = 0{,}178\ MPa$$

$$\tau_2^{Tz} = -\frac{5\cdot10^3(-150\cdot10^{-6})}{14000\cdot10^{-8}\cdot2\cdot10^{-2}} = 0{,}268\ MPa$$

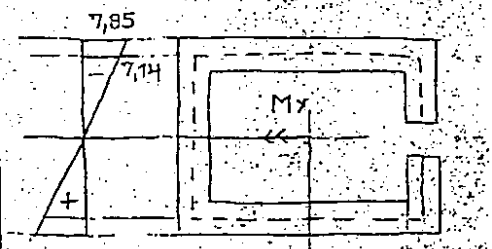
$$\tau_3^{Tz} = -\frac{5\cdot10^3(-650\cdot10^{-6})}{14000\cdot10^{-8}\cdot2\cdot10^{-2}} = 1{,}16\ MPa$$

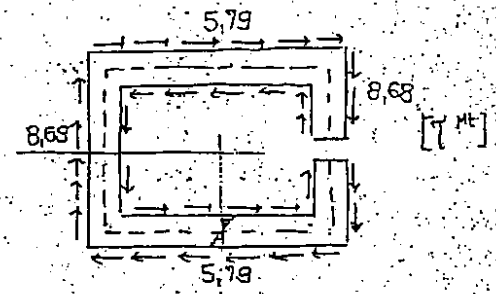
$$\tau_4^{Tz} = -\frac{5\cdot10^3(-650\cdot10^{-6})}{14000\cdot10^{-8}\cdot3\cdot10^{-2}} = 0{,}774\ MPa$$

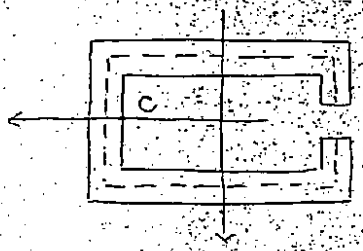
$$\tau_5^{Tz} = -\frac{5\cdot10^3(-800\cdot10^{-6})}{14000\cdot10^{-8}\cdot3\cdot10^{-2}} = 0{,}952\ MPa$$



$$\sigma_x = \frac{M_y z}{I_y} = \frac{10\cdot10^3\cdot10\cdot10^{-2}}{14000\cdot10^{-8}} = 7{,}14\ MPa$$

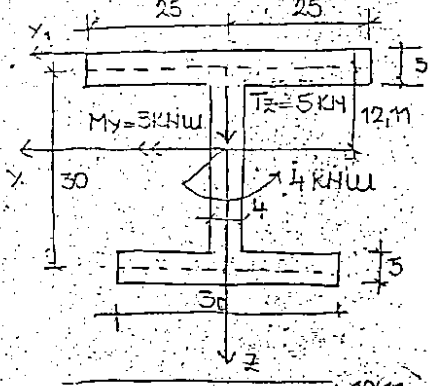






$$\tau_c = \tau_{max} = 8{,}68 + 0{,}952 = 9{,}632\ MPa$$

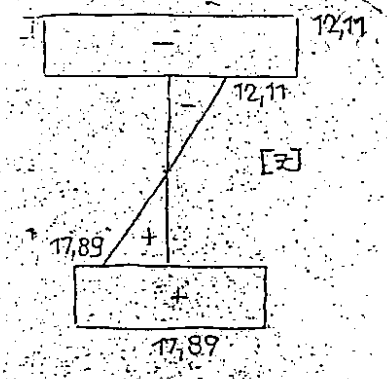
26.09.2005. W



$$\tau_{xz}^{Tz} = -\frac{T_z S_y^*}{I_y \cdot t} \qquad F = 50\cdot 5 + 30\cdot 4 + 30\cdot 5 = 520\ cm^2$$

$$\tau^{Mt} = \frac{M_t}{I_t} \cdot t$$

$$z_T = \frac{250\cdot 2{,}5 + 120\cdot 17{,}5 + 150\cdot 32{,}5}{520} = 14{,}61\ cm$$



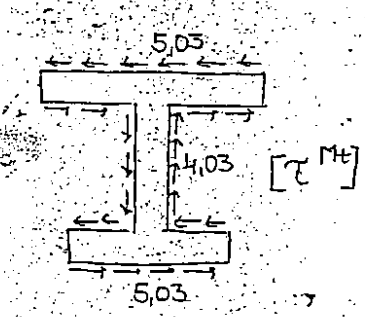
$$I_y = 50\,12{,}11^2\cdot 5 + \left(\frac{1}{3}12{,}11^3 + \frac{1}{3}17{,}89^3\right)4 + 30\cdot 17{,}89^2\cdot 5$$

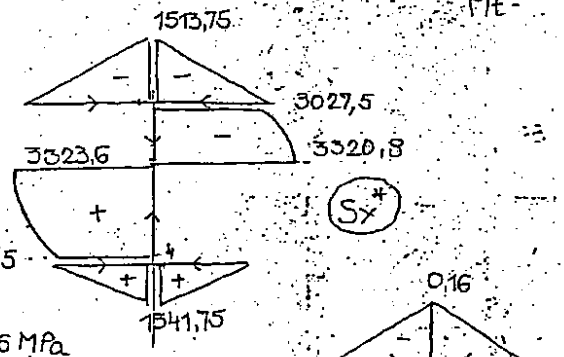
$$I_y = 94673{,}1\ cm^4$$

$$I_t = \frac{1}{3}\sum b_i t_i^3 = \frac{1}{3}\left(50\cdot 5^3 + 30\cdot 5^3 + 30\cdot 4^3\right) = 3973{,}33\ cm^4$$

$t = 5\ cm \quad \tau^{Mt} = \frac{M_t}{I_t}\cdot t = \frac{4\cdot 10^3 \cdot 5\cdot 10^{-2}}{3973{,}33\cdot 10^{-8}} = 5{,}03\ MPa$

$t = 4\ cm \quad \tau^{Mt} = \frac{M_t}{I_t}\cdot t = 4{,}03\ MPa$





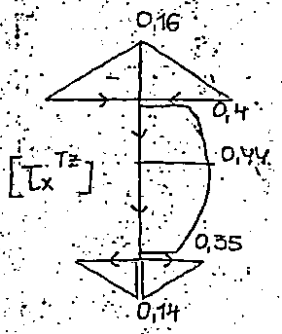
$$\tau_x^1 = +\frac{5\cdot 10^3 \cdot 1513{,}75\cdot 10^{-6}}{94673{,}7\cdot 10^{-8}\cdot 5\cdot 10^{-2}} = 0{,}16\ MPa$$

$\tau_x^2 = 0{,}4\ MPa$

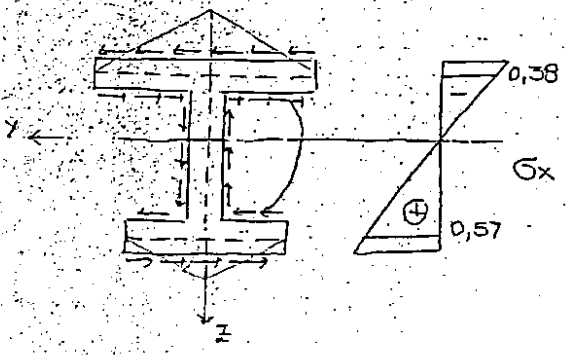
$\tau_x^3 = 0{,}44\ MPa$

$\tau_x^4 = -0{,}35\ MPa$

$\tau_x^5 = -0{,}14\ MPa$



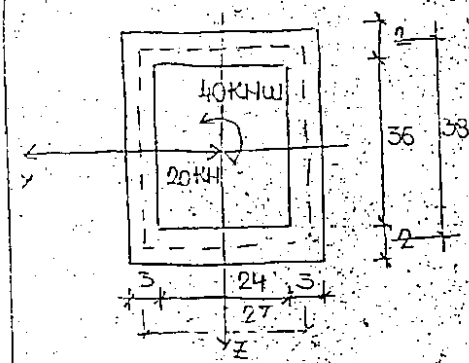
$$\sigma_x = \frac{M_y}{I_y} z \qquad z = -12{,}11 \quad \sigma_x = -0{,}38\ MPa \qquad z = 17{,}89 \quad \sigma_x = 0{,}57\ MPa$$



(52)

02.04.2004

$I_y = 66688\ cm^4$ $I_z = 48528\ cm^4$

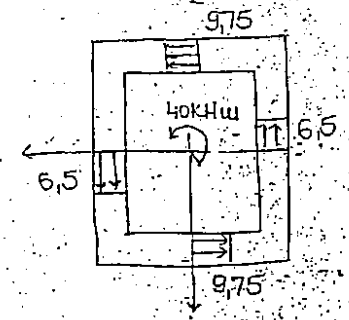


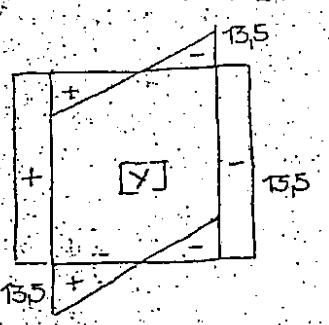
$$\tau^{Mt} = \frac{Mt}{2F \cdot ti} = \frac{40 \cdot 10^3}{2 \cdot 38 \cdot 27 \cdot 10^{-4} \cdot ti}$$

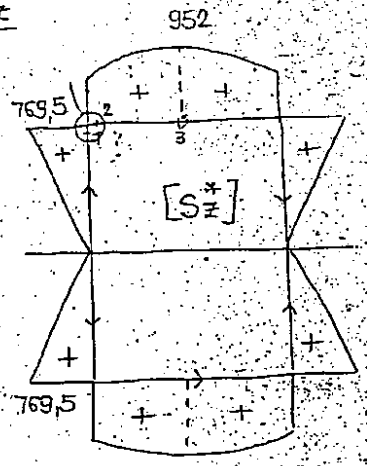
$t = 2\ cm$ $\tau^{Mt} = 9{,}75\ MPa$

$t = 3\ cm$ $\tau^{Mt} = 6{,}5\ MPa$

$$\tau^{Ty} = -\frac{T_x \cdot S_z^*}{I_z \cdot t}$$

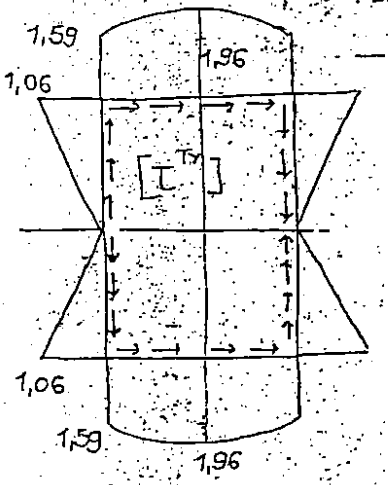






$$\tau_x^1 = -\frac{(-20 \cdot 10^{-3})\ 769{,}5 \cdot 10^{-6}}{48528 \cdot 10^{-8} \cdot 3 \cdot 10^{-2}} = 1{,}06\ MPa$$

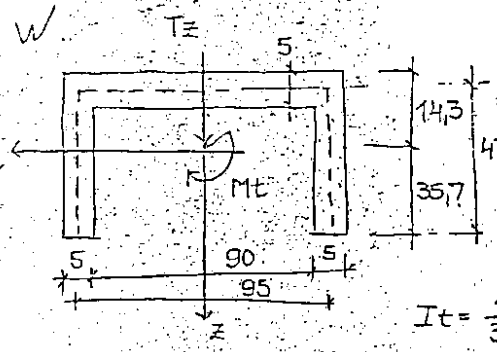
$\tau_x^2 = 1{,}59\ MPa$ $\tau_x^3 = 1{,}96\ MPa$



24.09.2004.

a/ НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ СИЛА.

b/ ОДРЕДИТИ τ^{max} И ТАЧКУ У КОЈОЈ СЕ ЈАВЉА

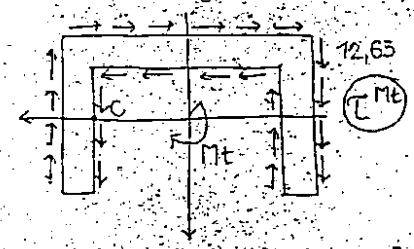


$|Mt| = 20\ kNm \quad |Tz| = 50\ kN$

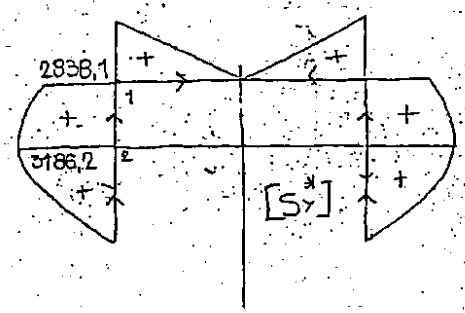
$\Rightarrow S_y^*, I_y$

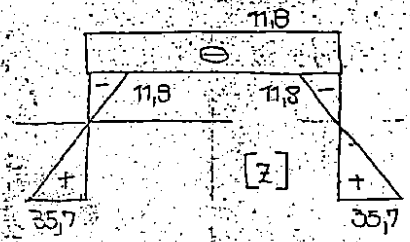
$$I_t = \frac{1}{3}\sum b_i t_i^3 = \frac{1}{3} 190 \cdot 5^3 = 7916{,}67\ cm^4$$

$$\tau^{Mt} = \frac{Mt}{I_t} t = \frac{20 \cdot 10^3 \cdot 5 \cdot 10^{-2}}{7916{,}67 \cdot 10^{-8}} = 12{,}63\ MPa$$



$$\tau^{Tz} = -\frac{Tz \cdot S_y^*}{I_y \cdot t}$$

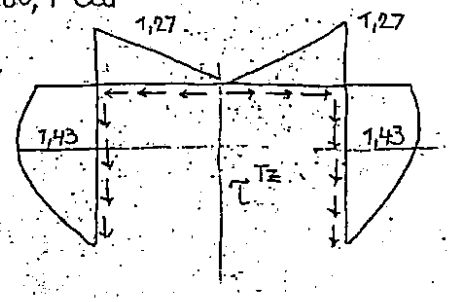




$$I_y = 2\cdot\frac{35{,}7^3}{3}\cdot 5 + 2\cdot\frac{11{,}8^3}{3}\cdot 5 + 11{,}8^2 \cdot 95 \cdot 5 = 223\,280{,}1\ cm^4$$

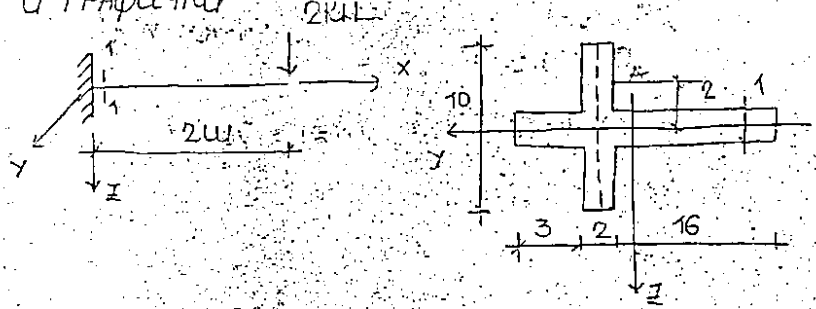
$$\tau_z^{(1)} = -\frac{50 \cdot 10^3 \cdot 2838{,}1 \cdot 10^{-6}}{223\,280{,}1 \cdot 10^{-8} \cdot 5 \cdot 10^{-2}} = -1{,}27\ MPa$$

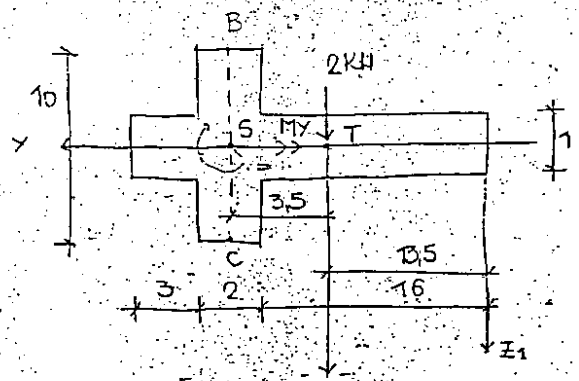
$$\tau_z^{(2)} = -\frac{50 \cdot 10^3 \cdot 3186{,}2 \cdot 10^{-6}}{223\,280{,}1 \cdot 10^{-8} \cdot 5 \cdot 10^{-2}} = -1{,}43\ MPa$$



$$\tau_c = \tau^{max} = 12{,}63 + 1{,}43 = 14{,}06\ MPa$$

4. ЗА ПРЕСЕК 1-1 НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА И ИЗВРШИТИ АНАЛИЗУ НАПОНСКОГ СТАЊА У ТАЧКИ А АНАЛИТИЧКИ И ГРАФИЧКИ



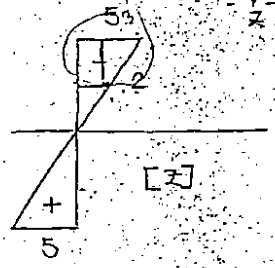


$$F = 3\cdot1 + 2\cdot10 + 16\cdot1 = 39\ cm^2$$

$$y_T = \frac{3\cdot1\cdot19{,}5 + 2\cdot10\cdot11 + 16\cdot1\cdot8}{39} = 13{,}5\ cm$$

$$\sigma_x = \frac{M_y}{I_y}\cdot z$$

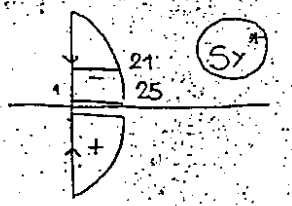
$$I_y = 2\cdot2\cdot\frac{5}{3}\cdot5^2 = 166{,}67\ cm^4$$



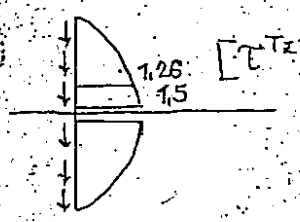
$$\sigma_{x,B}^{max} = -\frac{2\cdot10^3\cdot2\cdot(-5\cdot10^{-2})}{166{,}67\cdot10^{-8}} = 120\ MPa$$

$$\sigma_{x,A} = \frac{-2\cdot2\cdot2}{166{,}67}\cdot10^9 = 48\ MPa$$

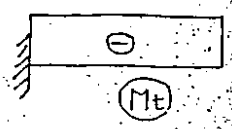
$$\tau^{T_z} = -\frac{T_z\cdot S_y^*}{I_y\cdot t}$$



$$\tau_x^1 = -\frac{2\cdot10^3\cdot(-25\cdot10^{-6})}{166{,}67\cdot10^{-8}\cdot2\cdot10^{-2}} = 1{,}5\ MPa$$



$$\tau_x^{\wedge} = -\frac{2\cdot10^3(-21\cdot10^{-6})}{166{,}67\cdot10^{-8}\cdot2\cdot10^{-2}} = 1{,}26\ MPa$$



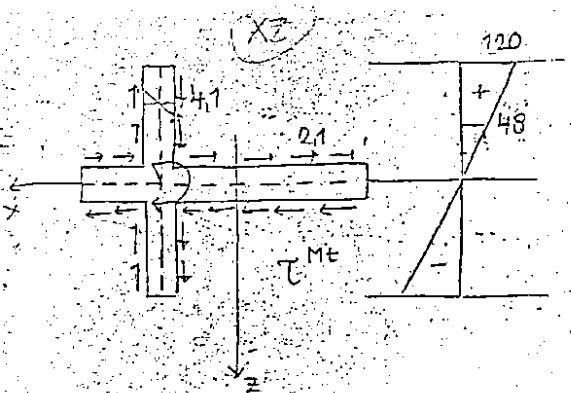
$$M_t = 2\cdot0{,}035 = 0{,}07\ kN\,m$$

$$I_t = \frac{1}{3}\Sigma b_i t_i^3 = \frac{1}{3}\left[21\cdot1^3 + 10\cdot2^3\right] = 33{,}67\ cm^4$$

$$\tau^{M_t} = \frac{M_t}{I_t}\cdot t$$

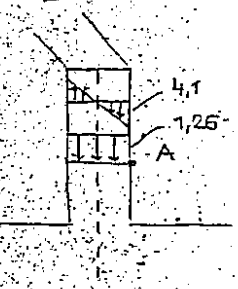
$$t = 2 \quad \tau^{M_t} = \frac{0{,}07\cdot10^3\cdot2\cdot10^{-2}}{33{,}67\cdot10^{-8}} = 4{,}1\ MPa$$

$$t = 1 \quad \tau^{M_t} = \frac{0{,}07\cdot10^3\cdot1\cdot10^{-2}}{33{,}67\cdot10^{-8}} = 2{,}1\ MPa$$



УКУПНИ СМИЧУЋИ НАПОН У А

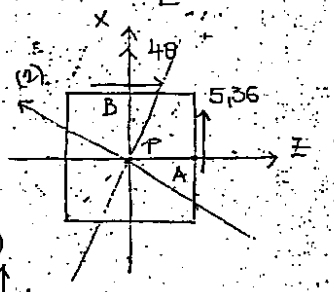
$\tau_A = 1{,}26 + 4{,}1 = 5{,}36$ MPa

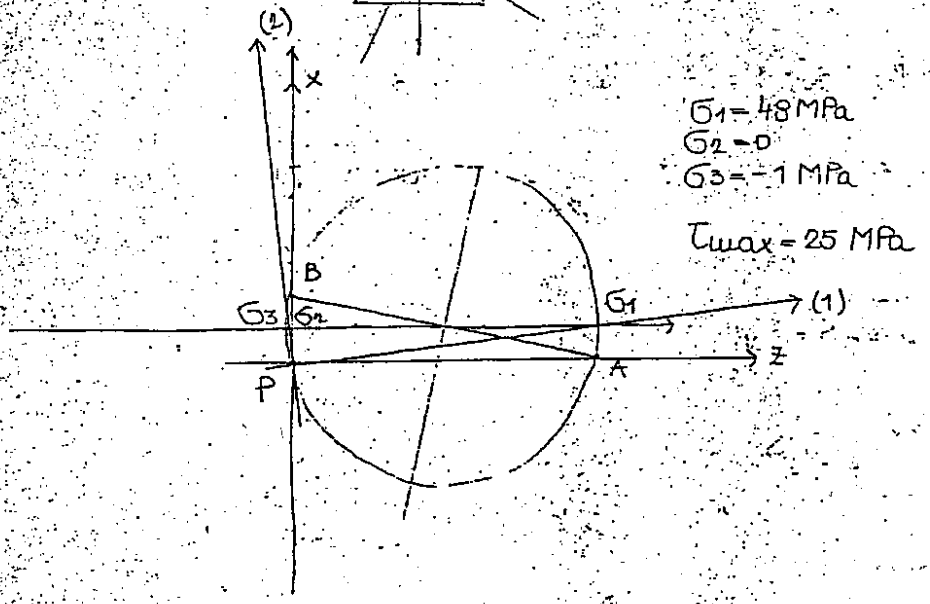


$\sigma_x^A = 48$ MPa

$$S = \begin{bmatrix} 48 & 0 & 5{,}36 \\ 0 & 0 & 0 \\ 5{,}36 & 0 & 0 \end{bmatrix} \text{MPa}$$

A(σ_{xx}, $-\sigma_{xz}$)
B(σ_{zz}, σ_{zx})



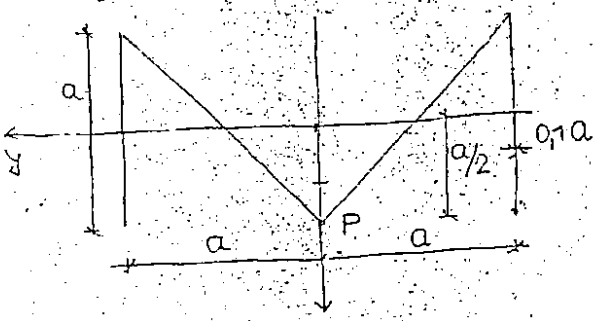


$\sigma_1 = 48$ MPa
$\sigma_2 = 0$
$\sigma_3 = -1$ MPa

$\tau_{max} = 25$ MPa

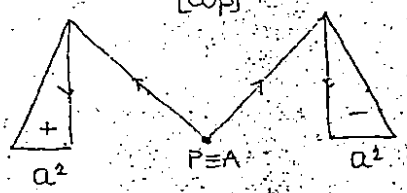
ВЕЖБЕ

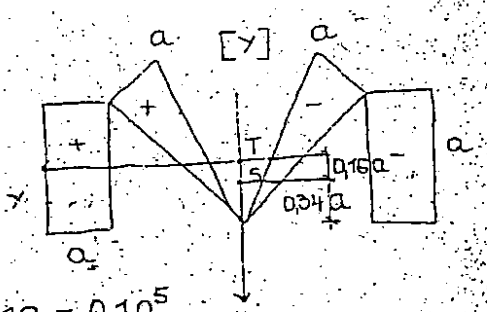
W ЗАДАТО ЈЕ $M\omega = 30\ KNm^2$, $Mt\omega = 250\ KNm$, $Mt_S = 20\ KNm$
НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА



ПОТРЕБНО ЈЕ:

$I\omega, I_\Xi, S\omega^*, \omega$

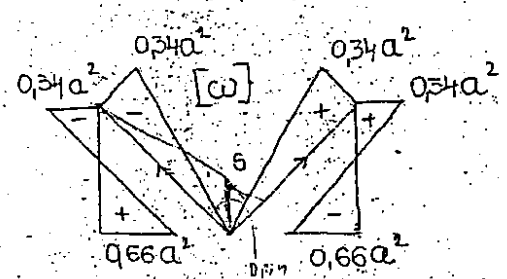




$$I_{y\omega p} = \left(\frac{a}{2}a^2 a + \frac{a}{2}a^2 a\right) 0,1a = 0,1a^5$$

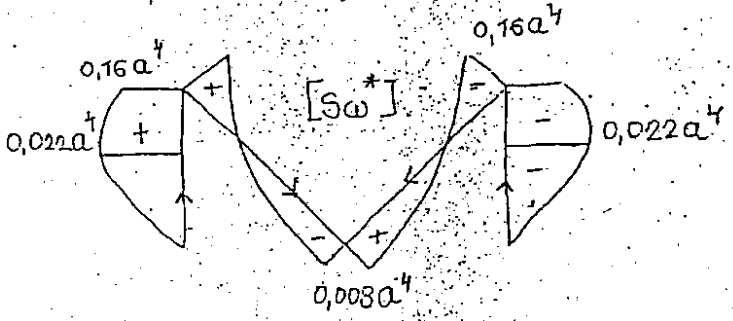
$$I_\Xi = \left[2\cdot a\cdot a^2 + 2\cdot\frac{a\sqrt{2}}{3}a\,a\right]0,1a = 0,2943a^4$$

$$z_S = z_p - \frac{I_{y\omega p}}{I_\Xi} = z_p - \frac{0,1a^5}{0,2943a^4} = \frac{a}{2} - 0,34a = 0,16a$$



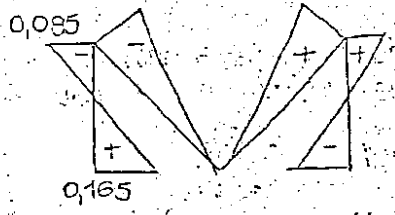
$$I\omega = t\int \omega^2 dS = 0,1a\left[2\cdot\frac{a}{3}\left(0,66^2a^4 - 0,66a^2\cdot 0,34a^2 + 0,34^2a^4\right) + 2\cdot\frac{a\sqrt{2}}{3}0,34^2a^4\right]$$

$$= 0,0327\cdot a^6$$



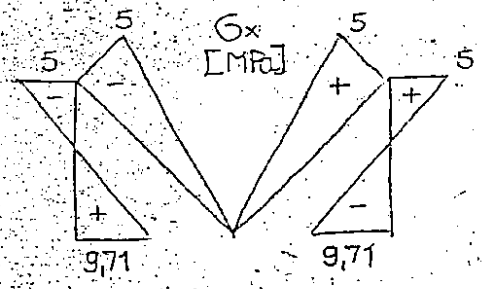
$$\sigma_x = \frac{M_\omega \cdot \omega}{I_\omega}$$

ω (a=0,5)



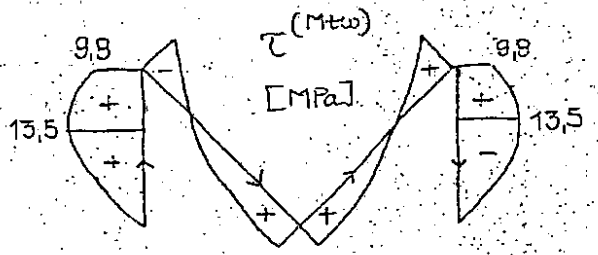
ЗА $a = 0{,}5\,m$

$$I_\omega = 0{,}00051\,m^6$$



$$\sigma_x = \frac{30 \cdot 10^3 \cdot \omega}{I_\omega}$$

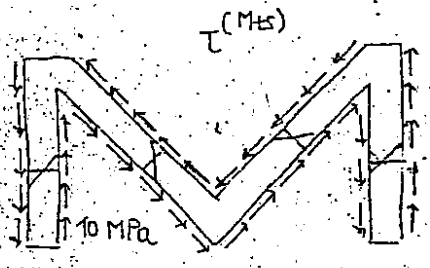
$$\tau^{(M_{t\omega})} = -\frac{M_{t\omega} \cdot S_\omega^*}{I_\omega \cdot t}$$



$$\tau^{(M_{t\omega})} = -\frac{250 \cdot 10^3 \cdot S_\omega^*}{I_\omega \cdot t}$$

$$\tau^{(M_{ts})} = \frac{M_{ts} \cdot t}{I_t} \qquad I_t = \frac{1}{3} \underbrace{2(a + a\sqrt{2})}_{\text{дължина на щапова}} \cdot (0{,}1a)^3 \overset{a=0{,}5}{\Longrightarrow} I_t = 0{,}0001\,m^4$$

$$\tau^{(M_{ts})} = \frac{20 \cdot 10^3 \cdot 0{,}5 \cdot 0{,}1}{0{,}0001} = 10\ MPa$$



$M_{t\omega}$ – МОМЕНТ ОГРАНИЧЕНО ТОРЗИЕ

M_{ts} – СЕНТ-ВЕНАНОВ МОМЕНТ

M_ω – БИМОМЕНТ

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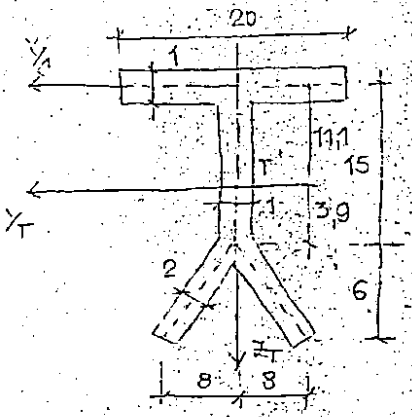
31.01.2004

2.1. НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ ПРЕСЕЧНИХ СИЛА КОЈЕ ДЕЛУЈУ НА ТАНКОЗИДНОМ НОСАЧУ

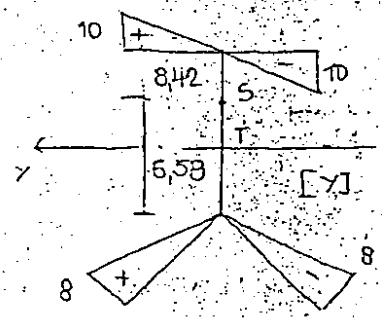


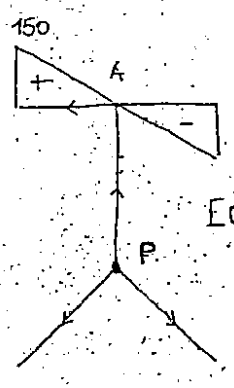
$M\omega = 2\,kNm^2$

$MTS = 3\,kNm$

$M_{T\omega} = 4\,kNm$

$$z_T = \frac{20\cdot1\cdot0 + 15\cdot1\cdot7{,}5 + 2\cdot2\cdot10\cdot18}{20+15+40} = 11{,}1\ cm$$





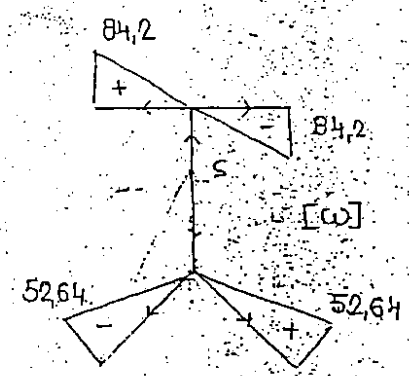
$$I_x = 1\left(2\cdot\frac{10}{3}10^2\right) + 2\left(2\cdot\frac{10}{3}\cdot8^2\right)$$

$$I_x = 1520\ cm^4$$

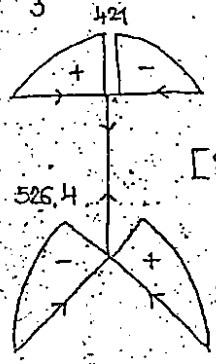
$$[\omega_P^*] \quad I_y\omega_P = 2\cdot1\left(\frac{10}{3}\cdot10\cdot150\right) = 10\,000\ cm^4$$

$$z_S = z_P - \frac{I_y\omega_P}{I_x}$$

$$z_S = 3{,}9 - \frac{10000}{1520} = -2{,}68$$

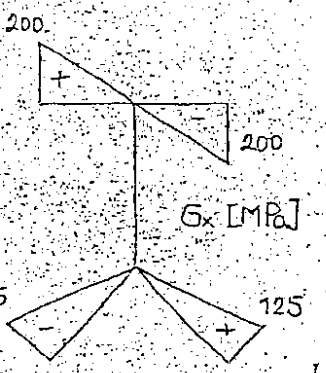


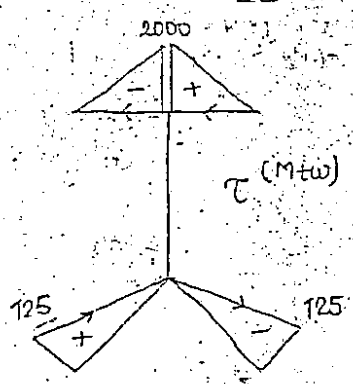
$$I_\omega = 1\cdot2\,\frac{10}{3}\,84{,}2^2 + 2\cdot2\,\frac{10}{3}\,52{,}64^2 = 84\,210{,}53\ cm^6$$



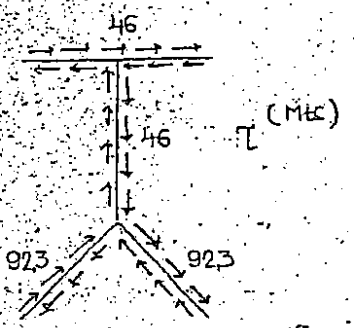
$[S\omega^*]$

$$\sigma_x = \frac{M\omega}{I\omega}\cdot\omega = \frac{2\cdot\omega\,10^{-4}}{84\,210{,}5\,10^{-12}} \qquad \tau^{(M+\omega)} = -\frac{M+\omega\cdot S\omega^*}{I\omega\cdot t} = -$$





$$\tau^{(M+s)} = \frac{M+s\cdot t}{I+} \qquad I+ = \frac{1}{3}(20\cdot 1^3 + 15\cdot 1^3 + 2\cdot 10\cdot 2^3) = 65\ cm^4$$

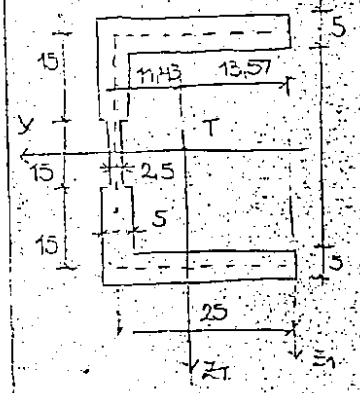


26.09.2005

$M\omega = -3\,KNm^2$

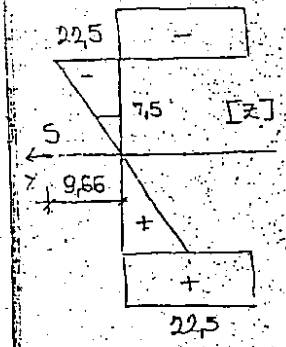
$M_{TS} = 4\,KNm$

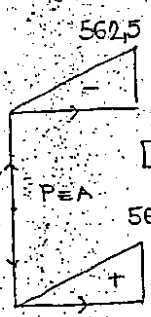
$M_{T\omega} = 5\,KNm$



$$y_T = \frac{25\cdot5\cdot12,5\cdot2 + 2\cdot15\cdot25\cdot5 + 15\cdot2,5\cdot25}{2\cdot25\cdot5 + 2\cdot15\cdot5 + 15\cdot2,5} = 13,57\ cm$$

$$y_S = y_P + \frac{I_{z\omega P}}{I_y}$$

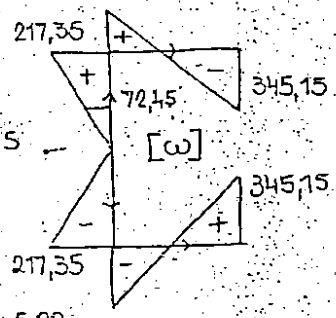




$$I_y = 2\cdot25\,22,5^2\cdot5 + 2\cdot\frac{15}{3}\left[22,5^2 + 22,5\cdot7,5 + 7,5^2\right]\cdot5 + 2\cdot\frac{7,5}{3}\cdot7,5^2\cdot2,5 = 163\,828,125\ cm^4$$

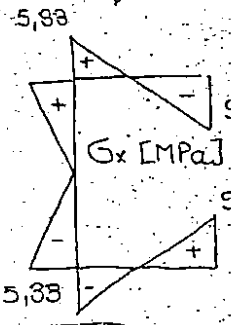
$$I_{z\omega P} = 2\cdot\frac{25}{2}\cdot22,5\cdot562,5\cdot5 = 1582031,25\ cm^4$$

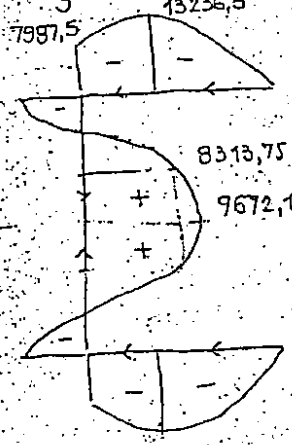
$$y_S = 11,43 + \frac{1582031,25}{163\,828,125} = 21,09\ cm$$



$$I_\omega = 2\cdot5\,\frac{25}{3}\left(217,35^2 - 217,35\cdot345,15 + 345,15^2\right) + 2\cdot5\cdot\frac{15}{3}\left(217,35^2 + 217,35\cdot72,45 + 72,45^2\right) + 2\cdot2,5\cdot\frac{7,5}{3}\,72,45^2 = 11\,090\,063,53\ cm^6$$

$$I_t = \frac{1}{3}\left[5^3(2\cdot25 + 2\cdot15) + 2,5^3\cdot15\right] = 3411,46\ cm^4$$

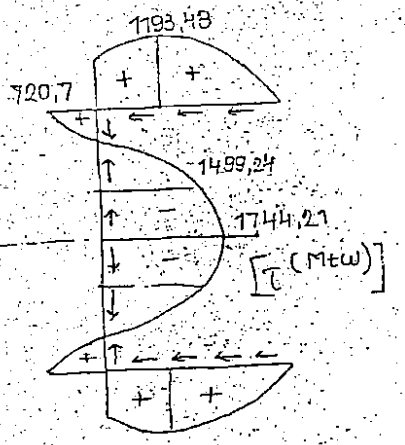


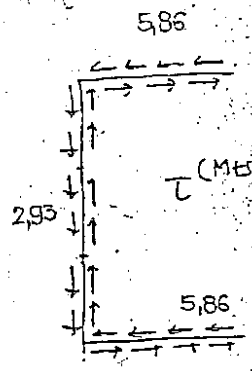


NIJE TAČNO !

$$\sigma_x = \frac{M_\omega}{I_\omega}\,\omega = \frac{3\cdot10^3\cdot\omega\,10^{-4}}{11\,090\,063,53\cdot10^{-12}}$$

$$\tau^{(M_{t\omega})} = -\frac{M_{t\omega}\, S_\omega^{\#}}{I_\omega\, t} = -\frac{M_{t\omega}\cdot 10^{3}\cdot S_\omega^{(*)}\cdot 10^{-6}}{I_\omega\cdot 10^{-12}\cdot t\cdot 10^{-2}}$$

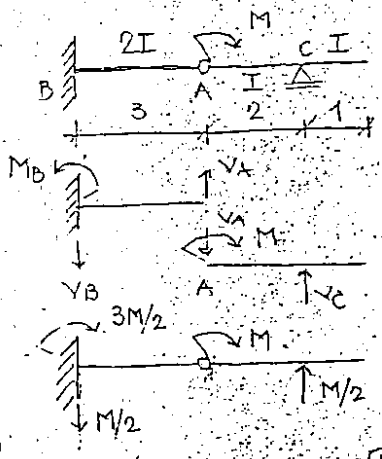




$$\tau^{(M_{ts})} = \frac{M_{ts}\, t}{I_t} = \frac{M_{ts}\cdot 10^{3}\cdot t\cdot 10^{-2}}{I_t\cdot 10^{-8}}$$

(62) Користећи Mohr-Maxwell-ову аналогију одредити величину момента M при коме је угиб у тачки A $w_A \leq 2$ mm.

$E = 200$ GPa $I = 100$ cm^4



$\curvearrowright \Sigma M_A = 0 \quad -M + V_C \cdot 2 = 0 \quad V_C = M/2$

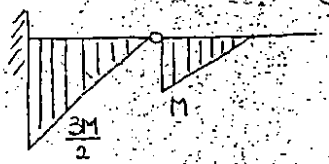
$V_A = M/2$

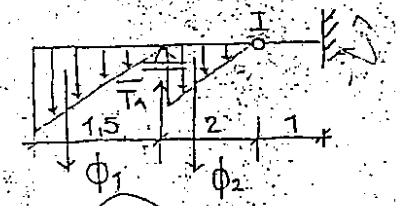
$V_B = M/2$

$\curvearrowright \Sigma M_A = 0 \quad M_B + 3V_B = 0$

$M_B = -3M/2$

(M)





$\phi_1 = \frac{1}{2} 1{,}5 \cdot \frac{3M}{2} = 1{,}125\,M$

$\phi_2 = \frac{1}{2} 2 \cdot M = M$

$\curvearrowright M_I = 0 \quad -\overline{T}_A \cdot 2 - \phi_2 \cdot 1{,}33 - \phi_2 \cdot 3 = 0$

$\overline{T}_A \cdot 2 = M \cdot 1{,}33 + 1{,}125\,3M$

$\overline{T}_A = 2{,}35\,M$

$M_A = \phi_1 \cdot 2 = 1{,}125\,M \cdot 2 = 2{,}25\,M$

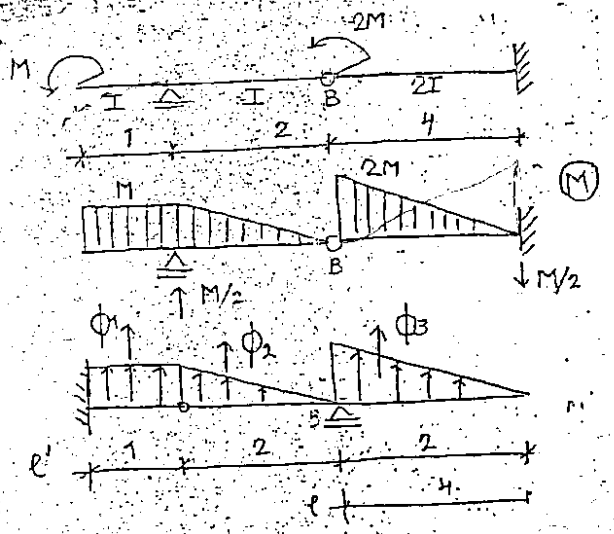
$w_A = \frac{M_A}{EI} = \frac{2{,}25M}{EI} \leq 2$ mm

$2{,}25\,M \leq 2 \cdot 10^{-3} \cdot 200 \cdot 10^9 \cdot 100 \cdot 10^{-8}$

$M \leq 177{,}78$ Nm

2. Користећи Мохр-Maxwell-ову аналогију одредити величину момента M при коме угиб у тачки B $w_B \leq 2$ mm

$E = 40$ GPa, $I = 500$ cm^4



$\phi_1 = M$

$\phi_2 = \frac{1}{2} M \cdot 2 = M$

$\phi_3 = \frac{1}{2} M \cdot 2 \cdot 2 = 2M$

$M_B = \phi_3 \cdot \frac{1}{3} 4 = \frac{4}{3} 2M = \frac{8M}{3} = 2{,}67\,M$

$w_B = \frac{M_B}{EI} = \frac{2{,}67 \cdot M}{EI} \leq 2 \cdot 10^{-3}$

$2{,}67\,M \leq 2 \cdot 10^{-3} \cdot 40 \cdot 10^{9} \cdot 500 \cdot 10^{-8}$ $\quad M \leq 150$ Nm

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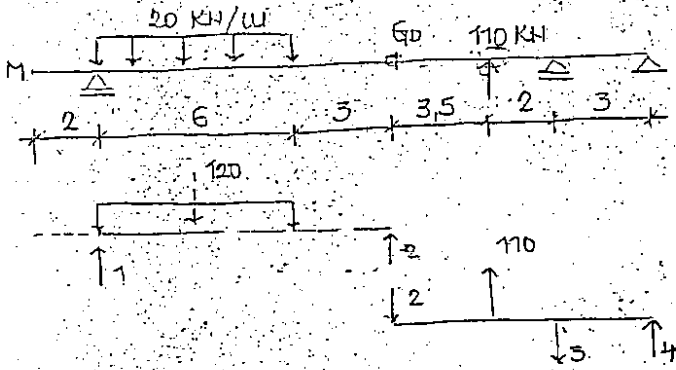
02.04.2004 B W

2.2. Користећи Мор - Максвелову аналогију одредити вертикално померање тачке M и нагиб тангенте на еластичну линију у тачки GD

$EI = const$



$\Sigma M_2 = 0 \quad I \cdot 9 - 120 \cdot 6 = 0$

$I = 80 \quad II = 40$

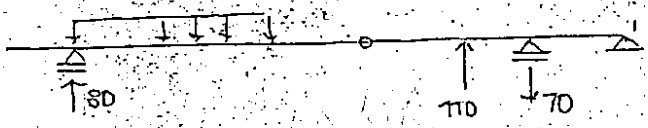
$I + II - 120 = 0 \quad II = 120 - 80$

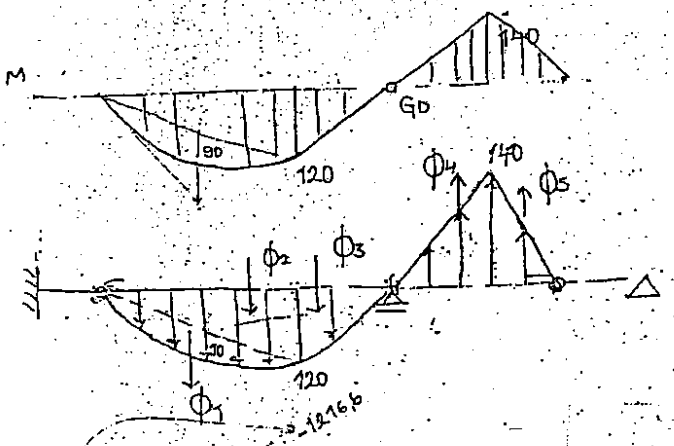
$\Sigma M_4 = 0 \quad III \cdot 3 + 110 \cdot 5 - 40 \cdot 8,5 = 0$

$III = -70$

$-II + 110 - III + IV = 0$

$IV = 0$





$\phi_1 = \frac{2}{3} fL = \frac{2}{3} 90 \cdot 6 = 360\ kNm^2$

$\phi_2 = \frac{1}{2} 120 \cdot 6 = 360\ kNm^2$

$\phi_3 = \frac{1}{2} 3 \cdot 120 = 180$

$\phi_4 = \frac{1}{2} 3,5 \cdot 140 = 245$

$\phi_5 = \frac{1}{2} 2 \cdot 140 = 140$

$w_M = \frac{M_D}{EI} = \frac{y_D}{EI}\ m$

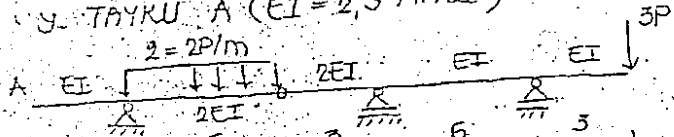
$y_{GD} = \frac{T_{GD}}{EI} = -\frac{385}{EI}\ (245+140)$

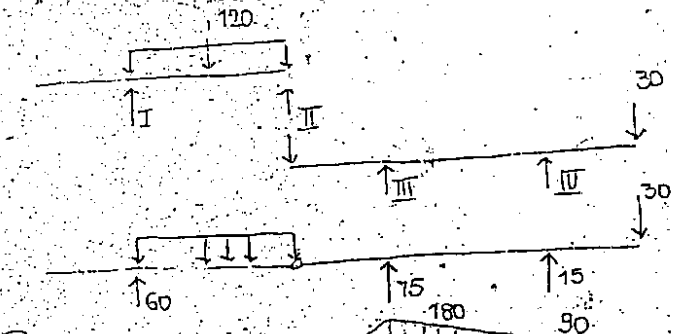
(угиб) $w = \frac{M_A}{EI}$

(нагиб) $\varphi_A = \frac{T_A}{EI}$

30.01.2004.

2.2. - Мор - Maxwell - Обом аналогијом одредити угиб и нагиб у тачки А ($EI = 2{,}5\ MNm^2$)





$\sum M_{II} = 0$

$-120 \cdot 3 + I \cdot 6 = 0$

$I = 60$

$I + II - 120 = 0$

$II = 60$

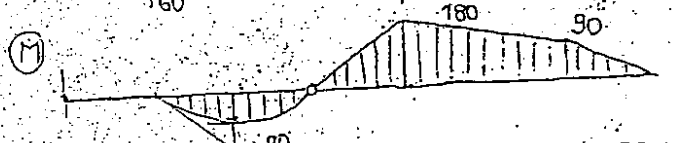
$\sum M_{I} = 0 \quad III \cdot 3 + IV \cdot 9 - 30 \cdot 12 = 0$

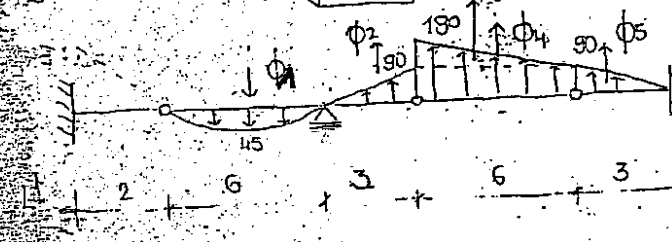
$-60 + III + IV - 30 = 0$

$III = 90 - IV \qquad III = 75$

$270 - 3IV + 9IV - 360 = 0$

$IV = 15$





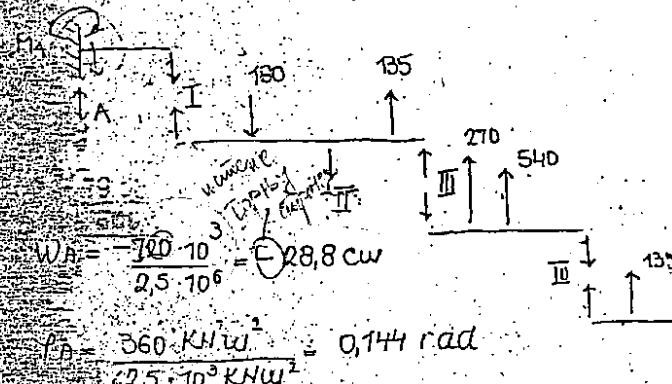
$\phi_1 = \frac{2}{3} \cdot 45 \cdot 6 = 180$

$\phi_2 = \frac{1}{2} \cdot 90 \cdot 3 = 135$

$\phi_3 = \frac{1}{2} \cdot 90 \cdot 6 = 270$

$\phi_4 = 90 \cdot 6 = 540$

$\phi_5 = \frac{1}{2} \cdot 90 \cdot 3 = 135$



$III = 450$

$II = 315$

$I = 360 \qquad A = 360$

$M_A = 360 \cdot 2 = 720$

$w_A = \frac{-720 \cdot 10^3}{2{,}5 \cdot 10^6} = -28{,}8\ cm$

$\varphi_A = \frac{360\ KNm^2}{2{,}5 \cdot 10^3\ KNm^2} = 0{,}144\ rad$

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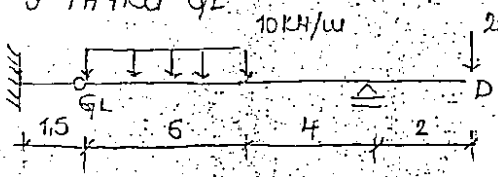
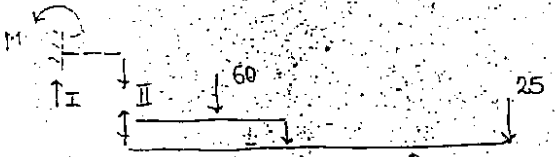
02.04.2004 > A W

КОРИСТЕЋИ МОЈР-MAXWELL-ОВУ АНАЛОГИЈУ ОДРЕДИТИ ВЕРТИКАЛНО ПОМЕРАЊЕ ТАЧКЕ D И НАГИБ ТАНГЕНТЕ НА ЕЛАСТИЧНУ ЛИНИЈУ У ТАЧКИ GL.



$EI = const$

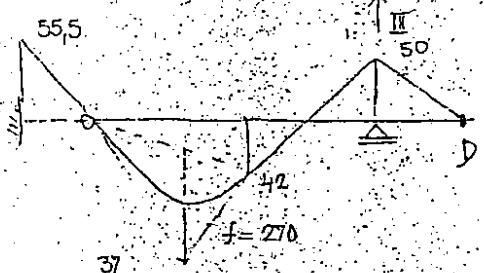


$\sum M_{II} = 0 \quad III \cdot 10 - 25 \cdot 12 - 60 \cdot 3 = 0$

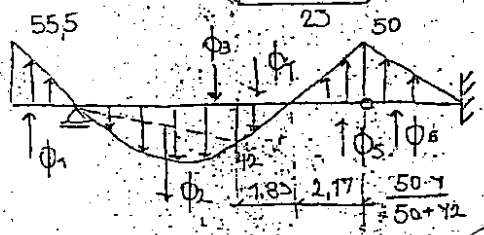
$III = 78$

$III - 25 - 60 - II = 0 \quad II = -37$

$M = 55,5 \quad \Rightarrow I = 37$



$50 : Y = Y2 : x$



$\phi_1 = \frac{1}{2} 1,5 \cdot 55,5 = Y1,62 \quad \phi_4 = \frac{1}{2} Y2 \cdot 1,83 = 38 Y3$

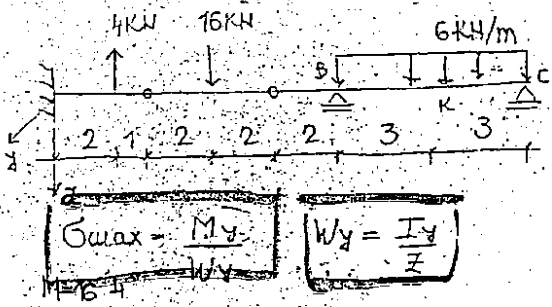
$\phi_2 = \frac{2}{3} 245 \cdot 6 = 180 \quad \phi_5 = \frac{1}{2} 2,17 \cdot 50 = 54,25$

$\phi_3 = \frac{1}{2} Y2 \cdot 6 = 126 \quad \phi_6 = \frac{1}{2} 2 \cdot 50 = 50$

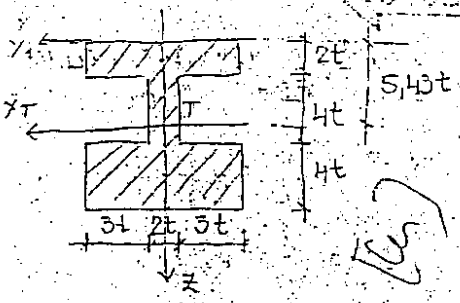
Да ли овде да нађе
једнаке ϕ-е, а за моме
моменат узме
ала: дар

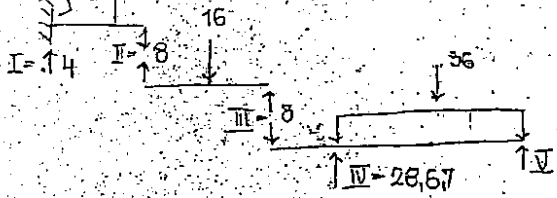
05.10.2003.

$\max \sigma \leq \sigma_{dop} = 14\ MPa$



$\sigma_{max} = \frac{M_y}{W_y}$ $W_y = \frac{I_y}{z}$





$\sum M_{II} = 0 \quad 16 \cdot 2 - III \cdot 4 = 0 \quad III = 8$

$-II - 16 + 8 = 0 \quad II = -8$

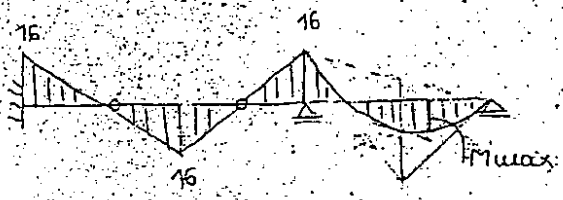
$\sum M_V = 0 \quad -8 \cdot 8 + IV \cdot 6 - 36 \cdot 3 = 0$

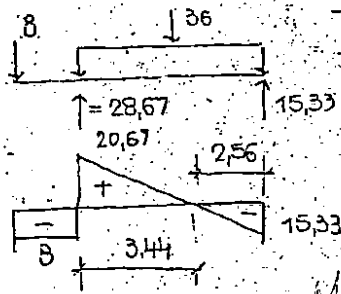
$IV = 28,67$

$-8 + 28,67 - 36 + V = 0$

$V = 15,33$

$M = 8 \cdot 3 - 4 \cdot 2 = 16$





$M_{max} = 15,33 \cdot 2,56 - 6 \frac{2,56^2}{2} = 39,58\ KNm$

$F = 2t \cdot 8t + 4t \cdot 2t + 4t \cdot 8t = 56t^2\ cm^2$

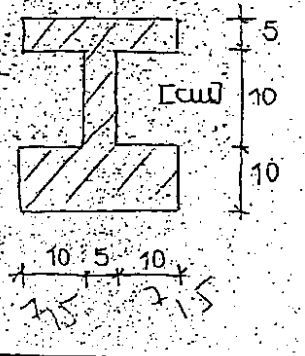
$z_T = \frac{16t^2 \cdot t + 8t^2 \cdot 4t + 32t^2 \cdot 8t}{56t^2} = 5,43t\ cm$

$\max \sigma = \frac{M_{max}}{I_y / z_t}$

$I_y = \frac{1}{12} 8t (2t)^3 + (4,43t)^2 16t^2 + \frac{1}{12} 2t (4t)^3 + 8t^2 (1,43t)^2 + \frac{1}{12} 8t (4t)^3 + 32t^2 (2,57t)^2$

$I_y = 600,38\ t^4\ cm^4$

$\frac{19,58 \cdot 10^3}{\frac{600,38 t^4 \cdot 10^{-8}}{5,43t \cdot 10^{-2}}} \leq 14 \cdot 10^6 \qquad \frac{19,58 \cdot 10^3 \cdot 5,43 \cdot 10^{-2}}{600,38\ t^3 \cdot 10^{-8}} \leq 14 \cdot 10^6$



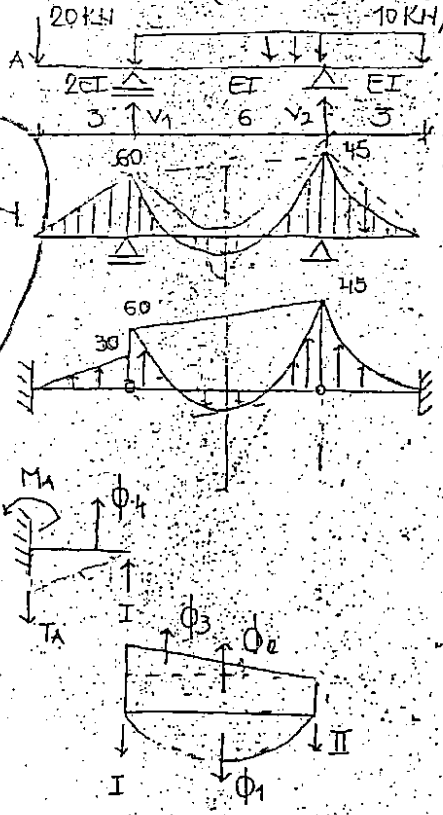
$t \geq 2,33\ cm$ USVOJENO $t = 2,5\ cm$.

$I_y = 23\,452,34\ cm^4$

$W_y = 4319,03\ cm^3$

27.06.2005.

Мор-Maxwell-овом аналогијом одредити угиб и нагиб у тачки A ($EI = 1{,}5\ MNm^2$)



$\omega_A = ?\quad \varphi_A = ?$

$\sum M_A^{+\circlearrowleft} = 0 \quad -V_1 \cdot 3 + 90 \cdot 7{,}5 - V_2 \cdot 9 = 0$

$3V_1 + 9V_2 = 675$

$\sum M^{+\circlearrowleft} = 0 \quad V_2 \cdot 3 - 90 \cdot 4{,}5 + V_1 \cdot 9 - 20 \cdot 12 = 0$

$9V_1 + 3V_2 = 645$

$V_2 = 57{,}5$

$V_1 = 52{,}5 \qquad V_1 = \frac{645 - 3V_2}{9}$

$\frac{645 - 3V_2}{3} + 9V_2 = 675$

$645 - 3V_2 + 27V_2 = 2025$

$\phi_1 = \frac{2}{3} f \cdot l = \frac{2}{3} 45 \cdot 6 = 180\ KN$

$\phi_2 = 6 \cdot 45 = 270$

$\phi_3 = \frac{1}{2} 6 \cdot 15 = 45 \qquad \phi_4 = \frac{1}{2} \cdot 3 \cdot 30 = 45$

$\sum M_I^{+\circlearrowleft} = 0 \quad 45 \cdot \frac{6}{3} + 270 \cdot 3 - 180 \cdot 3 - II \cdot 6 = 0$

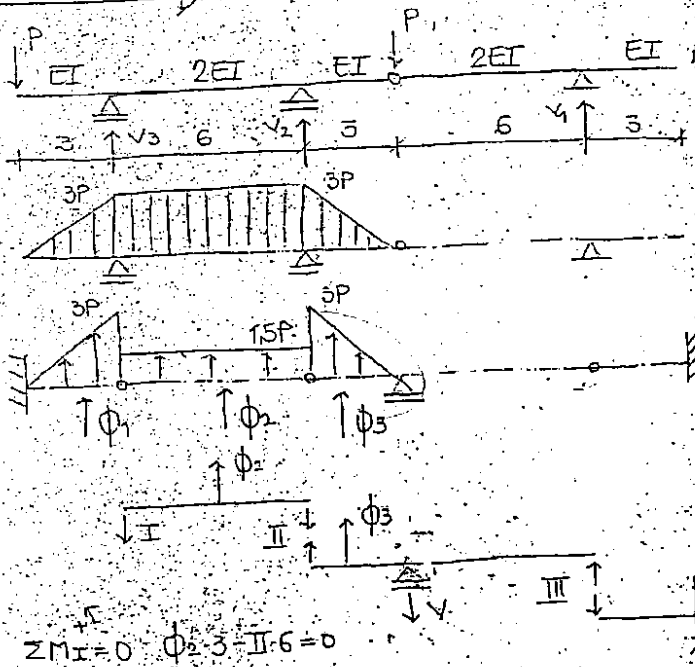
$I = 75 \qquad II = 60$

$\omega_A = \frac{M_A}{EI} = \frac{315 \cdot 10^3}{1{,}5 \cdot 10^6} = 0{,}21\ m \quad \Rightarrow T_A = 120\ KN \quad M_A = 315\ KNm$

$\varphi_A = \frac{T_A}{EI} = \frac{120 \cdot 10^3}{1{,}5 \cdot 10^6} = 0{,}08\ rad$

Замишљено

25.09.2005. W $\omega_A = ?$ $\varphi_A = ?$ $EI = 1,5\ MN\,m^2$



$\Sigma M_{(P)} = 0 \quad -V_1 = 0$

$-V_3 = P \quad V_2 = P$

$\phi_1 = \frac{1}{2} 3P \cdot 3 = 4,5P$

$\phi_2 = 1,5P \cdot 6 = 9P$

$\phi_3 = \frac{1}{2} 3P \cdot 3 = 4,5P$

$\Sigma M_I = 0 \quad \phi_2 \cdot 3 - II \cdot 6 = 0$

$II = 4,5P$

$4,5P + 4,5P - 12,75P + III = 0$

$\Sigma M_{III} = 0 \quad II \cdot 9 + \phi_3 \cdot 8 - V \cdot 6 = 0$

$III = 3,75P \Rightarrow \boxed{T_A = 3,75P}$

$V = 12,75P$

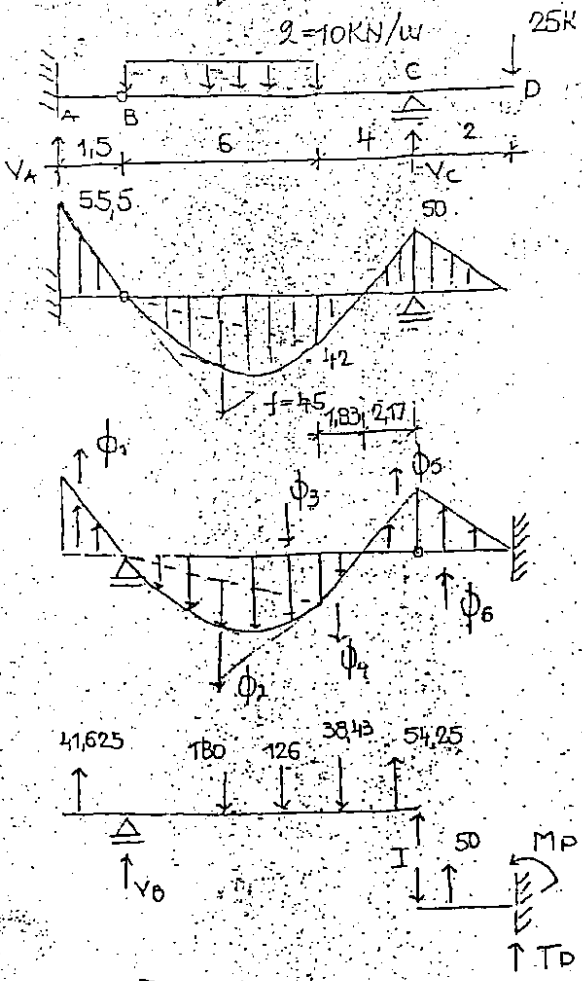
$M_A = 11,25P$

$P = 10\ KN \Rightarrow T_A = 3,75 \cdot 10 = 37,5\ KN$

$M_A = 11,25 \cdot 10 = 112,5\ KN\,m$

$\omega_A = \frac{M_A}{EI} = \frac{112,5 \cdot 10^3}{1,5 \cdot 10^6} = 0,075\,m \qquad \varphi_A = \frac{T_A}{EI} = \frac{-37,5 \cdot 10^3}{1,5 \cdot 10^6} = -0,025\ rad$

02.04.2004. w $V_D = ?$ $\varphi_C = ?$



$\curvearrowright \sum M_B = 0 \quad 60 \cdot 3 - V \cdot 10 + 25 \cdot 12 = 0$

$V_C = 48$

$V_A - 60 + V_C - 25 = 0 \quad V_A = 37$

$\phi_1 = \frac{1}{2} 55,5 \cdot 1,5 = 41,625$

$\phi_2 = \frac{2}{3} 45 \cdot 6 = 180$

$\phi_3 = \frac{1}{2} 42 \cdot 6 = 126$

$\phi_4 = \frac{1}{2} 42 \cdot 1,83 = 38,43$

$\phi_5 = \frac{1}{2} 50 \cdot 2,17 = 54,25$

$\phi_6 = \frac{1}{2} 50 \cdot 2 = 50 \text{ KN}$

$\curvearrowright \sum M_I = 0 \quad -54,25 \cdot 0,72 + 38,43 \cdot 3,39 + 126 \cdot 6 + 180 \cdot 7 - V_B \cdot 10 - 41,625 \cdot 11 = 0$

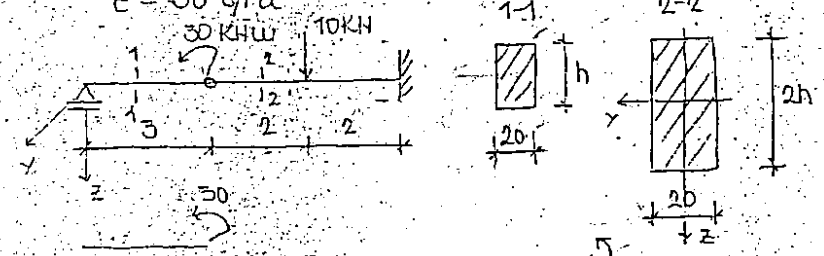
$V_B = 164,93 \qquad I = 83,63 \qquad T_D = -83,63 + 50 = -33,63$

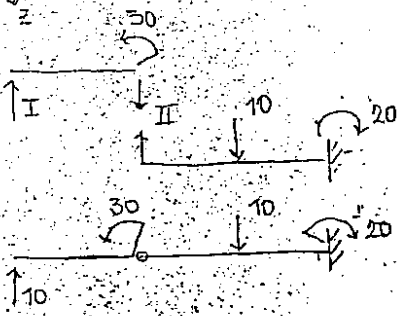
$M_D = 83,63 \cdot 2 - 50 \cdot 1,33 = 100,76$

$V_D = \frac{-100,76}{EI} \qquad \varphi_C = -\frac{83,63}{EI}$

28.01.2005. W

ДИМЕНЗИОНИСАТИ ДАТИ НОСАЧ НА ОСНОВУ УСЛОВА ДА ЈЕ:
$\sigma_{max} \le \sigma_{doz} = 10$ MPa и $v_{max} \le v_{doz} = 5$ cm
ПРИ РЕШАВАЊУ КОРИСТИТИ Mohr-Maxwell-ову АНАЛОГИЈУ
$E = 30$ GPa

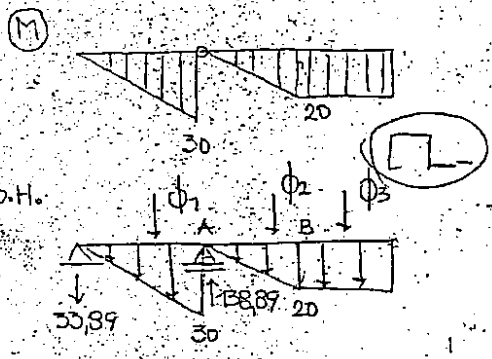




$\sum M_I^{+\circlearrowleft} = 0 \quad 30 - II \cdot 3 = 0, \quad II = 10$

$II - 10 + III = 0 \quad III = 0$

$M_3 = -10 \cdot 4 + 10 \cdot 2 \quad M_3 = -20$



$\phi_1 = \frac{1}{2} 30 \cdot 3 = 45$

$\phi_2 = \frac{1}{2} 20 \cdot 2 = 20$

$\phi_3 = 20 \cdot 2 = 40$

$\sum M = 0 \quad \phi_1 \cdot \frac{2}{3} 3 + \phi_2 \cdot (\frac{2}{3} 2 + 3) + \phi_3 \cdot 6 = V_2 \cdot 3$

$V_2 = 138,89$

$V_1 + 138,89 - 45 - 20 - 40 = 0 \quad V_1 = -33,89$

$M_A = 33,89 \cdot 3 + 45 \cdot 1 = 146,67$

$\sigma_{max}^{1-1} = \frac{M_{max}}{W_y} \qquad W_y = \frac{20 h^2}{6} \qquad I_y = \frac{20 h^3}{12}$

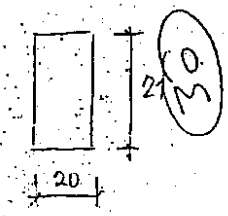
$\sigma_{max}^{1-1} = \frac{146,67 \cdot 10^3}{\frac{20 \cdot 10^{-2} h^2}{6}} \le 10^6 \cdot 10^6$

$h \ge 0,21$ m

$v_{max}^{1-1} = \frac{M_{max}}{EI_y} = \frac{146,67 \cdot 10^3}{30 \cdot 10^9 \frac{20 h^3}{12}} \le 3 \cdot 10^{-2}$

$h \ge 0,046$ m

$h_{usv} = 21$ cm



$M_B = 40 \cdot 1 = 40$ KNm

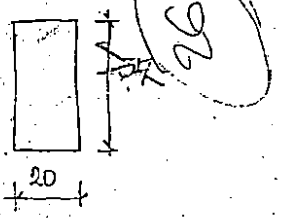
$\sigma_{max}^{2-2} = \frac{40 \cdot 10^3}{\frac{10^{-2} \cdot 20 \cdot 4h^2}{6}} \le 10 \cdot 10^6$

$h^{2-2} \ge 0,17$ m

$v_{max}^{2-2} = \frac{40 \cdot 10^3}{30 \cdot 10^9 \frac{20 \cdot 8h^3}{12} \cdot 10^{-2}} \le 3 \cdot 10^{-2}$

$h^{2-2} \ge 0,069$ m

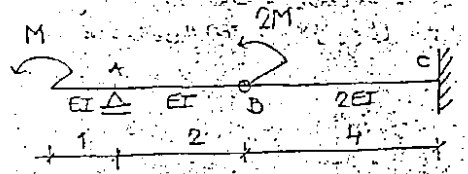
$h_{usv}^{2-2} = 17$ cm

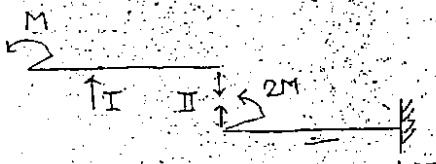


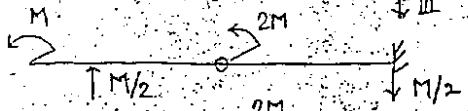
72

Mhor-Maxwell W $M=?$ из услов да је угиб у B $w_B \leq 2$ mm

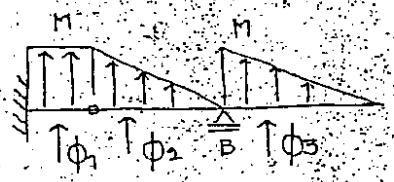
$E = 40$ GPa $I = 500$ cm^4







$\phi.H_o$



$\overset{+}{\circlearrowleft} \Sigma M_{II} = 0 \quad M - I \cdot 2 = 0 \quad I = M/2$

$I + II = 0 \quad II = -M/2$

$II + III = 0 \quad III = -M/2$

$\overset{+}{\circlearrowleft} \Sigma M_B = 0 \quad 2M - M_x - III \cdot 4 = 0$

$M_x = 2M - \frac{M}{2} \cdot 4^2 = 0$

$\phi_1 = 1 \cdot M = M$

$\phi_2 = \frac{1}{2} M \cdot 2 = M$

$\phi_3 = \frac{1}{2} M \cdot 4 = 2M$

$M_B = \phi_3 \cdot \frac{4}{3} = 2{,}67M \qquad w_B = \frac{2{,}67M}{EI} = \frac{2{,}67M}{40 \cdot 10^9 \cdot 500 \cdot 10^{-8}} \leq 2 \cdot 10^{-3}$

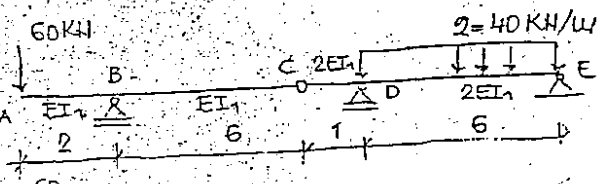
$M \leq 150$ Nm

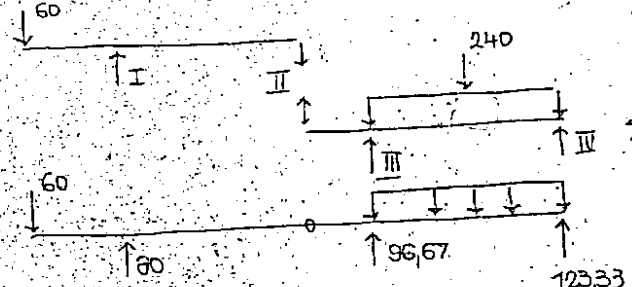
02.10.2004. W

Mohr - Maxwell - овом аналитичком одредити обртање пресека са десне стране тачке C. E = 10 GPa

Део A-B-C: $I = 85\,000\ cm^4$ $EI_1 = 850\,000$ $2EI_1 = EI_2$

Део C-D-E: $I = 170\,000\ cm^4$ $EI_2 = 1\,700\,000$





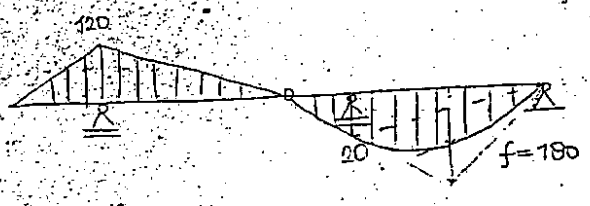
$\sum M_{II}^{+\circlearrowleft} = 0 \quad 60 \cdot 8 - I \cdot 6 = 0 \quad I = 80$

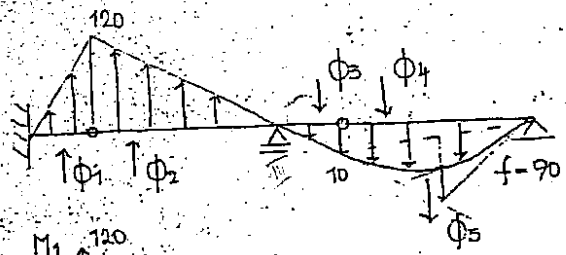
$I + II - 60 = 0 \quad II = -20$

$\sum M_{III}^{+\circlearrowleft} = 0 \quad 240 \cdot 3 - III \cdot 6 - 20 \cdot 7 = 0$

$III = 96{,}67$

$20 + 96{,}67 - 240 + IV = 0 \quad IV = 123{,}33$





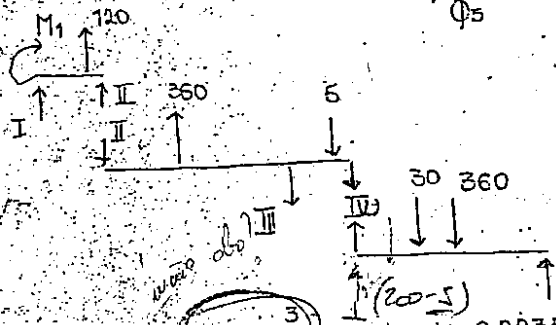
$\phi_1 = \frac{1}{2} 120 \cdot 2 = 120$

$\phi_2 = \frac{1}{2} 120 \cdot 6 = 360$

$\phi_3 = \frac{1}{2} 1 \cdot 10 = 5$

$\phi_4 = \frac{1}{2} 6 \cdot 10 = 30$

$\phi_5 = \frac{2}{3} 90 \cdot 6 = 360$



$\sum M_{III}^{+\circlearrowright} = 0 \quad 30 \cdot \frac{6}{3} + 360 \cdot 3 - V \cdot 6 = 0$

$V = 190$

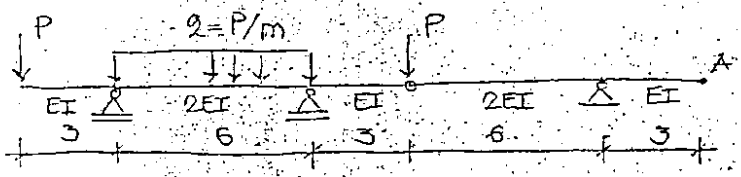
$190 - 30 - 360 - IV = 0$

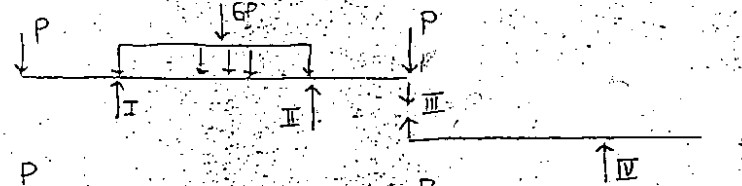
$IV = -200$

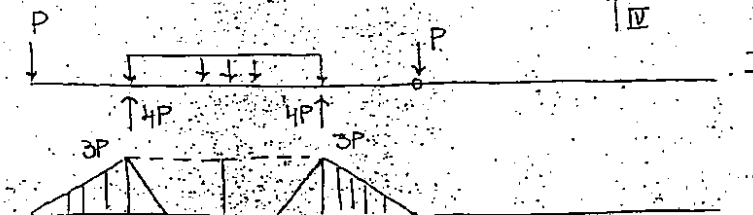
$\varphi_B = \frac{T_{BD}}{EI} = \frac{200 \cdot 10^3}{10 \cdot 10^9 \cdot 85000 \cdot 10^{-8}} = 0{,}0235\ rad$

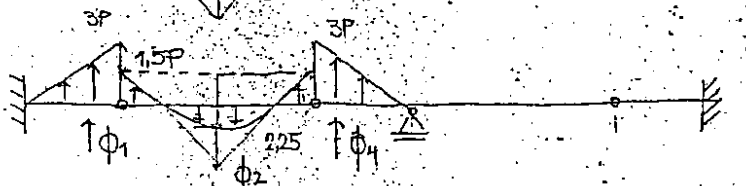
(74) 30.01.2004 W

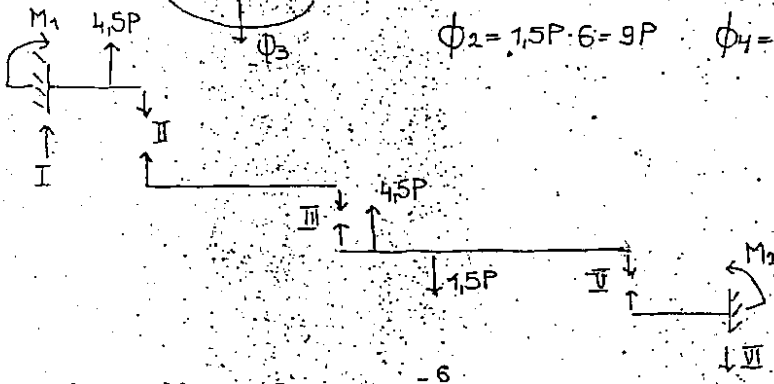
Мор-Maxwell-овом аналогијом одредити угао нагиба пресека у тачки A ($EI = 1,5\ MN\,m^2$)











$\Sigma M_{III}^{+\circlearrowleft} = 0 \quad IV \cdot 6 = 0 \quad IV = 0$

$III + IV = 0 \quad III = 0$

$\Sigma M_L^{+\circlearrowleft} = 0$

$I \cdot 3 + II \cdot 9 - 6P \cdot 6 - P \cdot 12 = 0$

$3I + 9II = 48P$

$\Sigma M_D^{+\circlearrowright} = 0 \quad I = 4P$

$II \cdot 3 + I \cdot 9 - 6P \cdot 6 - P \cdot 12 = 0$

$3II + 9I = 48P \quad I = \frac{48P - 3II}{9}$

$\frac{48P - 3II}{3} + 9II = 48P$

$II = 4P$

$\phi_1 = \frac{1}{2} 3P \cdot 3 = 4,5P \qquad \phi_3 = \frac{2}{3} 2,25 \cdot 6^2 = 9P$

$\phi_2 = 1,5P \cdot 6 = 9P \qquad \phi_4 = \frac{1}{2} 3P \cdot 3 = 4,5P$

$\Sigma M_{III}^{+\circlearrowleft} = 0 \quad 4,5P \cdot 1 + IV \cdot 3 - V \cdot 9 = 0$

$9V - 3IV = 4,5P$

$\Sigma M_{IV}^{+\circlearrowright} = 0 \quad V = 1P$

$4,5P \cdot 2 + IV \cdot 6 = 0 \quad IV = -1,5P$

$V - VI = 0 \quad VI = 1P$

$M_2 = V \cdot 3 = 3P$

(1,5)

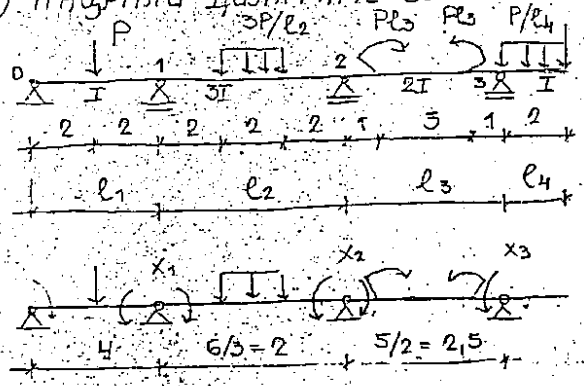
$\omega_A = \frac{M_A}{EI} = \frac{3P}{1,5 \cdot 10^6} = 2P \cdot 10^{-6}\ m = 0,002\ cm$

$\varphi_A = \frac{T_A}{EI} = \frac{P}{1,5 \cdot 10^6} = 0,666P \cdot 10^{-6} \doteq 6,66 \cdot 10^{-6}\ rad$

ВЕЖБЕ

$$l_i' = l_i \frac{I_c}{I_i} \qquad x_{i-1} l_i' + 2x_i(l_i' + l_{i+1}') + x_{i+1} l_{i+1}' = -6\left(\bar{R}_i^L \frac{I_c}{I_i} + \bar{R}_i^D \frac{I_c}{I_{i+1}}\right)$$

① НАЦРТАТИ ДИЈАГРАМЕ СИЛА У ПРЕСЕКУ



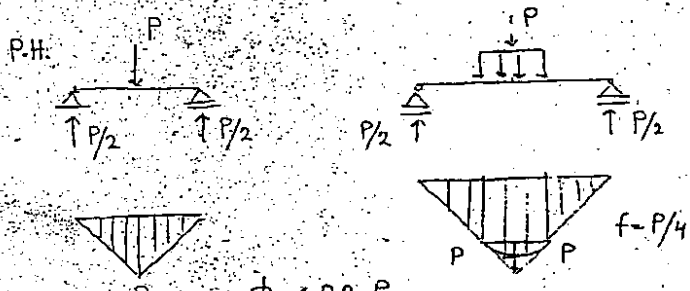
$I_c = I$

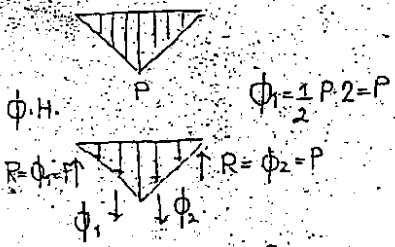
$X_3 = -(P \cdot 2 + P/2 \cdot 2 \cdot 1) = -3P$

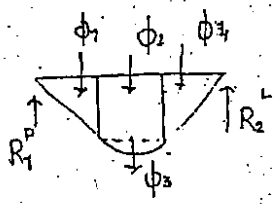
$$X_1 \cdot 4 + 2X_1(4+2) + X_2 \cdot 2 = -6\left(\bar{R}_1^L \frac{1}{1} + \bar{R}_1^P \frac{1}{3}\right)$$

$$X_2 \cdot X_1 \cdot 2 + 2X_2(2+2,5) + X_3 \cdot 2,5 = -6\left(\bar{R}_2^L \frac{1}{3} + \bar{R}_2^D \frac{1}{2}\right)$$

P.H.





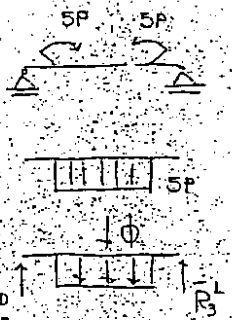


$\phi_1 = \frac{1}{2} \cdot 2 \cdot P = P \qquad \phi_4 = \phi_1 = P$

$\phi_2 = P \cdot 2 = 2P$

$\phi_3 = \frac{2}{3} \cdot \frac{P}{4,2} \cdot 2 = P/3$

$$R_1^D = R_2^L = \frac{1}{2}(2P + 2P + P/3) = \frac{13}{6}P$$



$\bar{R}_2^D = \bar{R}_3^L = \frac{1}{2} \cdot 15P = 7,5P$

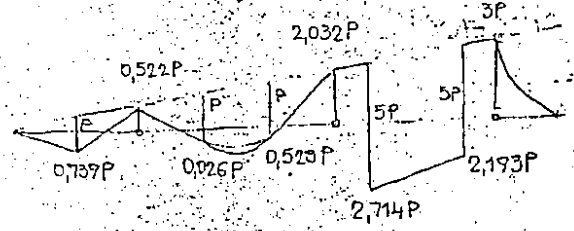
$$12X_1 + 2X_2 = -6\left(P \cdot \frac{1}{1} + \frac{13}{6}P \cdot \frac{1}{3}\right)$$

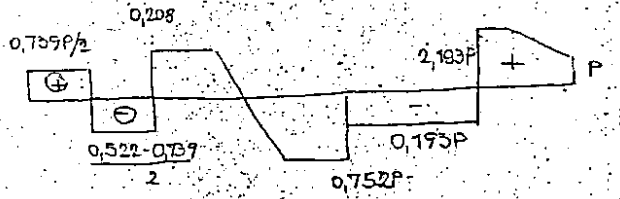
$$2X_1 + 9X_2 - 7,5P = -6\left(\frac{13}{6}P\frac{1}{3} + \frac{15}{2}P\frac{1}{2}\right)$$

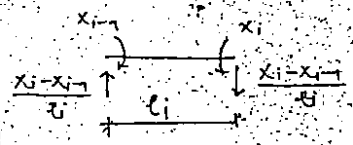
$X_1 = -0,522\,P$

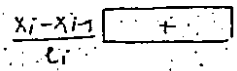
$X_2 = -2,032\,P$


-0,522P
-2,032P
-3P


0,522P
2,032P
3P
P
P
5P
5P
0,739P
0,026P
0,523P
2,714P
2,193P

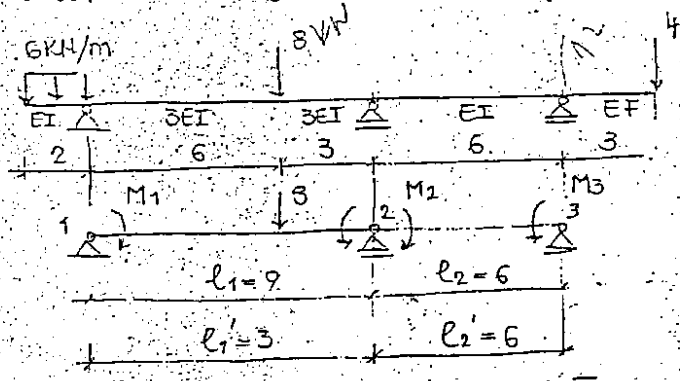

0,208
0,739P/2
2,193P
P
0,522-0,739
2
0,193P
0,752P


x_{i-1}
x_i
$\frac{x_i - x_{i-1}}{l_i}$
l_i
$\frac{x_i - x_{i-1}}{l_i}$


$\frac{x_i - x_{i-1}}{l_i}$
+

30.01.2004. W

2.3. КОРИСТЕЋИ ТРИМОМЕНТНУ ЈЕДНАЧИНУ, НАЦРТАТИ ДИЈАГРАМЕ СИЛА У ПРЕСЕЦИМА

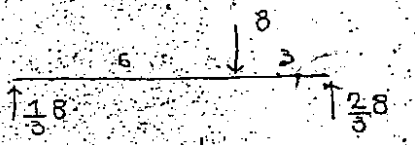


$M_1 = -6 \cdot 2 \cdot 1 = -12$ kNm

$M_0 = -4 \cdot 3 = -12$ kNm

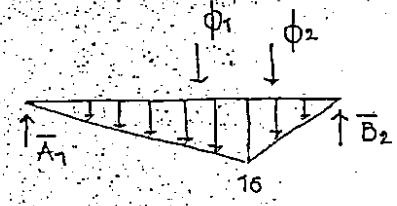
$$M_1 l_1' + 2M_2(l_1' + l_2') + M_3 l_2' = -6\left[\bar{B}_2 \frac{I_c}{I_2} + \bar{A}_2\right]$$

$$-12 \cdot 3 + 2M_2(3+6) - 12 \cdot 6 = -6\left[\bar{B}_2 \frac{1}{3} + \bar{A}_2 \cdot 1\right]$$



$\phi_1 = \frac{1}{2} \cdot 16 \cdot 6 = 48$

$\phi_2 = \frac{1}{2} \cdot 16 \cdot 3 = 24$

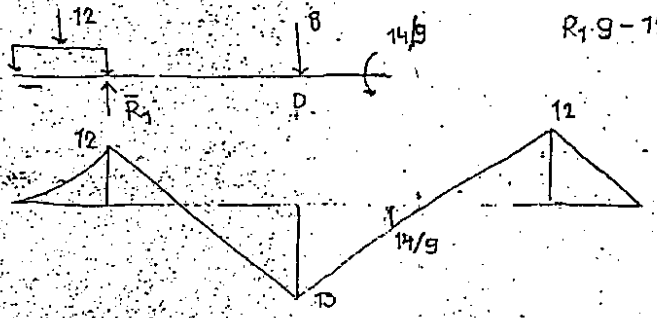


$\bar{B}_2 \cdot 9 - \phi_1 \cdot 4 + \phi_2 \cdot 7 = 0$

$\bar{B}_2 = 40$

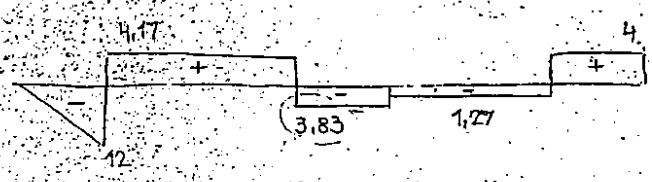
$$-36 + 18M_2 - 72 = -6\left[40 \frac{1}{3} + 0 \cdot 1\right]$$

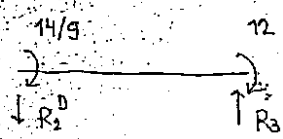
$M_2 = 14/9$



$R_1 \cdot 9 - 12 \cdot 10 + 8 \cdot 3 - 14/9 = 0$ $R_1 = 16,17$

$M_0 = R_1 \cdot 6 - 12 \cdot 7 = 13$





$R_2 \cdot 6 - 14/9 + 12 = 0$ $R_2 = 2,56$

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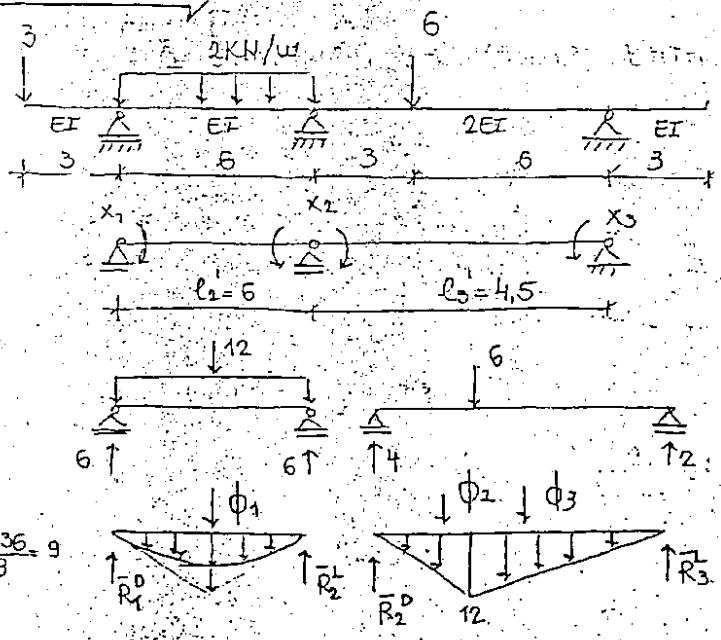


$X_1 = -9$ KNm

$X_3 = 0$

$\phi_1 = \frac{2}{3} 9 \cdot 6 = 36$

$\bar{R}_1^D = \bar{R}_2^L = 18$

$\phi_2 = \frac{1}{2} 12 \cdot 3 = 18$

$\phi_3 = \frac{1}{2} 12 \cdot 6 = 36$

$\bar{R}_3^L \cdot 9 - 18 \frac{2}{3} 3 - 36 \cdot 5 = 0$

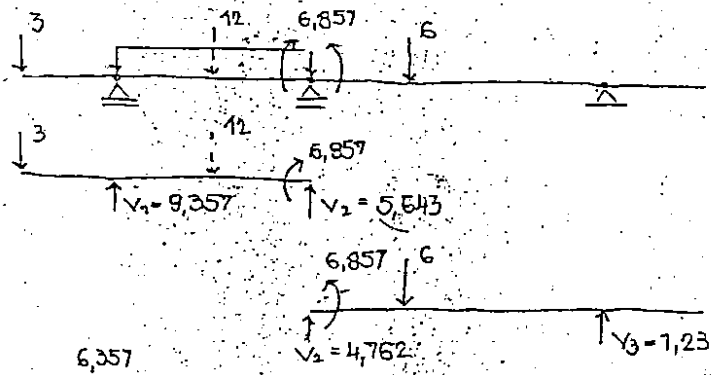
$\bar{R}_3^L = 24 \qquad \bar{R}_2^D = 30$

$$X_{i-1} l_i' + 2 X_i (l_i' + l_{i+1}') + X_{i+1} l_{i+1}' = -6 \left[\bar{R}_i^L \frac{I_c}{I_i} + \bar{R}_i^D \frac{I_c}{I_{i+1}} \right]$$

$$X_1 l_2' + 2 X_2 (l_2 + l_3') + X_3 l_3' = -6 \left[\bar{R}_2^L \frac{I_c}{I_2} + R_2^D \frac{I_c}{I_3} \right]$$

$$X_1 \cdot 6 + 2 X_2 (6 + 4{,}5) + X_3 \cdot 4{,}5 = -6 \left[18 \frac{1}{1} + 30 \frac{1}{2} \right]$$

$$-9 \cdot 6 + 21 X_2 = -198 \Rightarrow \underline{X_2 = -6{,}857}$$



$-V_1 \cdot 6 + 3 \cdot 9 + 12 \cdot 3 + 6{,}857 = 0$

$V_1 = 9{,}357$

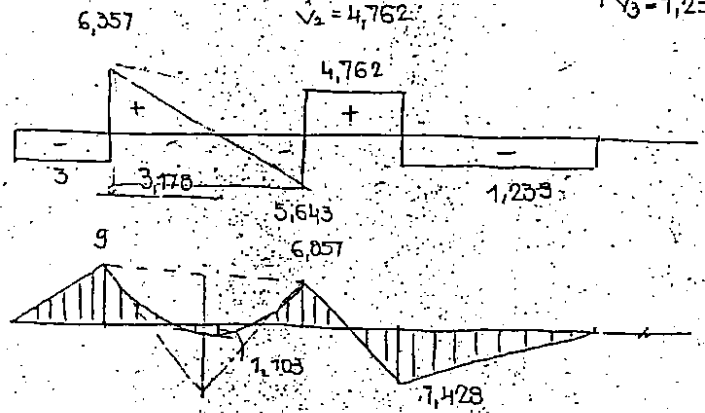
$9{,}357 + V_2 - 3 - 12 = 0$

$V_2 = 5{,}643$

$V_3 \cdot 9 - 6 \cdot 3 + 6{,}857 = 0$

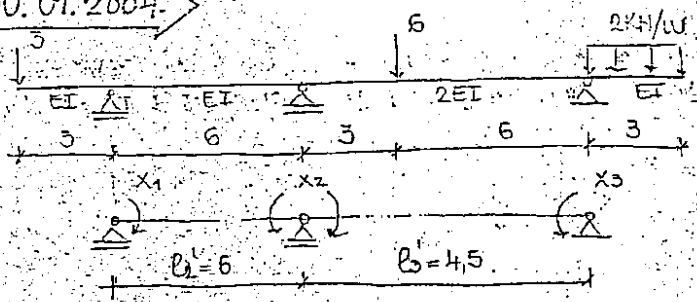
$V_3 = 1{,}238$

$V_2 + 1{,}238 - 6 = 0 \qquad V_2 = 4{,}762$



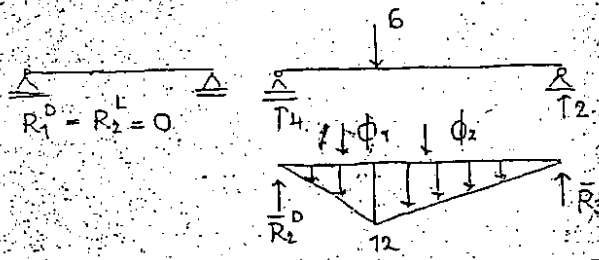
$$M = 3 \cdot 6{,}178 - 9{,}357 \cdot 3{,}178 + 2 \frac{3{,}178^2}{2} = -7{,}103$$

30.01.2004



$X_1 = -9\ kNm$

$X_3 = -\frac{2 \cdot 3^2}{2} = -9\ kNm$



$-6 \cdot 3 + V_2 \cdot 9 = 0$

$\phi_1 = \frac{1}{2} \cdot 12 \cdot 3 = 18$

$\phi_2 = \frac{1}{2} \cdot 12 \cdot 6 = 36$

$-18 \cdot 2 - 36 \cdot 5 + \bar{R}_3^L \cdot 9 = 9$

$\bar{R}_3^L = 24 \quad \bar{R}_2^D = \phi_1 - \phi_2 + \bar{R}_3^L = 0$

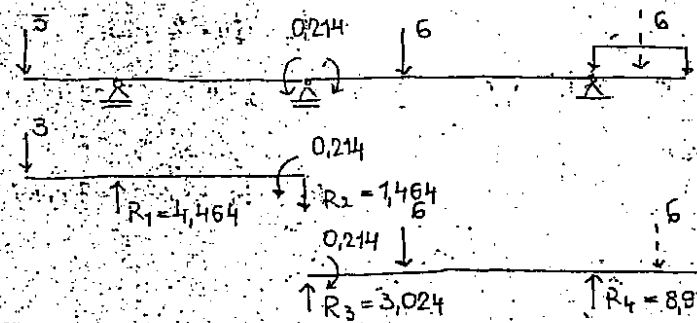
$R_2^D = 30$

$$x_{i-1} \cdot l_i' + 2x_i(l_i' + l_{i+1}') + x_{i+1} \cdot l_{i+1}' = -6\left[\bar{R}_i^L \frac{I_c}{I_i} + R_i^D \frac{I_c}{I_{i+1}}\right]$$

$i = 2$

$$x_1 \cdot l_2' + 2x_2(l_2' + l_3') + x_3 \cdot l_3' = -6\left[\bar{R}_2^L \frac{I_c}{I_2} + R_2^D \frac{I_c}{I_3}\right]$$

$-9 \cdot 6 + 2x_2(6 + 4{,}5) - 9 \cdot 4{,}5 = -6\left[30 \cdot \frac{1}{2}\right] \quad X_2 = 0{,}214$



$-3 \cdot 3 + R_1 \cdot 6 + 0{,}214 = 0$

$R_1 = 4{,}464$

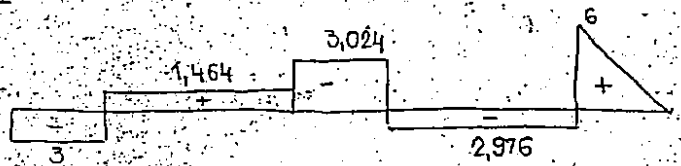
$R_1 - 3 - R_2 = 0 \quad R_2 = 1{,}464$

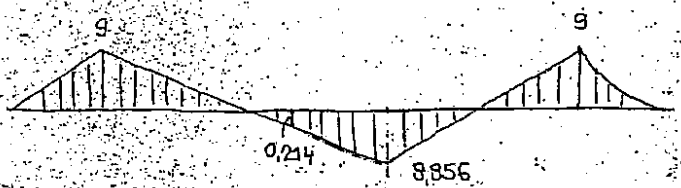
$-6 \cdot 3 + R_4 \cdot 9 - 6 \cdot 10{,}5 + 0{,}214 = 0$

$R_4 = 8{,}976$

$R_3 - 6 - 6 + 8{,}976 = 0$

$R_3 = 3{,}024$

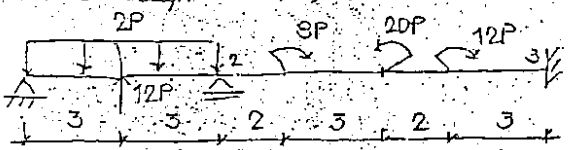


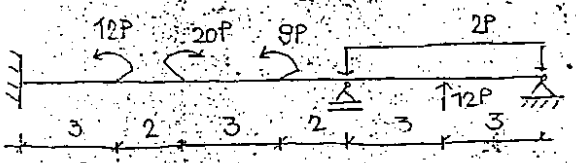


(80)

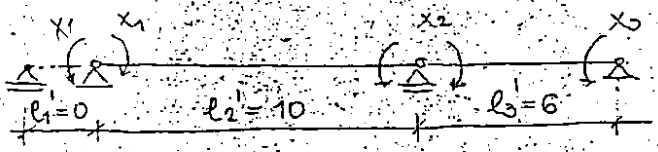
24.09.2004

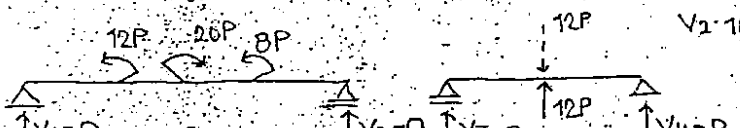
c/ ОДРЕДИТИ ВРЕДНОСТИ МОМЕНАТА У ТАЧКАМА 2 И 3 ДАТОГ КОНТИНУАЛНОГ НОСАЧА (ПРИ РЕШАВАЊУ КОРИСТИТИ ТРИМОМЕНТНЕ ЈЕДНАЧИНЕ) $EI = const.$





$X_3 = 0$

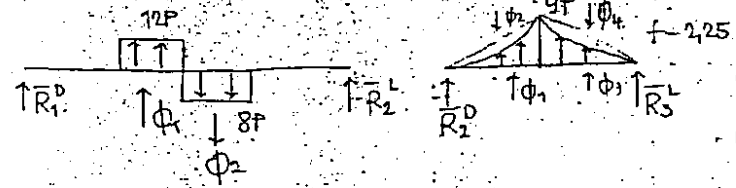




$V_1 \cdot 10 + 12P - 20P + 8P = 0$

$V_1 = 0 \quad V_2 = 0$

$V_3 = 0 \quad V_4 = 0$



$\phi_1 = 12P \cdot 2 = 24P$

$\phi_2 = 8P \cdot 3 = 24P$

$\phi_1 = \frac{1}{2} 9 \cdot 3 = 13,5P = \phi_3$

$\phi_2 = \phi_4 = \frac{2}{3} 2,25 \cdot 3 = 3P$

$24P \cdot 4 - 24P \cdot 6,5 + \bar{R}_2^L \cdot 10 = 0$

$\bar{R}_2^L = 6P$

$\bar{R}_1^D + 24P - 24P + \bar{R}_2^L = 0$

$R_1^D = -6P$

$R_3^L \cdot 6 + 13,5 \cdot 2 - 5 \cdot 1,5 - 5 \cdot 4,5 + 13,5 \cdot 4 = 0$

$R_3^L = -8,5P$

$R_2^D + \phi_1 + \phi_3 - \phi_2 - \phi_4 + \bar{R}_3^L = 0$

$R_2^D = -8,5P$

$$x_{i-1}\ell_i' + 2x_i(\ell_i' + \ell_{i+1}') + x_{i+1}\ell_{i+1}' = -6\left[\bar{R}_i^L \frac{I_c}{I_i} + R_i^D \frac{I_c}{I_{i+1}}\right]$$

$i = 1$

$$x_0\ell_1^0 + 2x_1(\ell_1^0 + \ell_2') + x_2\ell_2' = -6\left[\bar{R}_1^L \frac{1}{1}^0 + R_1^D \frac{1}{1}\right]$$

$2x_1 \cdot 10 + x_2 \cdot 10 = -6(-6P)$ $\quad \boxed{20x_1 + 10x_2 = +36P} \Rightarrow 2x_1 + x_2 = 3,6P$

$x_2 = 3,6P - 2x_1$

$i = 2$ $\quad x_1\ell_2' + 2x_2(\ell_2' + \ell_3') + x_3\ell_3'^0 = -6\left[\bar{R}_2^L \frac{I_c}{I_2} + \bar{R}_2^D \frac{I_c}{I_2}\right]$

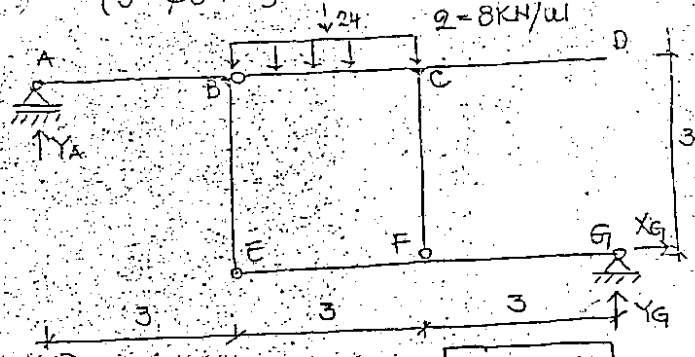
$x_1 \cdot 10 + 2x_2(10 + 6) = -6(6P + (-8,5P))$

$\boxed{10x_1 + 32x_2 = 15P}$

$10x_1 + 115,2P - 64x_1 = 15P$ $\quad x_1 = 1,85P$ $\quad x_2 = -0,1P$

30.01.2004. W

3.1. ОДРЕДИТИ ВЕРТИКАЛНО ПОМЕРАЊЕ И РОТАЦИЈУ ТАЧКЕ D (У ФУНКЦИЈИ КРУТОСТИ EI)



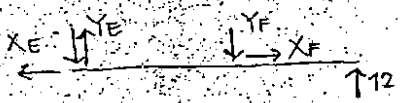
$\sum M_G^{+\circlearrowleft} = 0 \quad Y_A \cdot 9 - 24 \cdot 4{,}5 = 0$

$Y_A + Y_G - 24 = 0$

$Y_A = 12\,KN$

$Y_G = 12\,KN$

$X_G = 0$



$\sum M_E^{+\circlearrowleft} = 0 \quad Y_F \cdot 3 - 12 \cdot 6 = 0$

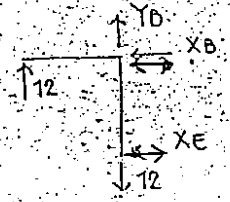
$-Y_E - Y_F + 12 = 0$

$X_F - X_E = 0$

$Y_F = 24\,KN$

$Y_E = -12\,KN$

$X_F = X_E$



$\sum Y = 0 \quad Y_B + 12 - 12 = 0$

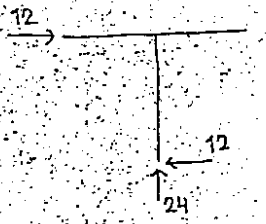
$Y_B = 0$

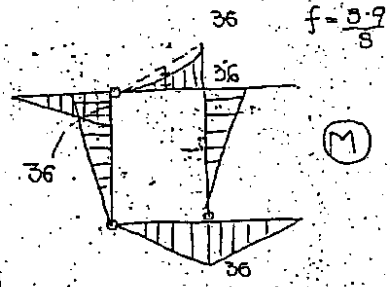
$\sum M_A^{+\circlearrowleft} = 0 \quad X_E \cdot 3 - 12 \cdot 3 = 0$

$\sum X = 0 \quad X_E - X_B = 0$

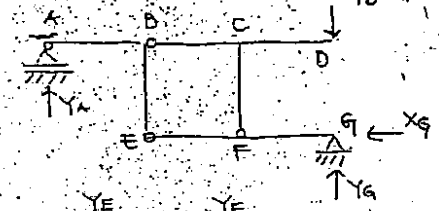
$X_E = 12\,KN \quad X_F = 12\,KN$

$X_B = -12\,KN$



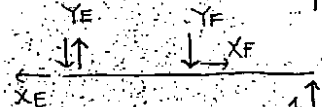


$f = \frac{3 \cdot 9}{8} = 9$



$\sum M_G^{+\circlearrowleft} = 0 \quad Y_A = 0 \quad Y_A + Y_G - 1 = 0$

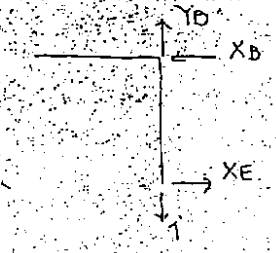
$X_G = 0 \quad Y_G = 1$



$\sum M_E^{+\circlearrowleft} = 0 \quad Y_F \cdot 3 - 1 \cdot 6 = 0 \quad Y_F = 2$

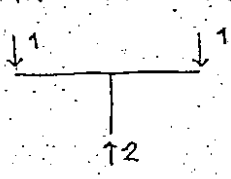
$-Y_E - Y_F + 1 = 0 \quad Y_E = 1 - 2 \quad Y_E = -1$

$X_F = X_E$



$\sum Y = 0 \quad Y_B - 1 = 0 \quad Y_B = 1$

$\sum M_A^{+\circlearrowleft} = 0 \quad 1 \cdot 3 + X_E \cdot 3 - 1 \cdot 3 = 0 \quad X_E = 0 \quad X_F = 0 \quad X_B = 0$



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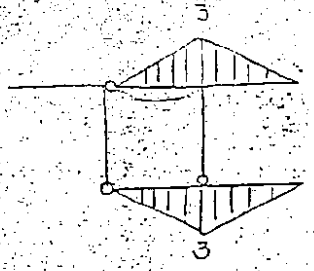
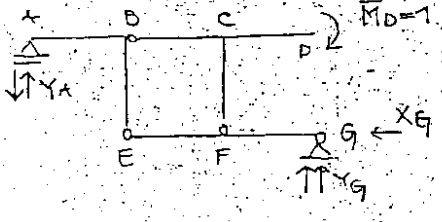
$\overline{Y_D = 1}$



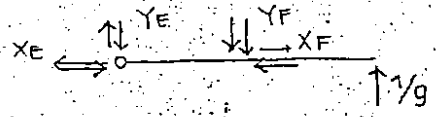
$\overline{M_1}$



$\Sigma M_G^{+\circlearrowright} = 0 \quad Y_A \cdot 9 + 1 = 0$

$\boxed{Y_A = -1/9}$

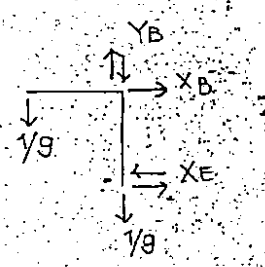
$Y_A + Y_G = 0 \quad \boxed{Y_G = 1/9} \quad \boxed{X_G = 0}$



$\Sigma M_E^{+\circlearrowright} = 0 \quad Y_F \cdot 3 - \frac{1}{9} \cdot 6 = 0$

$Y_F = \frac{2}{3} \cdot \frac{1}{3} \quad \boxed{Y_F = 2/9}$

$-Y_E - Y_F + \frac{1}{9} = 0 \quad Y_E = \frac{1}{9} - \frac{2}{9} \quad \boxed{Y_E = -1/9}$



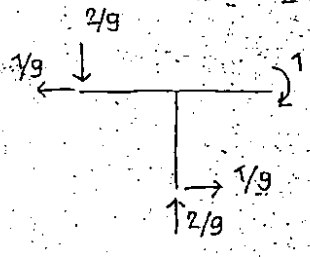
$\Sigma M_E^{+\circlearrowright} = 0$

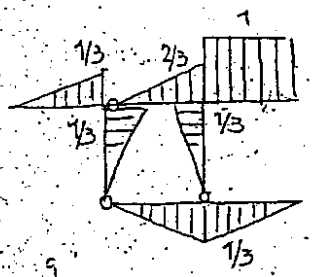
$X_B \cdot 3 - \frac{1}{9} 3 = 0 \quad X_B = \frac{1}{9} \frac{1}{3} \quad \boxed{X_B = 1/9}$

$\boxed{X_F = X_E}$

$X_B + X_E = 0 \quad \boxed{X_E = -1/9} \quad \boxed{X_F = -1/9}$

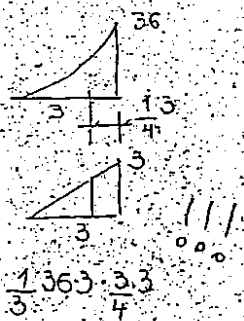
$-1/9 - \frac{1}{9} - Y_B = 0 \quad \boxed{Y_B = -2/9}$





$\overline{M_B = 1}$

$\overline{M_2}$



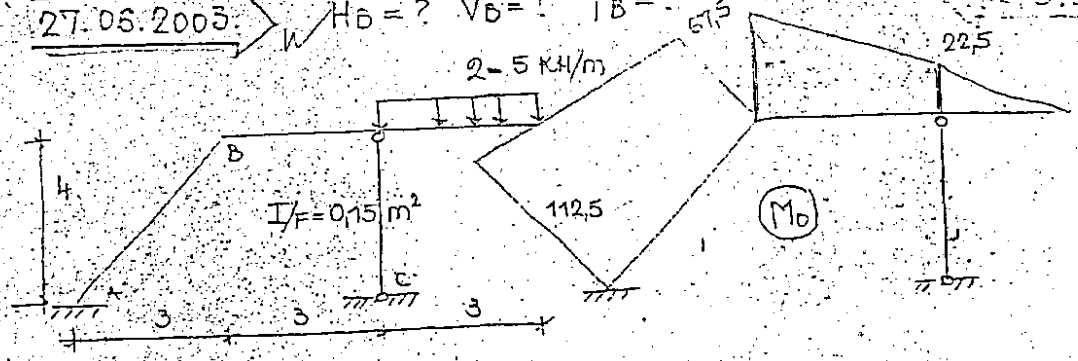
$\frac{1}{3} 36 \cdot 3 \cdot \frac{3}{4} 3$

$$W_D = \frac{1}{EI} \int M \overline{M}_1 \, ds = \frac{1}{EI} \left[\frac{1}{3} 36 \cdot 3 \cdot \frac{3}{4} 3 + 2 \cdot \frac{3}{3} 36 \cdot 3 \right] = \frac{297}{EI}$$

$$Y_D = \frac{1}{EI} \int M \overline{M}_2 \, ds = \frac{1}{EI} \left[\frac{1}{3} 36 \cdot 3 \cdot \frac{3}{4} \frac{2}{3} - \frac{3}{3} 36 \cdot \frac{1}{3} \cdot 2 + 2 \frac{3}{3} \frac{1}{3} 36 - \frac{3}{3} \frac{1}{3} 36 \right] = \frac{6}{EI}$$

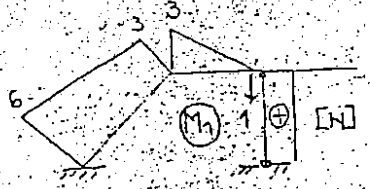
27.05.2003

$H_D = ?$ $V_D = ?$ $\varphi_B = ?$



$$\delta_{11} X_1 + \delta_{10} = 0$$

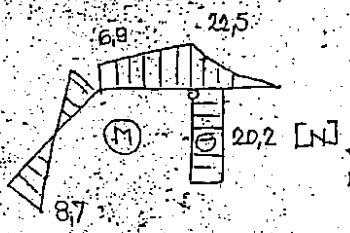
$$\delta_{11} = \frac{1}{EI}\left(\frac{3}{3}3^2 + \frac{5}{3}(6^2 + 6\cdot 3 + 3^2)\right) + \frac{1}{EI}\frac{I}{F}\cdot 5\cdot 1^2 = \frac{114{,}75}{EI}$$

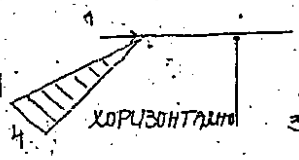


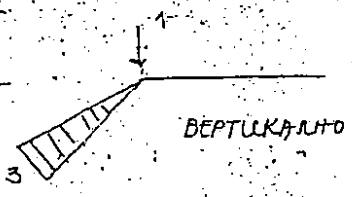
$$\delta_{10} = \frac{1}{EI}\left[\left(\frac{5}{6}\left(6(2\cdot 112{,}5 + 67{,}5) + 3(2\cdot 67{,}5 + 112{,}5)\right)\right.\right.$$
$$\left.\left. + \frac{3}{6}\cdot 3(2\cdot 67{,}5 + 22{,}5)\right] = \frac{2317{,}5}{EI}$$

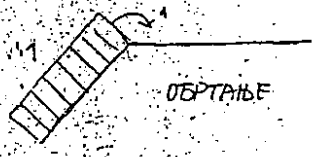
$$X_1 = -\frac{\frac{2317{,}5}{EI}}{\frac{114{,}75}{EI}} = -20{,}2$$

$M = M_1 X_1 + M_0$









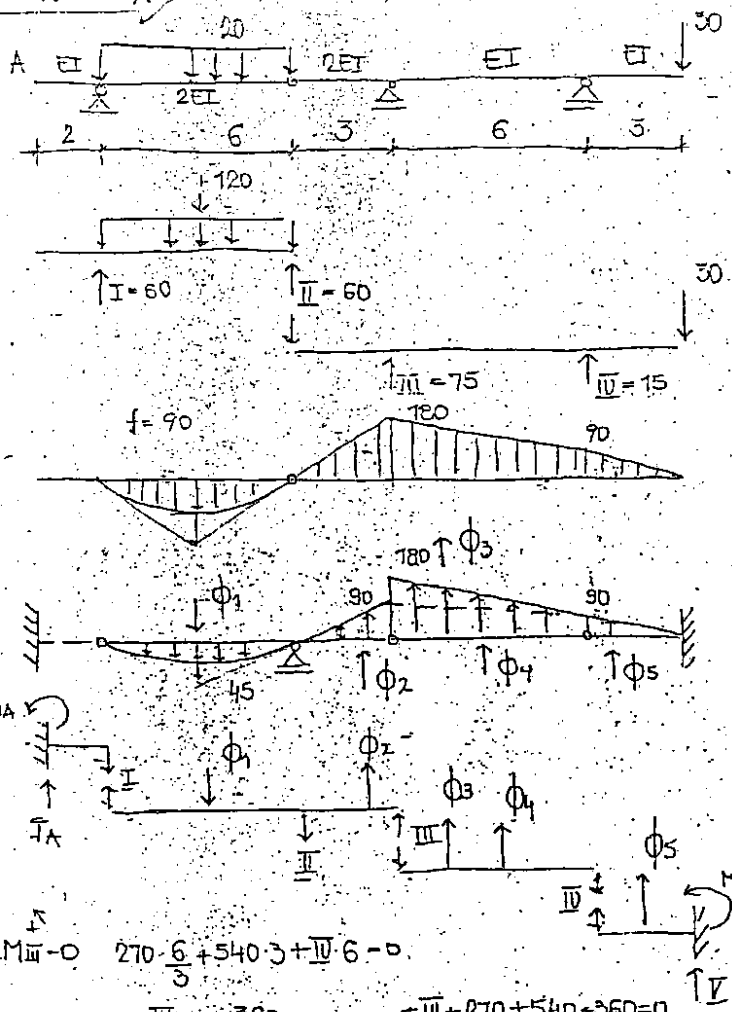
$EI = \text{const.}$

$$H_D = \frac{1}{EI}\cdot\frac{5}{6}\,4\,(2(-8{,}7) + 6{,}9) = -\frac{35}{EI}$$

$$V_B = \frac{1}{EI}\cdot\frac{5}{6}\,3\,(2(-8{,}7) + 6{,}9) = -\frac{26{,}25}{EI}$$

$$\varphi_B = \frac{1}{EI}\cdot\frac{5}{2}\,(-8{,}7 + 6{,}9) = -\frac{4{,}5}{EI}$$

30.01.2004. $w_A = ?$ $\varphi_A = ?$ $EI = 2{,}5\ MNm^2$



$\Sigma M_{II} = 0 \quad -120 \cdot 3 + I \cdot 6 = 0$

$I = 60$

$60 + II - 120 = 0 \quad II = 60$

$\Sigma M_{II} = 0 \quad -III \cdot 3 - IV \cdot 9 + 30 \cdot 12 = 0$

$3III + 9IV = 360$

$\Sigma M_K = 0 \quad III \cdot 3 + III \cdot 9 - 60 \cdot 12 = 0$

$III = \frac{720 - 3III}{9}$

$\frac{720 - 3IV}{3} + 9IV = 360$

$720 - 3IV + 27IV = 1080$

$IV = 15$

$III = 75$

$\phi_1 = \frac{2}{3} 45 \cdot 6 = 180$

$\phi_2 = \frac{1}{2} 90 \cdot 3 = 135$

$\phi_3 = \frac{1}{2} 90 \cdot 6 = 270$

$\phi_4 = 6 \cdot 90 = 540$

$\phi_5 = \frac{1}{2} 90 \cdot 3 = 135$

$\Sigma M_{III} = 0 \quad 270 \cdot \frac{6}{3} + 540 \cdot 3 + IV \cdot 6 = 0$

$IV = -360 \qquad -III + 270 + 540 - 360 = 0$

$III = 450$

$\Sigma M_I = 0 \quad 180 \cdot 3 - II \cdot 6 - 135 \cdot 8 - 450 \cdot 9 = 0 \qquad II = -765$

$-I - 180 + 135 + 450 = 765 = 0 \qquad I = -360$

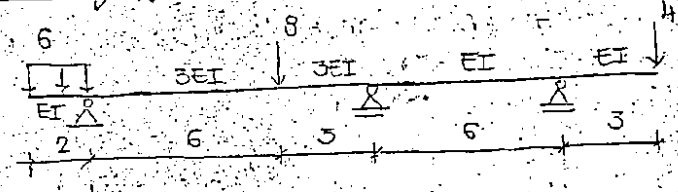
$\Rightarrow \quad T_A = 360\ KN$

$M_A = 720\ KNm$

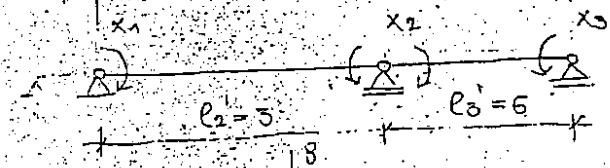
$$w_A = \frac{M_A}{EI} = \frac{720 \cdot 10^3}{2{,}5 \cdot 10^6} = 0{,}288\ m$$

$$\varphi_A = \frac{T_A}{EI} = \frac{-360 \cdot 10^3}{2{,}5 \cdot 10^6} = 0{,}144\ rad$$

30.01.2004 // ТРИМОМЕНТНА ЈЕДНАЧИНА ⇒ ДИЈАГРАМИ СИЛА У2 ПРЕСЕЧНИКА

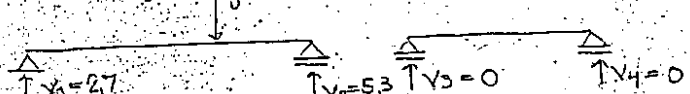


$X_1 = -12$ KNm
$X_3 = -12$ KNm

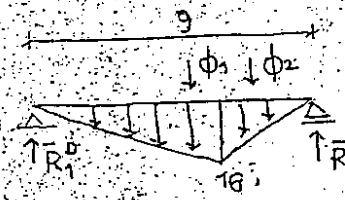


$8 \cdot 6 - V_2 \cdot 9 = 0$
$V_2 = 5{,}3$

$V_3 = V_4 = 9$



$\bar{R}_2^D = \bar{R}_3^L = 0$



$\Phi_1 = \frac{1}{2} 16 \cdot 6 = 48$ $\quad \Sigma M^{+} = 0$

$\Phi_2 = \frac{1}{2} 16 \cdot 3 = 24$ $\quad \Phi_1 \cdot \frac{2}{3} 6 + \Phi_2 \cdot 7 - \bar{R}_2^L \cdot 9 = 0$

$\bar{R}_2^L = 40$ $\quad \bar{R}_1^D = 32$

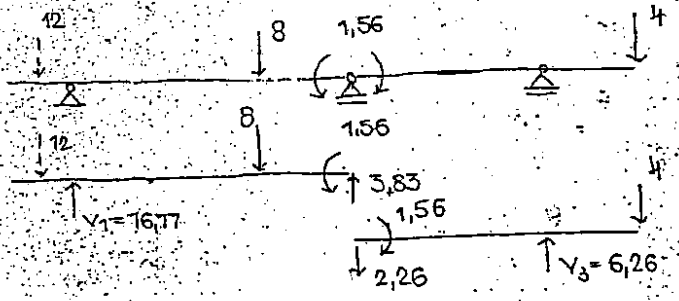
$$X_{i-1} l_i' + 2X_i (l_i' + l_{i+1}') + X_{i+1} l_{i+1}' = -6 \left[\bar{R}_i^L \frac{I_c}{I_i} + \bar{R}_i^D \frac{I_c}{I_{i+1}} \right]$$

$i = 2$

$$X_1 l_2' + 2X_2 (l_2' + l_3') + X_3 l_3' = -6 \left[\bar{R}_2^L \frac{1}{3} + \bar{R}_2^D \frac{1}{1} \right]$$

$$-12 \cdot 3 + 2X_2 (3 + 6) + 12 \cdot 6 = -6 \left[40 \cdot \frac{1}{3} \right]$$

$X_2 = 1{,}56$



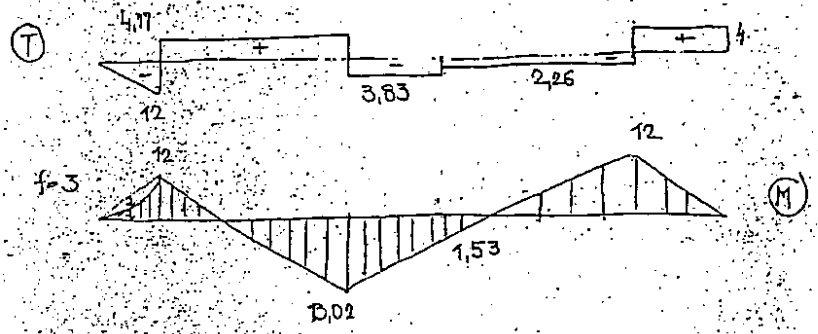
$\Sigma M_2 = 0$

$-1{,}56 + V_3 \cdot 6 - 4 \cdot 9 = 0$
$V_3 = 6{,}26$

$V_2 + V_3 - 4 = 0$
$V_2 = -2{,}26$

$\Sigma M_2 = D$

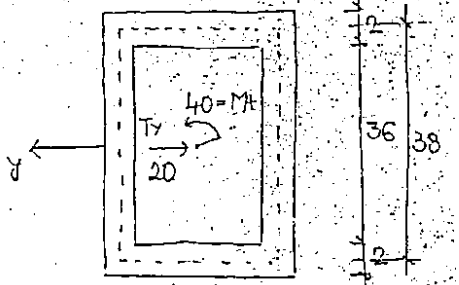
$12 \cdot 10 - V_1 \cdot 9 + 8 \cdot 3 + 1{,}56 = 0$
$V_1 = 16{,}77$



(86) 02.04.2004. W

НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА У ПРЕСЕКУ, УСЛЕД ЗАДАТЕ ТРАНСФЕРЗАЛНЕ СИЛЕ И МОМЕНТА ТОРЗИЈЕ

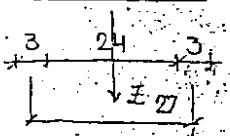
$I_y = 06688\ cm^4$ $I_z = 48528\ cm^4$



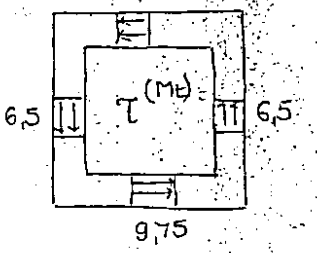
$$\tau^{(Mt)} = \frac{Mt}{2F \cdot t_i}$$

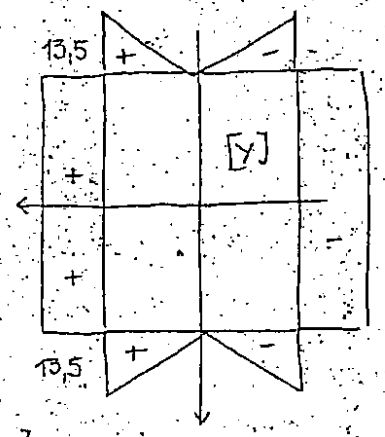
$t = 2\ cm \quad \tau^{(Mt)} = \frac{40 \cdot 10^3}{2 \cdot 27{,}38 \cdot 10^{-4} \cdot 2 \cdot 10^{-2}} = 9{,}75\ MPa$

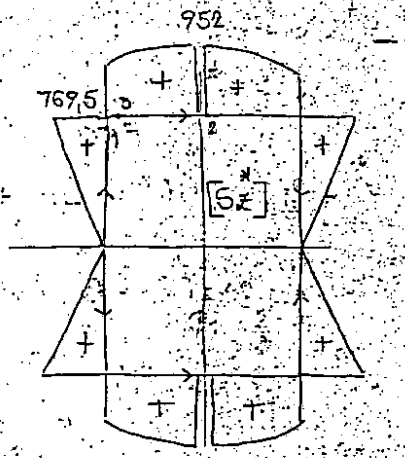
$t = 3\ cm \quad \tau^{(Mt)} = \frac{40 \cdot 10^3}{2 \cdot 27{,}38\ 10^{-4} \cdot 3 \cdot 10^{-2}} = 6{,}5\ MPa$



$$\tau^{Ty} = -\frac{T_y S_z^*}{I_z \cdot t}$$

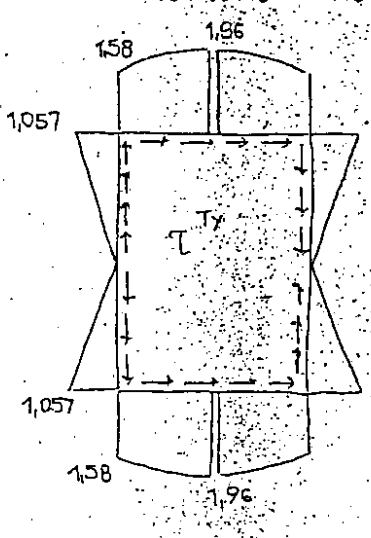






$$\tau_{x1} = -\frac{(-20 \cdot 10^3) \cdot 769{,}5 \cdot 10^{-6}}{48528 \cdot 10^{-8} \cdot 3 \cdot 10^{-2}} = 1{,}057\ MPa$$

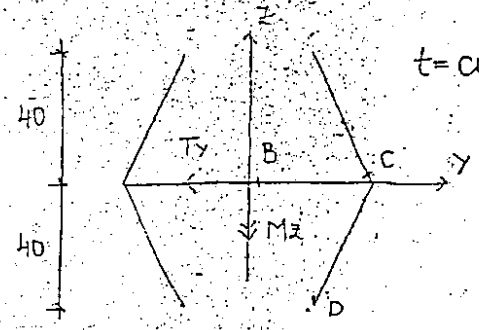
$$\tau_{x2} = -\frac{(-20 \cdot 10^3) \cdot 952 \cdot 10^{-6}}{48528 \cdot 10^{-8} \cdot 2 \cdot 10^{-2}} = 1{,}96\ MPa \qquad \tau_{x3} = -\frac{(-20 \cdot 10^3) \cdot 769{,}2 \cdot 10^{-6}}{48528 \cdot 10^{-8} \cdot 2 \cdot 10^{-2}} = 1{,}58\ MPa$$



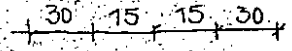
02.09.2005.

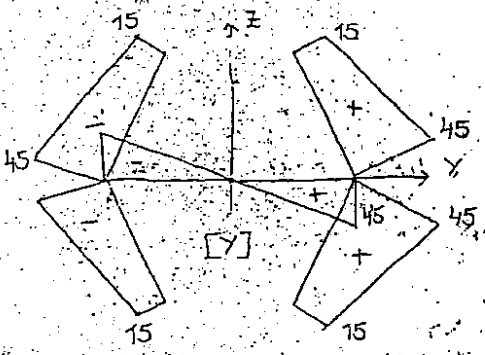
2.1. НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА И НАПИСАТИ ТЕНЗОРЕ НАПОНА ЗА ТАЧКЕ B, C И D У СИСТЕМУ XYZ, АКО НА ПОТАРЕЧНИ ПРЕСЕК ДЕЛУЈУ СИЛЕ:

$M\omega = 20\ KN\,cm^2$
$|T_y| = 100\ KN$
$M_z = 100\ KN\,cm$
$t = const = 1\ cm$



$$\tau^{T_y} = -\frac{T_y S_z^*}{I_z t_i}$$

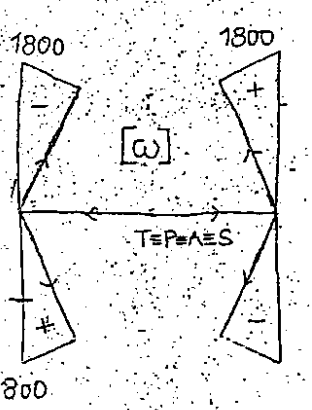


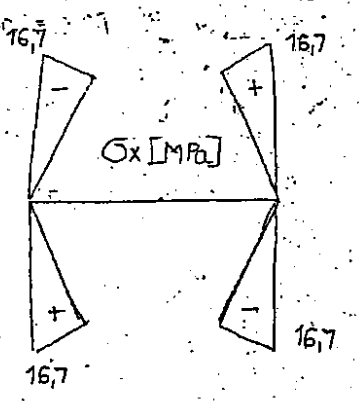


$$I_z = t\int y^2 ds = 1\cdot 4\cdot \frac{50}{6}(15^2 + 15\cdot 45 + 45^2)\cdot \frac{50}{3}\ \text{NIJE TAČNO}$$
$$+ 1\cdot 2\cdot \frac{45}{3}\,45^2 = 158\,250\ cm^4$$
ISPIT 20.9.05

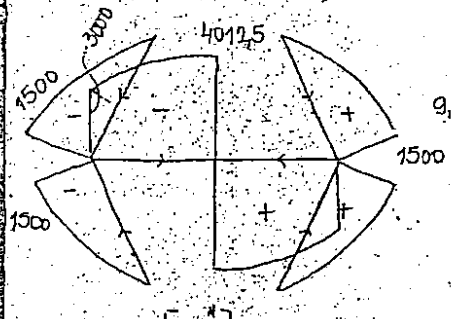
$$I_\omega = t\int \omega^2 ds = 1\cdot 4\cdot \frac{50}{3}\cdot 1800^2 = 216\cdot 10^6\ cm^6$$

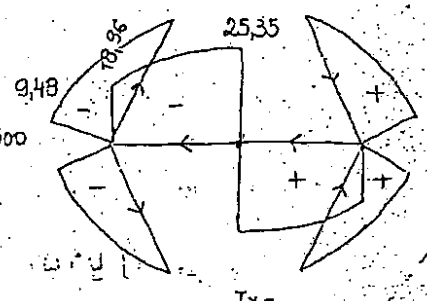
$$\sigma_x = \frac{M\omega}{I\omega}\cdot\omega = \frac{20\cdot 10^3}{216\cdot 10^6\cdot 10^{-12}}\cdot\omega\cdot 10^{-4} =$$



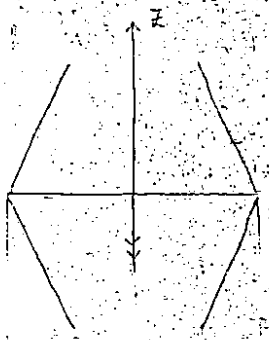


$$\tau^{T_x} = \frac{100\cdot 10^3\cdot S_z^*\cdot 10^{-6}}{158\,250\cdot 10^{-8}\cdot 1\cdot 10^{-2}}$$





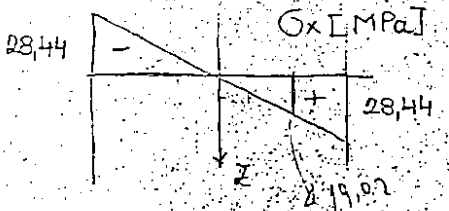
$\sigma_x = -\frac{M_z}{I_z} y \Rightarrow \sigma_x = \frac{100 \cdot 10^3}{158250 \cdot 10^{-8}} \cdot 45 \cdot 10^{-2} = 28,44\ MPa$



$$S_B = \begin{bmatrix} 0 & 25,35 & 0 \\ 25,35 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$$

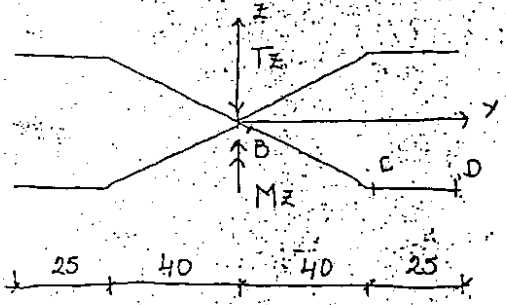
$$S_C = \begin{bmatrix} 28,44 & 9,48 & 0 \\ 9,48 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$S_D = \begin{bmatrix} 11,74 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



02.09.2005

2.1. НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА И НАПИСАТИ ТЕНЗОРЕ НАПОНА ЗА ТАЧКЕ B, C, D У СИСТЕМУ xyz, АКО НА ПОПРЕЧНИ ПРЕСЕК ДЕЛУЈУ СИЛЕ У ПРЕСЕКУ



$M\omega = 20\ KNcm^2$ $\quad t = const = 1\ cm$

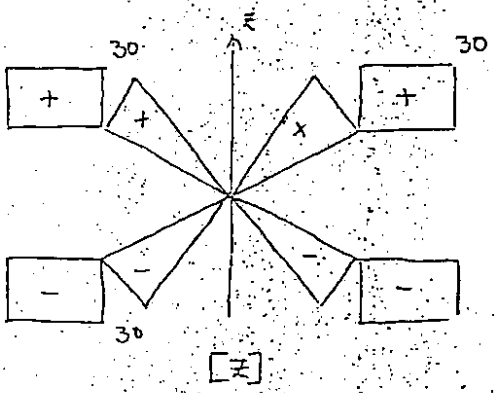
$|Tz| = 500\ KN$

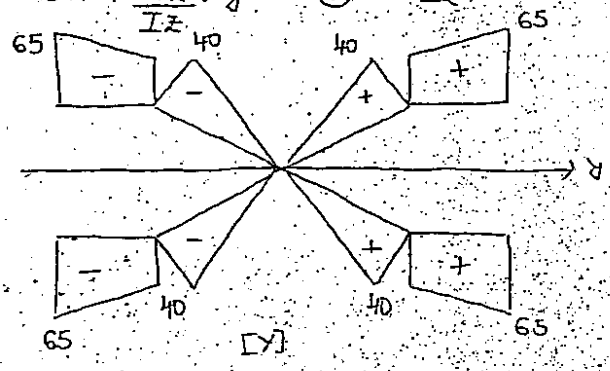
$Mz = 300\ KNcm$

$\sigma_x = \frac{M\omega}{I\omega} \cdot \omega \Rightarrow (\omega) \Rightarrow I\omega$

$\tau^{Tz} = -\frac{Tz\, Sy^*}{Iy \cdot t} \Rightarrow (z) \Rightarrow Iy, (Sy^*)$

$\sigma_x = \frac{-Mz}{Iz} \cdot y \Rightarrow (y) \Rightarrow Iz$





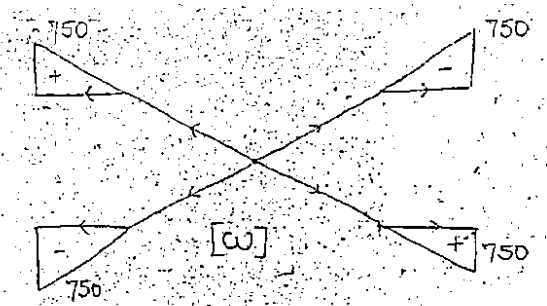
$Iy = t\int z^2 ds = 1 \cdot 4 \cdot 25\, 30^2 + 1 \cdot 4 \frac{50}{3} 30^2$

$Iy = 150\,000\ cm^4$

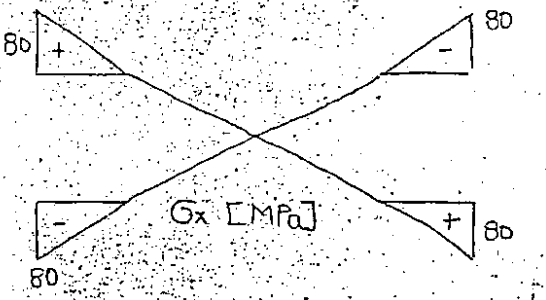
$Iz = t\int y^2 ds = 1 \cdot 4 \cdot \frac{25}{3}(65^2 + 65\,40 + 40^2)$

$+ 1 \cdot 4 \cdot \frac{50}{3} 40^2$

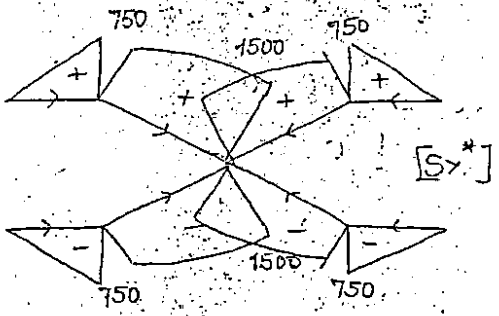
$Iz = 387\,500\ cm^4$

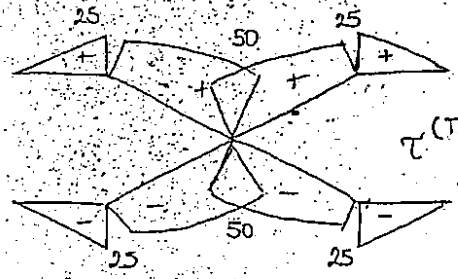


$$I_\omega = t\int \omega^2 ds = 1 \cdot 4 \cdot \frac{25}{3} \cdot 750^2 = 18750000\ cm^6$$

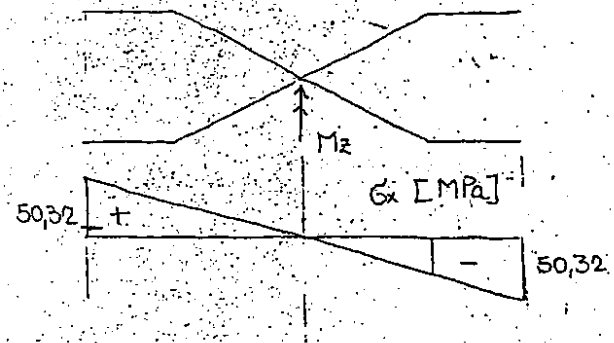


$$\sigma_x = \frac{M_\omega \cdot \omega}{I_\omega} = \frac{20 \cdot 10^3 \cdot \omega \cdot 10^{-4}}{18750000 \cdot 10^{-12}}$$





$$\tau^{(T_z)} = -\frac{T_z \cdot S_y^*}{I_y \cdot t} = \frac{500 \cdot 10^3 \cdot S_y^* \cdot 10^{-6}}{150\,000 \cdot 10^{-8} \cdot 1 \cdot 10^{-2}}$$



$$\sigma_x = -\frac{(-300 \cdot 10^3)}{387\,500 \cdot 10^{-8}} \cdot 65 \cdot 10^{-2} = 50{,}32\ MPa$$

$$50{,}32 : 65 = x : 40 \qquad x = 30{,}97$$

$$S_C = \begin{bmatrix} 30{,}97 & 0 & 25 \\ 0 & 0 & 0 \\ 25 & 0 & 0 \end{bmatrix} \qquad S_B = \begin{bmatrix} 0 & 0 & 50 \\ 0 & 0 & 0 \\ 50 & 0 & 0 \end{bmatrix} \qquad S_D = \begin{bmatrix} 29{,}68 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

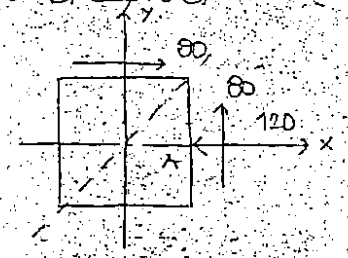
02.09.2005.

2.2. ДАТ ЈЕ ТЕНЗОР НАПОНА S

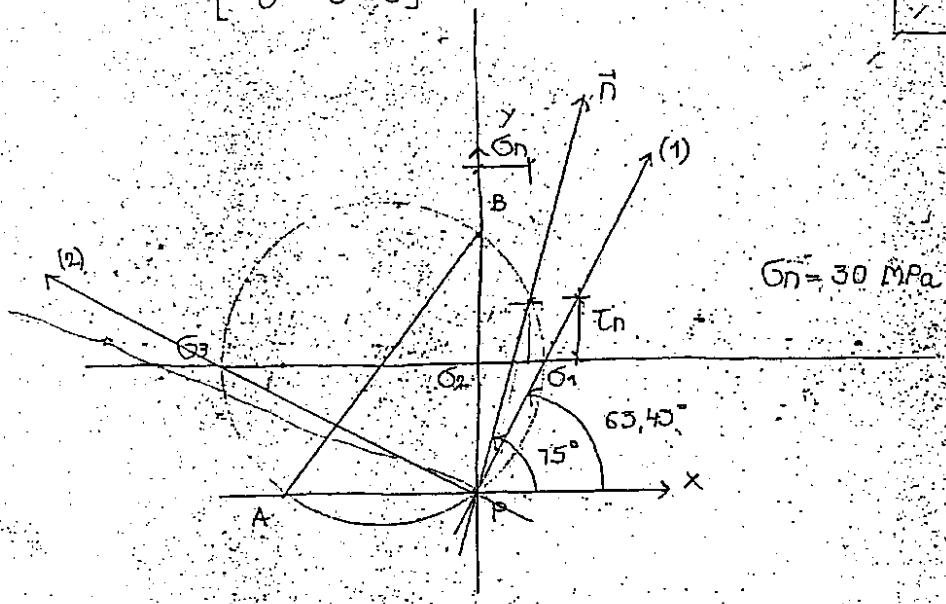
a/ ПОМОЋУ МОРОВОГ КРУГА ОДРЕДИТИ ВРЕДНОСТИ КОМПОНЕНТАЛНИХ НАПОНА ЗА РАВАН ЧИЈА НОРМАЛА ЛЕЖИ У РАВНИ XY И СА ОСОМ X ЗАКЛАПА УГАО ОД 75°

b/ АНАЛИТИЧКИМ ПУТЕМ ОДРЕДИТИ ПРАВАЦ АЛГЕБАРСКИ НАЈВЕЋЕГ НОРМАЛНОГ НАПОНА И ДОБИЈЕНУ ВРЕДНОСТ ПРОВЕРИТИ НА КРУГУ

$$S_{xyz} = \begin{bmatrix} -120 & 80 & 0 \\ 80 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$



A(−120, −80)
B(0, 80)



$\sigma_n = 30$ MPa $\quad \tau_n = 40$ MPa

$$\sigma_{1,2} = \frac{-120+0}{2} \pm \sqrt{\left(\frac{-120-0}{2}\right)^2 + 80^2}$$

$\sigma_1 = 40$ MPa $\quad \sigma_2 = 0 \quad \sigma_3 = -160$ MPa

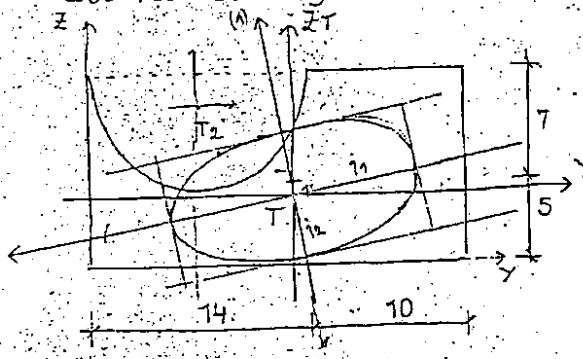
$$\operatorname{tg} 2\alpha = \frac{2\sigma_{12}}{\sigma_{11}-\sigma_{22}} = \frac{2 \cdot 80}{-120}$$

$\sin 2\alpha > 0$, $\cos 2\alpha < 0$

$$\alpha = \frac{1}{2}(180 + \operatorname{arctg}(\,)) = 63{,}43°$$

02.09.2005

1.1. За пресек определи вредности главних централних момената инерције, правце главних централних оса инерције, вредности полупречника елипсе инерције и скицирати елипсу инерције



$F_1 = 12 \cdot 24 = 288\ cm^2$

$F_2 = \frac{7^2\pi}{2} = 76,97\ cm^2$

$F = F_1 - F_2 = 211,03\ cm^2$

$T_1(12; 6)$ $T_2(7; 9,03)$

$T(15,82; 4,89)$

$y_T = \frac{12 \cdot 288 - 7 \cdot 76,97}{211,03} = 15,82$ $z_T = \frac{6 \cdot 288 - 9,03 \cdot 76,97}{211,03} = 4,89$

$I_y = \frac{1}{12} 24 \cdot 12^3 + 1,11^2 \cdot 288 - 7^4 \cdot 2 \cdot 0,05488 = 4,14^2 \cdot 76,97 = 2228,08\ cm^4$

$I_z = \frac{1}{12} 12 \cdot 24^3 + 1,82^2 \cdot 288 - \frac{\pi \cdot 7^4}{8} - 6,82^2 \cdot 76,97 = 10\,255,04\ cm^4$

$I_{yz} = (-1,82)(1,11) \cdot 288 - (-6,82)(4,14) \cdot 76,97 = 1591,41\ cm^4$

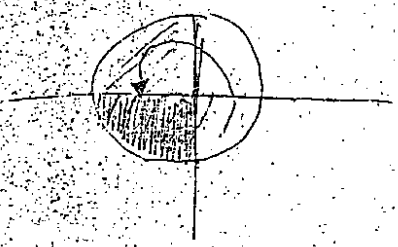
$I_{1,2} = \frac{2228,08 + 10\,255,04}{2} \pm \sqrt{\left(\frac{2228,08 - 10255,04}{2}\right)^2 + 1591,47^2}$

$I_1 = 10\,559,04\ cm^4$ $I_2 = 1924,08\ cm^4$

$tg\,2\alpha = -\frac{2 I_{yz}}{I_y - I_z} = -\frac{2 \cdot 1591,41}{2228,08 - 10255,04} < 0$

$\alpha = \frac{1}{2}(180 + arctg(\ )) = 100,81°$

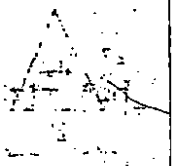
$i_1 = \sqrt{\frac{I_1}{F}} = 7,07\ cm$ $i_2 = \sqrt{\frac{I_2}{F}} = 3,02\ cm$



$\alpha = \frac{1}{2}(180 + Arctg(\ )) = 100,81°$

$i_1 = \sqrt{\frac{I_1}{F}} = 7,01$

$i_2 = \sqrt{F}\ 3,02$

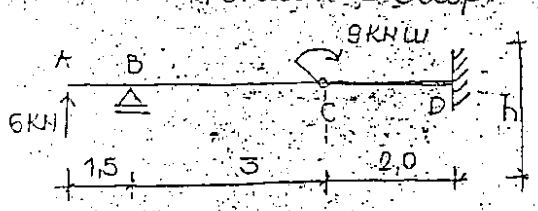


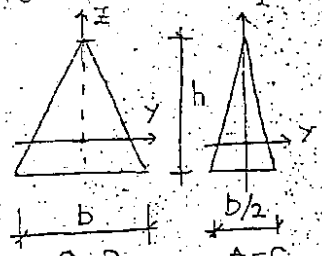
92

02.09.2005

1.2. Димензионисати дати носач на основу услова

$|\sigma|_{max} \le \sigma_{dop} = 16\ MPa$





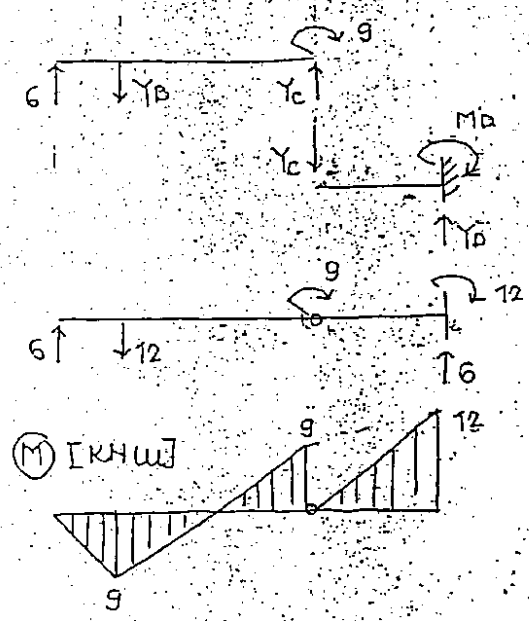
$h/b = 1$

$I_y = \frac{bh^3}{36}$

$I_z = \frac{b^3 h}{48}$

C-D ⇒ $I_y = \frac{bh^3}{36} - \frac{b^4}{36}$ $\quad I_z = \frac{b^3 h}{48} - \frac{b^4}{48}$

A-C ⇒ $I_y = \frac{\frac{b}{2} h^3}{36} - \frac{b^4}{96}$ $\quad I_z = \frac{b^3 h}{8 \cdot 48} - \frac{b^4}{384}$



$\sum M_C = 0 \quad 9 + Y_B \cdot 3 + 6 \cdot 4{,}5 = 0$

$Y_B = -12$

$6 - 12 + Y_C = 0 \quad Y_C = 6\ kN$

$Y_D - Y_C = 0 \quad Y_D = 6\ kN$

$M_D = 12\ kNm$

$\sigma_{max} = \frac{M_{max}}{W_y} \le \sigma_{dot}$

$W_y = \frac{I_y}{z_{max}}$

$\sigma_{max} = \frac{M_{max}}{I_y} \cdot z_{max} \le \sigma_{dot}$

DEO C-D : $z_{max} = \frac{2}{3} h = \frac{2}{3} b$

$\frac{12 \cdot 10^3}{\frac{b^4}{36}} \cdot \frac{2}{3} b \le 16 \cdot 10^6$

$\frac{12 \cdot 2 \cdot 36}{3 b^3} 10^3 \le 16 \cdot 10^6 \quad b \ge 0{,}2621\ m$

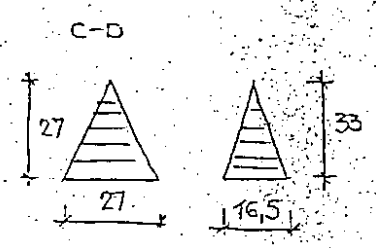
Усвојено : $b = h = 27\ cm$

DEO A-C : $z_{max} = \frac{2}{3} b$

$\frac{9 \cdot 10^3}{\frac{b^4}{96}} \cdot \frac{2}{3} b \le 16 \cdot 10^6$

$\frac{9 \cdot 2 \cdot 96}{3 b^3} \le 16 \cdot 10^3 \quad b \ge 0{,}33\ m$

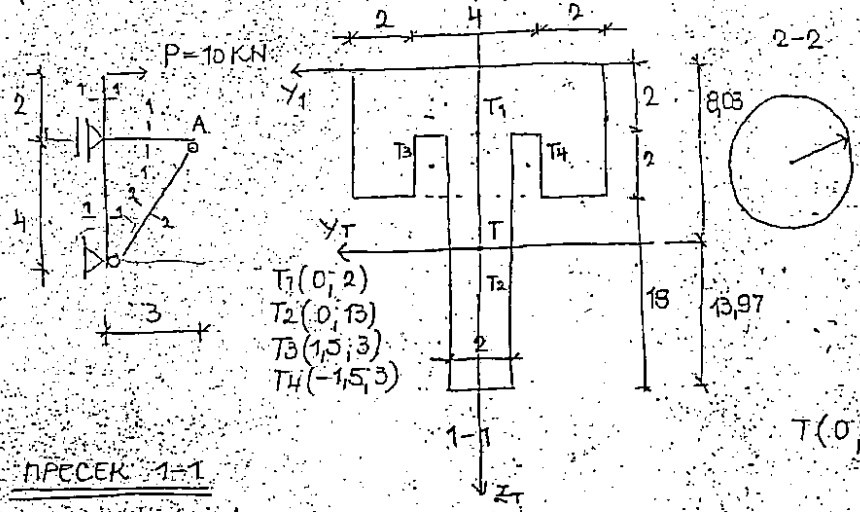
Усвојено : $b = h = 33\ cm$



05.09.2003

2.2. Одредити обртање пресека у тачки A за вредност силе

$P = 10\ KN \quad E = 210\ GPa$



ПРЕСЕК 1-1

$F_1 = 8 \cdot 4 = 32\ cm^2$

$F_2 = 18 \cdot 2 = 36\ cm^2$

$F_3 = F_4 = 2 \cdot 1 = 2\ cm^2$

$z_T = \frac{2 \cdot 32 + 13 \cdot 36 - 2 \cdot 3 \cdot 2}{64} = 8,03\ cm$

$F = F_1 + F_2 - F_3 - F_4 = 64\ cm^2$

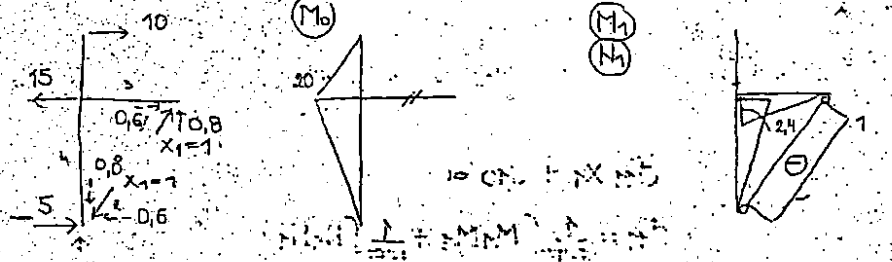
$I_y = \frac{1}{12} 8 \cdot 4^3 + 6,03^2 \cdot 32 + \frac{1}{12} 2 \cdot 18^3 + 4,97^2 \cdot 36 - \left(\frac{1}{12} 1 \cdot 2^3 + 5,03^2 \cdot 2\right) \cdot 2$

$I_y = 2964,91\ cm^4$

ПРЕСЕК 2-2

$F = R^2 \pi = 4\pi = 12,57\ cm^2$

$\frac{I_y}{F_2} = \frac{2964,91 \cdot 10^{-8}}{12,57 \cdot 10^{-4}} = 0,024\ m^2$

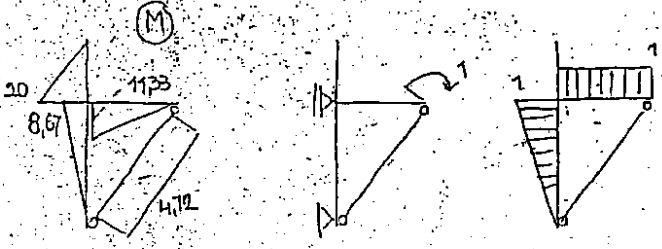


$EI\,\delta_{11} = \frac{3}{3} 2,4^2 + \frac{4}{3} 2,4^2 + 5 \cdot 1 \cdot 0,024 = 13,56$

$EI\,\delta_{10} = -\frac{4}{3} 20 \cdot 2,4 = -64$

$X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-64}{13,56} = 4,72$

$M = M_0 + X_1 M_1$

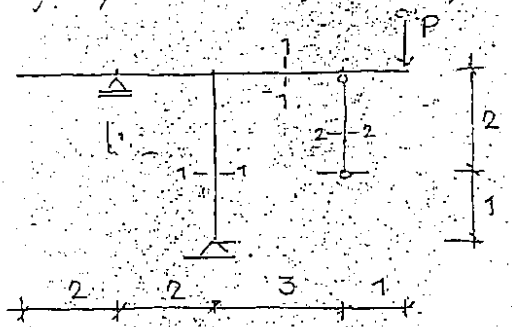


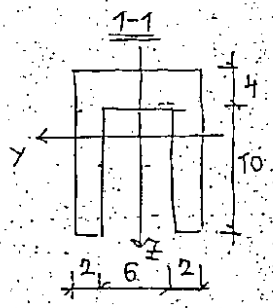
$EI\,\varphi_A = -\frac{3}{2} 1 \cdot 11,33 + \frac{4}{3} 1 \cdot 8,67 = -5,435$

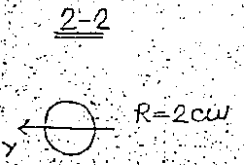
$\varphi_A = \frac{-5,435 \cdot 10^3}{210 \cdot 10^9 \cdot 2964,91 \cdot 10^{-8}} = -8,73 \cdot 10^{-4}\ rad$

(84) 05.10.2005 W

3.1 Одредити граничну вредност оптерећења P^*







$\sigma_T = 24\ MPa$

$F = 12{,}57\ cm^2$

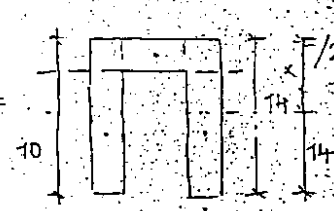
$I_y = 12{,}5\pi\ cm^2$

$\sigma_T = 240\ MPa$

$I_y = 1366{,}7\ cm^4$

$F = 80\ cm^2$

1-1



$F_1 = 2 \cdot x \cdot 2 + 6 \cdot 4 = 40$

$4x = 16$

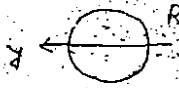
$x = 4$

$S_{pL} = 40 \cdot 2 + 2 \cdot 20 \cdot 5 = 80 + 200 = 280\ cm^3$

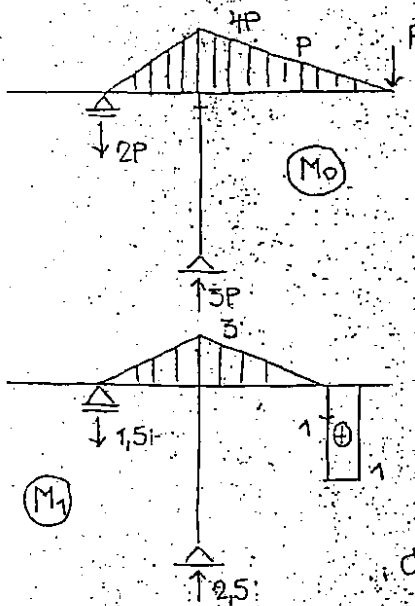
$\frac{I_1}{F_2} = \frac{1366{,}7 \cdot 10^{-8}}{12{,}57 \cdot 10^{-4}} = 0{,}01\ m^2$

$M^* = \sigma_T \cdot S_{pL} = 240 \cdot 10^6 \cdot 280 \cdot 10^{-6} = 67{,}2\ kNm$

2-2



$N^* = \sigma_T \cdot F = 24 \cdot 10^6 \cdot 12{,}57 \cdot 10^{-4} = 30{,}17\ kN$



$\sum M_2 \overset{+}{=} 0 \quad V_1 \cdot 2 + P \cdot 4 = 0$

$V_1 = \frac{-4P}{2} \quad V_1 = -2P$

$\sum M_2 \overset{+}{=} 0 \quad V_1 \cdot 2 + 1 \cdot 3 = 0$

$V_1 = -1{,}5P$

$\sigma_{11} X_1 + \sigma_{10} = 0$

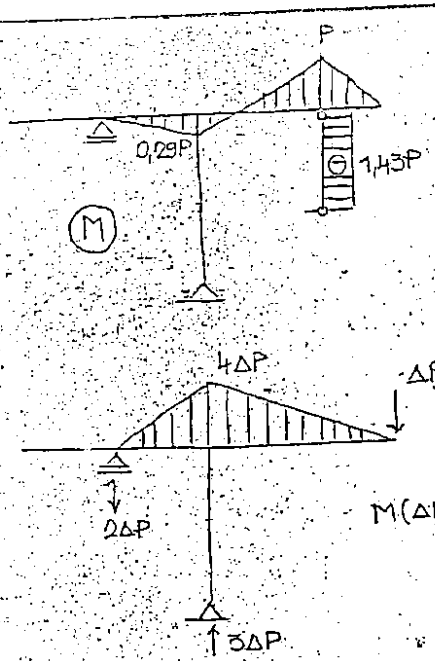
$\sigma_{11} = \frac{1}{EI} \int M_1 M_1 + \frac{1}{EF} \int N_1 N_1$

$\sigma_{11} = \frac{1}{EI} \left(\frac{2}{3} 3 \cdot 3 + \frac{3}{8} 3 \cdot 3 \right) + \frac{0{,}01}{EI} 2 \cdot 1^2 = \frac{15{,}02}{EI}$

$\sigma_{10} = \frac{1}{EI} \int M_1 M_0 + \frac{1}{EF} \int N_1 N_0$

$\sigma_{10} = \frac{1}{EI} \left(\frac{2}{3} 4P \cdot 3 + \frac{3}{6} \left[4P(2 \cdot 3 + 0) + P\left(2 \cdot 0 + 3\right) \right] \right) = \frac{21{,}5P}{EI}$

$X_1 = -\frac{\sigma_{10}}{\sigma_{11}} = -\frac{21{,}5P}{-15{,}02} = 1{,}43P$



$M = M_0 + X_1 M_1$

$1{,}43\,P_1' = M^*$ $\quad P_1' = 21{,}1\ KN$

$P_1'' = M^*$ $\quad P_1'' = 67{,}2\ KN$

$\Downarrow$

ПРВО ДОЛАЗИ ДО ПЛАСТИФИКАЦИОНЕ ШТАПА

$\boxed{P_1 = 21{,}1\ KN}$

САД САБИРАМ M И $M(\Delta P)$

$M(\Delta P)$ $\quad 4\Delta P + (-0{,}29P) = M^* = 67{,}2$

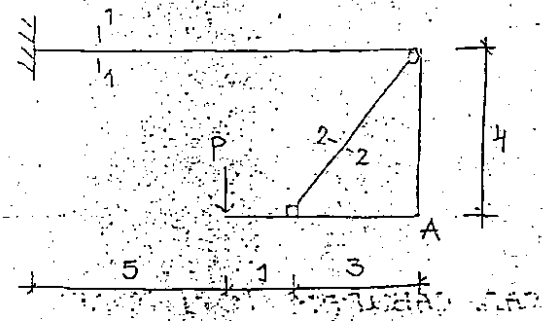
$\Delta P = \dfrac{67{,}2 + 0{,}29 \cdot 21{,}1}{4}$

$\Delta P = 18{,}33\ KN$

$P^* = P_1 + \Delta P = 21{,}1 + 18{,}33 = 39{,}43\ KN$

05.10.2005.

Одредити граничну вредност оптерећења P^+



1–1

$F = 768\ cm^2$

$I_y = 71376\ cm^4$

$W_y^* = 6336\ cm^3$

2–2

$F = 28\ cm^2$

$I_y = 64\ cm^4$

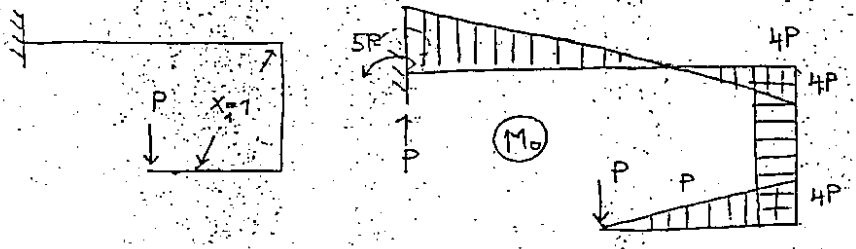
$W_y^* = 36\ cm^3$

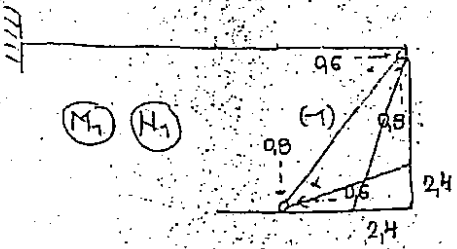
$\sigma_T = 30\ MPa \quad E = 20\ GPa$

ПРЕСЕК 1–1: $M^* = \sigma_T \cdot W_y^* = 30 \cdot 10^6 \cdot 6336 \cdot 10^{-6} = 190{,}08\ kNm$

ПРЕСЕК 2–2: $N^* = F \cdot \sigma_T = 28 \cdot 10^{-4} \cdot 30 \cdot 10^6 = 84\ kN$

$$\frac{I_1}{F_2} = \frac{71376 \cdot 10^{-8}}{28 \cdot 10^{-4}} = 0{,}255\ m^2$$





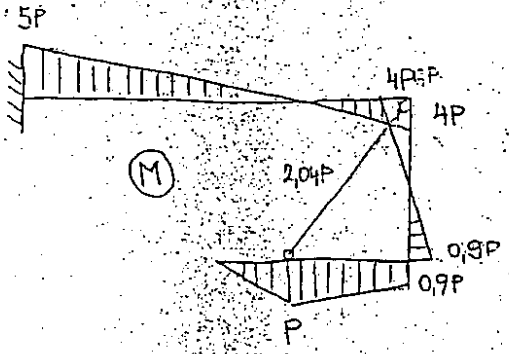
$$\delta_{11} = \frac{1}{EI}\left(\frac{3}{3} \cdot 2{,}4^2 + \frac{4}{3} 2{,}4^2\right) + \frac{0{,}255}{EI} \cdot 5 \cdot 1^2 = \frac{14{,}72}{EI}$$

$$\delta_{10} = \frac{1}{EI}\left(\frac{4}{2} 4P \cdot 2{,}4 + \frac{3}{6}\left[P(2{,}6 + 2{,}4) + 4P(2 \cdot 2{,}4 + 0)\right]\right)$$

$$\delta_{10} = \frac{1}{EI} \cdot 30P$$

$$\delta_{11} X_1 + \delta_{10} = 0 \qquad X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{30}{14{,}72} = -2{,}04P$$

$M = M_0 + X_1 M_1$

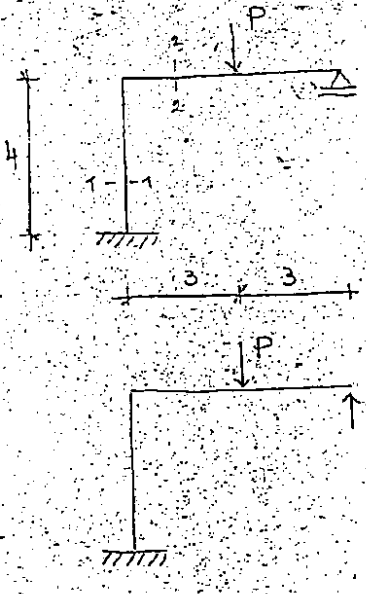


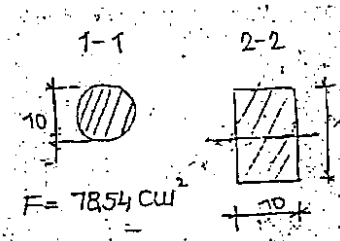
$2{,}04\,P_1' = N^* \qquad P_1' = \frac{84}{2{,}04} = 41{,}18\ KN$

$5P \cdot P_1'' = M^* \qquad P_1'' = \frac{190{,}08}{5} = 38{,}02\ KN \qquad \Rightarrow \underline{\underline{P_1 = 38{,}02\ KN}}$

26.09.2003 W

$\sigma_T = 10\ MPa$





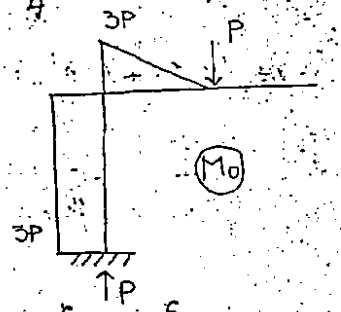
$F = 78{,}54\ cm^2$

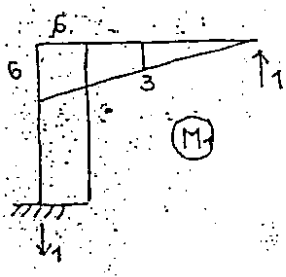
$F = 150\ cm^2$

$I_y = \frac{1}{12}\,10\,15^3 = 2812{,}5\ cm^4$

$S_{pL} = 75\left(\frac{7{,}5}{2}\cdot 2\right) = 562{,}5\ cm^3$

$I_y = \frac{\pi R^4}{4} = 490{,}87\ cm^4$





$2\text{-}2 \quad M_1^* = S_{pL} \cdot \sigma_T = 562{,}5 \cdot 10^{-6} \cdot 10 \cdot 10^6 = 5{,}625\ KNm$

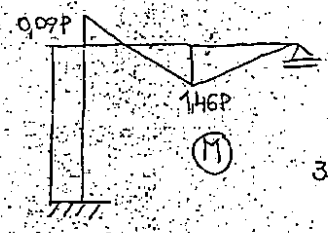
$\frac{I_2}{I_1} = \frac{2812{,}5 \cdot 10^{-8}}{490{,}87 \cdot 10^{-8}} = 5{,}73 \quad I_2 = 5{,}73 I_1 \rightarrow I_1 = \frac{I_2}{5{,}73}$

$\delta_{11} = \frac{1}{EI_1}\,4 \cdot 6^2 + \frac{1}{EI_2}\,\frac{6}{3}\,6^2 = \frac{1}{EI_2}\left(5{,}73 \cdot 4 \cdot 6^2 + 2 \cdot 6^2\right) = \frac{897{,}12}{EI_2}$

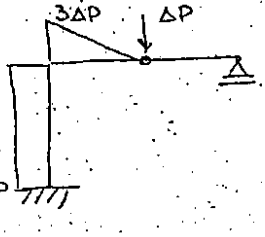
$\delta_{10} = \frac{1}{EI_1}(-4 \cdot 3P \cdot 6) + \frac{1}{EI_2}\left[-\frac{3}{6}\cdot\frac{1}{2}\left[3P(2 \cdot 6 + 3)\right]\right) = \frac{1}{EI_2}\left(-5{,}73 \cdot 4 \cdot 3P \cdot 6 - \frac{3P}{2} \cdot 15\right) = -\frac{435{,}06P}{EI_2}$

$\delta_{11} X_1 + \delta_{10} = 0 \qquad X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{435{,}06P}{897{,}12} = 0{,}485P$

$M = M_0 + X_1 M_1$



$1{,}46\,P_1 = M^* \qquad P_1 = \frac{5{,}625}{1{,}46} = 3{,}85\ KN$

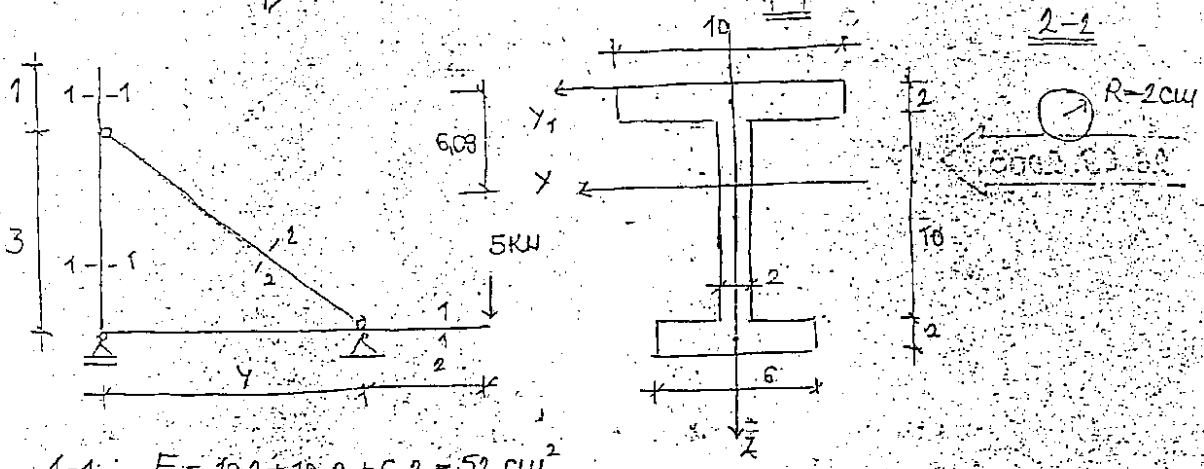


$3\Delta P + 0{,}09 P_1 = M^*$

$\Delta P = \frac{5{,}625 - 0{,}09 \cdot 3{,}85}{3} = 1{,}76$

$P^* = P_1 + \Delta P = 3{,}85 + 1{,}76 = 5{,}61\ KN$

(98) 05.09.2005. P = 5kN E = 210 GPa W = ? w

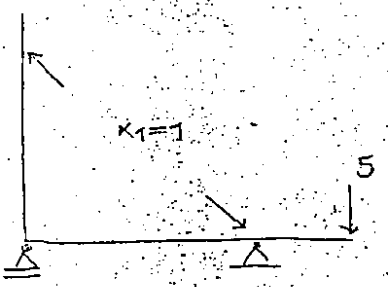


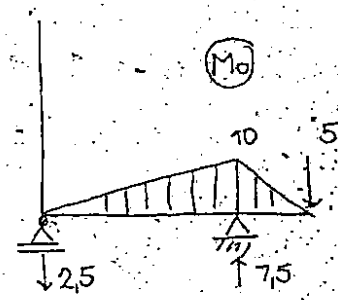
1-1 : $F = 10 \cdot 2 + 10 \cdot 2 + 6 \cdot 2 = 52\ cm^2$

$$z_T = \frac{10 \cdot 2 \cdot 1 + 10 \cdot 2 \cdot 7 + 6 \cdot 2 \cdot 13}{52} = 6{,}08\ cm$$

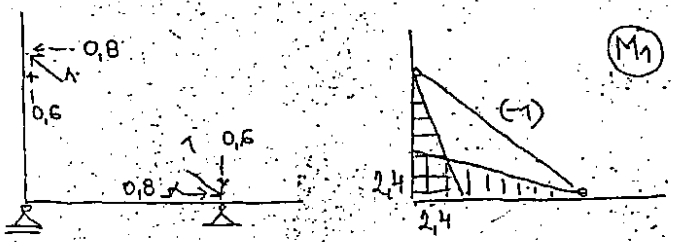
$$I_y = \frac{1}{12} 10 \cdot 2^3 + 5{,}08^2 \cdot 20 + \frac{1}{12} 2 \cdot 10^3 + 0{,}92^2 \cdot 20 + \frac{1}{12} 6 \cdot 2^3 + 6{,}92^2 \cdot 12 = 1285{,}03\ cm^4$$

2-2 : $F = 2^2 U = 12{,}57\ cm^2$ $\quad \frac{I_1}{F_2} = \frac{1285{,}03 \cdot 10^{-8}}{12{,}57 \cdot 10^{-4}} = 0{,}01\ cm^2$





$$\sum M_{+5} = 0$$
$$V_2 \cdot 4 - 5 \cdot 6 = 0$$
$$V_2 = 7{,}5$$

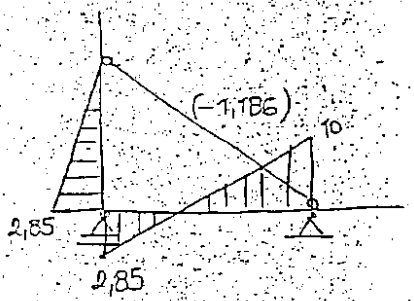


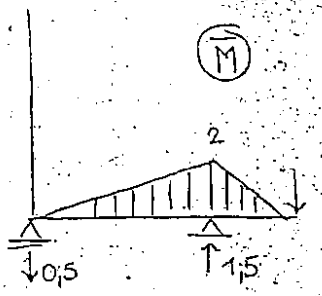
$$\delta_{11} = \frac{1}{EI_1}\left(\frac{4}{3} 2{,}4^2 + \frac{3}{3} 2{,}4^2\right) + \frac{1}{EF_2} 5 \cdot 1^2 = \frac{1}{EI_1} \cdot 13{,}49$$

$$\delta_{10} = \frac{1}{EI_1} \frac{4}{6}\left[10\left(\frac{2{,}0}{0}\right) + 2{,}4\right] = \frac{1}{EI_1} 16$$

$$\delta_{11} X_1 + \delta_{10} = 0 \qquad X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{16}{13{,}49} = -1{,}186$$

$M = M_0 + X_1 M_1$





$\sum M_{\curvearrowright} = 0$

$1 \cdot 6 - V_2 \cdot 4 = 0$

$V_2 = 1,5$

$V_1 + V_2 - 1 = 0$

$V_1 = 1 - 1,5$

$V_1 = -0,5$

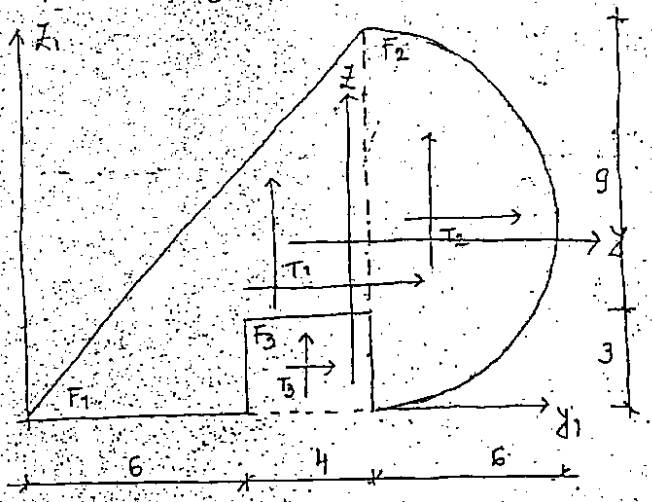
$$V = \frac{1}{EI_1} \int M\overline{M}$$

$$V = \frac{1}{EI_1} \frac{4}{6} \left[-2,85 (2 \cdot 0 + 2) + 10 (2 \cdot 2 + 0) \right]$$

$$V = \frac{1}{EI_1} 30,47 = \frac{30,47 \cdot 10^3}{210 \cdot 10^6 \cdot 1285,03 \cdot 10^{-8}} = 0,11 \text{ m}$$

ВЕЖБЕ

За задати пресек одредити:
главне централне моменте инерције
правце главних оса
нацртати Моров круг инерције и
елипсу инерције



$F_1 = \frac{1}{2} \cdot 10 \cdot 12 = 60\ cm^2$

$F_2 = \frac{6^2 \pi}{2} = 56{,}55\ cm^2$

$F_3 = 4 \cdot 3 = 12\ cm^2$

$F = F_1 + F_2 - F_3 = 104{,}55\ cm$

$T_1(6{,}67;\ 4)$

$T_2(12{,}55;\ 6)$

$T_3(8;\ 1{,}5)$

$R^4 \cdot 2 \cdot 0{,}05 \cdot 22$

$y_T = \frac{6{,}67 \cdot 60 + 12{,}55 \cdot 56{,}55 - 8 \cdot 12}{104{,}55} = 9{,}7\ cm \quad \therefore \quad T(9{,}7;\ 5{,}37)$

$z_T = \frac{4 \cdot 60 + 6 \cdot 56{,}55 - 1{,}5 \cdot 12}{104{,}5} = 5{,}37\ cm$

$935{,}27\ cm^4$

$I_y = \frac{1}{36} 10 \cdot 12^3 + 1{,}37^2 \cdot 60 + \frac{6^4 \pi}{8} + 0{,}63^2 \cdot 56{,}55 - \frac{1}{12} 4 \cdot 3^3 - 3{,}87^2 \cdot 12 = 11826{,}85\ cm^4$

$I_z = \frac{1}{36} 10^3 \cdot 12 + 3{,}03^2 \cdot 60 + 6^4 \cdot 2 \cdot 0{,}05488 + 2{,}85^2 \cdot 56{,}55 - \frac{1}{12} 4^3 \cdot 3 - 1{,}7^2 \cdot 12 = 1435{,}08\ cm^4$

$I_{yz} = \frac{1}{72} 10^2 \cdot 12^2 + (-3{,}03)(-1{,}37) \cdot 60 + 0{,}63 \cdot 2{,}85 \cdot 56{,}55 - 3{,}87 \cdot 1{,}7 \cdot 12 = 471{,}65\ cm^4$

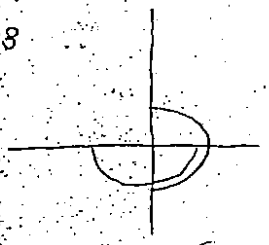
$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$

$I_1 = 11848{,}21\ cm^4$

$I_2 = 1413{,}72\ cm^4$

$tg\,2\alpha = \frac{-2 I_{yz}}{I_y - I_z} = \frac{-2 \cdot 471{,}65}{11826{,}85 - 1435{,}08} = -0{,}0908$ ($\sin 2\alpha < 0$, $\cos 2\alpha > 0$)

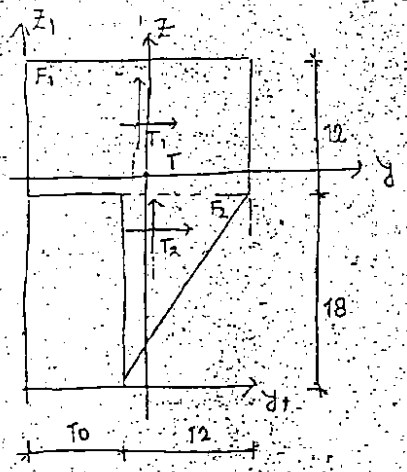
$\alpha = \frac{1}{2}(360 + \operatorname{arctg}(\cdot)) = 177{,}41°$



$i_1 = \sqrt{\frac{11848{,}21}{104{,}55}} = 10{,}64\ cm \qquad i_2 = \sqrt{\frac{1413{,}72}{104{,}55}} = 3{,}66\ cm$

(102) (18.06.2004)

1.1. ЗА ДАТИ ПРЕСЕК ОДРЕДИТИ ПРАВАЦ ТЕЖИШНЕ ОСЕ КОЈИ ОДГОВАРА НАЈВЕЋЕМ МОМЕНТУ ИНЕРЦИЈЕ И ВРЕДНОСТИ ГЛАВНИХ ЦЕНТРАЛНИХ МОМЕНАТА ИНЕРЦИЈЕ



$F_1 = 22 \cdot 12 = 264\ cm^2$

$F_2 = \frac{1}{2} 18 \cdot 12 = 108\ cm^2$

$F = 372\ cm^2$

$T_1(11; 24)$

$T_2(14; 12)$ — " —

$y_T = \frac{11 \cdot 264 + 14 \cdot 108}{372} = 11,87\ cm$ $\quad T(11,87; 20,52)$

$z_T = \frac{24 \cdot 264 + 12 \cdot 108}{372} = 20,52\ cm$

$I_y = \frac{1}{12} 22\, 12^3 + 3,48^2 \cdot 264 + \frac{1}{36} \cdot 12\, 18^3 + 8,52^2 \cdot 108 = 16\,148,97\ cm^4 \Rightarrow I_{max}$

$I_z = \frac{1}{12} 22^3 \cdot 12 + 0,87^2 \cdot 264 + \frac{1}{36} \cdot 12^3 \cdot 18 + 2,13^2 \cdot 108 = 12\,201,87\ cm^4 \Rightarrow I_{min}$

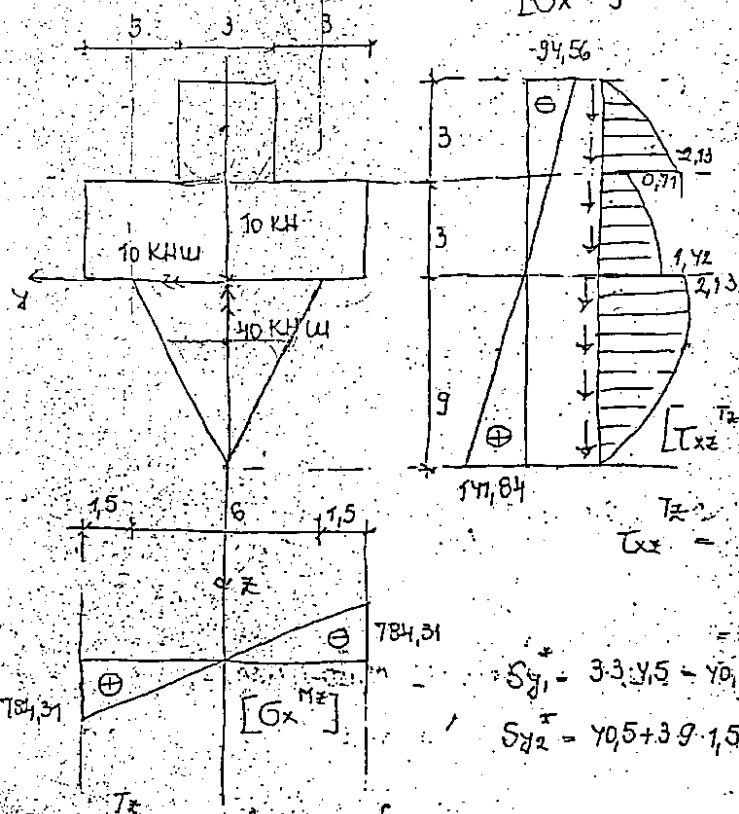
$I_{yz} = -3,48\ 0,87\ 264 + \frac{1}{72} \cdot 18^2\, 12^2 - 8,52 \cdot 2,13 \cdot 108 = -2\,111,23\ cm^4$

$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$ $\quad I_1 = 17\,065,37\ cm^4$

$I_2 = 11\,285,34\ cm^4$

02.04.2004

$I_y = 637,5\ cm^4$ $I_z = 229,5\ cm^4$



$$\sigma_x^{M_z} = \frac{M_z \cdot y}{I_z} = \frac{40 \cdot 10^3}{229,5\ 10^{-8}} \cdot 4,5 \cdot 10^{-2}$$

$$\sigma_x^{M_z} = 784,31\ MPa$$

$$\sigma_x^{M_y} = \frac{M_y \cdot z}{I_y} = \frac{10}{637,5} \cdot 6 \cdot 10^9 = 94,56\ MPa$$

$$\tau_{xz} = \frac{T_z \cdot S_y}{I_y \cdot b(z)}$$

$$S_{y_1}^* = 3 \cdot 3 \cdot 3,5 = 70,5\ cm^3$$

$$S_{y_2}^* = 70,5 + 3 \cdot 9 \cdot 1,5 = 81\ cm^3$$

$$\tau_{xz_1}^{T_z} = \frac{10 \cdot 10^3 \cdot 70,5 \cdot 10^{-6}}{637,5 \cdot 10^{-8} \cdot 3 \cdot 10^{-2}} = 2,13\ MPa$$

$$\tau_{xz_2}^{T_z} = \frac{10 \cdot 10^3 \cdot 70,5 \cdot 10^{-6}}{637,5 \cdot 10^{-8} \cdot 9 \cdot 10^{-2}} = 0,71\ MPa$$

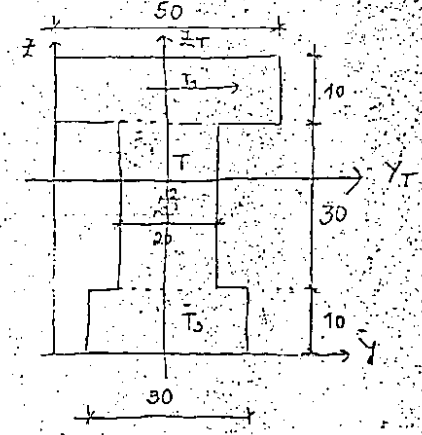
$$\tau_{xz_3}^{T_z} = \frac{10 \cdot 10^3 \cdot 81 \cdot 10^{-6}}{637,5 \cdot 10^{-8} \cdot 9 \cdot 10^{-2}} = 1,42\ MPa$$

$$\tau_{xz_4}^{T_z} = \frac{10 \cdot 10^3 \cdot 81 \cdot 10^{-6}}{637,5 \cdot 10^{-8} \cdot 6 \cdot 10^{-2}} = 2,13\ MPa$$

(104)

05. 09. 2005.

W 7.1. Одредити вредности главних централних момената инерције



$F_1 = 10 \cdot 50 = 500\ cm^2$
$F_2 = 20 \cdot 30 = 600\ cm^2$
$F_3 = 10 \cdot 30 = 300\ cm^2$
$F = 1400\ cm^2$

$T_1(25; 45)$ $T_2(25; 25)$ $T_3(25; 5)$

$y_T = 25\ cm$

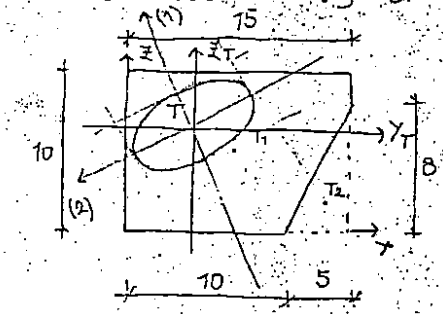
$z_T = \frac{500 \cdot 45 + 600 \cdot 25 + 300 \cdot 5}{1400} = 27,86\ cm$

$T(25; 27,86)$

$I_y = \frac{1}{12} 50 \cdot 10^3 + 500 \cdot 17,14^2 + \frac{1}{12} 20 \cdot 30^3 + 600 \cdot 2,86^2 + \frac{1}{12} 30 \cdot 10^3 + 300 \cdot 22,86^2 = 360238,11\ cm^4$

$I_z = \frac{1}{12} 50^3 10 + \frac{1}{12} 20^3 30 + \frac{1}{12} 30^3 10 = 146\,666,67\ cm^4$

W 7.2. Одредити полупречник елипсе инерције и нацртати елипсу инерције



$F_1 = 15 \cdot 10 = 150\ cm^2$
$F_2 = \frac{1}{2} 8 \cdot 5 = 20\ cm^2$
$F = 150 - 20 = 130\ cm^2$

$T_1(7,5; 5)$ $T_2(13,33; 2,67)$

$y_T = \frac{150 \cdot 7,5 - 20 \cdot 13,33}{130} = 6,6\ cm$

$z_T = \frac{150 \cdot 5 - 20 \cdot 2,67}{130} = 5,36\ cm$

$T(6,6; 5,36)$

$I_y = \frac{1}{12} 15\,10^3 + 150 \cdot 0,36^2 - \frac{1}{36} 5 \cdot 8^3 - 20 \cdot 2,69^2 = 1053,61\ cm^4$

$I_z = \frac{1}{12} 15^3 10 + 150 \cdot 0,9^2 - \frac{1}{36} 5^3 8 - 20 \cdot 6,73^2 = 2000,36\ cm^4$

$I_{yz} = 150(-0,36 \cdot 0,9) - \frac{1}{72} 5^2 8^2 + 20 \cdot 2,69 \cdot 6,73 = 291,25\ cm^4$

$I_{1,2} = \frac{1053,61 + 2000,36}{2} \pm \sqrt{\left(\frac{1053,61 - 2000,36}{2}\right)^2 + 291,25^2}$

$I_1 = 2082,78\ cm^4$ $I_2 = 971,19\ cm^4$

$i_1 = \sqrt{\frac{2082,78}{130}} = 4\ cm$ $i_2 = \sqrt{\frac{971,19}{130}} = 2,73$

$tg\,2\alpha = \frac{-2 \cdot 291,25}{-946,75}$ (<0)

$\alpha = \frac{1}{2}(180 + arctg(\ )) = 105,8°$ (<0)

1.3. Одредити интензитет тоталног напона за раван са нормалом n [$\vec{p}^{(n)}$], ако је задат тензор напона и правац нормале n

$$\vec{n} = \frac{1}{\sqrt{2}}\vec{e}_1 - \frac{1}{\sqrt{2}}\vec{e}_2$$

$$S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \text{MPa} \qquad \underline{\underline{p_i^{(n)} = \sigma_{ij} n_j}}$$

$$p_1^{(n)} = \sigma_{11} n_1 + \sigma_{12} n_2 + \sigma_{13} n_3$$

$$p_1^{(n)} = 1 \cdot \frac{1}{\sqrt{2}} + 2\left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{2}} = -1/\sqrt{2}$$

$$p_2^{(n)} = 2\frac{1}{\sqrt{2}} - 4\frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} - \frac{4}{\sqrt{2}} = -\frac{2}{\sqrt{2}}$$

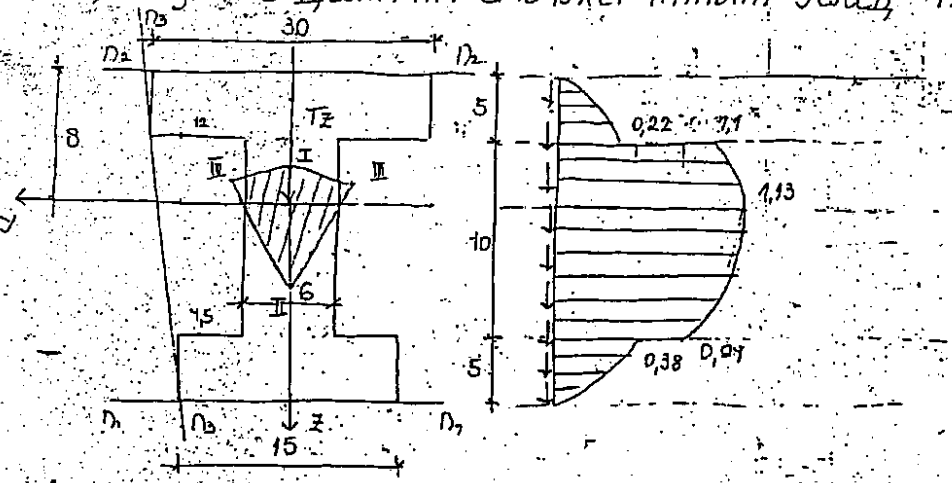
$$p_3^{(n)} = 3\frac{1}{\sqrt{2}} - 6\frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}} - \frac{6}{\sqrt{2}} = -\frac{3}{\sqrt{2}}$$

$$\vec{p}^{(n)} = \left\{-\frac{1}{\sqrt{2}}; -\frac{2}{\sqrt{2}}; -\frac{3}{\sqrt{2}}\right\}$$

$$|\vec{p}^{(n)}| = \sqrt{\frac{1}{2} + \frac{4}{2} + \frac{9}{2}} = \sqrt{7}$$

1.4 a) Нацртати језгро пресека, задато је:
$I_y = 12514{,}8\ cm^4$ $I_z = 12\,836{,}3\ cm^4$ $F = 285\ cm^2$

b) Нацртати дијаграм смичућег напона услед $T_z = 10$ kN



$$i_y^2 = \frac{I_y}{F} = \frac{12\,514{,}8}{285} = 43{,}91\ cm^2 \qquad i_z^2 = \frac{I_z}{F} = \frac{12\,836{,}3}{285} = 45{,}04\ cm^2$$

$\frac{n_1 - n_1}{(I)}$ $p_y = \infty$ $p_z = -12$

$$e_y = -i_z^2/p_y = 0$$

$$e_z = -\frac{i_y^2}{p_z} = -43{,}91/12 = -3{,}66$$

$\frac{n_2 - n_2}{(II)}$ $p_y = \infty$ $p_z = -8$

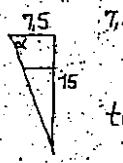
$$e_y = 0$$

$$e_z = -i_y^2/p_z = -43{,}91/-8 = 5{,}49$$

$\frac{n_3 - n_3}{(III, IV)}$ $p_y = 13{,}5$ $p_z = 27$

$$e_y = -i_z^2/p_y = -3{,}37$$

$$e_z = -i_y^2/p_z = -1{,}63$$



$7{,}5 : 15 = x : 12$

$x = 6$

$\operatorname{tg}\alpha = \frac{15}{7{,}5}$ $\alpha = 63{,}43°$

$\operatorname{tg}\alpha = \frac{x}{13{,}5}$

$x = 27$

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$$\tau^{T_z} = \frac{T_z \cdot S_y}{I_y \, b(z)}$$

$S_{y_1} = 30 \cdot 5 \cdot 5{,}5 = 825 \text{ cm}^3$

$S_{y_2} = 825 + 3 \cdot 6 \cdot 1{,}5 = 852 \text{ cm}^3$

$S_{y_3} = 852 - 7 \cdot 6 \cdot 3{,}5 = 705 \text{ cm}^3$

$$\tau_1 = \frac{10 \cdot 10^3 \cdot 825 \cdot 10^{-6}}{12514{,}8 \cdot 10^{-8} \cdot 30 \cdot 10^{-2}} = 0{,}22 \text{ MPa}$$

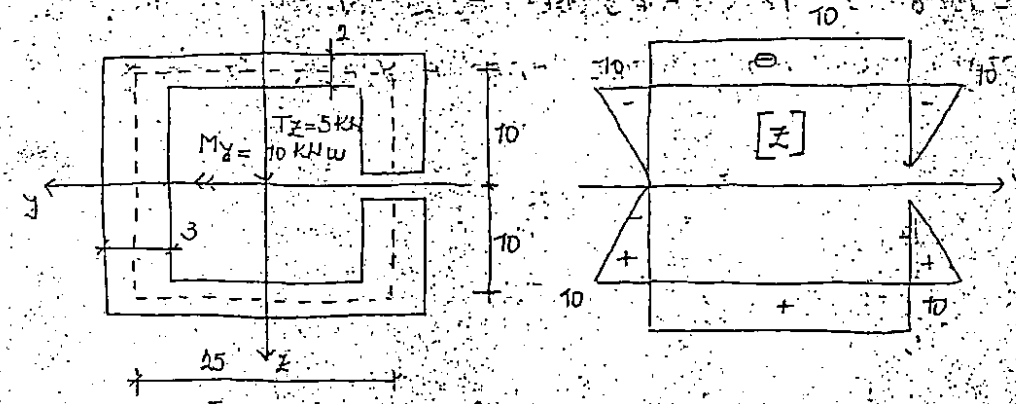
$$\tau_2 = \frac{10 \cdot 825 \cdot 10^7}{12514{,}8 \cdot 6} = 1{,}1 \text{ MPa}$$

$$\tau_3 = \frac{10 \cdot 852 \cdot 10^7}{12514{,}8 \cdot 6} = 1{,}13 \text{ MPa}$$

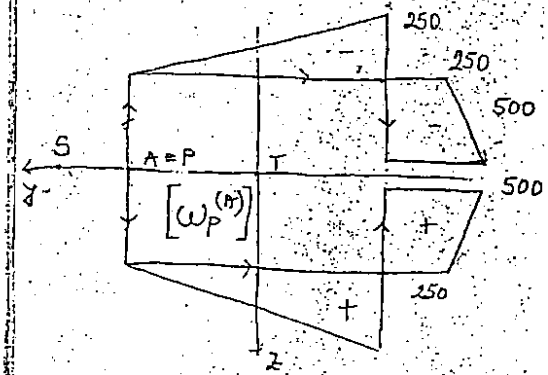
$$\tau_4 = \frac{10 \cdot 705 \cdot 10^7}{12514{,}8 \cdot 6} = 0{,}94 \text{ MPa}$$

$$\tau_5 = \frac{10 \cdot 705 \cdot 10^7}{12514{,}8 \cdot 15} = 0{,}38 \text{ MPa}$$

1.5. НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ ПРЕСЕЧНИХ СИЛА КОЈЕ ДЕЛУЈУ У ТЕЖИШТУ И ОДРЕДИТИ МАКСИМАЛНУ ВРЕДНОСТ КОМПОНЕНТАЛНОГ СМИЧУЋЕГ НАПОНА $\tau_{xz,max}$ И ТАЧКУ У КОЈОЈ СЕ ЈАВЉА



$$I_y = 2\left[25 \cdot 10^2 \cdot 2 + \frac{10}{3} \cdot 10^2 \cdot 3 \cdot 2\right] = 14\,000 \text{ cm}^4$$



$$I_{z\omega_P}^{(A)} = 2 \cdot \left[\frac{25}{2} \cdot 10 \cdot 250 \cdot 2 + \right.$$

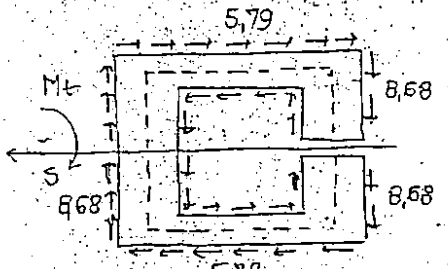
$$\left. + \frac{10}{6}\left(-10\left(2 \cdot (-250) - 500\right)3\right)\right] = 225\,000 \text{ cm}^4$$

$$y_S = y_P + \frac{I_{z\omega_P}^{(A)}}{I_y} = 12{,}5 + \frac{225\,000}{14\,000} = \boxed{28{,}57 \text{ cm}}$$

$M_t = T_z \cdot y_s = 5\ KN \cdot 28{,}57 \cdot 10^{-2}\ m = 1{,}428\ KN\,m$

$I_t = \frac{1}{3} \sum b_i t_i^3 = \frac{1}{3}(2 \cdot 20 \cdot 3^3 + 2 \cdot 25 \cdot 2^3) = 493{,}33\ cm^4$

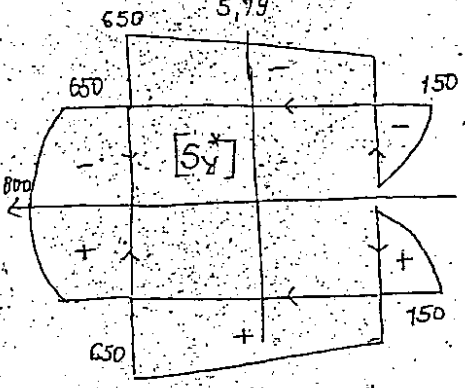
$\tau^{M_t} = \frac{M_t}{I_t} t_i = \frac{1{,}428 \cdot 10^3}{493{,}33 \cdot 10^{-8}} \cdot t_i \cdot 10^{-2}$



τ^{M_t} [MPa]

$\tau = \frac{T_z \cdot S_y^*}{I_y \cdot b(z)}$

стрелище?

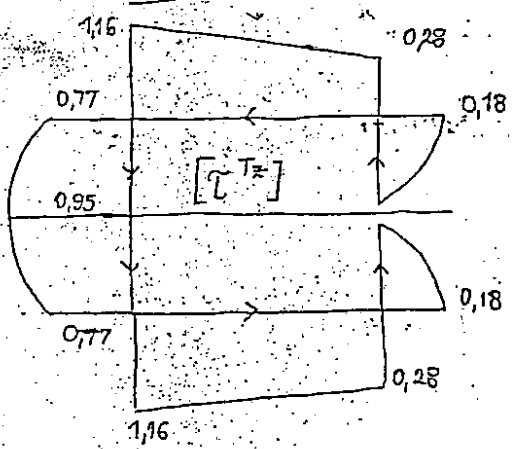


$S_y^* = t \int_s z\,ds$

$S_{y1} = \frac{1}{2} \cdot 10 \cdot 10 \cdot 3 = 150$

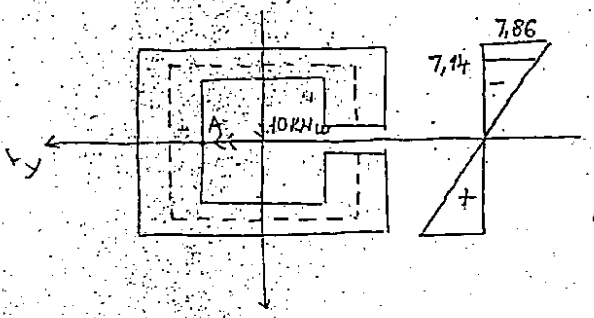
$S_{y2} = 150 + 25 \cdot 10 \cdot 2 = 650$

$S_{y3} = 650 + \frac{1}{2} \cdot 10 \cdot 10 \cdot 3 = 800$



$\tau_1^{T_z} = -\frac{5 \cdot 10^3 (-150 \cdot 10)^{-6}}{14\,000 \cdot 10^{-8} \cdot 3 \cdot 10^{-2}} = 0{,}18\ MPa$

$\sigma_x = \frac{M_y}{I_y} \cdot z = \frac{10 \cdot 10^3}{14\,000 \cdot 10^{-8}} \cdot z \cdot 10^{-2} =$



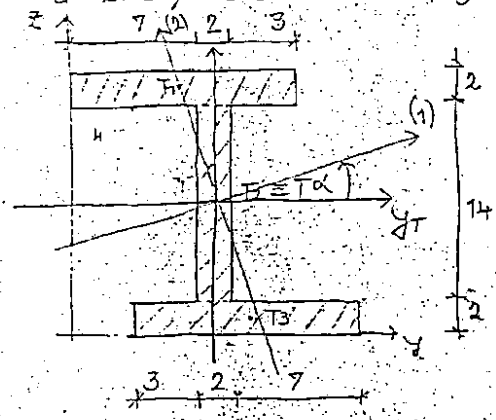
[σx]

$\tau_{x,max} = 8{,}68 + 0{,}95 = 9{,}63\ MPa$

Где цртам тачку А?

(108) 02.04.2004.

1.1. ОДРЕДИТИ ПРАВЦЕ ГЛАВНИХ ЦЕНТРАЛНИХ ОСА ИНЕРЦИЈЕ И ВРЕДНОСТИ ГЛАВНИХ ЦЕНТРАЛНИХ МОМЕНАТА ИНЕРЦИЈЕ



$F_1 = 12 \cdot 2 = 24\ cm^2$
$F_2 = 14 \cdot 2 = 28\ cm^2$
$F_3 = 12 \cdot 2 = 24\ cm^2$
$F = 76\ cm^2$

$T_1(6;17)$ $T_2(8;9)$ $T_3(10;1)$

$T(8;9)$

$y_T = \frac{24 \cdot 6 + 28 \cdot 8 + 24 \cdot 10}{76} = 8\ cm$ $z_T = \frac{24 \cdot 17 + 28 \cdot 9 + 24 \cdot 1}{76} = 9\ cm$

$I_y = \frac{1}{12} 12 \cdot 2^3 + 8^2 \cdot 24 + \frac{1}{12} 2 \cdot 14^3 + \frac{1}{12} 12 \cdot 2^3 + 8^2 \cdot 24 = 3545{,}33\ cm^4$

$I_z = 2\left(\frac{1}{12} 12^3 \cdot 2 + 2^2 \cdot 24\right) + \frac{1}{12} 2 \cdot 14^3 = 777{,}33\ cm^4$

$I_{yz} = (-2) \cdot 8 \cdot 24 + 2 \cdot (-8) \cdot 24 = -768\ cm^4$

$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$

$I_1 = 3744{,}14\ cm^4$

$I_2 = 578{,}52\ cm^4$

$tg\,2\alpha = \frac{-2 I_{yz}}{I_y - I_z} = \frac{-2 \cdot (-768)}{3575{,}33 - 777{,}33}$ ($I_y > 0$, $I_z > 0$)

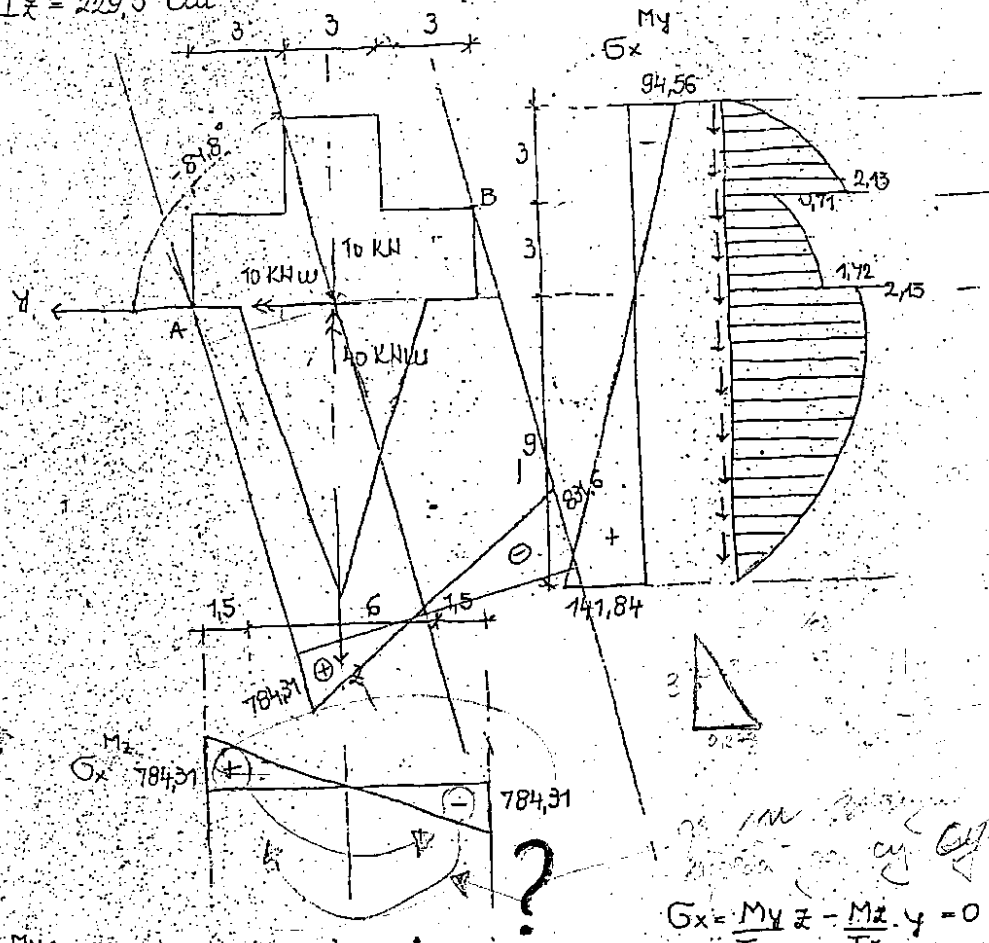
$\alpha = \frac{1}{2} arctg(\cdot)$

$\alpha = 14{,}5°$

1.2. Нацртати дијаграме компоненталних напона услед задатих сила. У пресеку одредити тачку у којој се јавља $\tau_{xz,max}$ и у тој написати тензор напона.

$I_y = 634,5\ cm^4$

$I_z = 229,5\ cm^4$



$$\sigma_x^{M_y} = \frac{M_y}{I_y} \cdot z_{max} = \frac{10 \cdot 10^3 \cdot 6 \cdot 10^{-2}}{634,5 \cdot 10^{-8}} = 94,56\ MPa$$

$$\sigma_x^{M_z} = \frac{M_z}{I_z} \cdot y_{max} = \frac{-40 \cdot 10^3 \cdot 4,5 \cdot 10^{-2}}{229,5 \cdot 10^{-8}} = 784,31\ MPa$$

$$\tau^{T_z} = \frac{T_z \cdot S_y^*}{I_y \cdot b}$$

$S_{y1} = 3 \cdot 3 \cdot 4,5 = 40,5\ cm^3$

$S_{y2} = 40,5 + 3 \cdot 9 \cdot 1,5 = 81\ cm^3$

$A(4,5;0)$

$B(-4,5;-3)$

$\sigma_x = \frac{-40 \cdot 10^3}{229,5 \cdot 10^{-8}} \cdot 4,5 \cdot 10^{-2} = 784,31$

$\sigma_x = \frac{10 \cdot 10^3}{634,5 \cdot 10^{-8}} \cdot (-3 \cdot 10^{-2}) + \frac{40 \cdot 10^3}{229,5 \cdot 10^{-8}} \cdot (-4,5 \cdot 10^{-2}) = -831,6$

$$\sigma_x = \frac{M_y}{I_y} z - \frac{M_z}{I_z} \cdot y = 0$$

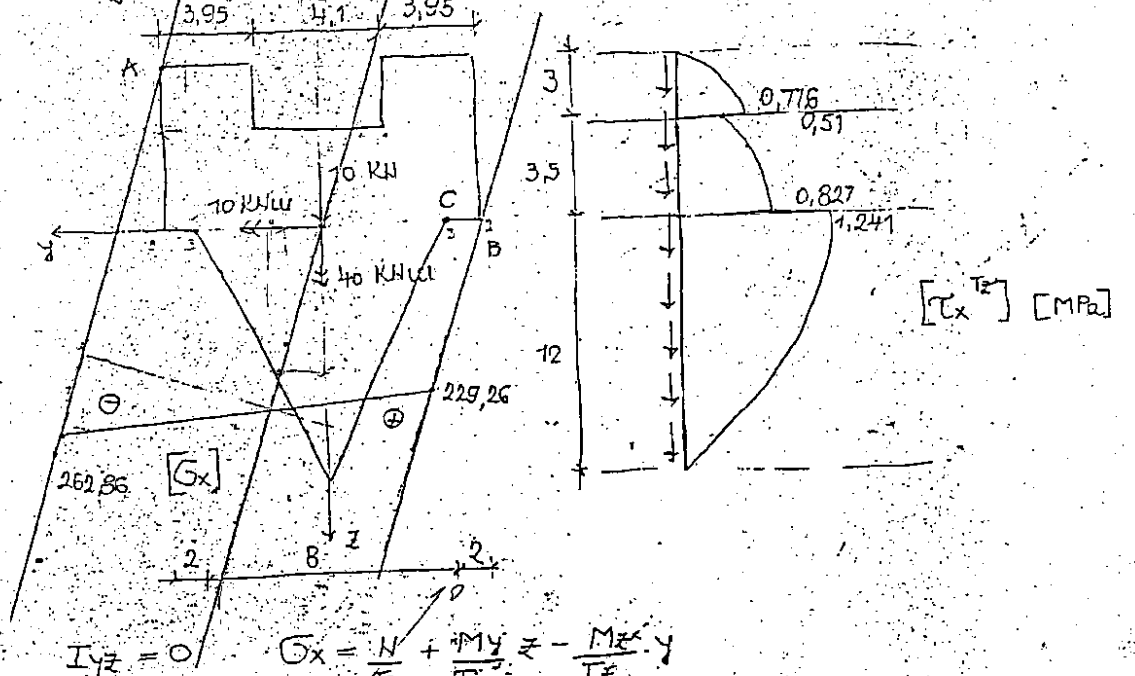
$$z = \frac{M_z}{M_y} \frac{I_y}{I_z} \cdot y = tg\alpha \cdot y$$

$$tg\alpha = \frac{-40}{10} \cdot \frac{634,5}{229,5} = -11,1$$

$\alpha = -84,8°$

НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ N СИЛА У ПРЕСЕКУ. ОДРЕДИТИ ТАЧКУ У КОЈОЈ СЕ ЈАВЉА τ_x^{max} И У ЊОЈ НАПИСАТИ ТЕНЗОР НАПОНА

$I_y = 1933,8\ cm^4$ $I_z = 1046,8\ cm^4$



$I_{yz} = 0$ $\quad \sigma_x = \frac{N}{F} + \frac{M_y}{I_y} z - \frac{M_z}{I_z} y$

$\sigma_x = \frac{10 \cdot 10^2}{1933,8} z - \frac{40 \cdot 10^2}{1046,8} y$

$\sigma_x = 0,517\, z - 3,821\, y \quad [kN/cm^2] \times 10 \Rightarrow [MPa]$

$5,17\, z - 38,21\, y = 0$

$z = 7,391\, y$

y	0	1
z	0	7,391

A(6, -6,5) B(-6, 0)

$\sigma_x^A = 5,17 \cdot (-6,5) - 38,21 \cdot 6 = -262,86\ MPa$

$\sigma_x^B = -38,21 \cdot (-6) = 229,26\ MPa$

$\tau^{Tz} = \frac{T_z \cdot S_y}{I_y \cdot b(z)} = \frac{10 \cdot S_y}{1933,8 \cdot b(z)}$ [$\times 10\ MPa$]

$S_{y1} = 3,95 \cdot 3,5 \cdot 2 = 118,5\ cm^3$

$S_{y2} = 118,5 + 3,5 \cdot 12 \cdot 1,75 = 192\ cm^3$

$\Rightarrow C(-4, 0)$ $\quad S_C = \begin{bmatrix} 152,8 & 12 \cdot 1,24 \\ 10 & 0 & 0 \\ 1,24 & 0 & 0 \end{bmatrix}$

$\tau_1^{Tz} = 0,0517 \cdot 118,5 / 7,9 = 0,776\ MPa$

$\tau_2^{Tz} = 0,0517 \cdot 118,5 / 12 = 0,51\ MPa$

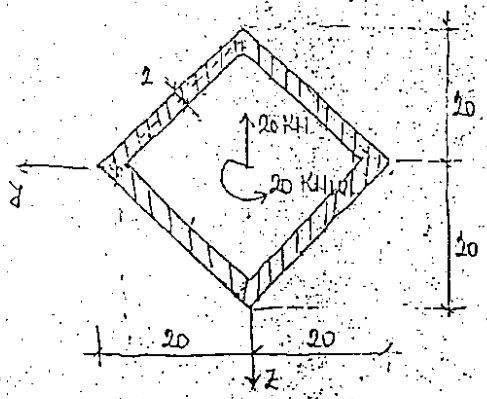
$\tau_3^{Tz} = 0,0517 \cdot 192 / 12 = 0,827\ MPa$

$\tau_4^{Tz} = 0,0517 \cdot 192 / 8 = 1,241\ MPa$

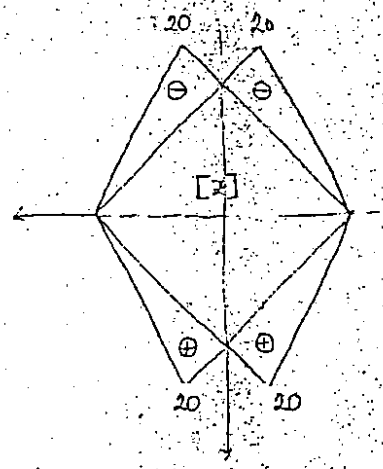
$\sigma_x^C = 5,17 \cdot 0 + 38,21 \cdot 4 = 152,8\ MPa$

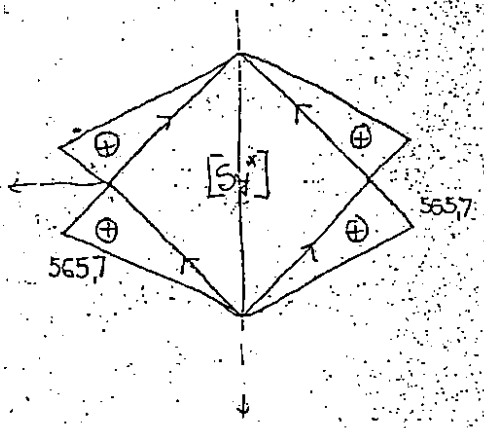
(11.2) НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА У ПРЕСЕКУ, УСЛЕД ЗАДАТЕ ТРАСФЕРЗАЛНЕ СИЛЕ И МОМЕНТА ТОРЗИЈЕ

$I_y = 24395{,}5\ cm^4$ $I_z = 24395{,}5\ cm^4$

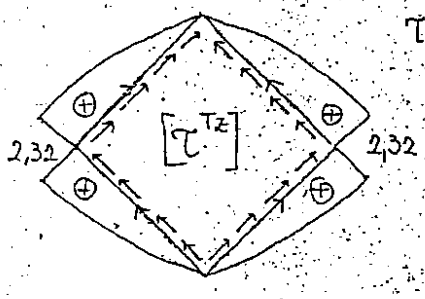


$$\tau^{Tz} = -\frac{T_z S_y^*}{I_z \cdot t}$$



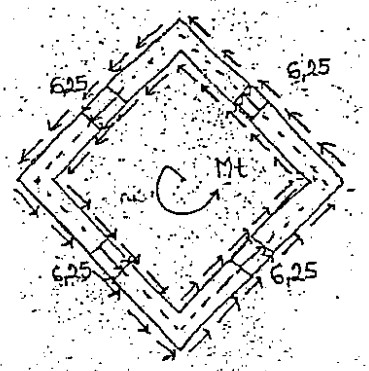


$$\tau_x = -\frac{(-20)\cdot 565{,}7}{24395{,}5 \cdot 2} \cdot 10 = 2{,}32\ MPa$$



$$\tau^{Mt} = \frac{Mt}{2F \cdot ti} = \frac{20 \cdot 10^2}{2 \cdot 2 \cdot 140{,}20 \cdot 2} = 6{,}25\ MPa$$

ОВО NIJE F



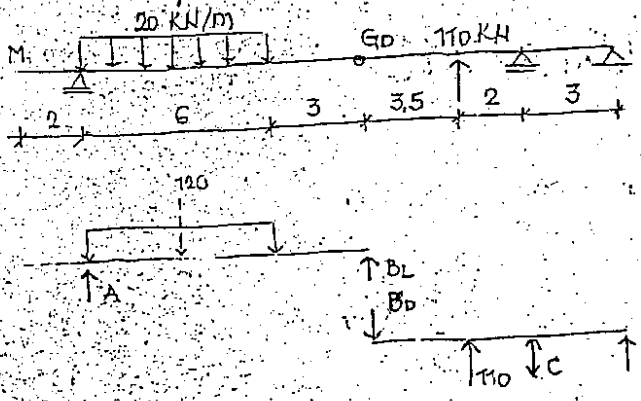
$[\tau^{Mt}]$

Mohr-Maxwelova analogija

02.04.2004. W ? zaHN?

Користећи Mohr-Maxwell-ову аналогију одредити вертикално померање тачке M и нагиб тангенте на еластичну линију у тачки G_D

$EI = const.$

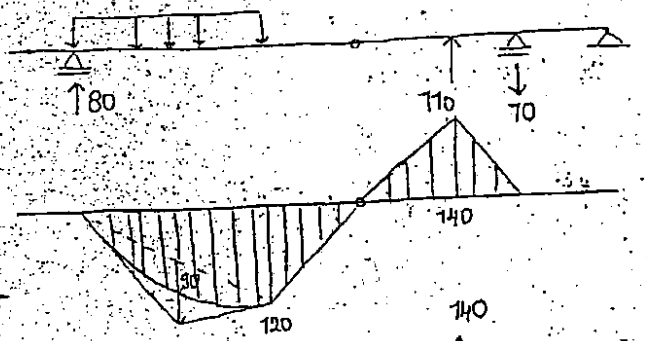


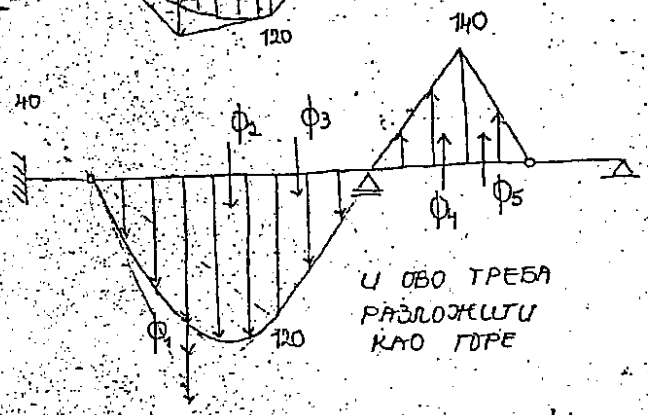
$\Sigma M_B = 0 \quad A \cdot 9 - 120 \cdot 6 = 0$

$A = 80 \qquad A - 120 + B_L = 0 \qquad B_L = 40$

$\Sigma M_D = 0 \quad -40 \cdot 8,5 + 110 \cdot 5 + C \cdot 3 = 0 \qquad C = -70$

$-40 + 110 - 70 + D = 0 \qquad D = 0$





И ово треба разложити као горе

$\phi_1 = \frac{2}{3} f L = \frac{2}{3} 90 \cdot 6 = 360 \text{ KN } w^2$

$\phi_2 = \frac{1}{2} 120 \cdot 6 = 360 \text{ KN } w^2$

$\phi_3 = \frac{1}{2} 3 \cdot 120 = 180 \text{ KN } w^2$

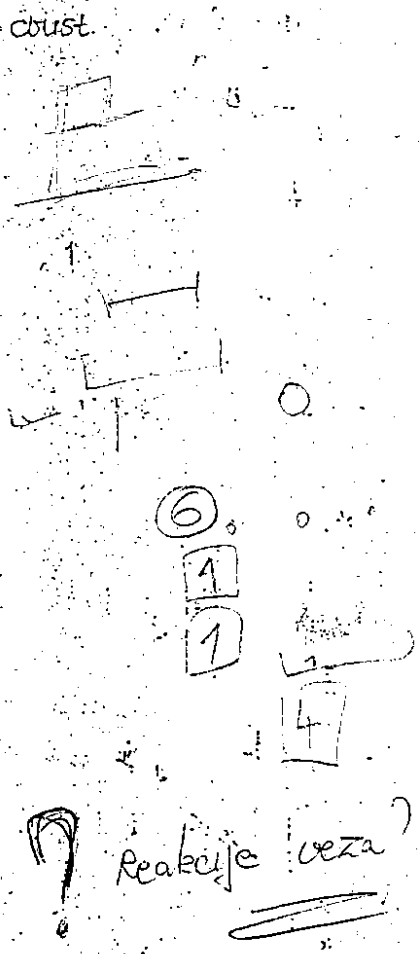
$\phi_4 = \frac{1}{2} 3,5 \cdot 140 = 245$

$\phi_5 = \frac{1}{2} 2 \cdot 140 = 140$

$120 : 6 = x : 2 \qquad x = 40$

Померање у M : $W_M = \frac{M_M}{EI} = \frac{40}{EI} w$

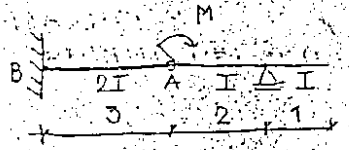
Нагиб у G_D : $\varphi_{G^D} = \frac{T_G{}^D}{EI} = -\frac{385}{EI}$

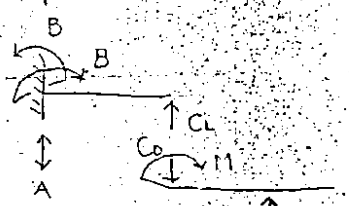


Реакције веза

(114) Користећи Мор-Maxwell-ову аналогију одредити величину момента M при коме је угиб у тачки A $w_A \leq 2$ mm

$E = 200$ GPa $I = 700$ cm^4





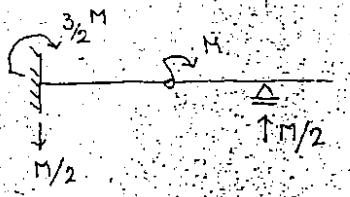
$\overset{+}{\curvearrowright} \Sigma M_{C_D} = 0 \quad M - D \cdot 2 = 0 \quad D = M/2$

$C_D = M/2$

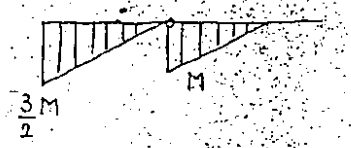
$\Sigma Y = 0 \quad A + C_L = 0 \quad A = -M/2$

$\overset{+}{\curvearrowright} \Sigma M_{C_L} = 0 \quad -A \cdot 3 - B = 0$

$B = -M/2 \cdot 3 \quad B = -\frac{3}{2}M$

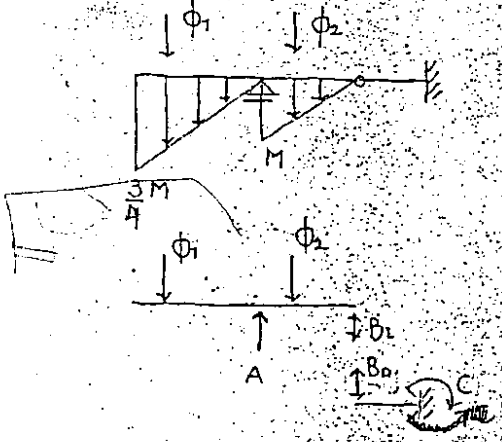


(M)



$\phi_1 = \frac{1}{2}\frac{3}{4}M \cdot 3 = \frac{9}{8}M$

$\phi_2 = \frac{1}{2} 2M = M$



$\overset{+}{\curvearrowleft} \Sigma M_{B_L} = 0 \quad -\frac{9}{8}M \cdot 4 - \phi_2 \cdot \frac{4}{3}M + A \cdot 2 = 0$

$A = 2{,}92M$

$\Sigma Y = 0 \quad -\frac{9}{8}M - M + 2{,}92M + B_L = 0$

$B_L = -0{,}8M$

$M_A^L = \phi_1 \cdot 2 - \frac{9}{8}M \cdot 2 = 9/4\,M = 2{,}25M$

$\frac{N}{m^2} = Pa$

$w_A = \frac{M_A}{EI} = \frac{2{,}25M}{EI} \leq 2$ mm

$2{,}25M \leq 2 \cdot 10^{-3} \cdot 200 \cdot 10^9 \cdot 100 \cdot 10^{-8}$

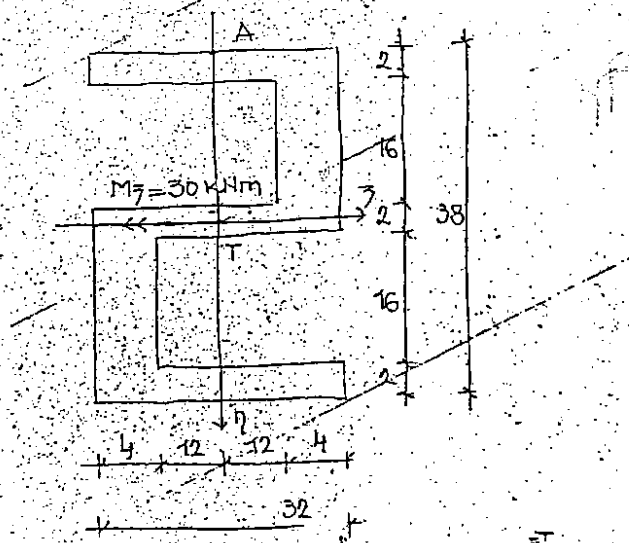
$M \leq 177{,}78$ Nm

27. 06. 2003.

За дати попречни пресек познати су моменти инерције за тежишне осе y и z.

$I_\eta = 41643\ cm^4$ $I_\zeta = 54635\ cm^4$ $I_{\eta\zeta} = -16128\ cm^4$

Услед задатог момента $M_\zeta = 30\ kNm$ одредити положај тачака у којима је компонентални нормални напон једнак нули, и вредност нормалног компонеталног напона у тачки.



$I_{\eta\zeta} \neq 0$ $\quad \sigma_\xi = \dfrac{M_\zeta I_\eta + M_\eta I_{\eta\zeta}}{J_2}\eta - \dfrac{M_\eta I_\zeta + M_\zeta I_{\eta\zeta}}{J_2}\zeta$

$J_2 = I_\zeta I_\eta - I_{\eta\zeta}^2$

$J_2 = 54635 \cdot 41643 - (-16128)^2 = 2015052921\ cm^8$

$\sigma_\xi = \dfrac{-30 \cdot 10^2 \cdot 41643 \cdot 10}{J_2}\eta - \dfrac{-30 \cdot 10^2 \cdot (-16128) \cdot 10}{J_2}\zeta$

$\sigma_\xi = -0{,}62\eta - 0{,}24\zeta$

$\sigma_\xi = 0 \Rightarrow \boxed{\eta = -0{,}387\zeta}$

$A(0, -19)$ $\quad \zeta = 0$, $\eta = -19$

$\sigma_\xi = -0{,}62 \cdot (-19)$

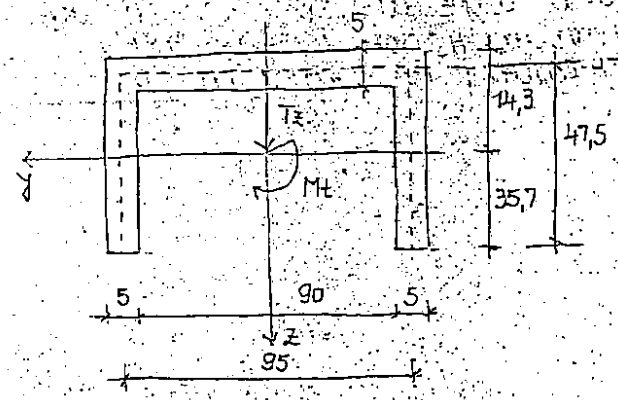
$\sigma_\xi = 11{,}78\ MPa$

24.09.2004

a) НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ СИЛА

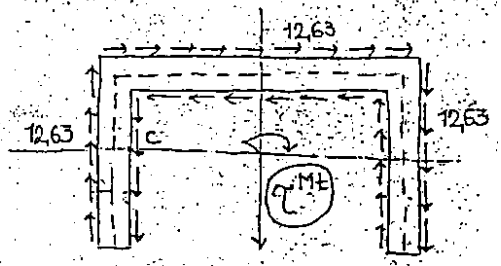
b) ОДРЕДИТИ τ_{max} И ТАЧКУ У КОЈОЈ СЕ ЈАВЉА

$|Mt| = 20\ KNm \qquad |Tz| = 50\ KN$

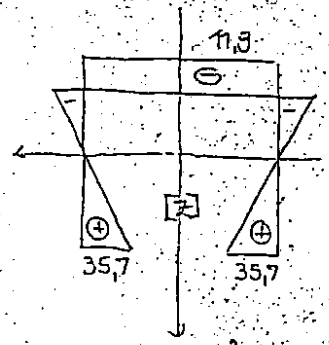


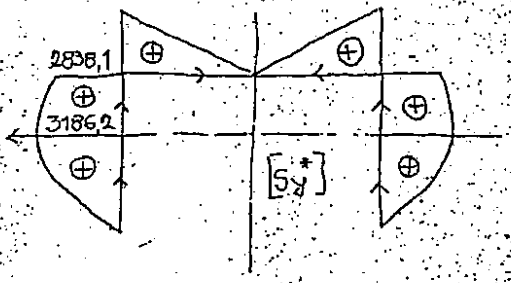
$$I_t = \frac{1}{3}\sum b_i t_i^3 = \frac{1}{3}(2\,47{,}5\cdot 5^3 + 95\cdot 5^3) = 7916{,}67\ cm^4$$

$$\tau^{Mt} = \frac{Mt}{I_t}\cdot t = \frac{20\cdot 10^1}{7916{,}67}\cdot 5\cdot 10 = 12{,}63\ MPa$$



$$\tau^{Tz} = -\frac{Tz\,S_y^*}{I_y\cdot t}$$



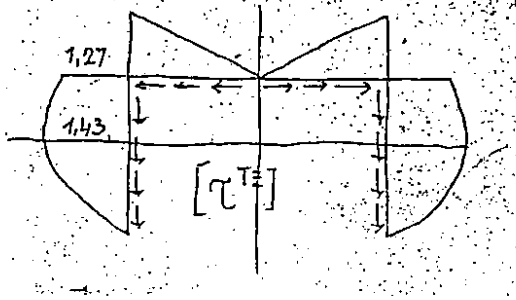


$$I_y = 2\cdot\frac{35{,}7}{3}35{,}7^2\cdot 5 + 2\cdot\frac{11{,}8}{3}11{,}8^2\cdot 5 + 95\cdot 11{,}8^2\cdot 5 = 223280{,}1\ cm^4$$

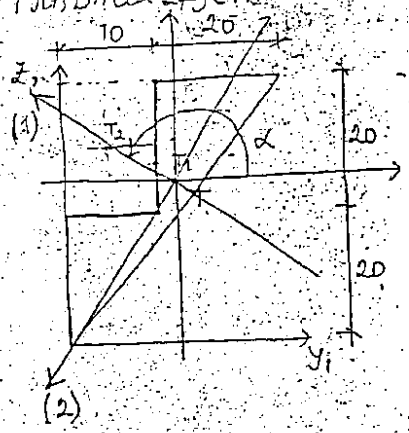
$$\tau_1^{Tz} = -\frac{50\cdot 3186{,}2}{223280{,}1\cdot 5}\cdot 10 = -1{,}43\ MPa$$

$$\tau_2^{Tz} = -1{,}27\ MPa$$

$$\tau_{max}^{c} = 12{,}63 + 1{,}43 = 14{,}06\ MPa$$



Одредити главне централне моменте инерције и правце главних централних оса инерције



$F_1 = \frac{1}{2} 30 \cdot 40 = 600\ cm^2$

$F_2 = 10 \cdot 20 = 200\ cm^2$

$F = 400\ cm^2$

$y_T = \frac{600 \cdot 10 - 200 \cdot 5}{400} = 12,5\ cm$

$z_T = \frac{600 \cdot 26,67 - 200 \cdot 30}{400} = 25\ cm$

$T(12,5; 25)$

$I_y = \frac{1}{36} 30 \cdot 40^3 + 600 \cdot 1,67^2 - \frac{1}{12} 10 \cdot 20^3 - 200 \cdot 5^2 = 43\,340\ cm^4$

$I_z = \frac{1}{36} 30^3 \cdot 40 + 600 \cdot 2,5^2 - \frac{1}{12} 10^3 \cdot 20 - 200 \cdot 7,5^2 = 20\,833,33\ cm^4$

$I_{yz} = \frac{1}{72} 30^2 \cdot 40^2 + 600 \cdot (-1,67 \cdot 2,5) + 200 \cdot 5 \cdot 7,5 = 24\,995\ cm^4$

$I_{yz} \neq 0 \quad I_{1,2} = \frac{43340 + 20\,833,33}{2} \pm \sqrt{\left(\frac{43340 - 20833,33}{2}\right)^2 + 24995^2}$

$I_1 = 59\,498,11\ cm^4$

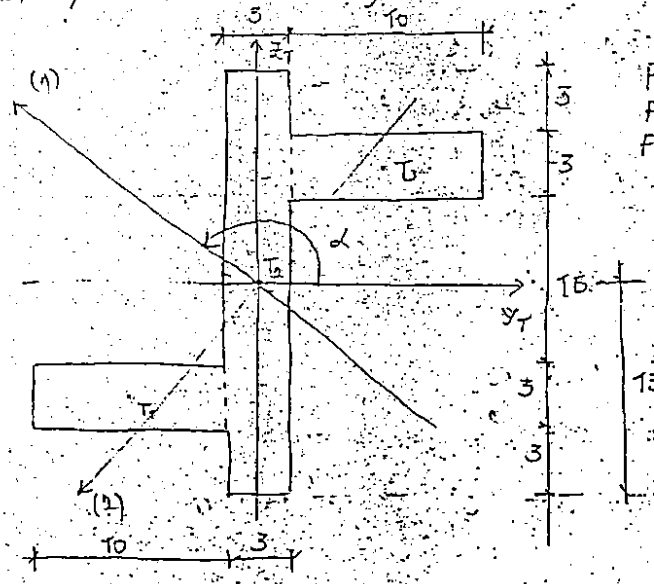
$I_2 = 4675,22\ cm^4$

$tg\ 2\alpha = \frac{-2 \cdot 24995}{43340 - 20833,33}$ ($-2 \cdot 24995 < 0$, $43340 - 20833,33 > 0$)

$\alpha = \frac{1}{2}(360 + arctg(\ )) = 147,12°$

(118) 02.04.2004

W Одредити правце главних централних оса инерције и вредности главних централних момената инерције



$F_1 = 10 \cdot 3 = 30\ cm^2$
$F_2 = 27 \cdot 3 = 81\ cm^2$
$F_3 = 10 \cdot 3 = 30\ cm^2$
$F = 141\ cm^2$

$$I_y = \frac{1}{12} 10 \cdot 3^3 + 9^2 \cdot 30 + \frac{1}{12} 3 \cdot 27^3 + 0 + \frac{1}{12} 10 \cdot 3^3 + 9^2 \cdot 30 = 9825{,}75\ cm^4$$

$$I_z = \frac{1}{12} 10^3 \cdot 3 + 6{,}5^2 \cdot 30 + \frac{1}{12} 3^3 \cdot 27 + 0 + \frac{1}{12} 10^3 \cdot 3 + 6{,}5^2 \cdot 30 = 3095{,}75\ cm^4$$

$$I_{yz} = 0 + 9 \cdot 6{,}5 \cdot 30 + 0 + 0 + 0 + 9 \cdot 6{,}5 \cdot 30 = 3510\ cm^4$$

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

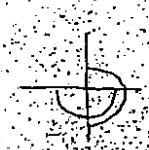
$$= \frac{9825{,}75 + 3095{,}75}{2} \pm \sqrt{\left(\frac{9825{,}75 - 3095{,}75}{2}\right)^2 + 3510^2} =$$

$I_1 = 11\,323{,}19\ cm^4$

$I_2 = 1598{,}31\ cm^4$

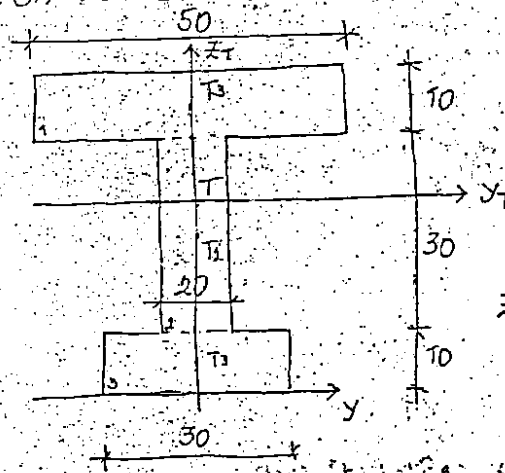
$$tg\,2\alpha = -\frac{2 I_{yz}}{I_y - I_z} = -\frac{2 \cdot 3510}{9825{,}75 - 3095{,}75} \quad \begin{matrix} <0 \\ >0 \end{matrix}$$

$$\alpha = \frac{1}{2}(360^\circ + arctg(\ )) = 156{,}9^\circ$$



05.09.2003

1.1. Определи вредности главних централних момената инерције за попречни пресек на слици.



$F_1 = 10 \cdot 50 = 500\ cm^2$

$F_2 = 20 \cdot 30 = 600\ cm^2$

$F_3 = 30 \cdot 10 = 300\ cm^2$

$F = 1400\ cm^2$

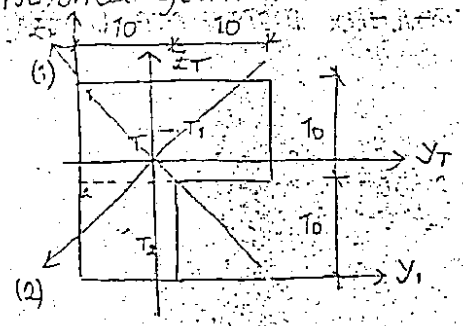
$z_T = \frac{500 \cdot 45 + 600 \cdot 25 + 300 \cdot 5}{1400} = 27,86\ cm$

$I_y = \frac{1}{12} 50 \cdot 10^3 + 17,14^2 \cdot 500 + \frac{1}{12} 20 \cdot 30^3 + 2,86^2 \cdot 600 + \frac{1}{12} 30 \cdot 10^3 + 22,86^2 \cdot 300 = 360\,238,11\ cm^4$

$I_z = \frac{1}{12} 50^3 \cdot 10 + \frac{1}{12} 20^3 \cdot 30 + \frac{1}{12} 30^3 \cdot 10 = 146\,666,67\ cm^4$

$I_{yz} = 0$

(120) W ODREDITI GLAVNE CENTRALNE MOMENTE INERCIJE I PRAVCE GLAVNIH CENTRALNIH OSA INERCIJE



$F_1 = 20 \cdot 10 = 200\ cm^2$

$F_2 = 10 \cdot 10 = 100\ cm^2$

$F = 300\ cm^2$

$$y_T = \frac{200 \cdot 10 + 100 \cdot 5}{300} = 8{,}33\ cm$$

$$z_T = \frac{200 \cdot 15 + 100 \cdot 5}{300} = 11{,}67\ cm$$

$$T(8{,}33;\ 11{,}67)$$

$$I_y = \frac{1}{12} 20 \cdot 10^3 + 200 \cdot 3{,}33^2 + \frac{1}{12} 10^4 + 100 \cdot 6{,}67^2 = 9166{,}67\ cm^4$$

$$I_z = \frac{1}{12} 20^3 \cdot 10 + 200 \cdot 1{,}67^2 + \frac{1}{12} 10^4 + 100 \cdot 3{,}33^2 = 9166{,}67\ cm^4$$

$$I_{yz} = 200 \cdot 3{,}33 \cdot 1{,}67 + 100 \cdot 6{,}67 \cdot 3{,}33 = 3333{,}33\ cm^4$$

$I_{yz} \neq 0 \Rightarrow$

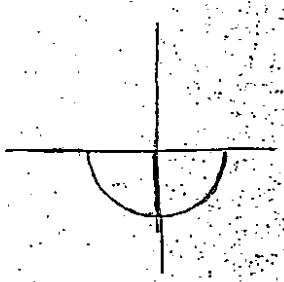
$$I_{1,2} = \frac{9166{,}67 + 9166{,}67}{2} \pm \sqrt{0^2 + 3333{,}33^2}$$

$$I_1 = 12\,500\ cm^4$$

$$I_2 = 5833{,}34\ cm^4$$

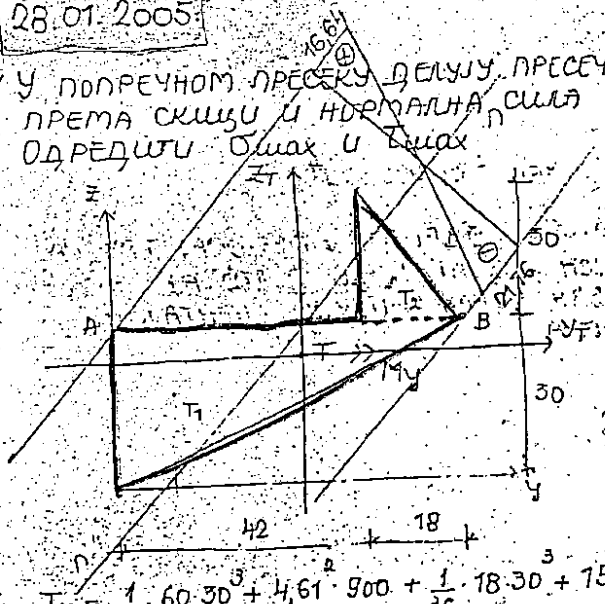
$$tg\,2\alpha = -\frac{2 I_{yz}}{I_y - I_z} = \frac{-2 \cdot 3333{,}33}{0} = -\infty$$

$$2\alpha = 270^\circ \qquad \alpha = 135^\circ$$



28.01.2005

У ПОПРЕЧНОМ ПРЕСЕКУ ДЕЛУЈУ ПРЕСЕЧНЕ СИЛЕ $M_y = 100$ kNm
ПРЕМА СКИЦИ И НОРМАЛНА СИЛА $N = -400$ kN
ОДРЕДИТИ σ_{max} И τ_{max}



$F_1 = \frac{1}{2} 60 \cdot 30 = 900$ cm²

$F_2 = \frac{1}{2} 18 \cdot 30 = 270$ cm²

$F = 1170$ cm²

$y_T = \frac{900 \cdot 20 + 270 \cdot 48}{1170} = 26{,}46$ cm

$z_T = \frac{900 \cdot 20 + 270 \cdot 40}{1170} = 24{,}61$ cm

$I_y = \frac{1}{36} 60 \cdot 30^3 + 4{,}61^2 \cdot 900 + \frac{1}{36} 18 \cdot 30^3 + 15{,}39^2 \cdot 270 = 141\,576{,}96$ cm⁴

$I_z = \frac{1}{36} 60^3 \cdot 30 + 6{,}46^2 \cdot 900 + \frac{1}{36} 18^3 \cdot 30 + 21{,}54^2 \cdot 270 = 347\,690{,}77$ cm⁴

$I_{yz} = \frac{1}{72} 60^2 \cdot 30^2 + 4{,}61 \cdot 6{,}46 \cdot 900 - \frac{1}{72} 18^2 \cdot 30^2 + 15{,}39 \cdot 21{,}54 \cdot 270 = 157\,257{,}7$ cm⁴

$I_{yz} \neq 0 \Rightarrow \sigma_x = \frac{N}{F} + \frac{M_y I_y + M_z I_{yz}}{J_2} z - \frac{M_z I_z + M_y I_{yz}}{J_2} y$ ← НЕ ВАЉА ФОРМУЛА

$J_2 = I_y \cdot I_z - I_{yz}^2 = 141\,576{,}96 \cdot 347\,690{,}77 - 157\,257{,}7^2 = 2{,}45 \cdot 10^{10}$ cm⁸

$\sigma_x = \frac{-400}{1170} \cdot 10 + \frac{100 \cdot 141\,576{,}96 \cdot 10^2}{2{,}45 \cdot 10^{10}} z - \frac{100 \cdot 157\,257{,}7 \cdot 10^2}{2{,}45 \cdot 10^{10}} y$

$\sigma_x = -3{,}42 + 0{,}58 z - 0{,}64 y$

$\sigma_x = 0 \quad \boxed{z - 1{,}1 y = 5{,}9}$

y	0	17
z	5,9	24

$A(-26{,}46 ; 5{,}39) \qquad B(33{,}54 ; 5{,}39)$

$\sigma_x^A = -3{,}42 + 0{,}58 \cdot 5{,}39 - 0{,}64 \cdot (-26{,}46) = 16{,}64$ MPa

$\sigma_x^B = -3{,}42 + 0{,}58 \cdot 5{,}39 - 0{,}64 \cdot 33{,}54 = -21{,}76$ MPa

$\tau_{max} = \frac{1}{2} |16{,}64 - (-21{,}76)| = 19{,}2$ MPa

опкле обр

27.05.2003

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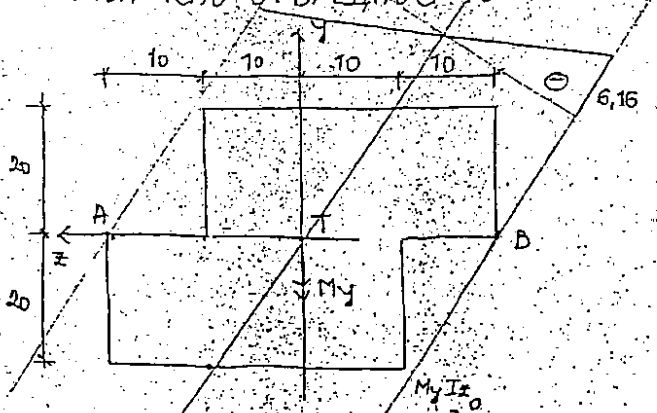
ЗА ДАТИ ПОПРЕЧНИ ПРЕСЕК ПОЗНАТИ СУ МОМЕНТИ ИНЕРЦИЈЕ ЗА ЊЕГОВЕ ТЕЖИШНЕ ОСЕ

$I_y = 120\,000\ cm^4$ $I_z = 160\,000\ cm^4$ $I_{yz} = -60\,000\ cm^4$

УСЛЕД ЗАДАТОГ МОМЕНТА $M_y = 40\ kNm$ ОДРЕДИТИ ПОЛОЖАЈ ТАЧАКА У КОЈИМА ЈЕ КОМПОНЕНТАЛНИ НАПОН ЈЕДНАК НУЛИ, ПОЛОЖАЈ ТАЧКЕ У КОЈОЈ СЕ ЈАВЉА НАЈВЕЋИ НЕГАТИВАН НОРМАЛНИ КОМПОНЕНТАЛНИ НАПОН КАО И ВРЕДНОСТ ТОГ НАПОНА



$M_y I_{z0}$ $M_z I_y$

$I_{yz} \neq 0$ $\sigma_x = \frac{N}{F} + \frac{M_z I_z + M_y I_{yz}}{J_2} y - \frac{M_y I_y + M_z I_{yz}}{J_2} z$ НЕ ВАЉА ФОРМУЛА

$J_2 = I_y I_z - I_{yz}^2 = 1{,}56 \cdot 10^{10}$

$\sigma_x = \frac{-40 \cdot 10^2 \cdot (-60\,000)}{1{,}56 \cdot 10^{10}} y - \frac{-40 \cdot 10^2 \cdot 120\,000}{1{,}56 \cdot 10^{10}} z$

$\sigma_x = 0{,}154\, y + 0{,}308\, z$

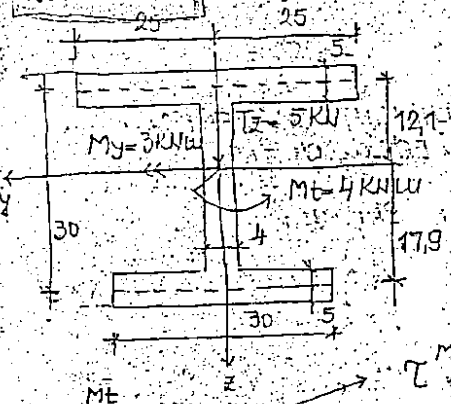
$\sigma_x = 0 \Rightarrow \boxed{y = -2z}$

z	0	10
y	0	-20

A(20; 0) $\sigma_x^A = 0{,}154 \cdot 0 + 0{,}308 \cdot 20 = 6{,}16\ MPa$

B(-20; 0) $\sigma_x^B = 0{,}154 \cdot 0 + 0{,}308 (-20) = -6{,}16\ MPa$

26-09-2003 W



$F_1 = 50 \cdot 5 = 250\ cm^2$

$F_2 = 30 \cdot 4 = 120\ cm^2$

$F_3 = 30 \cdot 5 = 150\ cm^2$

$F = 520\ cm^2$

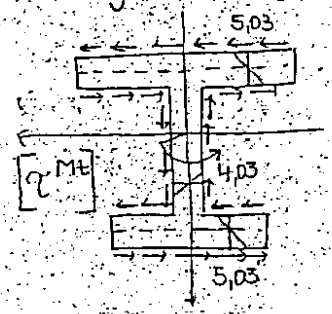
$z_T = \frac{250 \cdot 32,5 + 120 \cdot 17,5 + 150 \cdot 2,5}{520} = 14,6\ cm$

$\tau^{Mt} = \frac{Mt}{It} \cdot t$

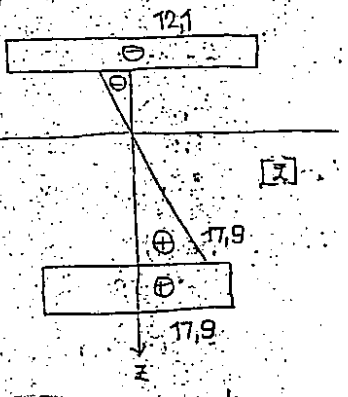
$\tau_1^{Mt} = \frac{4 \cdot 10^2}{3973,33} \cdot 5 \cdot 10 = 5,03\ MPa$

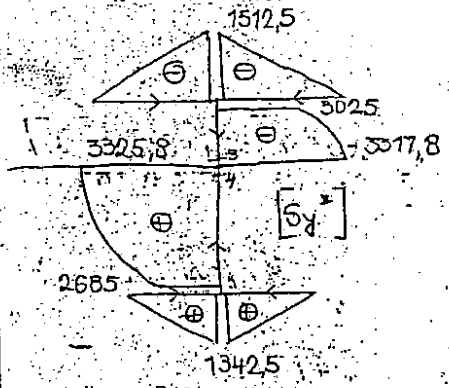
$\tau_2^{Mt} = \frac{4 \cdot 10^2}{3973,33} \cdot 4 \cdot 10 = 4,03\ MPa$

$It = \frac{1}{3} \sum bi\, ti^3 = \frac{1}{3}(50 \cdot 5^3 + 30 \cdot 5^3 + 30 \cdot 4^3) = 3973,33\ cm^4$



$\tau^{Tz} = -\frac{Tz\, Sy^*}{Iy \cdot t}$



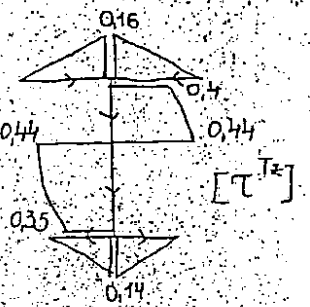


$Iy = 50 \cdot 12,1^2 \cdot 5 + 30 \cdot 17,9^2 \cdot 5 + \frac{12,1}{3} \cdot 12,1^3 + \frac{17,9}{3} \cdot 3^3 \cdot 4 = 94\,673,2\ cm^4$

$\tau^{Tz} = -\frac{5 \cdot Sy^*}{94\,673,2 \cdot t} 10$

$\tau_1^{Tz} = 0,16\ MPa \quad \tau_2^{Tz} = 0,4\ MPa \quad \tau_3^{Tz} = 0,44\ MPa$

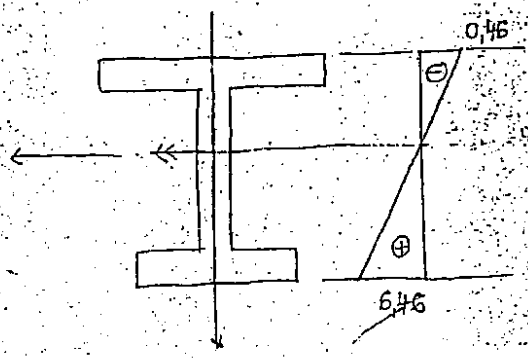
$\tau_4^{Tz} = -0,44\ MPa \quad \tau_5^{Tz} = -0,35\ MPa \quad \tau_6^{Tz} = -0,14\ MPa$



$\sigma_x = \frac{My \cdot z}{Iy}$

$\sigma_{x,6} = \frac{3 \cdot 10^2}{94\,673,2}(-14,6)10 = -0,46\ MPa$

$\sigma_{x,D} = \frac{3 \cdot 10^2}{94\,673,2} \cdot 20,4 \cdot 10 = 6,46\ MPa$

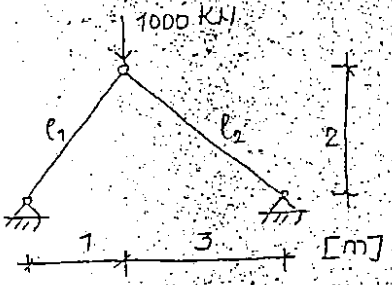


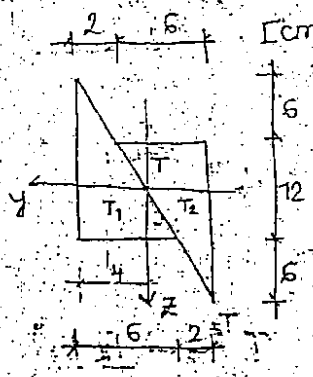
(124) (25) ИЗВИЈАЊЕ

05.09.2005

ОДРЕДИТИ КОЈИ ШТАП ЧИМА МАЊИ КОЕФИЦИЈЕНТ СИГУРНОСТИ НА ИЗВИЈАЊЕ И ОДРЕДИТИ ВРЕДНОСТ ТОГ КОЕФИЦИЈЕНТА

$E = 210\ GPa$ — $\sigma_{KR,T} = 310 - 1{,}14\lambda\ [MPa]$ $\sigma_T = 240\ MPa$



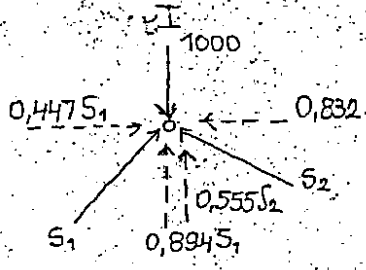


$l_1 = \sqrt{1^2 + 2^2} = 2{,}236\ m$

$l_1' = l_1 = 2{,}236\ m$

$l_2 = \sqrt{3^2 + 2^2} = 3{,}606\ m$

$l_2' = l_2 = 3{,}606\ m$



$0{,}894S_1 + 0{,}555S_2 - 1000 = 0$

$0{,}447S_1 - 0{,}832S_2 = 0$

$S_1 = 1{,}861\ S_2$ $\quad 1{,}664\ S_2 + 0{,}555S_2 - 1000$ $\quad \boxed{S_2 = 450{,}65\ KN \quad S_1 = 838{,}67\ KN}$

$F = 2 \cdot \frac{1}{2} 6 \cdot 18 = 108\ cm^2$

$I_y = \frac{1}{36} 6 \cdot 18^3 : 2 = 1944\ cm^4$

$I_z = \frac{1}{36} 18 \cdot 6^3 + 2^2 \cdot 54 + \frac{1}{36} 18 \cdot 6^3 + 2^2 \cdot 54 = 648\ cm^4$

$I_{yz} = -\frac{1}{72} 18^2 6^2 - \frac{1}{72} 18^2 6^2 = -324\ cm^4$

$I_{1,2} = \frac{1944 + 648}{2} \pm \sqrt{\left(\frac{1944 - 648}{2}\right)^2 + 324^2}$

$I_1 = 2020{,}49\ cm^4$

$I_2 = 571{,}51\ cm^4$

$I_{min} = I_2 = 571{,}51\ cm^4$ $\quad i_{min} = \sqrt{\frac{I_{min}}{F}} = \sqrt{\frac{571{,}51}{108}} = 2{,}3\ cm$

$\lambda_1 = \frac{l_1'}{i_{min}} = \frac{223{,}6}{2{,}3} = 97{,}22$

$\lambda_2 = \frac{l_2'}{i_{min}} = \frac{360{,}6}{2{,}3} = 156{,}78$

Штап S_1

$$\sigma_{KR} - \text{min} \begin{cases} \sigma_{KR,E} = \left(\frac{\bar{u}}{\lambda_1}\right)^2 \cdot E = \left(\frac{\bar{u}}{97{,}22}\right)^2 \cdot 210 \cdot 10^3 = 219{,}28 \text{ MPa} \\ \sigma_{KR,T} = 310 - 1{,}14\,\lambda_1 = 310 - 1{,}14 \cdot 97{,}22 = 199{,}17 \text{ MPa} \\ \sigma_T = 240 \text{ MPa} \end{cases}$$

$\sigma_{KR} = 199{,}17$ MPa $\quad P_{KR} = \sigma_{KR} \cdot F = 199{,}17 \cdot 108 \frac{1}{10} = 2151{,}03$ kN

$$\gamma_1 = \frac{P_{KR}}{S_1} = \frac{2151{,}03}{838{,}67} = 2{,}565$$

Штап S_2

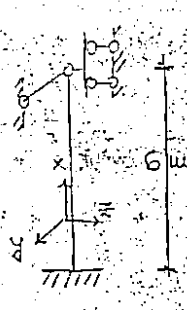
$$\sigma_{KR} - \text{min} \begin{cases} \sigma_{KR,E} = \left(\frac{\bar{u}}{\lambda_2}\right)^2 \cdot E = \left(\frac{\bar{u}}{156{,}78}\right)^2 \cdot 210 \cdot 10^3 = 84{,}32 \text{ MPa} \\ \sigma_{KR,T} = 310 - 1{,}14 \cdot \lambda_2 = 310 - 1{,}14 \cdot 156{,}78 = 131{,}27 \text{ MPa} \\ \sigma_T = 240 \text{ MPa} \end{cases}$$

$\sigma_{KR} = 84{,}32$ MPa $\quad P_{KR} = \sigma_{KR} \cdot F = 84{,}32 \cdot 108 = 910{,}66$ kN

$$\gamma_2 = \frac{P_{KR}}{S_2} = \frac{910{,}66}{450{,}65} = 2{,}021$$

$$\gamma_{min} = \text{min}\{\gamma_1, \gamma_2\} = 2{,}021$$

(126) Одредити вредност критичне силе извијања F_{KR}



$E = 210\ GPa$

$\sigma_T = 240\ MPa$

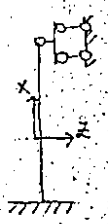
$\sigma_{KR}^T = 310 - 1{,}92\lambda\ [MPa]$

$I_y = 8000\ cm^4$

$I_z = 15\,000\ cm^4$

$F = 240\ cm^2$

* ИЗВИЈАЊЕ ОКО y-ОСЕ (посматрам xOz)



$L = 6\ m$ $\quad L_i = 0{,}5 \cdot 600 = 300\ cm$

$\mu = 0{,}5$ $\quad i_y = \sqrt{\frac{I_y}{F}} = \sqrt{\frac{8000}{240}} = 5{,}77\ cm$

$$\lambda_y = \frac{L_i}{i_y} = \frac{300}{5{,}77} = 51{,}99$$

* ИЗВИЈАЊЕ ОКО z-ОСЕ (посматрам xOy)

$L = 6\ m$ $\quad L_i = 0{,}7 \cdot 600 = 420\ cm$

$\mu_z = 0{,}7$ $\quad i_z = \sqrt{\frac{I_z}{F}} = \sqrt{\frac{15\,000}{240}} = 7{,}91$

$$\lambda_z = \frac{L_i}{i_z} = \frac{420}{7{,}91} = 53{,}1$$

$$\lambda = \max\{\lambda_y, \lambda_z\} = 53{,}1$$

$$\sigma_{KR} = \min \begin{cases} \sigma_{KR,E} = \left(\frac{\pi}{\lambda}\right)^2 E = \left(\frac{\pi}{53{,}1}\right)^2 210 \cdot 10^3 = 735{,}07\ MPa \\ \sigma_{KR,T} = 310 - 1{,}92\lambda = 310 - 1{,}92 \cdot 53{,}1 = 208{,}05\ MPa \\ \sigma_T = 240\ MPa \end{cases}$$

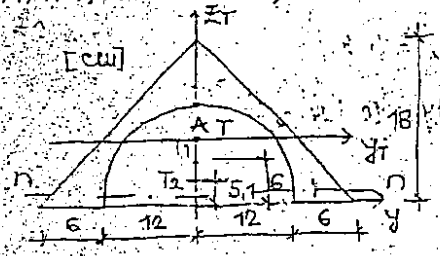
$$P_{KR} = \sigma_{KR} F = 208{,}05 \cdot \frac{240}{10} = 4993{,}15\ kN$$

ЈЕЗГРО ПРЕСЕКА

W ЗА ПОПРЕЧНИ ПРЕСЕК СА СЛИКЕ ОПРЕДЕЛИТИ

a) ВРЕДНОСТИ ГЛАВНИХ ЦЕНТРАЛНИХ МОМЕНАТА ИНЕРЦИЈЕ

b) ДА ЛИ ТАЧКА A ПРИПАДА ЈЕЗГРУ ПРЕСЕКА



$F_1 = \frac{36 \cdot 18}{2} = 324\ cm^2$

$F_2 = \frac{12^2 \pi}{2} = 226,19\ cm^2$

$F = F_1 - F_2 = 97,81\ cm^2$

a)

$z_T = \frac{6 \cdot 324 - 5,1 \cdot 226,19}{97,81} = 8,08\ cm$

$I_y = \frac{1}{36} 36 \cdot 18^3 + 5,98^2 \cdot 324 - 12^4 \cdot 0,0548 + 2,98^2 \cdot 226,19 = 2949,11\ cm^4$

$I_z = \frac{1}{48} 36^3 \cdot 18 - \frac{\pi \cdot 12^4}{8} = 9352,99\ cm^4$

$I_{yz} = 0$

$\Rightarrow I_1 = 9352,99\ cm^4 \quad I_2 = 2949,11\ cm^4$

b) $A(0, -3,92) \quad e_y = 0 \quad e_z = -3,92\ cm$

$i_y^2 = \frac{I_y}{F} = \frac{2949,11}{97,81} = 30,15\ cm^2 \qquad e_y = -\frac{i_z^2}{p_y}$

$i_z^2 = \frac{I_z}{F} = \frac{9352,99}{97,81} = 95,62\ cm^2 \qquad e_z = -\frac{i_y^2}{p_z}$

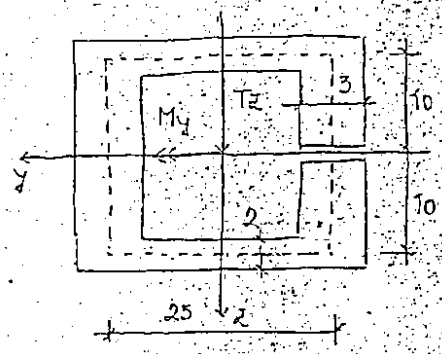
$p_y = -\frac{i_z^2}{e_y} = -\frac{95,62}{0} = -\infty$

$p_z = -\frac{i_y^2}{e_z} = -\frac{30,15}{-3,92} = 7,69$

$\Rightarrow$ НЕУТРАЛНА ОСА (n) СЕЧЕ ПОПРЕЧНИ ПРЕСЕК, ПА ТАЧКА A НЕ ПРИПАДА ЈЕЗГРУ

(128) 05. 09. 2005

VV НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ ПРЕСЕЧНИХ СИЛА КОЈЕ ДЕЛУЈУ У ТЕЖИШТУ И ОДРЕДИТИ τ_{max}, КАО И ТАЧКУ У КОЈОЈ СЕ ЈАВЉА



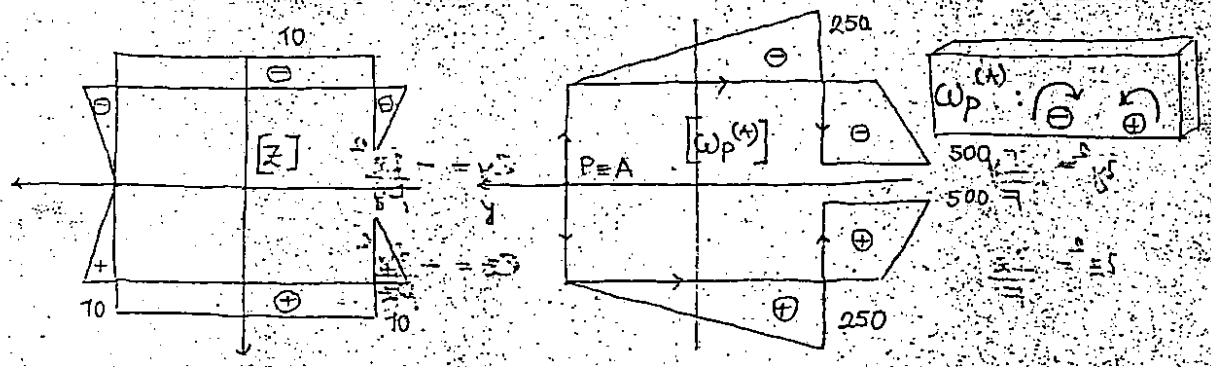
$T_z = 5$ kN

$M_y = 10$ kNcm

ПРЕСЕК СА ЈЕДНОМ ОСОМ СИМЕТРИЈЕ: $T, A \equiv P, S \rightarrow$ НА ОСИ СИМЕТРИЈЕ

$$y_S = y_P + \frac{I_z \omega_P^{(A)}}{I_y} \qquad z_S = z_P - \frac{I_y \omega_P^{(A)}}{I_z}$$

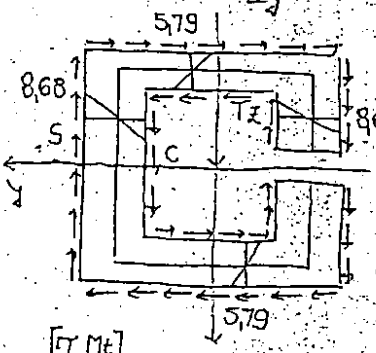
$\Rightarrow$ ОСА СИМЕТРИЈЕ ЈЕ $y \Rightarrow$ ДИЈАГРАМИ: z, $\omega_P^{(A)}$



$$I_y = t\int z^2 ds = 3 \cdot 4 \cdot \frac{10}{3} 10^2 + 2 \cdot 2 \cdot 25 \cdot 10^2 = 14\,000 \text{ cm}^4$$
$$5000$$

$$I_z \omega_P^{(A)} = t\int z \cdot \omega_P^{(A)} ds = 2 \cdot 2 \cdot \frac{25}{2} \cdot 250 \cdot 10 + 2 \cdot 3 \cdot \frac{10}{6}\left[10(2 \cdot 250 + 500)\right] = 225\,000 \text{ cm}^6$$

$$y_S = y_P + \frac{I_z \omega_P^{(A)}}{I_y} = 12{,}5 + \frac{225\,000}{14\,000} = 28{,}57 \text{ cm}$$ (РАСТОЈАЊЕ ОД ТЕЖИШТА)



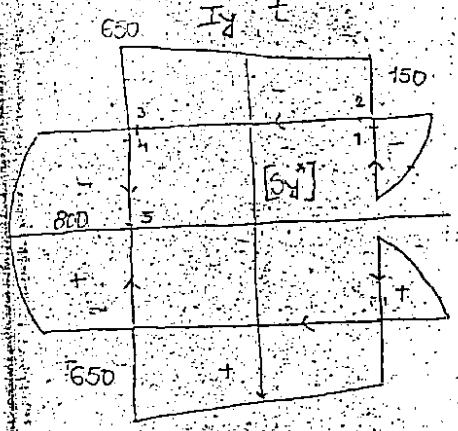
$M_t = T_z \cdot y_S = 5 \text{ kN} \cdot 0{,}2857 \text{ m} = 1{,}428 \text{ kN m}$

$$I_t = \frac{1}{3}\sum b_i t_i^3 = \frac{1}{3}(2 \cdot 25 \cdot 2^3 + 2 \cdot 20 \cdot 3^3) = 493{,}33 \text{ cm}^4$$

$$\tau^{Mt} = \frac{M_t}{I_t} \cdot t \quad \nearrow t = 2 \text{ cm} \quad \tau^{Mt} = \frac{1{,}428 \cdot 10^2}{493{,}33} \cdot 2 \cdot 10 = 5{,}79 \text{ MPa}$$

$$\searrow t = 3 \text{ cm} \quad \tau^{Mt} = \frac{1{,}428 \cdot 10^2}{493{,}33} \cdot 3 \cdot 10 = 8{,}68 \text{ MPa}$$

$$\tau^{Tz} = -\frac{T_z \cdot S_y^*}{I_y \cdot t} \qquad S_y^* = t\int z\,ds$$



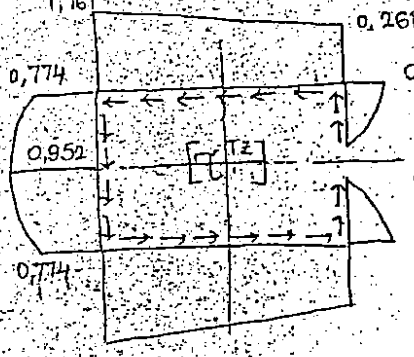
$$\tau_1^{Tz} = -\frac{5\cdot(-150)}{14000\cdot 3}\cdot 10 = 0{,}178\ MPa$$

$\tau_2^{Tz} = 0{,}268$ MPa

$\tau_3^{Tz} = 1{,}161$ MPa

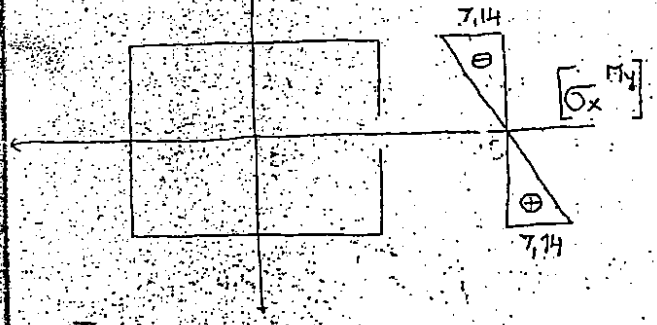
$\tau_4^{Tz} = 0{,}774$ MPa

$\tau_5^{Tz} = 0{,}952$ MPa



$$\sigma_x = \frac{M_z \cdot z}{I_y}$$

$$\sigma_x = \frac{10\cdot 10^2}{14000}\cdot 10\cdot 10 = 7{,}14\ MPa$$

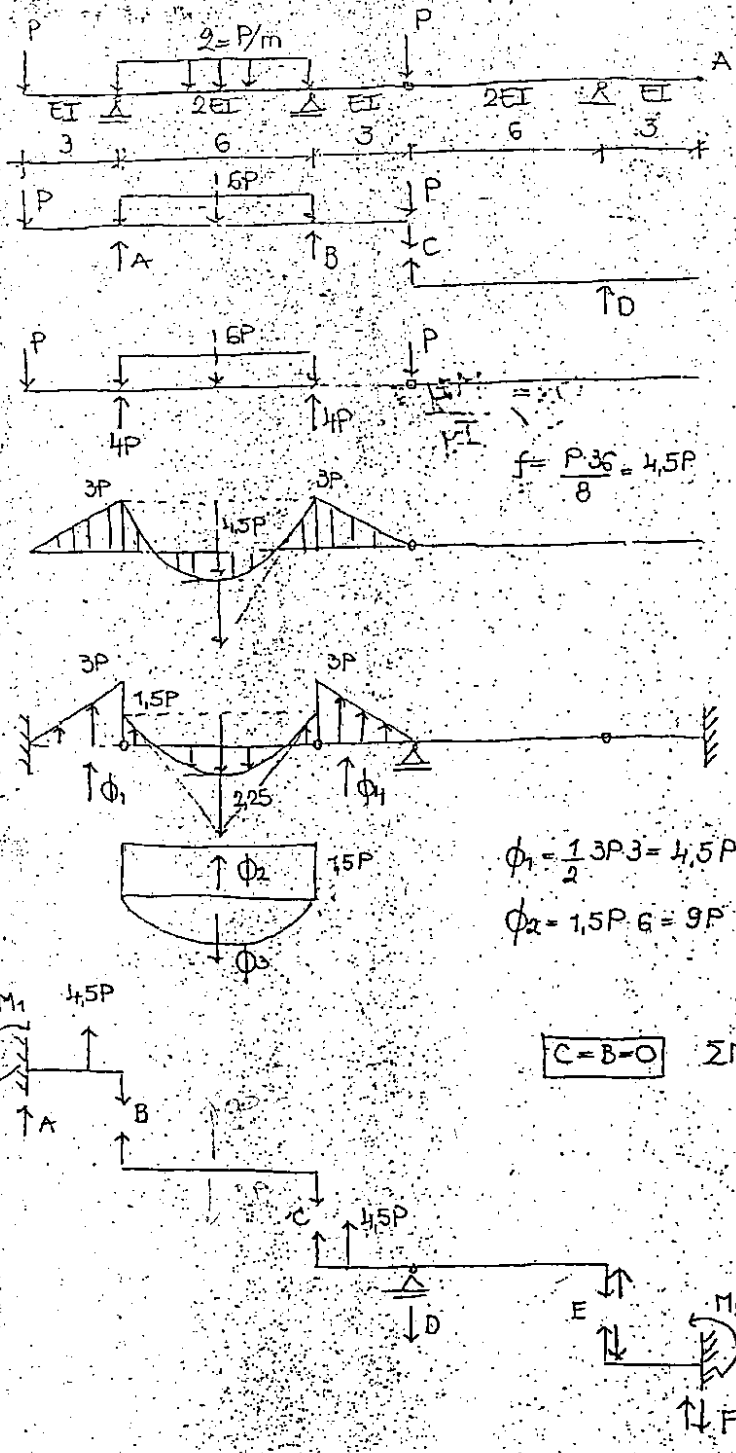


C

$\tau_{max} = 8{,}68 + 0{,}952 = 9{,}632$ MPa

(130) 30.01.2004. IV

Мор-Maxwell-овом аналогијом одредити угиб и нагиб пресека у тачки A ($EI = 1{,}5\ MNm^2$)



$\Sigma M_{CD}^{+\circlearrowleft} = 0 \quad D \cdot 6 = 0 \quad \boxed{D = 0}$

$C + D = 0 \quad \boxed{C = 0}$

$\Sigma M_{CL}^{+\circlearrowright} = 0$

$-P \cdot 12 + A \cdot 9 - 6P \cdot 6 + B \cdot 3 = 0$

$9A + 3B = 48P \quad /:3$

$-P - 6P - P + A + B = 0$

$A + B = 8P \quad A = 8P - B$

$3(8P - B) + B = 16P \quad \boxed{A = 4P}$

$24P - 3B + B = 16P$

$-2B = -8P \quad \boxed{B = 4P}$

$\phi_1 = \frac{1}{2} 3P \cdot 3 = 4{,}5P \qquad \phi_3 = \frac{2}{3} 2{,}25P \cdot 6 = 9P$

$\phi_2 = 1{,}5P \cdot 6 = 9P \qquad \phi_4 = \frac{3P}{2} \cdot 3 = 4{,}5P$

$\boxed{C = B = 0} \qquad \Sigma M_E^{+\circlearrowleft} = 0 \quad -D \cdot 6 + 4{,}5P \cdot 8 = 0$

$\boxed{D = 6P}$

$4{,}5P - 6P - E = 0 \quad \boxed{E = -1{,}5P}$

$\boxed{F = 1{,}5P}$

$M_2 = E \cdot 3 = 1{,}5P \cdot 3 = 4{,}5P$

$\omega_A = \frac{M_A}{EI} = \frac{4{,}5P}{1{,}5 \cdot 10^6} = 3P \cdot 10^{-6}\ m = 0{,}003\ cm$

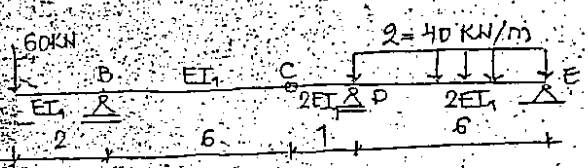
$\varphi_A = \frac{T_A}{EI} = \frac{1{,}5P}{1{,}5 \cdot 10^6} = P \cdot 10^{-6}\ m = 10^{-5}\ rad$

02.10.2004. W

Mohr-Maxwell-овом аналогијом одредити обртање пресека са десне стране греде. $E = 10$ GPa

Део A–B–C: $I = 85\,000$ cm^4 $EI_1 = 850\,000$ $2EI_1 = EI_2$

Део C–D–E: $I = 170\,000$ cm^4 $EI_2 = 1700\,000$



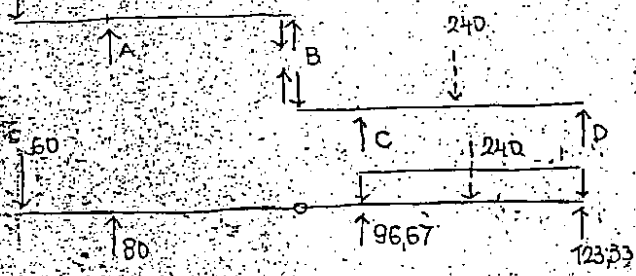
$\Sigma M_B = 0 \quad -60 \cdot 8 + A \cdot 6 = 0 \quad A = 80$

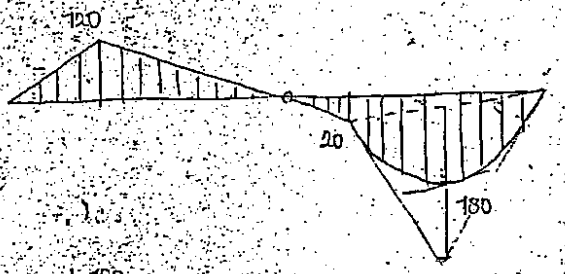
$-60 + 80 + B = 0 \quad B = -20$

$\Sigma M_D = 0 \quad 20 \cdot 7 + C \cdot 6 - 240 \cdot 3 = 0$

$C = 96{,}67$

$20 + 96{,}67 - 240 + D = 0 \quad D = 123{,}33$





$f = \frac{40 \cdot 36}{8}$

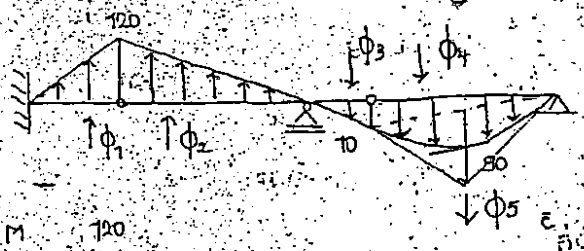
$\phi_1 = \frac{1}{2} \cdot 120 \cdot 2 = 120$

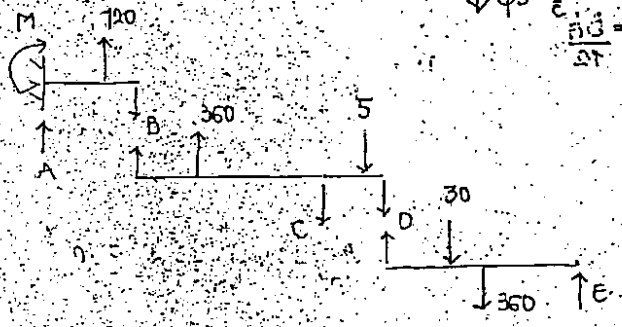
$\phi_2 = \frac{1}{2} \cdot 120 \cdot 6 = 360$

$\phi_3 = \frac{1}{2} \cdot 10 \cdot 1 = 5$

$\phi_4 = \frac{1}{2} \cdot 10 \cdot 6 = 30$

$\phi_5 = \frac{2}{3} \cdot 90 \cdot 6 = 360$





$\frac{dd}{dt} = \Sigma I \quad \frac{dd}{dW} = \Sigma M$

$\Sigma M_{DD} = 0 \quad 30 \cdot 2 + 360 \cdot 3 - E \cdot 6 = 0$

$E = 190$

$D - 30 - 360 + 190 = 0 \quad D = 200$

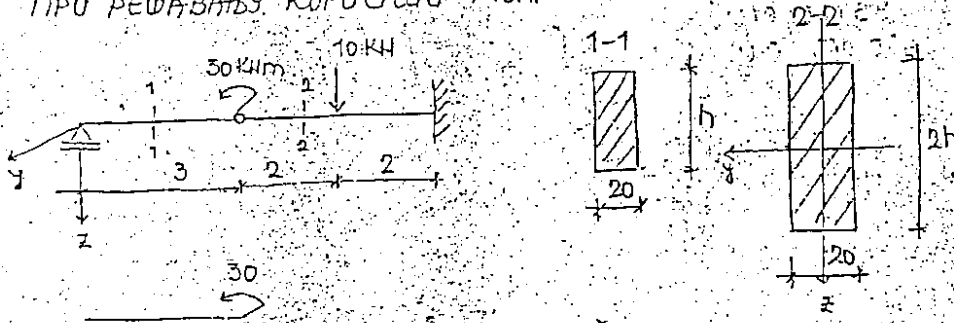
$\Sigma M_{BD} = 0 \quad 360 \cdot 2 + C \cdot 6 + 5 \cdot 6{,}67 + 200 \cdot 7 = 0$

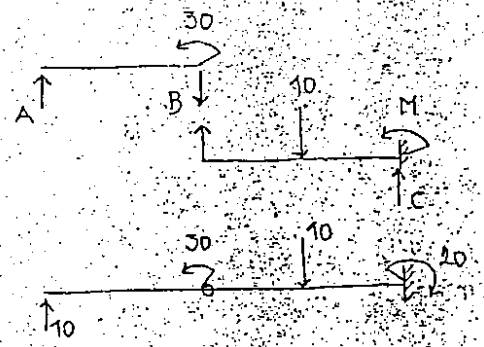
$C = -118{,}89$

$\gamma_{CD} = \frac{T}{EI} = \frac{195 \cdot 10^3}{10 \cdot 10^9 \cdot 85\,000 \cdot 10^{-8}} = 0{,}0229$ rad

(132) 28.01.2005.

Димензионисати дати носач на основу услова да је:
$\sigma_{max} \le \sigma_{doz} = 10\,MPa$ и $v_{max} \le v_{doz} = 3\,cm$
При решавању користити Mohr-Maxwell-ову аналогију $E = 30\,GPa$



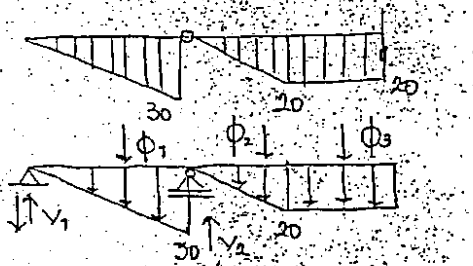


$\sum M_A^{+\circlearrowleft} = 0 \quad 30 - B \cdot 3 = 0 \quad \underline{B = 10}$

$A - B = 0 \quad \underline{A = 10}$

$B - 10 + C = 0 \quad \underline{C = 0}$

$M = 10 \cdot 4 - 10 \cdot 2 = 20 \quad \underline{M = 20}$



$\phi_1 = \frac{1}{2} 30 \cdot 3 = 45$

$\phi_2 = \frac{1}{2} 20 \cdot 2 = 20$

$\phi_3 = 2 \cdot 20 = 40$

$\sum M_{\overset{+}{D}}^{\circlearrowleft} = 0 \quad 45 \cdot \frac{2}{3} 3 + 20 \left(\frac{2}{3} 2 + 3 \right) + 40 \cdot 6 = V_2 \cdot 3 \quad V_2 = 138{,}9$

$V_1 - 45 - 20 - 40 + 138{,}9 = 0 \quad V_1 = -33{,}9$

$M_{max}^{1-1} = 33{,}9 \cdot 3 + 45 \cdot 1 = 146{,}7$

$M_{max}^{2-2} = 40 \cdot 1 = 40\,kNm$

Запамти

$\sigma_{max} = \frac{M_{max}}{W_y} \quad W_y = \frac{bh^2}{6} \quad I_y = \frac{bh^3}{12}$

$\sigma_{max}^{1-1} = \frac{146{,}7 \cdot 10^3}{\frac{20 \cdot 10^{-2} \cdot h^2}{6}} \le 10 \cdot 10^6$

$h^{1-1} \ge 0{,}663\,m$

$v_{max}^{1-1} = \frac{M_{max}}{EI} = \frac{146{,}7 \cdot 10^3}{30 \cdot 10^9 \cdot \frac{20 \cdot 10^{-2} \cdot h^3}{12}} \le 3 \cdot 10^{-2}$

$h^{1-1} \ge 0{,}214\,m$

$h_{usv}^{1-1} = 66{,}5\,cm$

$\sigma_{max}^{2-2} = \frac{40 \cdot 10^3}{\frac{20 \cdot 10^{-2} \cdot h^2}{6}} \le 10 \cdot 10^6$

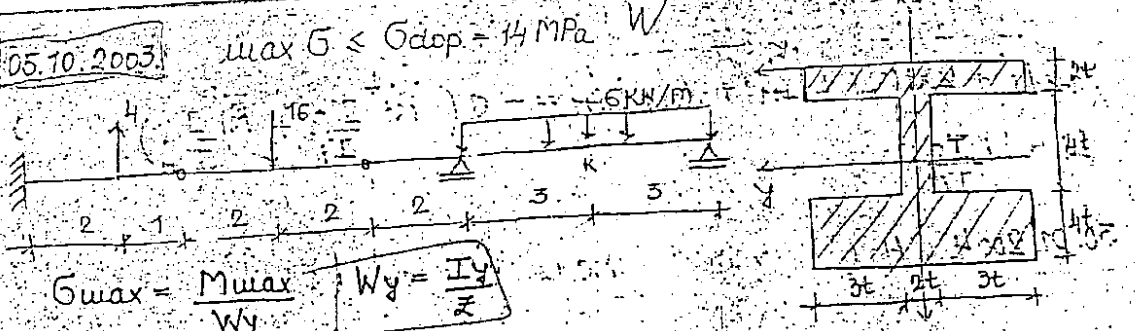
$h^{2-2} \ge 0{,}173\,m$

$v_{max}^{2-2} = \frac{40 \cdot 10^3}{30 \cdot 10^9 \cdot \frac{20 \cdot 10^{-2} \cdot 8 h^3}{12}} \le 3 \cdot 10^{-2}$

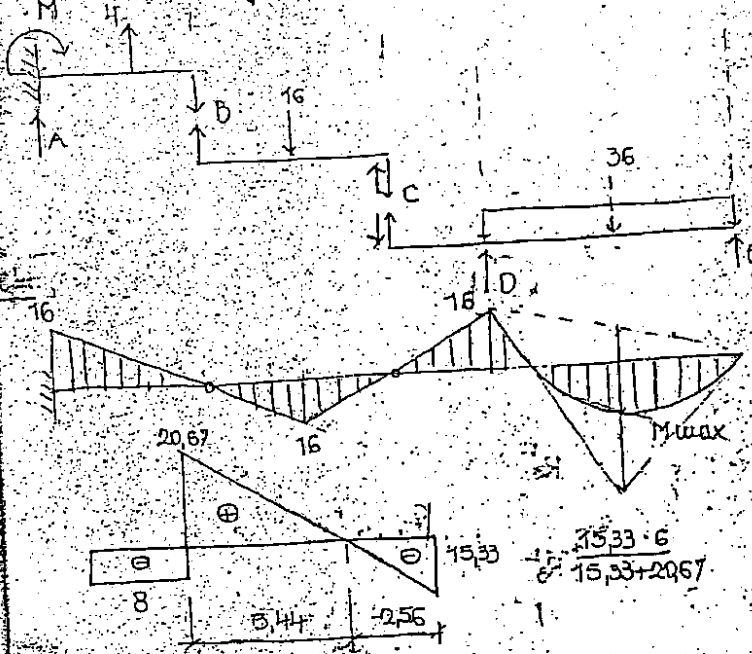
$h^{2-2} \ge 0{,}069\,m$

$h_{usv}^{2-2} = 17{,}5\,cm$

05.10.2003 $\sigma_{max} \leq \sigma_{dop} = 14\ MPa$ W



$\sigma_{max} = \frac{M_{max}}{W_y}$ $W_y = \frac{I_y}{z}$



$\Sigma M_B = 0 \quad 16 \cdot 2 + C \cdot 4 = 0$ $C = -8$

$B - 16 + 8 = 0$ $B = 8$

$A + 4 - 8 = 0$ $A = 4$

$M = -8 \cdot 3 + 4 \cdot 2 = 16$

$\Sigma M_E = 0 \quad -8 \cdot 8 + D \cdot 6 - 36 \cdot 3 = 0$ $D = 28,67$

$-8 + 28,67 - 36 + E = 0$ $E = 15,33$

$\frac{15,33 \cdot 6}{15,33 + 20,67}$

$M_{max} = 15,33 \cdot 2,56 - 6 \cdot 2,56 \cdot \frac{2,56}{2} = 19,58\ KN\,cm$

$F = 8t\ 2t + 4t\ 2t + 8t\ 4t = 56t^2\ cm^2$

$z_T = \frac{16t^2 \cdot t + 8t^2 \cdot 4t + 32t^2 \cdot 8t}{56t^2} = 5,43t$

$I_y = \frac{1}{12} 8t \cdot 8t^3 + 16t^2 \cdot 4,43^2 t^2 + \frac{1}{12} 2t \cdot 64t^3 + 8t^2 \cdot 2,04^2 t^2 + \frac{1}{12} 8t \cdot 64t^3 + 6,6^2 t^2 \cdot 32t^2$

$I_y = 600,18\ t^4$

$\sigma_{max} = \frac{19,58 \cdot 10^3}{\frac{600,18\ t^4}{5,43\ t}} \leq 14 \cdot 10^6$ $t \geq 0,02333\ m$ $t_{usv} = 2,5\ cm$

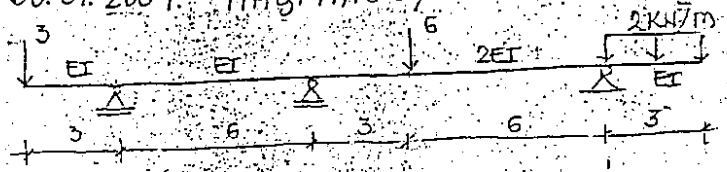
(139)

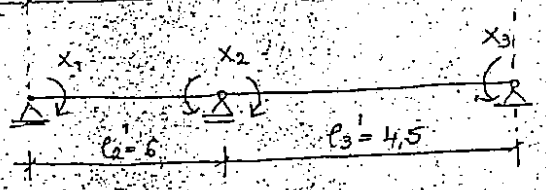
Тримоментна једначина

$$X_{i-1} L_i' + 2X_i (L_i' + L_{i+1}') + X_{i+1} L_{i+1}' = -6 \left(\bar{R}_i^L \frac{I_c}{I_i} + \bar{R}_i^D \frac{I_c}{I_{i+1}} \right)$$

$$L_i' = L_i \frac{I_c}{I_i}$$

30.01.2004. Нацртати дијаграме силе у пресеку [illegible]

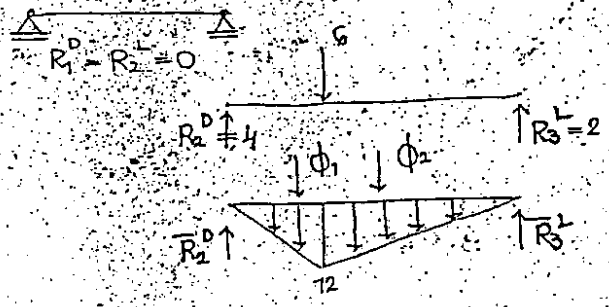




$X_1 = -9$ kN·m

$X_3 = -2 \cdot 3 - 1,5 = -9$ kN·m

$\Sigma M_2 \overset{+}{\circlearrowright} = 0 \quad 6 \cdot 3 - R_3^L \cdot 9 = 0 \quad R_3^L = 2$



$\Phi_1 = \frac{1}{2} 12 \cdot 3 = 18$

$\Phi_2 = \frac{1}{2} 12 \cdot 6 = 36$

$18 \cdot 2 + 36 \cdot 5 - R_3^L \cdot 9 = 0$

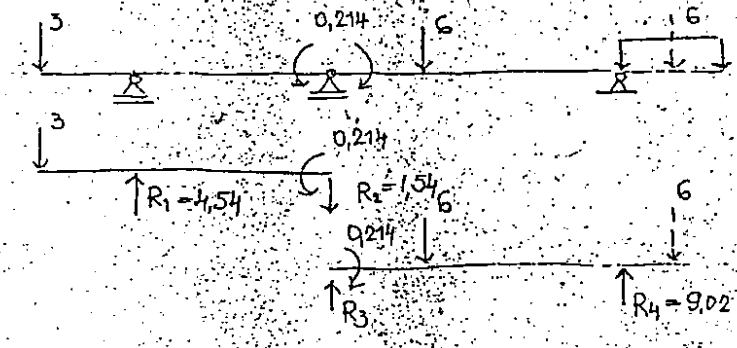
$R_3^L = 24 \quad R_2^D = 30$

$$X_{i-1} L_i' + 2X_i (L_i' + l_{i+1}') + X_{i+1} l_{i+1}' = -6 \left(\bar{R}_i^L \frac{I_c}{I_i} + R_i^D \frac{I_c}{I_{i+1}} \right)$$

$i = 2$

$$X_1 \cdot l_2' + 2 X_2 (l_2' + l_3') + X_3 \cdot l_3' = -6 \left[R_2^{L\,0} \frac{I_c}{I_2} + R_2^D \frac{I_c}{I_3} \right]$$

$-9 \cdot 6 + 2 X_2 (6 + 4,5) - 9 \cdot 4,5 = -6 \left[30 \cdot \frac{1}{2} \right] \quad X_2 = 0,214$



$\Sigma M \overset{+}{\circlearrowright} = 0 \quad -3 \cdot 9 + R_1 \cdot 6 - 0,214 = 0$

$R_1 = 4,54$

$R_1 - 3 - R_2 = 0 \quad R_2 = 1,54$

$6 \cdot 3 + 0,214 - R_4 \cdot 9 + 6 \cdot 10,5 = 0$

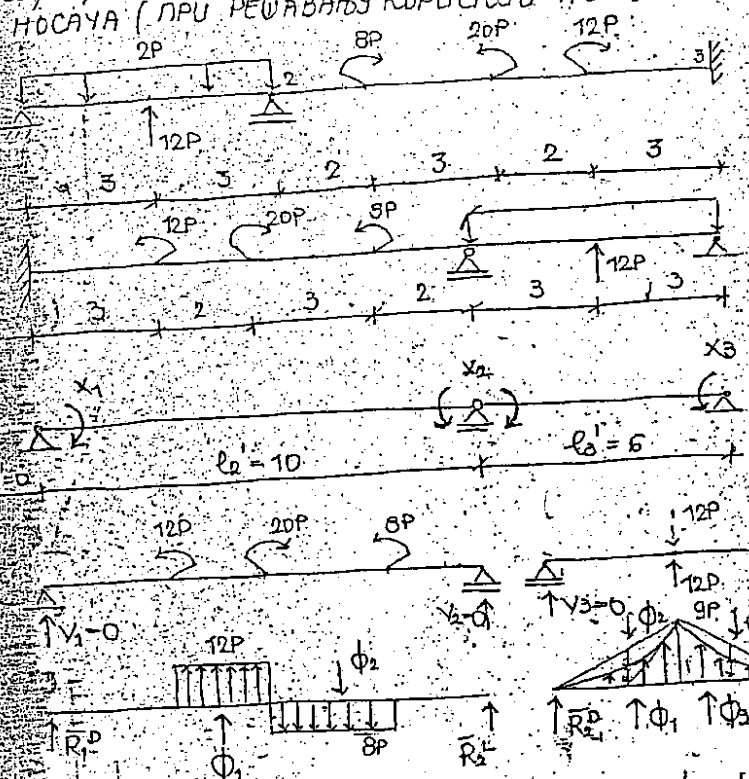
$R_4 = 9,02$

$R_3 - 6 - 6 + R_4 = 0$

$R_3 = 2,98$

24.09.2004.

ОДРЕДИТИ ВРЕДНОСТИ МОМЕНАТА У ТАЧКАМА 2 И 3 ДАТОГ КОНТИНУАЛНОГ НОСАЧА (ПРИ РЕШАВАЊУ КОРИСТИТИ ТРИМОМЕНТНУ ЈЕДНАЧИНУ) EI = const.



$X_3 = 0$

$\phi_1 = 12P \cdot 2 = 24P$

$\phi_2 = 8P \cdot 3 = 24P$

$24P \cdot 4 - 24P \cdot 6{,}5 + R_2^L \cdot 10 = 0$

$R_2^L = 6P$

$R_1^D + \phi_1 - \phi_2 + R_2^L = 0$

$R_1^D = -6P$

$\phi_3 = \phi_3 = \frac{1}{2} 9P \cdot 3 = 13{,}5P$

$\phi_2 = \phi_4 = \frac{2}{3} \cdot 2{,}25P \cdot 3 = 4{,}5P$

$13{,}5P \cdot 2 + 13{,}5P \cdot 4 + R_3^L \cdot 6 - 4{,}5P \cdot 1{,}5 - 4{,}5P \cdot 4{,}5 = 0$

$R_3^L = -9P$

$R_2^D + 2 \cdot 13{,}5P - 2 \cdot 4{,}5P - 9P = 0$

$R_2^D = -9P$

$i = 1 \quad X_1 \ell_1 + 2X_1(\ell_1 + \ell_2') + X_2 \ell_2' = -6\left[\bar{R}_1^L \frac{I_c}{I_1} + \bar{R}_1^D \frac{I_c}{I_2}\right]$

$2X_1 \cdot 10 + X_2 \cdot 10 = -6[-6P]$

$20X_1 + 10X_2 = 36P \quad \ldots (1)$

$i = 2 \quad X_1 \ell_2' + 2X_2(\ell_2' + \ell_3') + X_3 \ell_3' = -6\left[\bar{R}_2^L \frac{I_c}{I_2} + \bar{R}_2^D \frac{I_c}{I_3}\right]$

$X_1 \cdot 10 + 2X_2(10 + 6) + X_3 \cdot 6 = -6[6P + (-9P)]$

$10X_1 + 32X_2 = 18P \quad \ldots (2)$

$2X_1 + X_2 = 3{,}6P$

$X_2 = 3{,}6P - 2X_1$

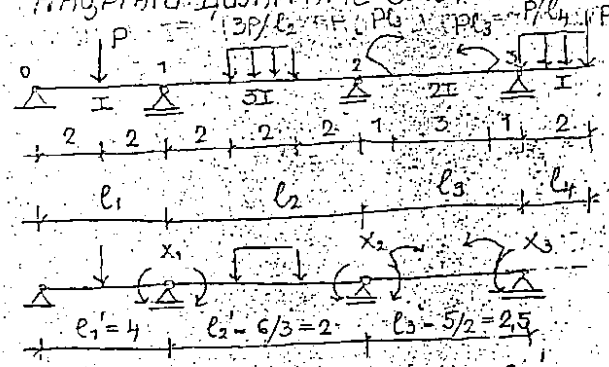
$10X_1 + 32(3{,}6P - 2X_1) = 18P$

$10X_1 + 115{,}2P - 64X_1 = 18P$

$54X_1 = 97{,}2P \quad X_1 = 1{,}8P \quad X_2 = 0$

136 ВЕЖБЕ

НАЦРТАТИ ДИЈАГРАМЕ СИЛА У ПРЕСЕКУ

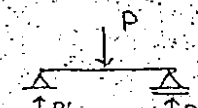


$X_3 = -P \cdot 2 - P/2 \cdot 1 = -3P$

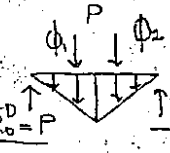
$$X_{i-1} l_i' + 2X_i (l_i' + l_{i+1}') + X_{i+1} l_{i+1}' = -6 \left(\bar{R}_i^L \frac{I_c}{I_i} + \bar{R}_i^D \frac{I_c}{I_{i+1}} \right)$$

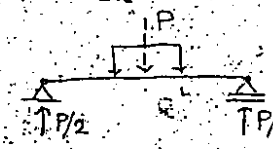
X_1: $0 \cdot 4 + 2X_1(4+2) + X_2 \cdot 2 = -6 \left(\bar{R}_1^L \frac{I_c}{I_1} + \bar{R}_1^D \frac{I_c}{I_2} \right)$

X_2: $X_1 \cdot 4 + 2X_2(2+2{,}5) + X_3 \cdot 2{,}5 = -6 \left(\bar{R}_2^L \frac{I_c}{I_2} + \bar{R}_2^D \frac{I_c}{I_3} \right)$

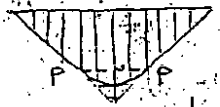


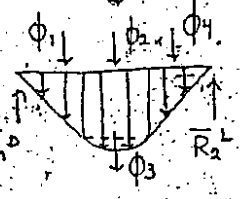
$\phi_1 = \phi_2 = \frac{1}{2} P \cdot 2 = P$





$f = \frac{P}{2} \cdot \frac{4^2}{8} = P/4$



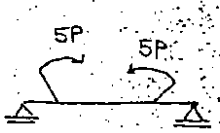


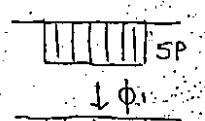
$\phi_1 = \phi_4 = \frac{1}{2} 2P = P$

$\phi_2 = P \cdot 2 = 2P$

$\phi_3 = \frac{1}{3} \frac{P}{4} \cdot 4 = P/3$

$\bar{R}_1^D = \bar{R}_2^L = \frac{1}{2} \left(2P + 2P + \frac{P}{3} \right) = \frac{13P}{6}$





$\phi_1 = 5P \cdot 3 = 15P$

$\bar{R}_2^D = \bar{R}_3^L = 7{,}5P$

$$12X_1 + 2X_2 = -6 \left[P + \frac{13}{6} P \frac{1}{3} \right]$$

$$4X_1 + 9X_2 - 7{,}5P = -6 \left[\frac{13}{6} P \frac{1}{3} + 7{,}5P \frac{1}{2} \right]$$

$12X_1 + 2X_2 = -10{,}33P$ $\quad X_2 = -5{,}165P - 6X_1$

$4X_1 + 9X_2 = -19{,}33P$

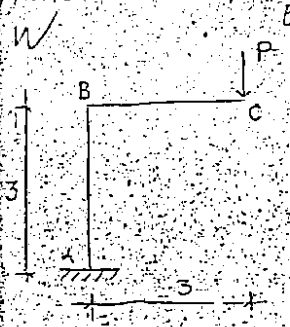
$4X_1 - 46{,}485P - 54X_1 = -19{,}33P$

$$X_1 = -0{,}543P$$

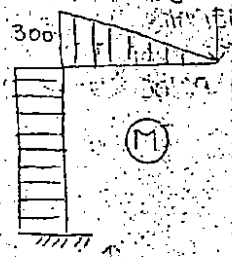
$$X_2 = -1{,}907P$$

Наћи обртање, хоризонтално и вертикално померање тачке C

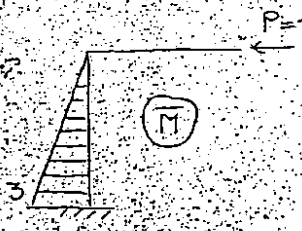
$EI = 10\ MN\,m^2$ $P = 100\ kN$



$$\delta_i = \int \frac{M\bar{M}}{EI}\,ds$$



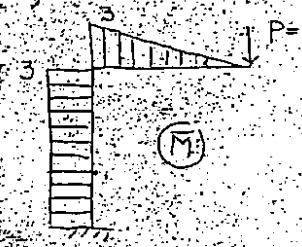
А) ХОРИЗОНТАЛНО ПОМЕРАЊЕ U_C



$$U_C = \frac{1}{EI}\int M\bar{M}\,ds =$$

$$= \frac{1}{EI}\cdot\frac{3}{2}\cdot 3\cdot 300 = \frac{1350\cdot 10^3}{10\cdot 10^6} = 0{,}135\ m$$

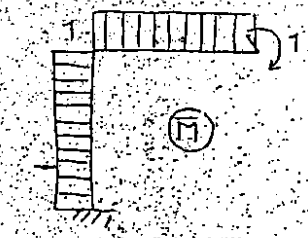
Б) ВЕРТИКАЛНО ПОМЕРАЊЕ V_C



$$V_C = \frac{1}{EI}\int M\bar{M}\,ds =$$

$$= \frac{1}{EI}\left[\frac{3}{3}\cdot 3\cdot 300 + 3\cdot 3\cdot 300\right] = \frac{3600\cdot 10^3}{10\cdot 10^6} = 0{,}36\ m$$

С) ОБРТАЊЕ φ_C



$$\varphi_C = \frac{1}{EI}\int M\bar{M}\,ds =$$

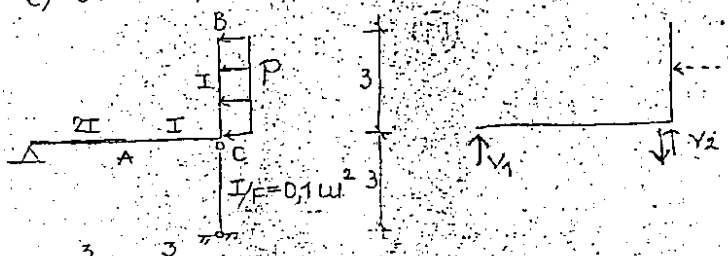
$$= \frac{1}{EI}\left(\frac{3}{2}\cdot 1\cdot 300 + 3\cdot 1\cdot 300\right) = \frac{1350\cdot 10^3}{10\cdot 10^6} = 0{,}135\ rad$$

ТОТАЛНО ПОМЕРАЊЕ: $\delta_C = \sqrt{U_C^2 + V_C^2} =$

$$= \sqrt{0{,}135^2 + 0{,}36^2} = 0{,}384\ m$$

(138)

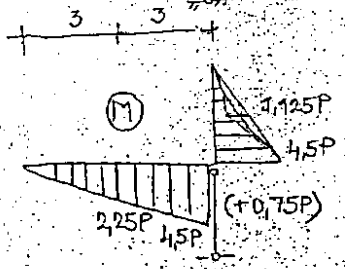
Услед задатог оптерећења у функцији крутости одредити:
a) РАЗМИЦАЊЕ ТАЧАКА A И B
b) ПРОМЕНУ УГЛА ИЗМЕЂУ A И B
c) ВЕРТИКАЛНО ПОМЕРАЊЕ ТАЧКЕ C
d) ХОРИЗОНТАЛНО ПОМЕРАЊЕ ТАЧКЕ B
e) ВЕРТИКАЛНО ПОМЕРАЊЕ ТАЧКЕ A



$\Sigma M_1^{\curvearrowleft}=0 \quad V_2 \cdot 6 + 3P \cdot 1,5 = 0$

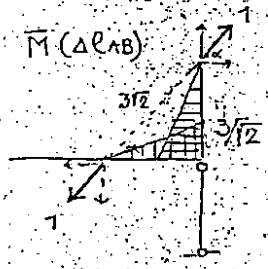
$V_2 = -0,75P$

$V_1 - 0,75P = 0 \quad V_1 = 0,75P$



$f = \frac{P \cdot 9}{8} = 1,125$

a) РАЗМИЦАЊЕ ТАЧАКА A И B Δl_{AB}

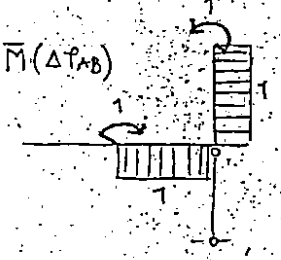


$\Delta l_{AB} = \frac{1}{EI}\int M\bar{M}\,ds =$

$= \frac{1}{EI}\left(-\frac{3}{6}\left(\frac{3}{12}(2\cdot 4,5P + 2,25P)\right) - \frac{3}{3}\,4,5P\,\frac{3}{12} + \frac{2}{3}\,1,125P\,3\,\frac{1}{2}\,\frac{3}{12}\right)$

$= -\frac{19,09P}{EI}$

b) ПРОМЕНА УГЛА ИЗМЕЂУ A И B

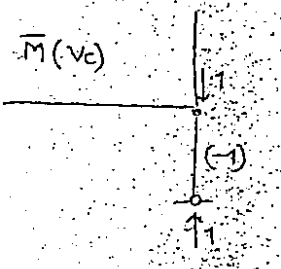


$\Delta\varphi_{AB} = \frac{1}{EI}\int M\bar{M}\,ds =$

$= \frac{1}{EI}\left[+\frac{3}{2}\,1\,(2,25P + 4,5P) + \frac{3}{2}\,1\cdot 4,5P - \frac{2}{3}\,3\,1,125P\,1\right]$

$= \frac{14,625P}{EI}$

c) ВЕРТИКАЛНО ПОМЕРАЊЕ C

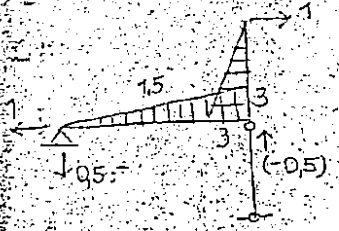


$V_C = \int \frac{N\bar{N}}{EF}\,ds = \frac{1}{EF}\,(-3\cdot 0,75P\cdot 1) = -\frac{2,25P}{EF}\Big/\frac{F}{I}$

$V_C\frac{F}{I} = -\frac{2,25P}{EI} \qquad V_C \cdot 10 = -\frac{2,25P}{EI}$

$V_C = -\frac{0,225P}{EI}$

a) VODORAVNO POMERANJE B



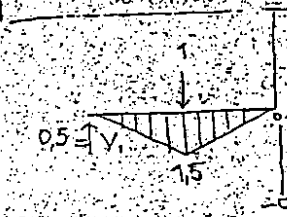
$\sum M^{\curvearrowleft} = 0 \quad -1 \cdot 3 + V_2 \cdot 6 = 0 \quad V_2 = 0,5$

$u_B = \int \frac{M\bar{M}}{EI} dS + \int \frac{N\bar{N}}{EF} =$

$= \frac{1}{EI}\left[-\frac{3}{3} 3 \cdot 4,5P + \frac{3}{3} 1,125P \cdot 3 - \frac{3}{6}\left[1,5(2 \cdot 2,25P + 4,5P) + 3(2 \cdot 4,5P + 2,25P)\right] + \frac{1}{EI}\left(-\frac{1}{2}\frac{1}{3} 1,5 \cdot 2,25P\right)\right.$

$\left. + \frac{0,1}{EI}(-3 \cdot 0,75P \cdot 0,5)\right) = -\frac{35,55}{EI}$

b) VERTIKALNO POMERANJE A



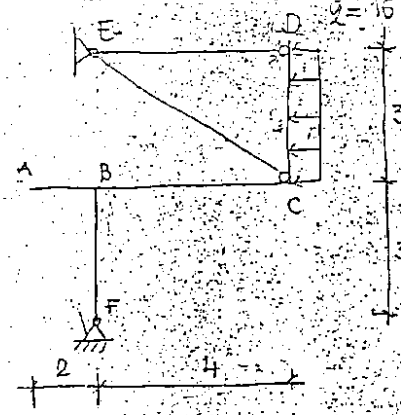
$\sum M_1^{\curvearrowright} = 0 \quad 1 \cdot 3 - V_2 \cdot 6 = 0 \quad V_2 = 0,5$

$V_A = \int \frac{M\bar{M}}{EI} dS + \int \frac{N\bar{N}}{EF} dS =$

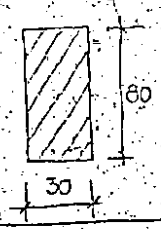
$= \frac{1}{EI}\left[\frac{1}{2}\frac{3}{3} 1,5 \cdot 2,25P + \frac{3}{6}\left[2,25P(2 \cdot 1,5) + 4,5P(1,5)\right]\right.$

$\left. - \frac{0,1}{EI} 3 \cdot 0,5 \cdot 0,75P = \frac{8,325}{EI}\right.$

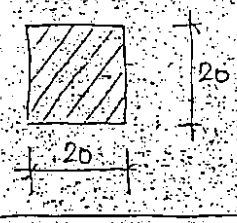
(17D) НАЙ-ГОЛЯМ ДИАГРАМ. МОМЕНТА СЪОРЪЖЕНИЕ



A-B-C-D ; B-F



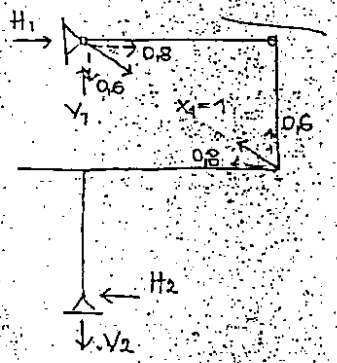
D-E ; C-E



$I = \frac{1}{12} 30 \cdot 80^3 = 128\,0000\ cm^4$

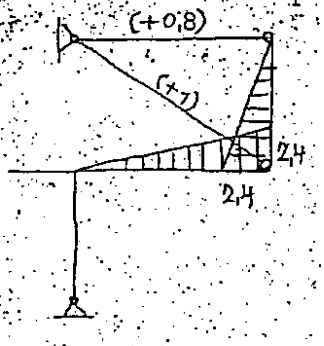
$F = 20^2 = 400\ cm^4$ $\quad EI_C = EI^{II}$

$\frac{I_C}{F} = \frac{128\,0000 \cdot 10^{-8}}{400 \cdot 10^{-4}} = 0{,}32\ m^2$

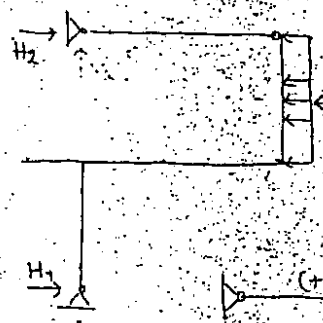


$\Sigma M_C^{+} = 0 \quad H_1 \cdot 6 + 0{,}8 \cdot 6 - 0{,}8 \cdot 3 - 0{,}6 \cdot 4 = 0 \quad H_1 = 0 \rightarrow H_2 = 0$

$\Sigma M_C^{+} = 0 \quad V_1 \cdot 4 - 0{,}6 \cdot 4 = 0 \quad V_1 = 0{,}6 \quad V_2 = 0{,}6$

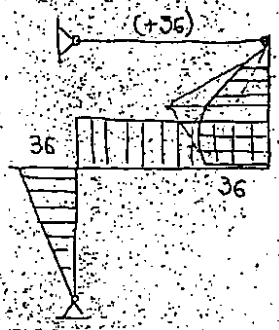


(M_1) (N_1)



$\Sigma M_1^{+} = 0 \quad H_2 \cdot 6 - 48 \cdot 4{,}5 = 0 \quad H_2 = 36$

$H_2 + H_1 - 48 = 0 \quad H_1 = 12$



$f = \frac{16 \cdot 9}{8} = 18$

(M_0) (N_0)

$$\delta_{11} X_1 + \delta_{10} = 0$$

$$\delta_{11} = \frac{1}{EI}\int M_1 M_1 \, ds + \frac{1}{EF}\int N_1 N_1 \, ds =$$

$$= \frac{1}{EI}\left(\frac{3}{3} 2{,}4^2 + \frac{4}{3} 2{,}4^2\right) + \frac{0{,}32}{EI}\left(4 \cdot 0{,}8^2 + 5 \cdot 1^2\right) = \frac{15{,}86}{EI}$$

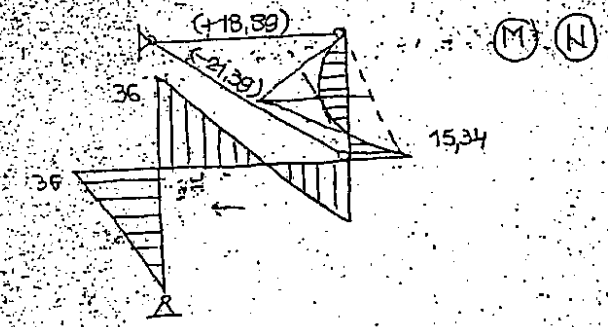
$$\delta_{10} = \frac{1}{EI}\int M_1 M_0 \, ds + \frac{1}{EF}\int N_1 N_0 \, ds =$$

$$= \frac{1}{EI}\left[\frac{3}{3} 2{,}4 \cdot 36 + \frac{3}{3} 18 \cdot 2{,}4 + \frac{4}{2} 36 \cdot 2{,}4\right] + \frac{0{,}32}{EI} 4 \cdot 0{,}8 \cdot 36 = \frac{339{,}26}{EI}$$

$$\delta_{11} X_1 + \delta_{10} = 0$$

$$\frac{15{,}86}{EI} X_1 + \frac{339{,}26}{EI} = 0 \qquad \boxed{X_1 = -21{,}39}$$

$$M = M_0 + M_1 X_1$$

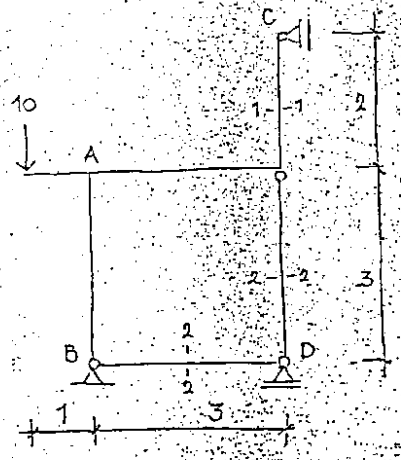


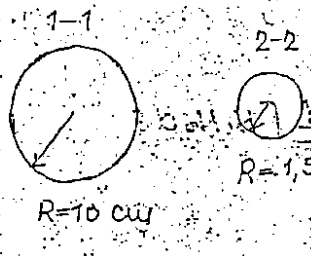
(142)

ЗА ДАТУ НОСАЧУ И ОПТЕРЕЋЕЊЕ:

А) НАЦРТАТИ ДИЈАГРАМ МОМЕНАТА САВИЈАЊА

В) ОДРЕДИТИ РОТАЦИЈУ А, КАО И ПРОМЕНУ РАСТОЈАЊА ИЗМЕЂУ В И С, ОДНОСНО ИЗМЕЂУ В И D, АКО ЈЕ $P=10$ kN $E=10$ GPa

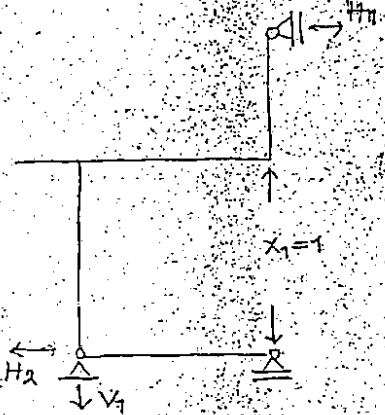




$$I_{1-1} = \frac{\pi R^4}{4} = \frac{\pi \cdot 10^4}{4} = 7853{,}98 \text{ cm}^4$$

$$F_{2-2} = R^2\pi = 1{,}5^2\pi = 7{,}07 \text{ cm}^2$$

$$\frac{I_{1-1}}{F_{2-2}} = \frac{7853{,}98 \cdot 10^{-8}}{7{,}07 \cdot 10^{-4}} = 0{,}111 \text{ m}^2$$

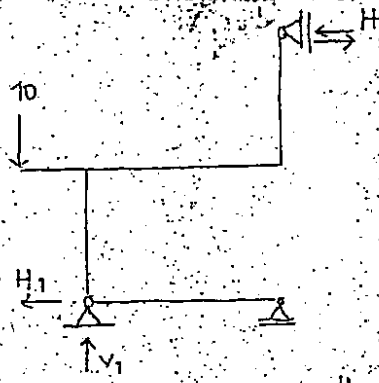


$\sum M_C = 0 \quad 1 \cdot 3 - V_2 \cdot 3 = 0 \quad V_2 = 1$

$V_1 + V_2 = 0 \quad V_1 = -1$

$\sum M_D = 0 \quad -H_1 \cdot 5 + V_1 \cdot 3 = 0 \quad H_1 = -3/5$

$H_2 = 3/5$



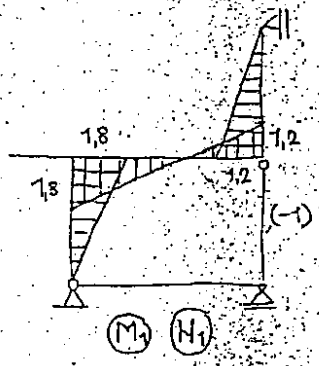
$\sum M_1 = 0$

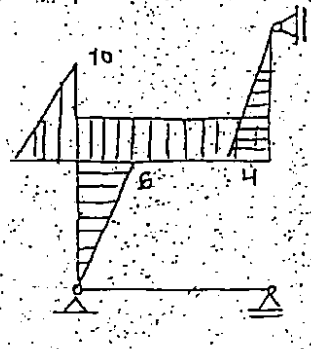
$-H_2 \cdot 5 - 10 \cdot 1 = 0$

$H_2 = -2$

$H_2 - H_1 = 0 \quad H_1 = 2$

$V_1 - 10 = 0 \quad V_1 = 10$





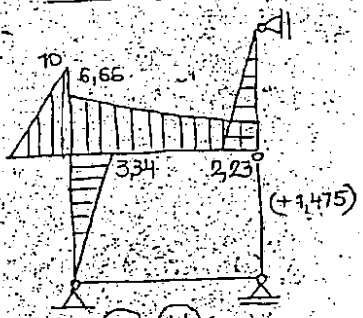
$$\delta_{11} X_1 + \delta_{10} = 0$$

$$\delta_{11} = \frac{1}{EI}\left[\frac{2}{3}1{,}2\cdot 1{,}2^2 + \frac{3}{3}1{,}8^2 + \frac{3}{3}(1{,}8^2 + 1{,}8\cdot 1{,}2 + 1{,}2^2)\right] + \frac{0{,}111\cdot 3\cdot 1^2}{EI} = \frac{7{,}05}{EI}$$

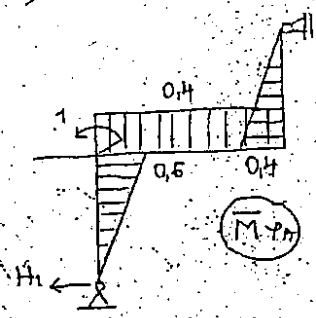
$$\delta_{10} = \frac{1}{EI}\left[\frac{2}{3}1{,}2\cdot 4 + \frac{3}{6}\left[1{,}2(2\cdot 4+4) - 1{,}8(2\cdot 4+4)\right] + \frac{3}{3}1{,}8\cdot 6\right] = \frac{10{,}4}{EI}$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{10{,}4}{7{,}05} = -1{,}475$$

$M = M_0 + X_1 M_1$



8) ROTACJA W TAKU A



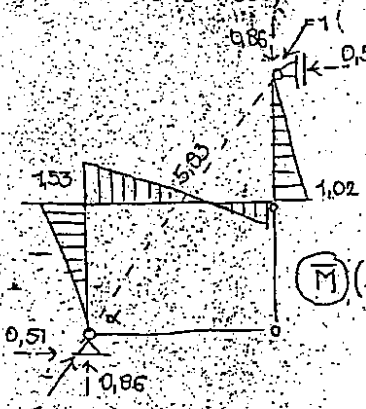
$\sum M_1^{\curvearrowright} = 0$

$H_2 \cdot 5 - 1 = 0$

$H_2 = H_1 = 1/5$

$$\varphi_A = \frac{1}{EI}\int M\bar{M}\,ds = \frac{1}{EI}\left[\frac{2}{3}2{,}23\cdot 0{,}4 + \frac{3}{2}0{,}4(6{,}66+2{,}23) + \frac{3}{3}3{,}34\cdot 0{,}6\right] = \frac{7{,}93}{EI}$$

$$\varphi_A = \frac{7{,}93\cdot 10^3\cdot 10^2}{10\cdot 10^9\cdot 7853{,}98\cdot 10^{-8}} = 1{,}01 \text{ rad}$$



$\cos\alpha = 0{,}51$

$\sin\alpha = 0{,}86$

$$\Delta l_{BC} = \frac{1}{EI}\int M\bar{M}\,ds = \frac{1}{EI}\left[-\frac{2}{3}2{,}23\cdot 1{,}02 + \frac{3}{6}\left[6{,}66(2\cdot 1{,}53 - 1{,}02) + 2{,}23(-2\cdot 1{,}02 + 1{,}53)\right]\right.$$
$$\left. - \frac{3}{3}1{,}53\cdot 3{,}34\right] = \frac{-0{,}402}{EI}$$

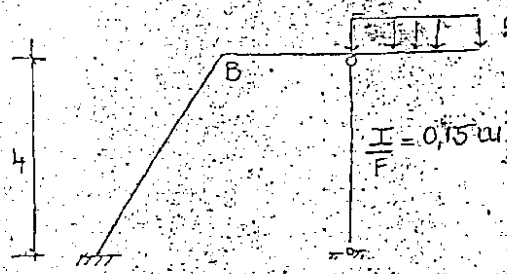
$$\Delta l_{BC} = \frac{-0{,}402\cdot 10^3\cdot 10^2}{10\cdot 10^9\cdot 7853{,}98\cdot 10^{-8}} = -0{,}051$$

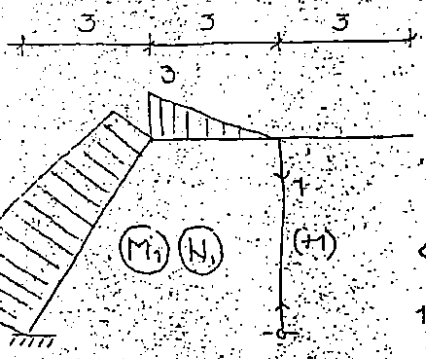
$\Delta l_{BD} = 0$

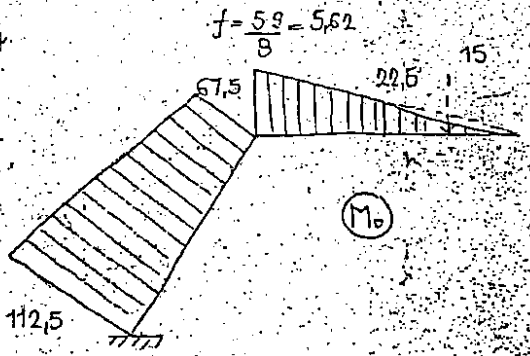
(14)

27. 05. 2005 W

ОДРЕДИТИ ВРЕДНОСТИ ХОРИЗОНТАЛНОГ И ВЕРТИКАЛНОГ ПОМЕРАЊА, ОДНОСНО РОТИРУЋЕ ПРЕСЕКА У ТАЧКИ B У ФУНКЦИЈИ КРУТОСТИ EI





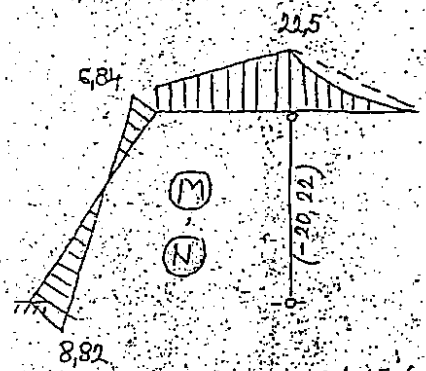


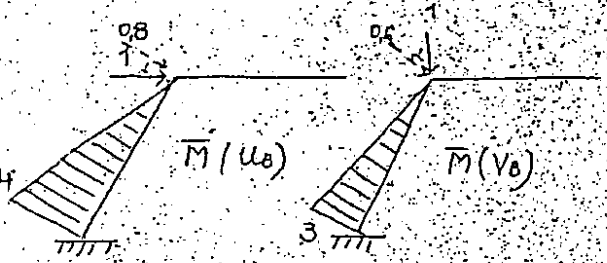
$$\delta_{11} X_1 + \delta_{10} = 0$$

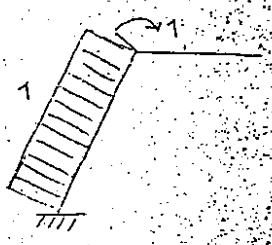
$$\delta_{11} = \frac{1}{EI}\int M_1 M_1 \, ds + \frac{1}{EF}\int N_1 N_1 \, ds = \frac{1}{EI}\left(\frac{5}{3}(6^2+6\cdot 3+3^2)+\frac{3}{3}3^2\right)+\frac{0{,}15}{EI}4\cdot 1^2 = \frac{114{,}6}{EI}$$

$$\delta_{10} = \frac{1}{EI}\left(\frac{5}{6}(6(2\cdot 112{,}5+67{,}5)+3(2\cdot 67{,}5+112{,}5))+\frac{3}{6}(3(2\cdot 67{,}5+22{,}5))\right) = \frac{2317{,}5}{1{,}1EI}$$

$$X_1 = -\frac{2317{,}5}{114{,}6} = -20{,}22 \qquad M = M_0 + X_1 M_1$$







$$u_B = \frac{1}{EI}\frac{5}{6}\left(6{,}84(2\cdot 0+4)-8{,}82(2\cdot 4-0)\right) = -\frac{36}{EI}$$

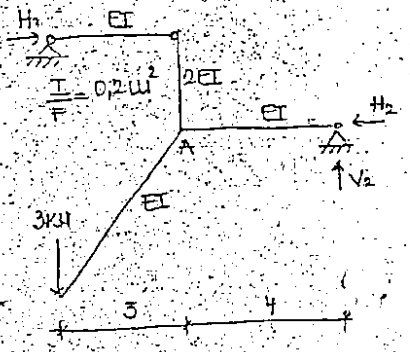
$$V_B = \frac{1}{EI}\frac{5}{6}\left(6{,}84(2\cdot 0+3)-8{,}82(2\cdot 3+0)\right) = -\frac{27}{EI}$$

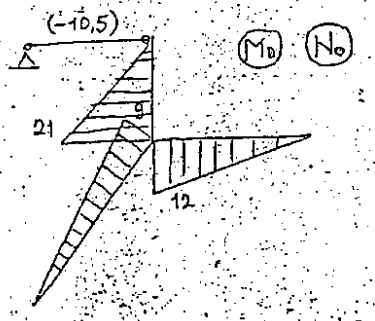
$$\varphi_B = \frac{1}{EI}\frac{5}{6}\left(6{,}84(2\cdot 1+1)-8{,}82(2\cdot 1+1)\right) = -\frac{5}{EI}$$

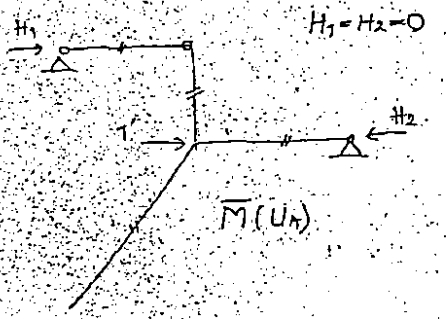
пе 09.2003. W

Одредити хоризонтално и вертикално померање у А, као и обртање

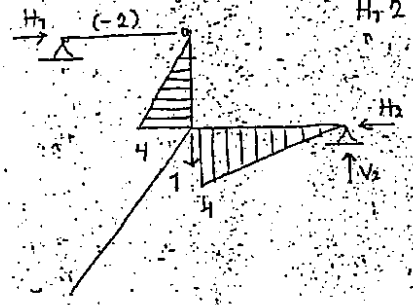
$\sum M_2 = 0 \quad H_1 \cdot 2 - 3 \cdot 7 = 0 \quad \underline{H_1 = 10,5} \quad \underline{H_2 = 10,5} \quad \underline{V_2 = 3}$







$H_1 = H_2 = 0$

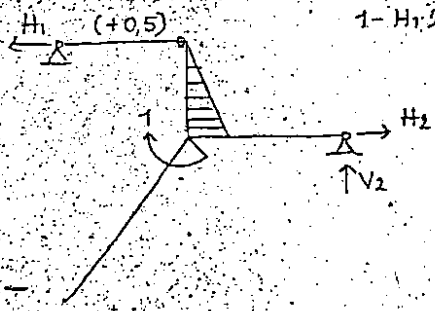


$H_1 \cdot 2 - 1 \cdot 4 = 0$

$H_1 = 2$

$H_2 = 2$

$V_2 = 1$



$1 - H_1 \cdot 2 = 0 \quad \underline{H_1 = 0,5}$

$\underline{H_2 = 0,5}$

$V_2 = 0$

$$U_A = \frac{1}{EI}\int M\bar{M}\,ds = 0$$

$$V_A = \frac{1}{EI}\left(\frac{1}{2}\cdot\frac{2}{3}\cdot 21\cdot 4 + \frac{4}{3}\cdot 12\cdot 4\right) + \frac{0,2}{EI}\,3\cdot 10,5\cdot 2 = \frac{104,6}{EI}$$

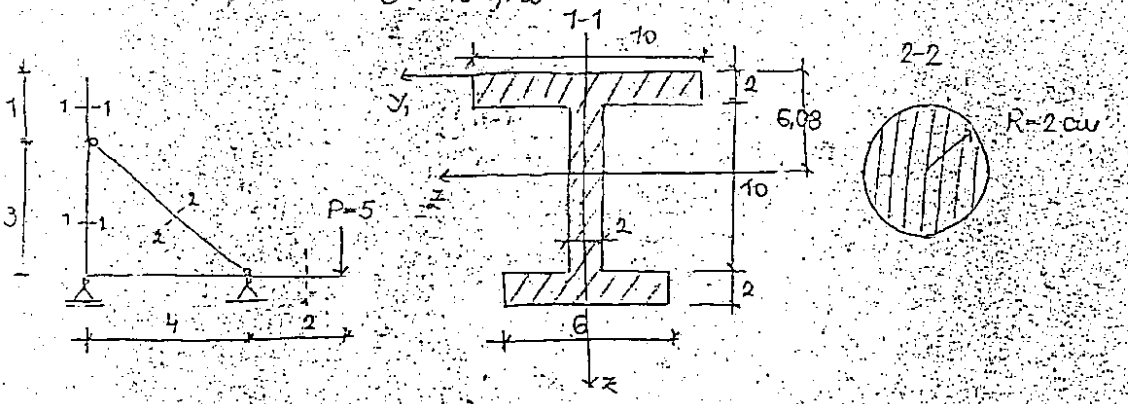
$$\varphi_A = \frac{1}{EI}\left(-\frac{1}{2}\cdot\frac{2}{3}\cdot 1\cdot 21\right) + \frac{0,2}{EI}\left(-0,5\cdot 10,5\cdot 3\right) = -\frac{10,15}{EI}$$

(14б)

05. 09. 2005.

Одредити вертикално померање нападне тачке силе $P = 5$ kN

$E = 210$ GPa



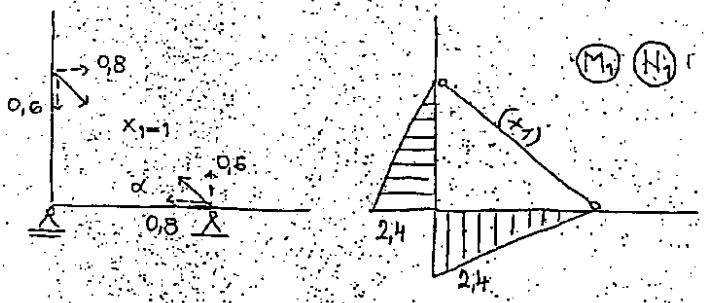
1-1 $F = 2 \cdot 10 + 10 \cdot 2 + 6 \cdot 2 = 52\ cm^2$

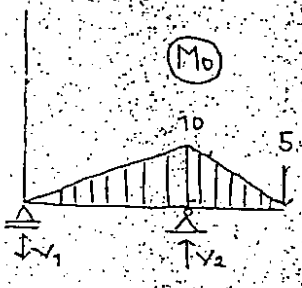
$z_T = \frac{20 \cdot 1 + 20 \cdot 7 + 12 \cdot 13}{52} = 6{,}08\ cm$

$I_y = \frac{1}{12} 10\, 2^3 + 5{,}08^2 \cdot 20 + \frac{1}{12} 2\, 10^3 + 0{,}92^2 \cdot 20 + \frac{1}{12} 6\, 2^3 + 6{,}92^2 \cdot 12 = 1285{,}03\ cm^4$

2-2 $F = R^2 \pi$

$F = 4\pi = 12{,}57\ cm^2$ $\quad \frac{I}{F} = \frac{1285{,}03 \cdot 10^{-8}}{12{,}57 \cdot 10^{-4}} = 0{,}01\ m^2$





$\Sigma M_{(+)} = 0 \quad 5 \cdot 6 - V_2 \cdot 4 = 0$

$V_2 = 7{,}5$

$V_1 = 2{,}5$

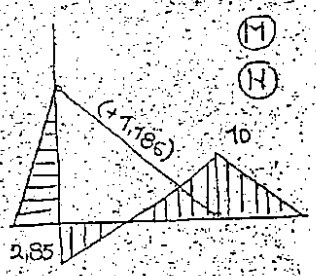
$\delta_{11} X_1 + \delta_{10} = 0$

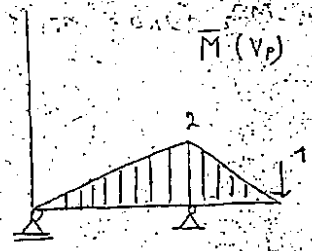
$\delta_{11} = \frac{1}{EI} \left(\frac{4}{3} 2{,}4^2 + \frac{3}{3} 2{,}4^2 \right) + \frac{0{,}01}{EI} 5 \cdot 1^2 = \frac{13{,}49}{EI}$

$\delta_{10} = \frac{1}{EI} \left(-\frac{4}{6} 10 \cdot 2{,}4 \right) = -\frac{36}{EI}$

$X_1 = -\frac{(-16)}{13{,}49} = 1{,}186$

$M = M_0 + M_1 X_1$





$$V = \frac{1}{EI}\int M\bar{M}\,ds = \frac{1}{EI}\left[\frac{2}{3}\,10\cdot 2 + \frac{4}{6}\left[2(2\cdot 10 - 2{,}85)\right]\right] = \frac{36{,}2}{EI}$$

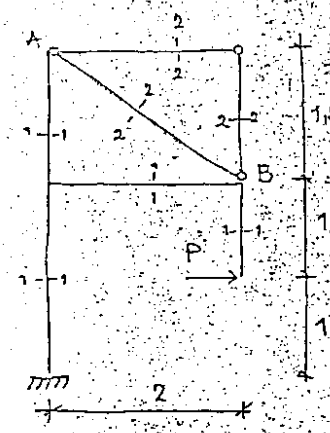
$$V = \frac{36{,}2\cdot 10^{3}}{210\cdot 10^{9}\cdot 1285{,}03\cdot 10^{-8}} = 0{,}073\ m$$

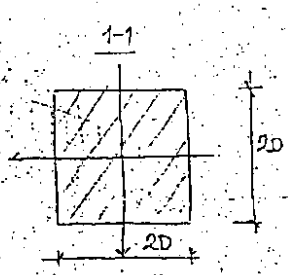
(148)

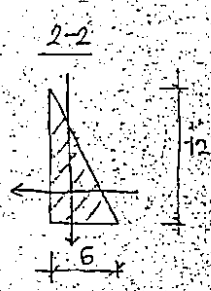
(A) ОДРЕДИТИ ВЕЛИЧИНУ СИЛЕ P ТАКО ДА КОЕФИЦИЈЕНТ СИГУРНОСТИ НА ИЗВИЈАЊЕ ШТАПА AB БУДЕ $n_S > 3,5$

W

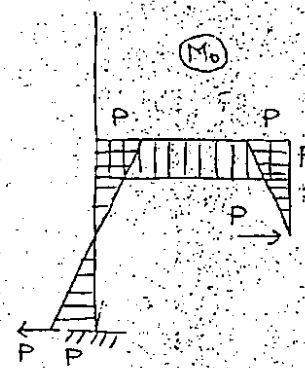
$E = 210\ GPa \quad \sigma_T = 240\ MPa \quad \sigma_{KR}^T = 310 - 1,14\lambda\ (MPa)$

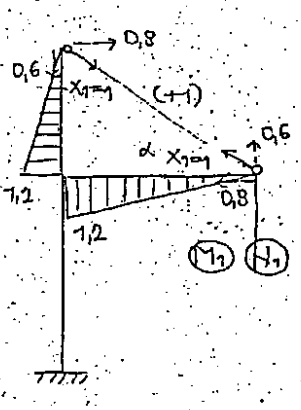






$\cos\alpha = 0,8$
$\sin\alpha = 0,6$





$I_{11} = \frac{1}{12} 20 \cdot 20^3 = 13333,33\ cm^4$

$F_{2-2} = \frac{1}{2} 6 \cdot 12 = 36\ cm^2$

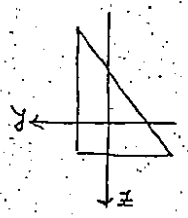
$\frac{I_{11}}{F} = \frac{13333,33 \cdot 10^{-8}}{36 \cdot 10^{-4}} = 0,037\ m^2$

$\delta_{11} = \frac{1}{EI}\left(\frac{1,5}{3} 1,2^2 + \frac{2}{3} 1,2^2\right) + \frac{0,037}{EI} \cdot 2,5 \cdot 1^2 = \frac{1,77}{EI}$

$\delta_{10} = \frac{1}{EI}\left(\frac{2}{2} P \cdot 1,2\right) = \frac{1,2P}{EI}$

$X_1 = -\frac{1,2P}{1,77} = -0,68P$

(N) $\Rightarrow \underline{\underline{-0,68P}}$



$I_y = \frac{1}{36} 6 \cdot 12^3 = 288\ cm^4$

$I_z = \frac{1}{36} 12 \cdot 6^3 = 72\ cm^4$

$I_{yz} = -\frac{1}{72} 6^2 \cdot 12^2 = -72\ cm^4$

$I_{1,2} = \frac{288+72}{2} \pm \sqrt{\left(\frac{288-72}{2}\right)^2 + (-72)^2}$

$I_1 = 309,8\ cm^4$

$I_2 = 50,2\ cm^4 \quad I_{min} = I_2$

$i_{min} = \sqrt{\frac{50,2}{36}} = 1,18\ cm \quad \lambda = \frac{l}{i_{min}} = \frac{250}{1,18} = 211,86$

$\sigma_{KR}^E = \left(\frac{\pi}{\lambda}\right)^2 \cdot E = 46,18\ MPa$

$\sigma_{KR}^T = 310 - 1,14\lambda = 68,48\ MPa$

$\sigma_{KR} = 46,18\ MPa$

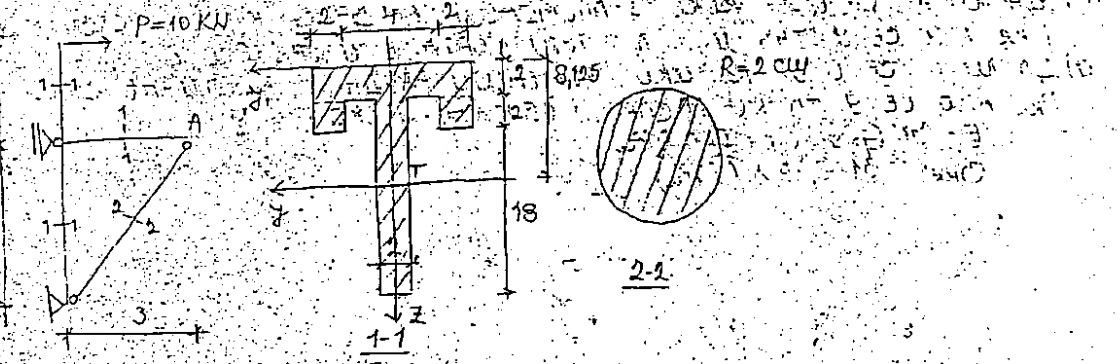
$P_{KR} = \sigma_{KR} \cdot F = \frac{46,18}{10} \cdot 36 = 166,25\ kN$

$n_S = \frac{P_{KR}}{P_S} > 3,5 \quad P_S < \frac{P_{KR}}{3,5} = 47,5\ kN \quad 0,68P < 47,5 \quad \underline{\underline{P < 70\ kN}}$

05.09.2005. IV

ОДРЕДИТИ ОБРТАЊЕ ПРЕСЕКА У ТАЧКИ A ЗА ВРЕДНОСТ СИЛЕ $P=10\,kN$

$E=210\,GPa$



ПРЕСЕК 1-1 : $F = 2\cdot2\cdot2+8\cdot2+20\cdot2 = 64\ cm^2$

$z_T = \dfrac{2\cdot2\cdot2\cdot3+2\cdot8\cdot1+20\cdot2\cdot12}{64} = 8{,}125\ cm$

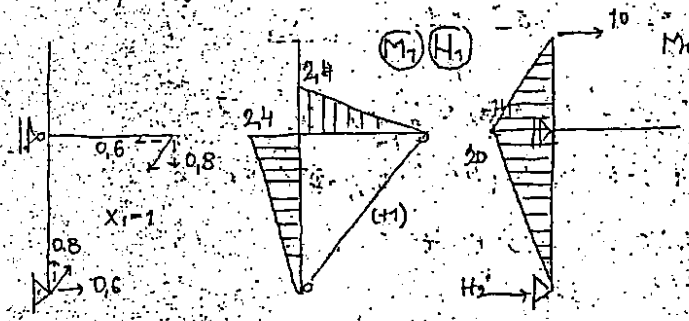
$I_y = \frac{1}{12}8\cdot2^3+8\cdot2\cdot7{,}125^2+2\left[\frac{1}{12}2\cdot2^3+5{,}125^2\cdot4\right]+\frac{1}{12}2\cdot20^3+3{,}875^2\cdot20\cdot2 = 2964{,}33\ cm^4$

ПРЕСЕК 2-2 : $F = R^2\bar{U} - 2^2\bar{U} = 12{,}57\ cm^2$

$\frac{I}{F} = \dfrac{2964{,}33\cdot10^{-8}}{12{,}57\cdot10^{-4}} = 0{,}0236\ m^2$

$\Sigma M = 0 \quad 10\cdot6-H_1\cdot4=0 \quad H_1=15$

$10-H_1-H_2=0 \quad H_2=-5$

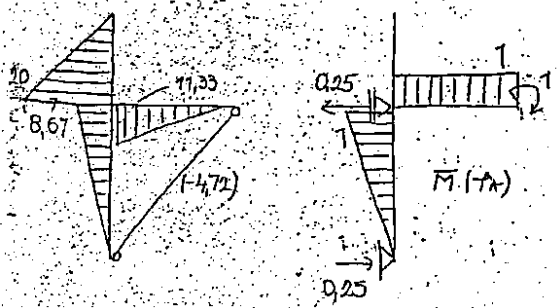


$\delta_{11} = \frac{1}{EI}\left(\frac{3}{3}2{,}4^2+\frac{4}{3}2{,}4^2\right)+0{,}0236\ 5\cdot1^2 = \frac{13{,}56}{EI}$

$X_1 = -\dfrac{64}{13{,}56} = -4{,}72$

$\delta_{10} = \frac{1}{EI}\frac{4}{3}20\cdot2{,}4 = \frac{64}{EI}$

$\Sigma M = 0 \quad 1+H_1\cdot4=0 \quad H_1=-0{,}25$



$f_A = \frac{1}{EI}\left(\frac{4}{3}8{,}67\cdot1-\frac{3}{2}11{,}33\cdot1\right) = -\dfrac{5{,}435}{EI}$

$f_A = \dfrac{-5{,}435\cdot10^{-8}}{210\cdot10^9\cdot2964{,}33\cdot10^{-8}} = -0{,}0873\ rad$

(150)

(*) a) ОДРЕДИТИ РАСТОЈАЊЕ b ТАКО ДА КОЕФИЦИЈЕНТ СИГУРНОСТИ ПРОТИВ ИЗВИЈАЊА ОКО ОСЕ z БУДЕ ИСТИ ЗА ТАКО ОДРЕЂЕНУ ВЕЛИЧИНУ b

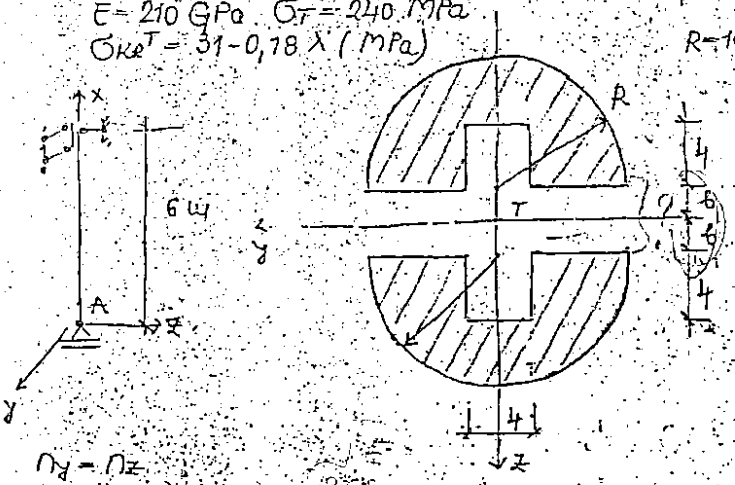
b) ОДРЕДИТИ ВРЕДНОСТ КРИТИЧНЕ СИЛЕ P_{KR}

c) ДА ЛИ ЋЕ СЕ И ЗА КОЛИКО ПРОМЕНИТИ ВРЕДНОСТ КРИТИЧНЕ СИЛЕ P_{KR} АКО СЕ У ТАЧКИ B ДОПУСТИ ОБРТАЊЕ И ОКО ОСЕ z?

d) ДА ЛИ ЋЕ СЕ И ЗА КОЛИКО ПРОМЕНИТИ ВРЕДНОСТ КРИТИЧНЕ СИЛЕ P_{KR} АКО СЕ У ТАЧКИ B СПРЕЧИ ОБРТАЊЕ И ОКО ОСЕ y?

$E = 210\ GPa$ $\sigma_T = 240\ MPa$

$\sigma_{KR}^T = 31 - 0{,}18\lambda\ (MPa)$



$l_y = 600 \cdot 1 = 600\ cm$

$l_z = 600 \cdot 0{,}7 = 420\ cm$

$F = R^2\bar{u} - 84 = 282{,}16\ cm^2$

$n_y = n_z$

$\frac{P_{KR,y}}{P} = \frac{P_{KR,z}}{P} \Rightarrow \frac{\bar{u}^2 E}{\lambda_y^2} = \frac{\bar{u}^2 E}{\lambda_z^2}$ $\frac{i_y^2}{l_y^2} = \frac{i_z^2}{l_z^2} \Rightarrow \frac{I_y}{F l_y^2} = \frac{I_z}{F l_z^2}$

$\Rightarrow \frac{I_y}{1} = \frac{I_z}{0{,}7^2}$ $0{,}7^2 I_y = I_z$ $0{,}5 I_y = I_z$ $\boxed{I_y = 2 I_z}$

$2\left[10^4 \cdot 2 \cdot 0{,}05488 + (4{,}24+b)^2 \frac{10^2\bar{u}}{2} - \frac{1}{12}4 \cdot 4^3 - 16(2+b)^2\right] = 2\left(\frac{1}{12}10^4\bar{u} - \frac{1}{12}4^4\right)$ I_b

$2\left[1097{,}6 + 157{,}08(17{,}98 + 8{,}48b + b^2) - 21{,}33 - 16(4 + 4b + b^2)\right] = 7832{,}65$

$\left[1097{,}6 + 2824{,}3 + 1332{,}04b + 157{,}08b^2 - 21{,}33 - 64 - 64b - 16b^2\right] = 7832{,}65$

$3836{,}57 + 1268{,}04b + 141{,}08b^2 = 7832{,}65$

$b^2 + 8{,}99b + 28{,}32 = 0$ $b_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8{,}99 \pm \sqrt{8{,}99^2 + 4 \cdot 28{,}32}}{2} \begin{cases} -2{,}47 \\ -11{,}46 \end{cases}$

$b = 2{,}47\ cm$

$I_z = 7832{,}65\ cm^4$ $I_y = 15\,657{,}88\ cm^4$ $i_z = \sqrt{\frac{I_z}{F}} = 5{,}27\ cm$ $i_y = \sqrt{\frac{I_y}{F}} = 7{,}45\ cm$

$\lambda_z = \frac{l_z}{i_z} = \frac{420}{5{,}27} = 79{,}7$ $\lambda_y = \frac{l_y}{i_y} = \frac{600}{7{,}45} = 80{,}54 \Rightarrow \lambda_{max} = 80{,}54$

$\sigma_{KR} = \sqrt{\left(\frac{\bar{u}}{\lambda}\right)^2 E} = \left(\frac{\bar{u}}{\lambda}\right)^2 \cdot 210 \cdot 10^3 = 319{,}52\ MPa$

$\sigma_{KR}^T = 310 - 0{,}8\lambda = 245{,}57\ MPa$

$\sigma_T = 240\ MPa$

$\sigma_{KR} = 240\ MPa$

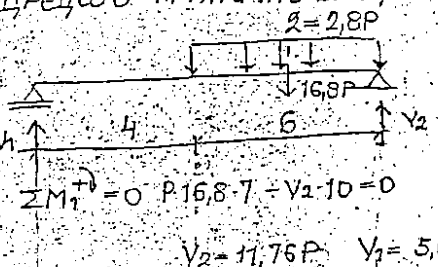
$P_{KR} = \sigma_{KR} F = 6771{,}84\ kN$

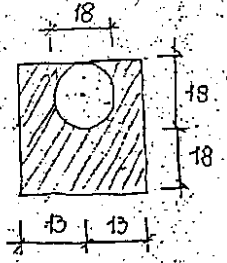
$\sigma_{KR,y} = \sigma_{KR,z} \Rightarrow \lambda$

ЕЛАСТОПЛАСТИЧНА АНАЛИЗА ГРЕДНОГ НОСАЧА

05.09.2003.

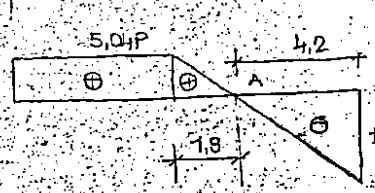
ОДРЕДИТИ ГРАНИЧНУ ВРЕДНОСТ ПАРАМЕТРА ОПТЕРЕЋЕЊА P^* $\sigma_T = 24\,MPa$



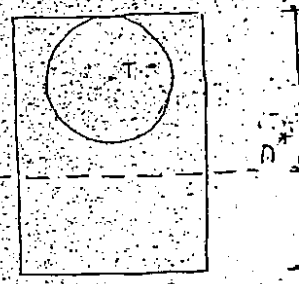


$\sum M_1^{+} = 0 \quad P\,16{,}8 \cdot 7 + V_2 \cdot 10 = 0$

$V_2 = 11{,}76P \qquad V_1 = 5{,}04P$



$M_{max} = 11{,}76P \cdot 4{,}2 - 2{,}8P \cdot \frac{4{,}2^2}{2} = 24{,}696\,P$



$F = 26 \cdot 36 - 9^2 U = 681{,}53\ cm^2 \qquad F/2 = 340{,}76\ cm^2 \qquad F_0 = 254{,}47\ cm^2$

$W^{+} \quad S_{pl} = 26 \cdot 13{,}106 \cdot 6{,}553 + 22{,}894 \cdot 26 \cdot 11{,}447$

$- 254{,}47 \cdot 13{,}894 = 5511{,}12\ cm^3$

$M^* = S_{pl} \cdot \sigma_T = 5511{,}12 \cdot 10^{-6} \cdot 24 \cdot 10^{6} = 132{,}27\ kNm$

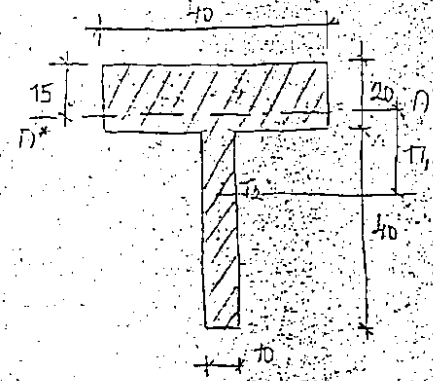
$M_{max} = 24{,}696\,P \qquad M^{+} = 132{,}27\ kNm$

$24{,}696\,P^{+} = 132{,}27 \qquad \underline{\underline{P^{+} = 5{,}35\ kN}}$

$M^* = M_{max} \cdot P^*$

147
146

(152) 27.06.2005 $M^* = ?$ $\sigma_T = 30\ MPa$ W



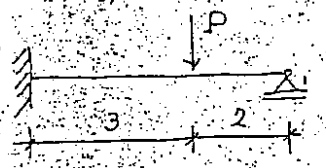
$F = 40 \cdot 20 + 40 \cdot 10 = 1200\ cm^2$ $F/2 = 600\ cm^2$

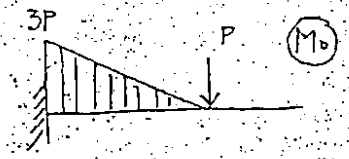
$z_{T_2} = \frac{400 \cdot 25 + 200 \cdot 2,5}{600} = 17,5\ cm$

$S_{pl} = 600 \cdot 17,5 + 600 \cdot 7,5 = 15\,000\ cm^3$

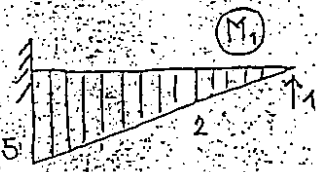
$M^* = S_{pl} \cdot \sigma_T = 15000 \cdot 10^{-6} \cdot 30 \cdot 10^6 = 450\ kNm$

$P^* = ?$



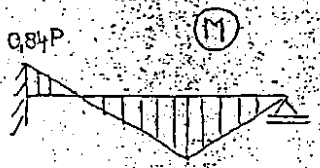


$\delta_{11} = \frac{1}{EI} \frac{5}{3} 5^2 = 41,67/EI$



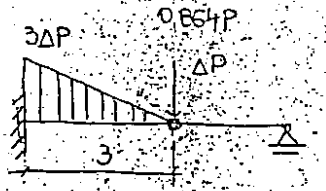
$\delta_{10} = \frac{1}{EI} \frac{3}{6} [3P(2 \cdot (-5) - 2)] = -18P/EI$

$X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{18P}{41,67} = 0,432P$



$M_{max} = 0,864\ P$

$P_1 = 1,157\ M^*$

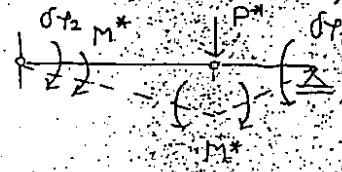


$M^* = 0,84P + 3\Delta P = 0,84 \cdot 1,157 M^* + 3\Delta P$

$\Delta P = 0,00937\ M^*$

$P^* = P_1 + \Delta P = (1,157 + 0,00937) M^* = 1,166\ M^*$

KINEMATIČKA METODA:



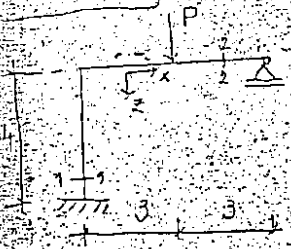
$3 \cdot \sigma\gamma_2 = 2 \cdot \sigma\gamma_1 \quad \sigma\gamma_2 = \frac{2}{3} \sigma\gamma_1$

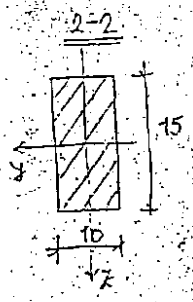
$P^* \sigma\gamma_1 \cdot 2 = 2M^* \sigma\gamma_2 + M^* \sigma\gamma_1$

$P^* \sigma\gamma_1 \cdot 2 = 2M^* \frac{2}{3} \sigma\gamma_1 + M^* \sigma\gamma_1$

$P^* = 1,166\ M^*$

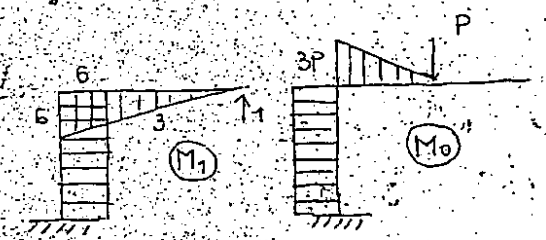
26.09.2003 $\sigma_T = 10$ MPa





ПРЕСЕК 1-1 $I_y^{1-1} = \frac{\bar{U} R^4}{4} = 490,87\ cm^4$ ПРЕСЕК 2-2 $I_y^{2-2} = \frac{1}{12} 10\ 15^3 = 2812,5\ cm^4$

$$\frac{I_y^{2-2}}{I_y^{1-1}} = \frac{2812,5}{490,87} \qquad \boxed{I_y^{2-2} = 5,73\ I_y^{1-1}}$$

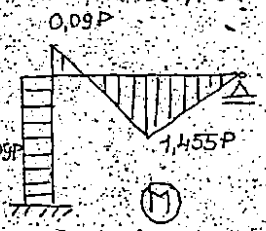


$$\delta_{11} = \frac{1}{EI_z} \frac{6}{3} 6^2 + \frac{5,73}{EI_z} 4 \cdot 6^2 = \frac{897,12}{EI_z}$$

$$\delta_{10} = \frac{1}{EI_z} \left[\frac{3}{6} (3P(2 \cdot (-6) - 3)) + 5,73 \cdot 4 \cdot 3P \cdot 6 \right]$$

$$\delta_{10} = -\frac{435,06P}{EI_z}$$

$$X_1 = \frac{435,06\ P}{897,12} = 0,485P$$

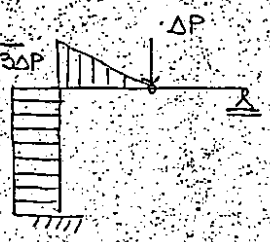


ПРЕСЕК 2-2 : $F = 150\ cm^2$ $F/2 = 75\ cm^2$

$S_{pl} = 75 \cdot 3,75 \cdot 2 = 562,5\ cm^3$

$M^* = S_{pl} \cdot \sigma_T = 562,5 \cdot 10^{-6} \cdot 10 \cdot 10^6 = 5,625\ kNm$

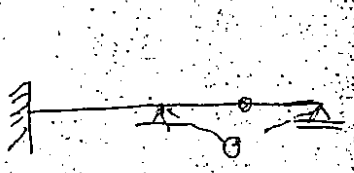
$1,455 P_1 = M^*$ $\quad P_1 = 3,866\ kN$



$3\Delta P + 0,09 P_1 = M^*$

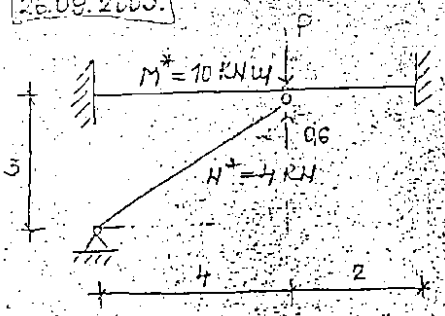
$3\Delta P + 0,348 = 5,625$ $\quad \Delta P = 1,759\ kN$

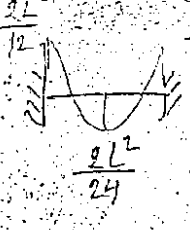
$P^* = P_1 + \Delta P = 3,866 + 1,759 = 5,625\ kN$

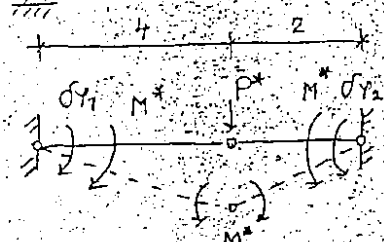


(157)

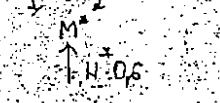
26.09.2005.







$4\,\delta\gamma_1 = 2\delta\gamma_2 \qquad \underline{\delta\gamma_2 = 2\delta\gamma_1}$



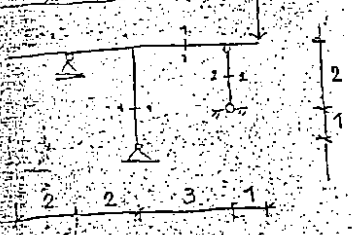
$$P^* \cdot 4\delta\gamma_1 - N^* 0{,}6 \cdot 4\delta\gamma_1 = 2M^*\delta\gamma_1 + 2M^*\delta\gamma_2$$

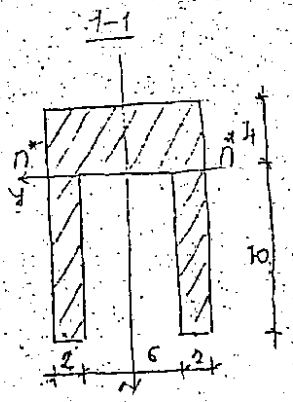
$$4P^*\delta\gamma_1 - 2{,}4N^*\delta\gamma_1 - 2M^*\delta\gamma_1 + 4M^*\delta\gamma_1$$

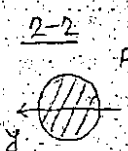
$$4P^* = 2{,}4 \cdot 4 + 6 \cdot 10$$

$$\underline{P^* = 17{,}4\ KN}$$

05.10-2003 W







$\sigma_T = 24\ MPa$
$F = 12,57\ cm^2$
$I_y = 12,57\ cm^4$

$\sigma_T = 240\ MPa$
$I_y = 1366,7\ cm^4$
$F = 80\ cm^2$

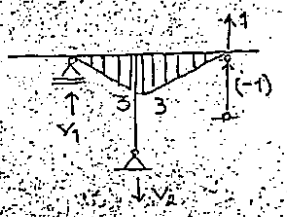
$\frac{I_y}{F_2} = \frac{1366,7 \cdot 10^{-8}}{12,57 \cdot 10^{-4}} = 0,01\ m^2$

PRESEK 1-1 $F/2 = 40\ cm^2$

$S_{pl} = 4 \cdot 10 \cdot 2 + 2 \cdot 2 \cdot 10 \cdot 5 = 280\ cm^3$

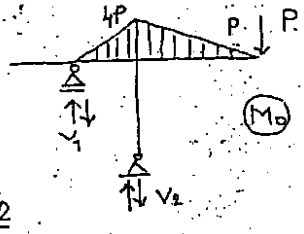
$M^* = S_{pl} \cdot \sigma_T = 280 \cdot 10^{-6} \cdot 240 \cdot 10^6 = 67,2\ kNm$

PRESEK 2-2 $N^* = F \cdot \sigma_T = 12,57 \cdot 10^{-4} \cdot 24 \cdot 10^6 = 30,17\ kN$



(M_1) (N_1)

$\Sigma M^{\curvearrowright} = 0 \quad V_1 \cdot 2 - 1 \cdot 3 = 0 \quad V_1 = 1,5$
$V_1 + 1 - V_2 = 0 \quad V_2 = 2,5$



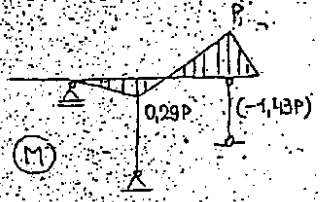
(M_0)

$\Sigma M^{\curvearrowright} = 0 \quad V_1 \cdot 2 + P \cdot 4 = 0$
$V_1 = -2P$
$V_1 - V_2 - P = 0$
$V_2 = -3P$

$\sigma_{11} = \frac{1}{EI}\left(\frac{2}{3}3^2 + \frac{3}{3}3^2\right) + \frac{0,01}{EI} 2 \cdot 1^2 = \frac{15,02}{EI}$

$\sigma_{10} = \frac{1}{EI}\left[-\frac{2}{3}3 \cdot 4P + \frac{3}{6}\left[-3(2 \cdot 4P + P)\right]\right] = -\frac{21,5P}{EI}$

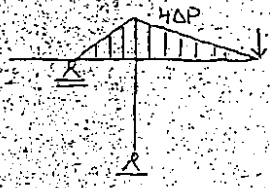
$X_1 = \frac{21,5P}{15,02} = 1,43P$



(M)

I PRVO NE DOĐU DO PLASTIFIKACIJE ŠTAPA

$1,43 P_1 = N^* \quad P_1 = \frac{30,17}{1,43} = 21,1\ kN$

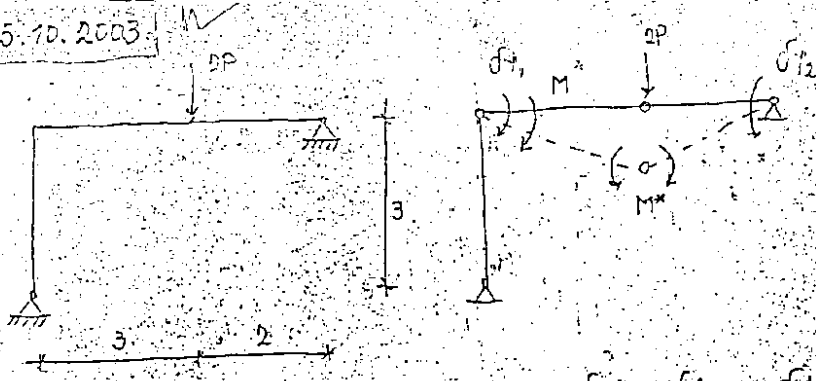


$4\Delta P - 0,29 P_1 = M^* \quad \Delta P = 18,33\ kN$

$P_1 + \Delta P = 21,1 + 18,33 = 39,43\ kN \quad P^* = 39,43\ kN$

(156)

05.10.2003



$3\delta\gamma_1 = 2\delta\gamma_2 \quad \delta\gamma_2 = 1{,}5\,\delta\gamma_1$

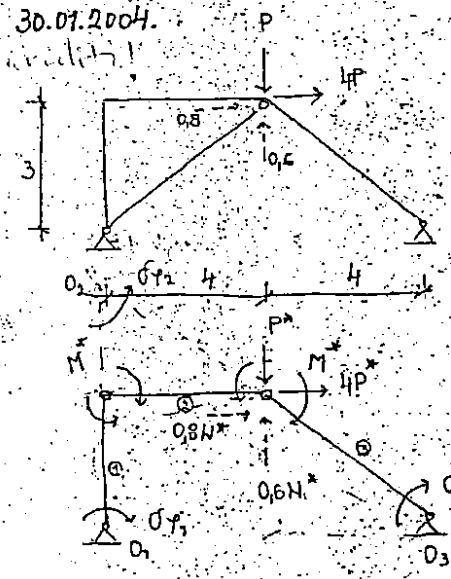
$2P \cdot 3\delta\gamma_1 = 2M^* \delta\gamma_1 + M^* \delta\gamma_2$

$6P\delta\gamma_1 = 2M^*\delta\gamma_1 + 1{,}5M^*\delta\gamma_1$

$P^* = 0{,}583\,M^*$

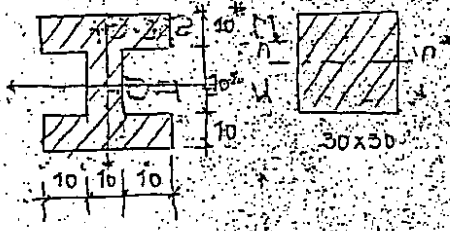
W

30.01.2004.



A-C

A-B
B-C
C-E



A-B, B-C

$F/2 = 350\ cm^2$

$Sp\ell = (300 \cdot 10 + 50 \cdot 2{,}5) \cdot 2 = 6250\ cm^3$

$M^* = 6250 \cdot 10^{-6} \cdot 10 \cdot 10^6 = 62{,}5\ KNm$

C-E

$F/2 = 450\ cm^2$

$Sp\ell = 450 \cdot 7{,}5 \cdot 2 = 6750\ cm^3$

$M^* = 6750 \cdot 10^{-6} \cdot 10 \cdot 10^6 = 67{,}5\ KNm$

A-C

$N^* = 225 \cdot 10^{-4} \cdot 10 \cdot 10^6 = 225\ KN$

$-P^* 4\delta\gamma_2 + 4P^* \delta\gamma_3 3 = M^* \delta\gamma_1 + 2M^* \delta\gamma_2 + M^* \delta\gamma_3 + 0{,}8N^* 3\delta\gamma_3 + 0{,}6N^* 4\delta\gamma_3$

$8P^* = 4M^* + 4{,}8N^*$

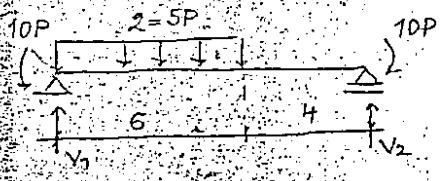
$P^* = \frac{4 \cdot 62{,}5 + 4{,}8 \cdot 225}{8}$

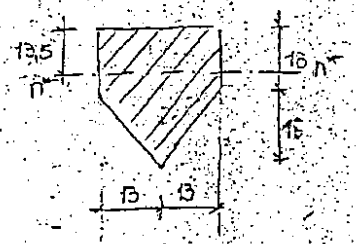
$P^* = 166{,}25\ KN$

05.09.2003. IV

Одредити граничну вредност параметра оптерећења P^*

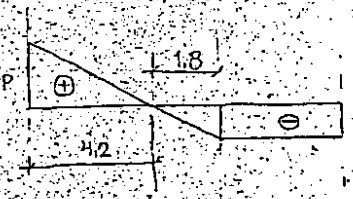
$\sigma_T = 24$ MPa





$\sum M_1 = 0 \quad 30P \cdot 3 - V_2 \cdot 10 = 0 \quad V_2 = 9P$

$V_1 - 30P + V_2 = 0 \quad V_1 = 21P$



$M_{max} = 21P \cdot 4,2 - 5 \cdot \frac{4,2^2}{2} = 70P = 34,1P$

$F = 18 \cdot 26 + \frac{1}{2} 26 \cdot 18 = 702 \text{ cm}^2 \qquad F/2 = 351 \text{ cm}^2$

$W^* = S_{pl} = 351 \cdot 6,75 + 4,5 \cdot 26 \cdot 2,25 + 234 \cdot 10,5 = 5089,5 \text{ cm}^3$

$M^* = S_{pl} \cdot \sigma_T = 5089,5 \cdot 10^{-6} \cdot 24 \cdot 10^6 = 122,15 \text{ KN m}$

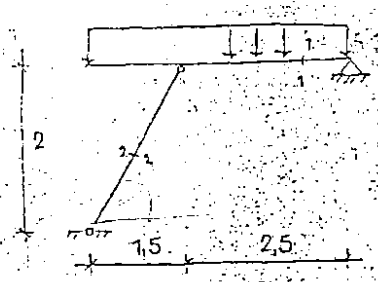
$34,1 P^* = 122,16 \qquad \underline{P^* = 3,58 \text{ KN}}$

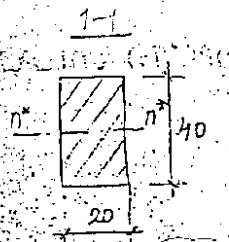
$M^* = P^* M_{max}$

$M^* = S_{pl} \cdot \sigma_T$

(15 8)

$2^T, 2^* = ?$ W





$\sigma_T = 24 MPa$

2-2

$\sigma_T = 240 MPa$
$R = 4 cm$

ПРЕСЕК 1-1

$F/2 = 400 cm^2$

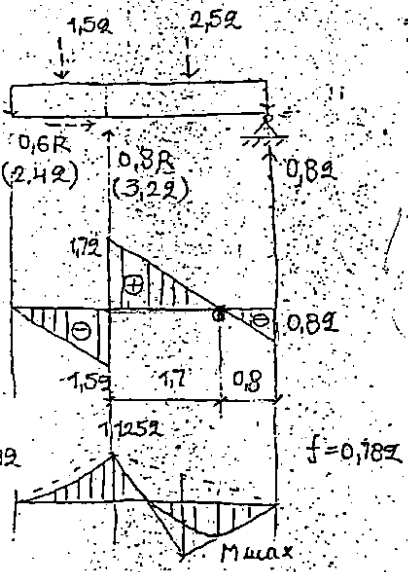
$S_{pl} = 400 \cdot 10 \cdot 2 = 8000 cm^3$

$M^* = S_{pl} \cdot \sigma_T = 8000 \cdot 24 = 192 KN cm$

ПРЕСЕК 2-2

$F = R^2 \pi = 50,26 cm^2$

$N^* = F \cdot \sigma_T = 50,26 \cdot 10^{-4} \cdot 240 \cdot 10^6 = 1206,24 KN$



$\Sigma M^{+\circlearrowright} = 0 \quad -4q \cdot 2 + 0,8R \cdot 2,5 = 0$

$R = 4q$

$M_{max} = 0,8q \cdot 0,8 - 2 \frac{0,8^2}{2} = 0,32q$

a) $1,125 q_1' = M^*$

$q_1' = \frac{192}{1,125} = 170,67 KN/m$

b) $4 q_1'' = N^*$

$q_1'' = \frac{1206,24}{4} = 301,56 KN/m$

$\Rightarrow \boxed{q^* = 170,67 KN/m}$

2^T: $M_{1-1}^T = W_y \sigma_T = \frac{1}{6} 20 \cdot 40^2 \cdot 10^{-6} \cdot 24 \cdot 10^6 = 128 KN cm$

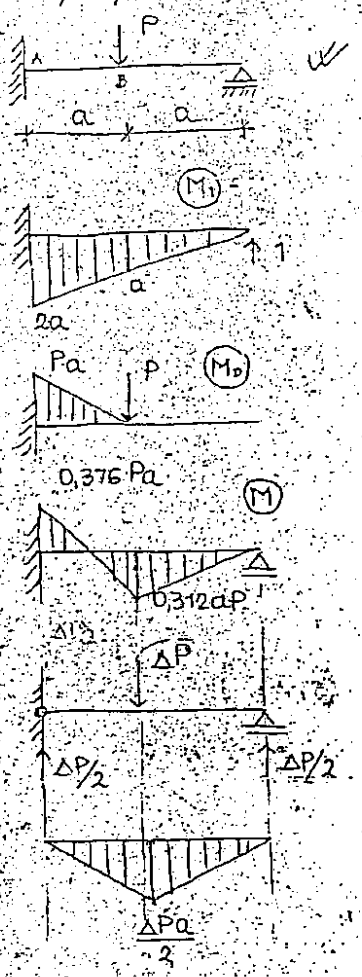
$N_{2-2}^T = F \cdot \sigma_T = 1206,24 KN$

$q_T' = \frac{128}{1,125} = 113,78 KN/m$

$q_T'' = \frac{1206,24}{4} = 301,56 KN/m$

$\boxed{q^T = 113,78 KN/m}$

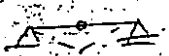
Наћи оптерећење при коме се формира први пластични зглоб W и одредити граничну оптерећење у функцији M^+



$$\delta_{11} = \frac{1}{EI}\cdot\frac{2a}{3}\,4a^2 = \frac{2{,}67a^3}{EI}$$

$$\delta_{10} = \frac{1}{EI}\cdot\frac{a}{6}\left[Pa(2(-2a)-a)\right] = -\frac{0{,}83a^3P}{EI}$$

$$X_1 = \frac{0{,}83a^3P}{2{,}67a^3} = 0{,}312P$$



$$M^+ = 0{,}376\,Pa$$

$$\boxed{P_1^+ = 2{,}66\frac{M^+}{a}}$$

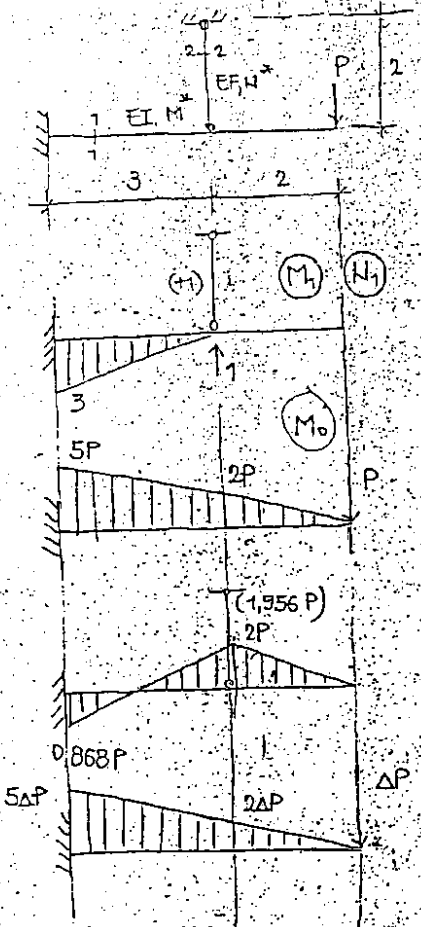
$$0{,}5\Delta Pa + 0{,}312Pa = M^+$$

$$0{,}5\Delta Pa = M^+ - 0{,}83M^+$$

$$\boxed{\Delta P = 0{,}34\frac{M^+}{a}}$$

$$P^+ = P_1 + \Delta P = 2{,}66\frac{M^+}{a} + 0{,}34\frac{M^+}{a} \qquad \boxed{P^+ = 3\frac{M^+}{a}}$$

(169) ОДРЕДИТИ ГРАНИЧНО ОПТЕРЕЋЕЊЕ НОСАЧА P^* W



$M^*_{H} = 150\ KN\,m$

$N^*_{2-2} = 60\ KN$

$\frac{I_{1-1}}{F_{2-2}} = 0{,}1\ m^2$

$\delta_{11} = \frac{1}{EI}\frac{3}{3}3^2 + \frac{0{,}1}{EI}2\cdot 1^2 = 9{,}2/EI$

$\delta_{10} = \frac{1}{EI}\frac{3}{6}\left[-5\left[2\cdot 5P + 2P\right]\right] = -\frac{18P}{EI}$

$X_1 = \frac{18P}{9{,}2} = 1{,}956\,P$

$2P_1' = M^* \quad P_1' = 75\ KN$

$1{,}956\,P_1' = N^* \quad P_1' = 30{,}67\ KN$ } ПЛАСТИФИКАЦИЈА ШТАПА

$P_1 = 30{,}67\ KN$

1° $2\Delta P' + 2P_1 = M^* \quad \Delta P' = \frac{150 - 2\cdot 30{,}67}{2} = 44{,}33\ KN$

2° $5\Delta P'' - 0{,}868P_1 = M^* \quad \Delta P'' = \frac{150 + 0{,}868\cdot 30{,}67}{5} = 35{,}32\ KN$

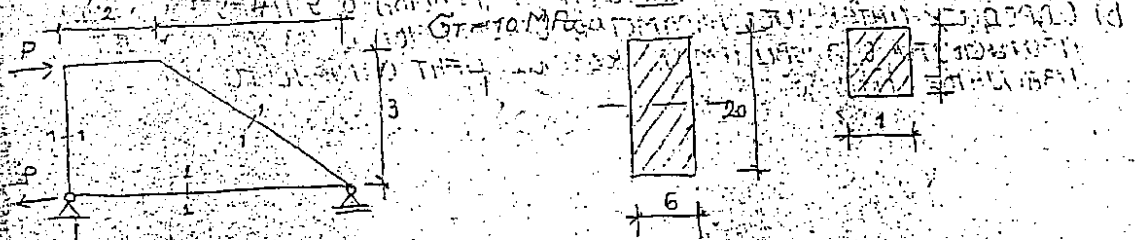
$\Rightarrow P^* = P_1 + \Delta P'' = 30{,}67 + 35{,}32 = 66\ KN$

$P^*\,5\delta_{11} - N^*\,3\delta_{11} = M^*\,\delta_{11}$

$5P^* = 150 + 3\cdot 60$

$P^* = 66\ KN$

ОДРЕДИТИ ГРАНИЧНУ ВРЕДНОСТ ОПТЕРЕЋЕЊА P^* И ИЗРАЧУНАТУ ВРЕДНОСТ ПРОВЕРИТИ КИНЕМАТИЧКОМ МЕТОДОМ



ПРЕСЕК 1-1. $F/2 = 60\ cm^2$

$S_{pl} = 60 \cdot 5 \cdot 2 = 600\ cm^3$

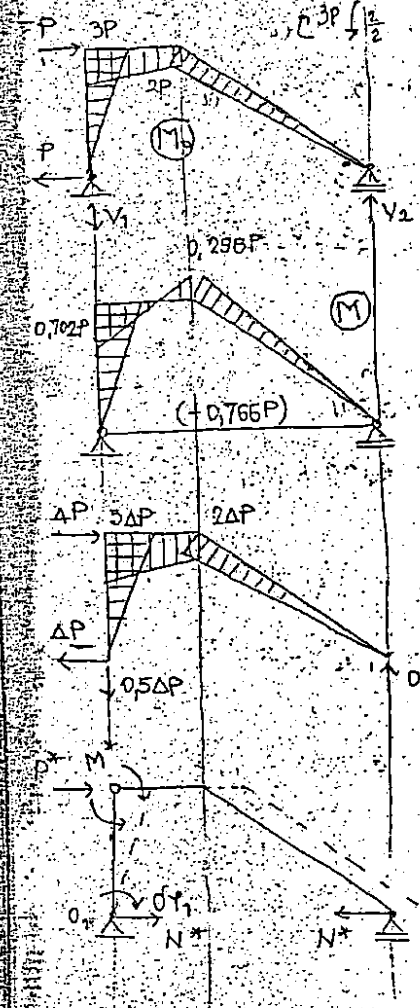
$M^* = 600 \cdot 10^{-6} \cdot 10 \cdot 10^6 = 6\ kNm$

ПРЕСЕК 2-2: $N^* = 1 \cdot 10^{-4} \cdot 10 \cdot 10^6 = 1\ kN$

$N^* = F \cdot \sigma_T$

$I_{1-1} = \frac{1}{12} 6 \cdot 20^3 = 4000\ cm^4$

$F_{2-2} = 1\ cm^2 \qquad \frac{I_{1-1}}{F_{2-2}} = \frac{4000 \cdot 10^{-8}}{10^{-4}} = 0{,}4\ m^2$



$\sum M_{(2)} = 0 \quad P \cdot 3 - V_2 \cdot 6 = 0 \quad V_2 = 0{,}5P$

$$\delta_{11} = \frac{1}{EI}\left[\frac{3}{2}3^2 + 2 \cdot 3^2 + \frac{5}{2}3^2\right) + \frac{0{,}4}{EI} 6 \cdot 1^2 = \frac{44{,}4}{EI}$$

$$\delta_{10} = \frac{1}{EI}\left[-\frac{3}{2} 3P \cdot 3 + \frac{2}{6}\left[3(-2 \cdot 3P - 2P) + 3(-2 \cdot 2P - 3P)\right] - \frac{5}{3} 3 \cdot 2P\right]$$

$$= -\frac{34P}{EI}$$

$$X_1 = \frac{34P}{44{,}4} = 0{,}766P$$

$$N^* = 0{,}766\, P_1 \qquad \boxed{P_1 = 1{,}31\ kN}$$

$$3\Delta P + 0{,}702\, P_1 = M^*$$

$$\Delta P = \frac{6 - 0{,}702 \cdot 1{,}31}{3} = 1{,}693\ kN$$

$$\underline{P^* = 1{,}31 + 1{,}693 = 3\ kN}$$

$$P^* \cdot 3\delta_{Y_1} + N^* \cdot 3\delta_{Y_1} = M^* \delta_{Y_1}$$

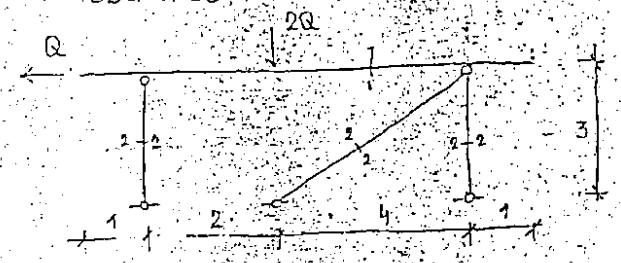
$$P^* = \frac{6 + 3 \cdot 1}{3}$$

$$\boxed{P^* = 3\ kN}$$

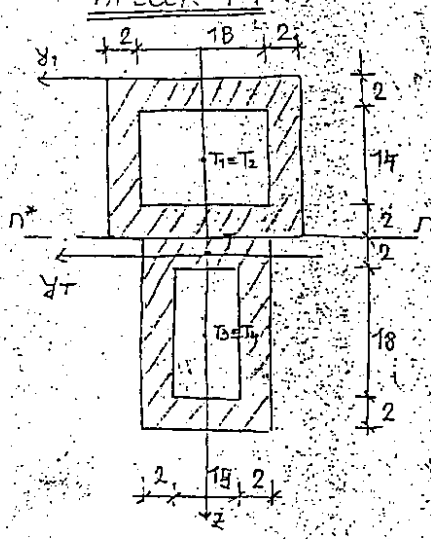
(162) (Н) 02.09.2005. IV

а) ОДРЕДИТИ ИНТЕНЗИТЕТ ПАРАМЕТРА ОПТЕРЕЋЕЊА Q У ТРЕНУТКУ ЛОМА
б) ОДРЕДИТИ ИНТЕНЗИТЕТ ПАРАМЕТРА ОПТЕРЕЋЕЊА Q ИЗ УСЛОВА ДА ПРИПИСНУТИ ШТАПОВИ ИМАЈУ КОЕФИЦИЈЕНТ СИГУРНОСТИ НА ИЗВИЈАЊЕ $n_S \geq 3{,}5$

$\sigma_T = 240$ MPa
$E = 210$ GPa



ПРЕСЕК 1-1



$F_1 = 22 \cdot 18 = 396$ cm^2
$F_2 = 18 \cdot 14 = 252$ cm^2
$F_3 = 22 \cdot 18 = 396$ cm^2
$F_4 = 18 \cdot 14 = 252$ cm^2
$F = 288$ cm^2
$F/2 = 144$ cm^2

$$z_T = \frac{9 \cdot 396 - 252 \cdot 9 + 396 \cdot 29 - 252 \cdot 29}{288} = 19 \text{ cm}$$

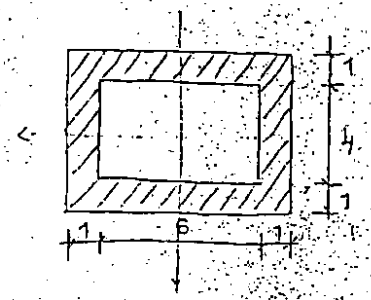
$$I_y = \frac{1}{12} 22 \cdot 18^3 + 10^2 \cdot 396 - \frac{1}{12} 18 \cdot 14^3 - 10^2 \cdot 252$$
$$+ \frac{1}{12} 18 \cdot 22^3 + 10^2 \cdot 396 - \frac{1}{12} 14 \cdot 18^3 - 10^2 \cdot 252$$

$I_y = 44\,544$ cm^4

$S_{pl} = 144 \cdot 9 + 144 \cdot 11 = 2880$ cm^3

$M^* = 2880 \cdot 10^{-6} \cdot 240 \cdot 10^6 = 691{,}2$ kN m

ПРЕСЕК 2-2



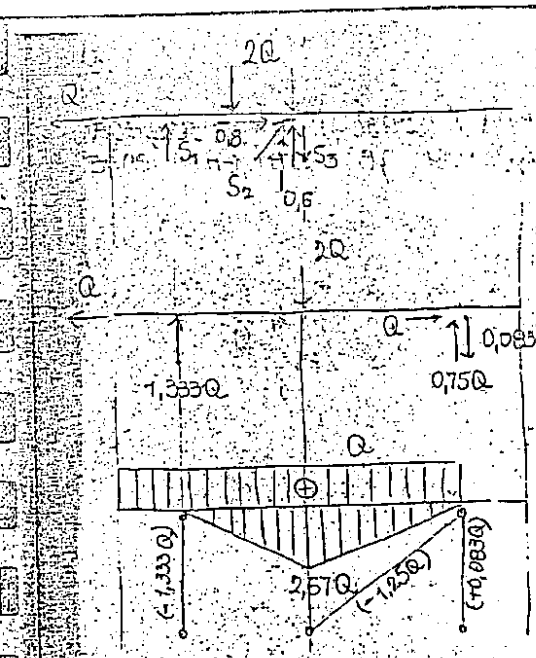
$F = 6 \cdot 8 - 6 \cdot 4 = 48 - 24 = 24$ cm^2

$N^* = 24 \cdot 10^{-4} \cdot 240 \cdot 10^6 = 576$ kN

$$\frac{I_y^{1-1}}{F_{2-2}} = \frac{44544 \cdot 10^{-8}}{24 \cdot 10^{-4}} = 0{,}186 \text{ m}^2$$

$$I_y = \frac{1}{12} 8 \cdot 6^3 - \frac{1}{12} 6 \cdot 4^3 = 112 \text{ cm}^4$$

$$I_z = \frac{1}{12} 8^3 \cdot 6 - \frac{1}{12} 6^3 \cdot 4 = 184 \text{ cm}^4$$



$\Sigma X = 0 \quad -Q + 0{,}8\,S_2 = 0 \quad \boxed{S_2 = 1{,}25Q}$

$\Sigma M_{+} = 0 \quad 2Q \cdot 3 - S_3 \cdot 3 - 0{,}6 \cdot 1{,}25Q \cdot 3 - S_1 \cdot 1 = 0$

$6 - 3S_3 + S_1 = 0{,}75Q \quad S_1 = 0{,}75Q - 3S_3$

$\Sigma Y = 0 \quad S_1 - 2Q + S_3 + 0{,}6 \cdot 1{,}25Q = 0$

$-2Q + 0{,}75Q - 3S_3 + S_3 + 0{,}6 \cdot 1{,}25Q = 0$

$\boxed{S_3 = -0{,}083\,Q \quad S_1 = 1{,}333\,Q}$

(M) (N)

$\boxed{2{,}67\,Q_1' = M^*} \quad Q_1' = 258{,}9\ KN$

$1{,}333\,Q_1'' = N^* \quad Q_1'' = 432{,}1\ KN$

$\boxed{Q^* = 258{,}9\ KN}$

ШТАП 1 $\boxed{1{,}333\,Q}$

$\sigma_{KR} = \left(\frac{\pi}{\lambda}\right)^2 E \qquad \lambda = \frac{l}{i_{min}} \qquad i_{min} = \sqrt{\frac{112}{24}} = 2{,}16\ cm$

$\lambda = \frac{300}{2{,}16} = 138{,}87$

$\sigma_{KR} = \left(\frac{\pi}{138{,}87}\right)^2 \cdot 210 \cdot 10^3 = 107{,}47\ MPa$

$P_{KR} = \sigma_{KR} \cdot F = 107{,}47 \cdot 24 \frac{1}{10} = 257{,}93\ KN$

$n_s = \frac{P_{KR}}{S_1} \geq 3{,}5 \quad S_1 \leq \frac{257{,}93}{3{,}5} \quad Q' \leq \frac{257{,}93}{3{,}5 \cdot 1{,}333} = 55{,}28\ KN$

ШТАП 2 $\boxed{1{,}25\,Q}$

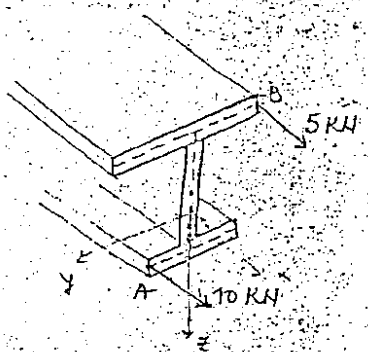
$n_s = \frac{P_{KR}}{S_2} \geq 3{,}5 \quad Q'' \leq \frac{257{,}93}{3{,}5 \cdot 1{,}25} = 58{,}96\ KN$

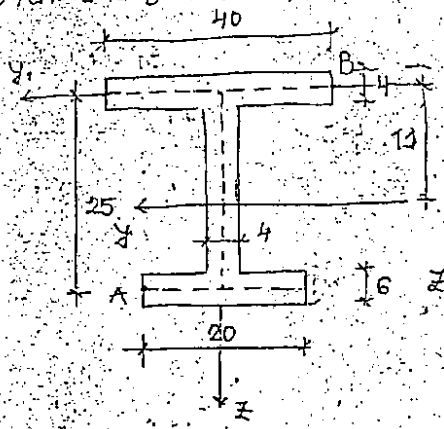
$\boxed{Q^* = 58{,}96\ KN}$

Пресм... да се... посредно P_{KR}... [illegible] (здої l)...

27.06.2003. !!! IV

Одредити вредности пресечних сила на крају конзоле танкозидног попречног пресека ако у тачкама A и B делују две концентрисане силе затезања $P_A = 10$ kN и $P_B = 5$ kN





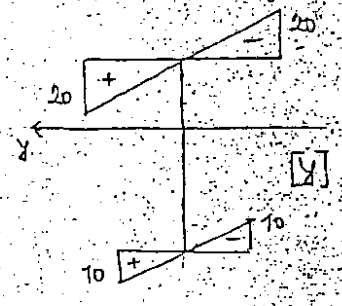
$F_1 = 40 \cdot 4 = 160$ cm^2

$F_2 = 20 \cdot 4 = 80$ cm^2

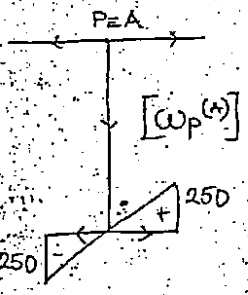
$F_3 = 6 \cdot 20 = 120$ cm^2

$F = 360$ cm^2

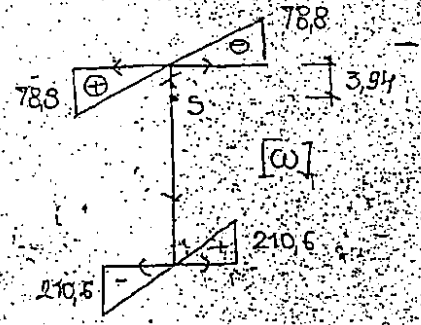
$z_T = \frac{80 \cdot 12 + 120 \cdot 25}{360} = 11$ cm



[y]



$[\omega_P^{(A)}]$



$[\omega]$

$$z_S = \underset{0}{z_P} - \frac{I_y \omega_P^{(A)}}{I_z}$$

$$I_z = 2 \cdot \frac{20}{3} 20^2 \cdot 4 + 2 \cdot \frac{10}{3} 10^2 \cdot 6 = 25333{,}33 \text{ cm}^4$$

$$I_y \omega_P^{(A)} = \left[-\frac{10}{3} 10 \cdot 250 - \frac{10}{3} 10 \cdot 250 \right] 6 = -100\,000 \text{ cm}^5$$

$$z_S = -\frac{(-100\,000)}{25333{,}33} = 3{,}94 \text{ cm}$$

ПРЕСЕЧНЕ СИЛЕ:

$N = 10 + 5 = 15$ kN

$M_y = 10 \cdot 14 - 5 \cdot 11 = 85$ kN cm

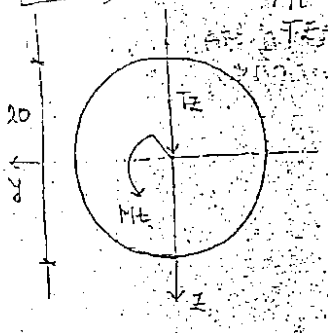
$M_z = -10 \cdot 10 + 5 \cdot 20 = 0$

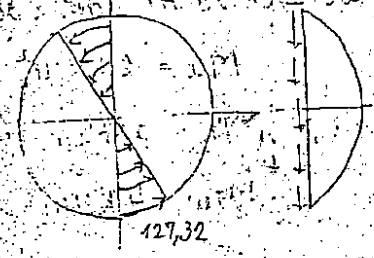
$M_\omega = 5 \cdot (-78{,}8) + 10 \cdot (-210{,}6) = -2500$ kN cm^2

26.09.2003

$M_t = 100$ KNcm

$T_z = 50$ KN





$F = R^2\pi = 314,16$ cm^2 $\quad S_y = \frac{R^2\pi}{2}\cdot\frac{4}{3\pi}R = R^3\frac{2}{3} = 666,67$ cm^3

$I_y = \frac{1}{4}R^4\pi = 7853,98$ cm^4

$\tau_{yz}^{T_z} = \frac{T_z S_y}{I_y\, b(z)} =$

$= \frac{50\cdot10^3\cdot 666,67\cdot10^{-6}}{7853,98\cdot10^{-8}\cdot 20\cdot10^{-2}} = 2,12$ MPa

$\tau_{max}^{T_z} = \frac{4}{3}\frac{T_z}{F}$ $\quad \frac{4}{3}\frac{50\cdot10^3}{314,16\cdot10^{-4}}$

$\tau^{Mt} = \frac{Mt}{Wt}$ $\quad W_t = \frac{\frac{1}{4}R^4\pi}{R} = \frac{R^3\pi}{4}$ $\quad \tau_{max} = 2,12 + 127,32 = 129,44$ MPa

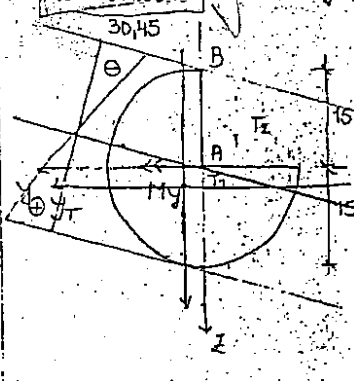
$\tau^{Mt} = \frac{100\cdot10^3\cdot4}{(10\cdot10^{-2})^3\pi} = 127,32$ MPa

$\tau_{max}^{Mt} = \frac{Mt}{Wt}$, $Wt = \frac{1}{2}R^3\pi$ $\quad \otimes 6,66$

$\tau = \frac{M_z}{\quad} z - \frac{M_z}{\quad}$

26.09.2003

$M_y = 50$ KNcm



$F_1 = R^2\pi = 706,86$ cm^2 $\quad \frac{4}{3}\frac{R}{\pi} = 6,366$ cm

$F_2 = 176,71$ cm^2

$F = 530,15$ cm^2

$y_T = +\frac{176,71\cdot6,366}{530,15} = +2,12$ cm

$z_T = +\frac{176,71\cdot6,366}{530,15} = +2,12$

$I_y = \frac{1}{4}15^4\pi + 2,12^2\cdot706,86 - 15^4\,0,05488 - 8,486^2\cdot176,71 = 27\,434,12$ cm^4

$I_z = \frac{1}{4}15^4\pi + 2,12^2\cdot706,86 - 15^4\,0,05488 - 8,486^2\cdot176,71 = 27\,434,12$ cm^4

$I_{yz} = 2,12^2\cdot706,86 + 15^4\,0,01647 - 8,486^2\cdot176,71 = -8714,57$ cm^4

$\sigma_x = \frac{M_y I_z + M_z I_{yz}}{J_z} z - \frac{M_z I_y + M_y I_{yz}}{J_z} y$ $\quad J_z = I_y I_z - I_{yz}^2 =$

$\sigma_x = 2,03\,z + 0,64\,y$ $\quad \sigma_x = 0$ $\quad z = -0,317\,y$

y	0	10
z	0	-317

$\sigma_x^B = 2,03\cdot(-15) = -30,45$ MPa

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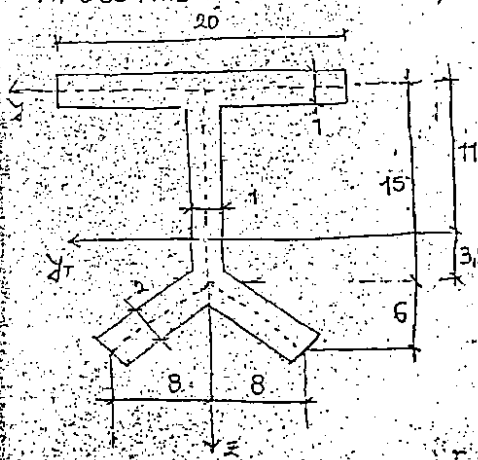
30.01.2004. W

НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА УСЛЕД ЗАДАТИХ ПРЕСЕЧНИХ СИЛА КОЈЕ ДЕЛУЈУ НА ТАНКОЗИДНОМ НОСАЧУ

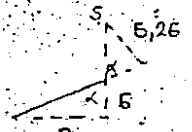


$M_\omega = 2\ KNm^2$

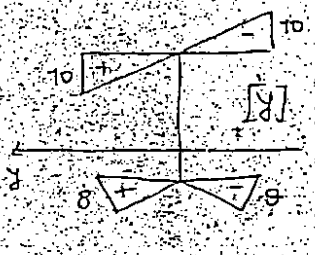
$M_{TS} = 3\ KNm$

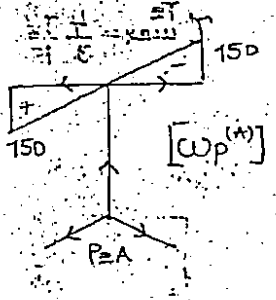
$M_{T\omega} = 4\ KNm$

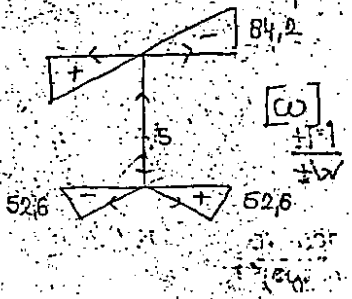
$z_T = \frac{15 \cdot 1 \cdot 7{,}5 + 2 \cdot 10 \cdot 2 \cdot 18}{15 + 40 + 20 \cdot 1} = 11{,}1\ cm$



$z_S = z_T - \frac{I_{y\omega T}}{I_x}$







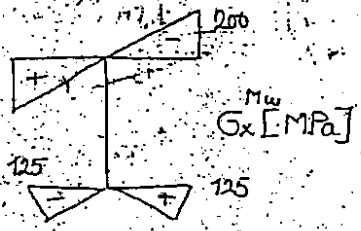
$I_x = 2 \frac{10}{3} 10^2 \cdot 1 + 2 \cdot \frac{10}{3} 8^2 \cdot 2 = 1520\ cm^4$

$I_{y\omega p} = -2 \frac{10}{3} \cdot 10 \cdot 150 \cdot 1 = 10\,000\ cm^5$

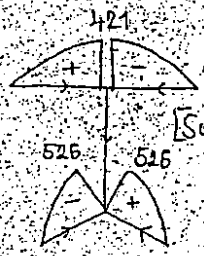
$z_S = 3{,}9 - \frac{10\,000}{1520} \approx 3{,}9 - 6{,}58 \approx -2{,}68\ cm$

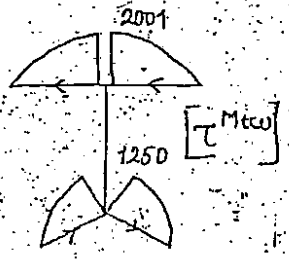
$I_\omega = 1 \cdot 2 \cdot \frac{10}{3} 84{,}2^2 + 2 \cdot 2 \cdot \frac{10}{3} 52{,}6^2 = 84\,154{,}4\ cm^6$

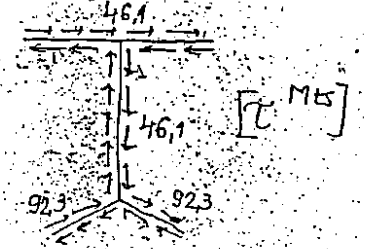
$\sigma_x^{M\omega} = \frac{M_\omega}{I_\omega} \cdot \omega = \frac{2 \cdot 10^3 \cdot \omega \cdot 10^{-4}}{84\,154{,}4 \cdot 10^{-12}} =$



$\tau^{M_{T\omega}} = -\frac{M_{T\omega} S_\omega^*}{I_\omega \cdot t} = -\frac{4 \cdot 10^3 \cdot S_\omega^* \cdot 10^{-6}}{84\,154{,}4 \cdot 10^{-12} \cdot t \cdot 10^{-2}}$







$\tau^{M_S} = \frac{M_S}{I_t} \cdot t = \frac{3 \cdot 10^3 \cdot t \cdot 10^{-2}}{65 \cdot 10^{-8}}$

$I_t = \frac{1}{3} \sum b_i t_i^3 = \frac{1}{3}(20 \cdot 1 + 15 \cdot 1 + 20 \cdot 2) = 65\ cm^4$

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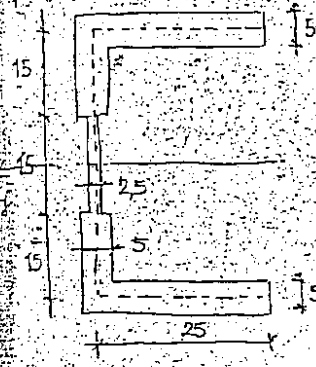
$M\omega = 3\ KHw^2$

$M_{ts} = 4\ KHw$

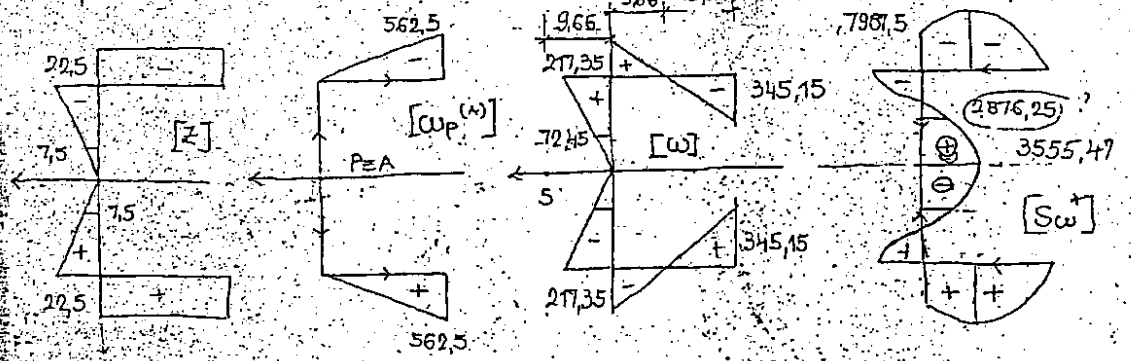
$M_{t\omega} = 5\ KHw$



$$\sigma_x = \frac{M\omega}{I\omega}\cdot\omega \rightarrow y_s,\ I_{\pm\omega_p^{(s)}},\ I_y,\ \omega_p^{(s)} \Rightarrow \omega$$

$$\tau^{Mts} = \frac{Mts}{It}\cdot t$$

$$\tau^{Mt\omega} = -\frac{Mt\omega\cdot S\omega^*}{I\omega\cdot t} \rightarrow S\omega^*$$



$$I_y = 2\cdot 5\,\frac{25}{3}\,22{,}5^2 + 2\cdot\frac{7{,}5}{3}\left(22{,}5^2+22{,}5\cdot 7{,}5+7{,}5^2\right)\cdot 55 + 2\cdot 2{,}5\cdot\frac{7{,}5}{3}\,7{,}5^2 = 163\,828{,}12\ cw^4$$

$$I_{\pm\omega_p^{(s)}} = 2\cdot\frac{25}{2}\,22{,}5\cdot 562{,}5\cdot 5 = 1582031{,}25\ cw^5$$

$$y_s = y_p^0 + \frac{I_{\pm\omega_p^{(s)}}}{I_y} = \frac{1582031{,}25}{163\,828{,}12} = 9{,}66\ cw$$

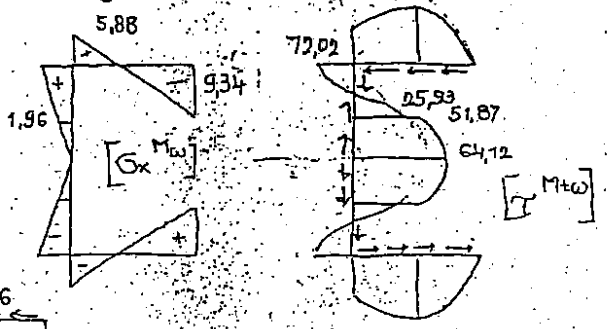
$$I\omega = 2\cdot 5\cdot\frac{25}{3}\left(345{,}15^2 - 345{,}15\cdot 217{,}35 + 217{,}35^2\right) + 2\cdot 5\cdot\frac{15}{3}\left(217{,}35^2 + 217{,}35\cdot 72{,}45 + 72{,}45^2\right)$$
$$+ 2\cdot 2{,}5\cdot\frac{7{,}5}{3}\cdot 72{,}45^2 = 11\,090063{,}53\ cw^6$$

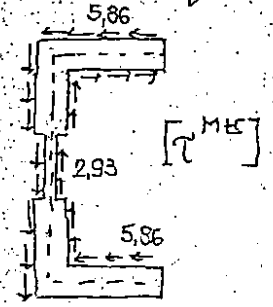
$$It = \frac{1}{3}\sum bi\,ti^3 = \frac{1}{3}\left[2\cdot 25\cdot 5^3 + 2\cdot 15\cdot 5^3 + 2\cdot 7{,}5\cdot 2{,}5^3\right] = 3411{,}46\ cw^4$$

$$\sigma_x = \frac{M\omega}{I\omega}\cdot\omega = \frac{3\cdot 10^3\cdot\omega\cdot 10^{-4}}{11090063{,}53\cdot 10^{-12}}$$

$$\tau^{Mt\omega} = -\frac{5\cdot 10^3\cdot S\omega^*\cdot 10^{-6}}{11090063{,}53\cdot 10^{-12}\cdot t\cdot 10^{-2}}$$

$$\tau^{Mts} = \frac{4\cdot 10^3\cdot t\cdot 10^{-2}}{3411{,}46\cdot 10^{-8}}$$





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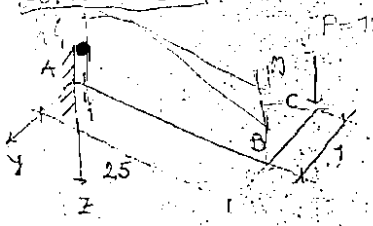
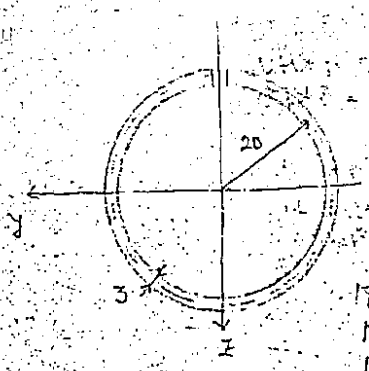
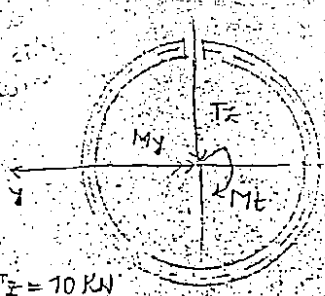






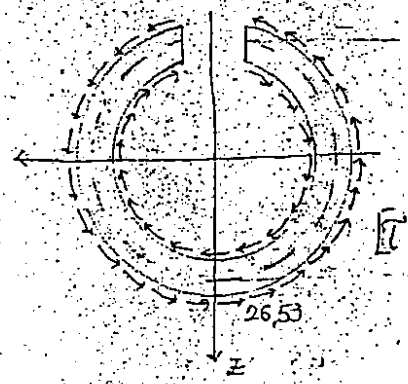
$T_z = 10\ KN$

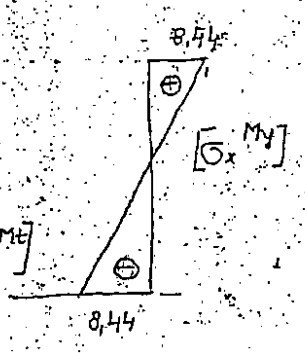
$M_y = 25\ KN\,m$

$M_t = 10\ KN\,m$

$$I_y = \frac{\bar{u}}{4}(\bar{u} R^3 t) = \frac{\bar{u}}{4}(\bar{u}\, 20^3 \cdot 3) = 59217{,}63\ cm^4$$

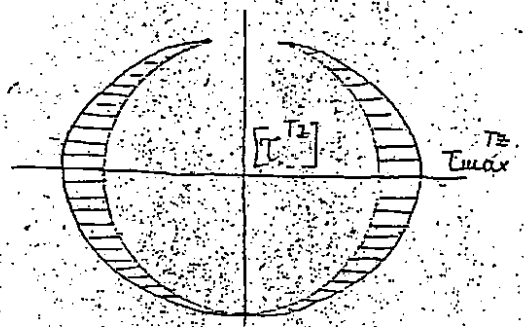
$$I_t = \frac{1}{3}\Sigma b_i t_i^3 = \frac{1}{3}(2R\bar{u}\cdot t_i^3) = \frac{1}{3}(2\cdot 20\,\bar{u}\cdot 3^3) = 1130{,}97\ cm^4$$





$$\tau^{Mt} = \frac{Mt}{It}\, t = \frac{10\cdot 10^3 \cdot 3\cdot 10^{-2}}{1130{,}97\cdot 10^{-8}} = 26{,}53\ MPa$$

$$\sigma_x = \frac{My}{Iy}\, z = \frac{-25\cdot 10^3 \cdot 20\cdot 10^{-2}}{59217{,}63\cdot 10^{-8}} = -8{,}44\ MPa$$



$$\tau_{max}^{Tz} = 4\,\frac{Tz}{F} = 4\,\frac{10\cdot 10^3}{376{,}99\cdot 10^{-4}} = 1{,}06\ MPa$$

$$F = 2R\bar{u}\cdot t = 376{,}99\ cm^2$$

ПРИМЕРИ ИЗ КЊИГЕ

① АНАЛИЗА НАПОНА W

СТАЊЕ НАПОНА У ДАТОЈ ТАЧКИ ТЕЛА ЈЕ ДЕФИНИСАНО ТЕНЗОРОМ НАПОНА
ОДРЕДИТИ ГЛАВНЕ НАПОНЕ И ГЛАВНЕ ПРАВЦЕ У ТОЈ ТАЧКИ ТЕЛА. ИЗВРШИТИ
ПРОВЕРИТИ ОРТОГОНАЛНОСТ ГЛАВНИХ ПРАВАЦА.

$$S = \begin{bmatrix} 30 & 12 & 12 \\ 12 & -4 & -8 \\ 12 & -8 & -4 \end{bmatrix} [MPa]$$

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33} = 22\ MPa$$

$$I_2 = \begin{vmatrix} -4 & -8 \\ -8 & -4 \end{vmatrix} + \begin{vmatrix} 30 & 12 \\ 12 & -4 \end{vmatrix} + \begin{vmatrix} 30 & 12 \\ 12 & -4 \end{vmatrix} = -48 - 264 - 264 = -576$$

$$I_3 = \begin{vmatrix} 30 & 12 & 12 \\ 12 & -4 & -8 \\ 12 & -8 & -4 \end{vmatrix} = -2592$$

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0 \qquad \sigma^3 - 22\sigma^2 - 576\sigma + 2592 = 0$$

$$\Rightarrow \boxed{\sigma_1 = 36;\ \sigma_2 = 4;\ \sigma_3 = -18}$$

$\sigma_1 = 36\ MPa$

$$S = \begin{bmatrix} -6 & 12 & 12 \\ 12 & -40 & -8 \\ 12 & -8 & -40 \end{bmatrix}$$

$$C_1^{(1)} = \begin{vmatrix} 12 & 12 \\ -40 & -8 \end{vmatrix} = 384 \qquad C_2^{(1)} = -\begin{vmatrix} 12 & 12 \\ 12 & -8 \end{vmatrix} = 96 \qquad C_3^{(1)} = \begin{vmatrix} 12 & -40 \\ 12 & -8 \end{vmatrix} = 96$$

$$\sqrt{384^2 + 96^2 + 96^2} = 407{,}29$$

$$n_1^{(1)} = \frac{C_1^{(1)}}{\sqrt{(C_1^{(1)})^2 + (C_2^{(1)})^2 + (C_3^{(1)})^2}}$$

$$n_1^{(1)} = 0{,}943 \qquad n_2^{(1)} = 0{,}236 \qquad n_3^{(1)} = 0{,}236$$

$$\boxed{\vec{n}^{(1)} = \{0{,}943;\ 0{,}236;\ 0{,}236\}}$$

АНАЛОГНО

$$\vec{n}^{(2)} = \{0;\ 0{,}707;\ -0{,}707\}$$

$$\vec{n}^{(3)} = \{0{,}333;\ 0{,}667;\ 0{,}667\}$$

ПРОВЕРА ОРТОГОНАЛНОСТИ

$$\vec{n}_1 \cdot \vec{n}_2 = 0 \quad (0{,}943;\ 0{,}236;\ 0{,}236)(0;\ 0{,}707;\ -0{,}707) = 0$$

$$\vec{n}_2 \cdot \vec{n}_3 = 0$$

$$\vec{n}_3 \cdot \vec{n}_1 = 0$$

(170) W/ У НЕКОЈ ТАЧКИ ТЕЛА ДАТО ЈЕ РАВАНСКО СТАЊЕ НАПОНА ДЕФИНИСАНО СА $\sigma_{11} = 15$ MPa, $\sigma_{22} = 30$ MPa, $\sigma_{12} = -10$ MPa

(a) ОДРЕДИТИ НОРМАЛНИ И СМИЧУЋИ НАПОН У РАВНИ ЧИЈА НОРМАЛА ГРАДИ СА ОСОМ X_1 УГАО $\gamma = 60°$

(b) ОДРЕДИТИ ВЕЛИЧИНЕ И ПРАВЦЕ ГЛАВНИХ НАПОНА

(c) ОДРЕДИТИ МАКСИМАЛНИ СМИЧУЋИ НАПОН ЗА ПРЕСЕЧНУ РАВАН ЧИЈА НОРМАЛА ЛЕЖИ У РАВНИ X_1, X_2

(d) НАЦРТАТИ MOHR-ОВ КРУГ НАПОНА

(a) $$\sigma = \frac{\sigma_{11}+\sigma_{22}}{2} + \frac{\sigma_{11}-\sigma_{22}}{2}\cos 2\gamma + \sigma_{12}\sin 2\gamma$$

$$\sigma = \frac{15+30}{2} + \frac{15-30}{2}\cos 120° - 10\sin 120° = 17{,}6 \text{ MPa}$$

$$\tau = \frac{\sigma_{11}-\sigma_{22}}{2}\sin 2\gamma - \sigma_{12}\cos 2\gamma$$

$$\tau = \frac{15-30}{2}\sin 120 + 10\cos 120 = -11{,}5 \text{ MPa}$$

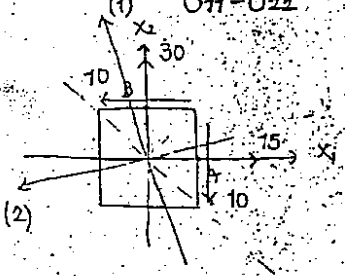
(b) $$\sigma_{1,2} = \frac{\sigma_{11}+\sigma_{22}}{2} \pm \sqrt{\left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2 + \sigma_{12}^2} = \frac{15+30}{2} \pm \sqrt{\left(\frac{15-30}{2}\right)^2 + (-10)^2}$$

$\sigma_1 = 35$ KN

$\sigma_2 = 10$ KN

$$\operatorname{tg} 2\alpha = \frac{2\sigma_{12}}{\sigma_{11}-\sigma_{22}} = \frac{2\cdot(-10)}{15-30} \quad (<0) \qquad \alpha = \frac{1}{2}(180 + \operatorname{arctg}(\ ))$$

$$\alpha = 116{,}57°$$

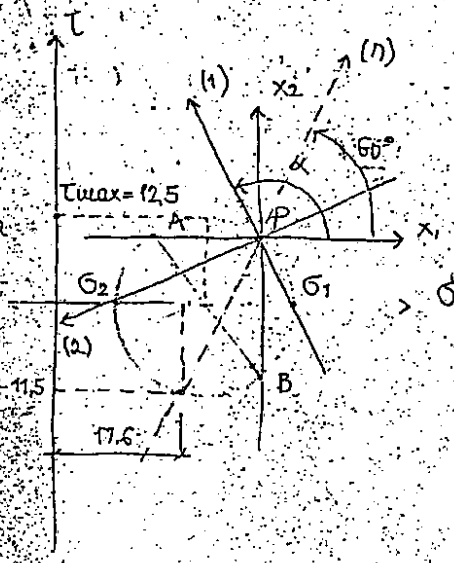


$A(\sigma_{11}; -\sigma_{12})$

$B(\sigma_{22}; \sigma_{21})$

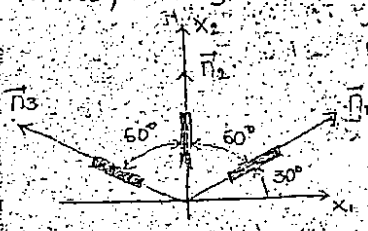
(c) $$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2)$$

$$\tau_{max} = \frac{1}{2}(35-10) = 12{,}5 \text{ MPa}$$



② АНАЛИЗА ДЕФОРМАЦИЈА

У НЕКОЈ ТАЧКИ ТЕЛА У КОЈОЈ ЈЕ СТАЊЕ ДЕФОРМАЦИЈЕ РАВАНСКО, ПОМОЋУ РОЗЕТЕ ДЕФОРМАЦИЈЕ ИЗМЕРЕНЕ СУ ДИЛАТАЦИЈЕ $\varepsilon^{(1)} = 7\cdot10^{-6}$ $\varepsilon^{(2)} = -10\cdot10^{-6}$, $\varepsilon^{(3)} = 33\cdot10^{-6}$ ЗА ТРИ ПРАВЦА $\vec{n}_1, \vec{n}_2, \vec{n}_3$ КОЈИ МЕЂУСОБНО ГРАДЕ УГЛОВЕ ОД 60°. ОДРЕДИТИ ВЕЛИЧИНЕ И ПРАВЦЕ ГЛАВНИХ ДИЛАТАЦИЈА У ТОЈ ТАЧКИ, И НАЦРТАТИ МОНР-ОВ КРУГ ДЕФОРМАЦИЈЕ



$\varepsilon^{(1)} = 7\cdot10^{-6}$

$\varepsilon^{(2)} = -10\cdot10^{-6}$

$\varepsilon^{(3)} = 33\cdot10^{-6}$

$$\varepsilon^{(k)} = \varepsilon_{11}\cos^2\varphi_k + \varepsilon_{22}\sin^2\varphi_k + \gamma_{12}\sin\varphi_k\cos\varphi_k$$

(1) $7\cdot10^{-6} = \varepsilon_{11}\cos^2 30 + \varepsilon_{22}\sin^2 30 + \gamma_{12}\sin 30\cos 30$

(2) $-10\cdot10^{-6} = \varepsilon_{22}$

(3) $33\cdot10^{-6} = \varepsilon_{11}\cos^2 150 + \varepsilon_{22}\sin^2 150 + \gamma_{12}\sin 150\cos 150$

$7\cdot10^{-6} = 0{,}75\varepsilon_{11} - 2{,}5\cdot10^{-6} + 0{,}433\gamma_{12}$

$-10\cdot10^{-6} = \varepsilon_{22}$

$33\cdot10^{-6} = 0{,}75\varepsilon_{11} - 2{,}5\cdot10^{-6} - 0{,}433\gamma_{12}$

$40\cdot10^{-6} = 1{,}5\varepsilon_{11} - 5\cdot10^{-6}$

$\varepsilon_{11} = 30\cdot10^{-6}$

$\gamma_{12} = -30\cdot10^{-6} \Rightarrow \varepsilon_{12} = \frac{1}{2}\gamma_{12}$

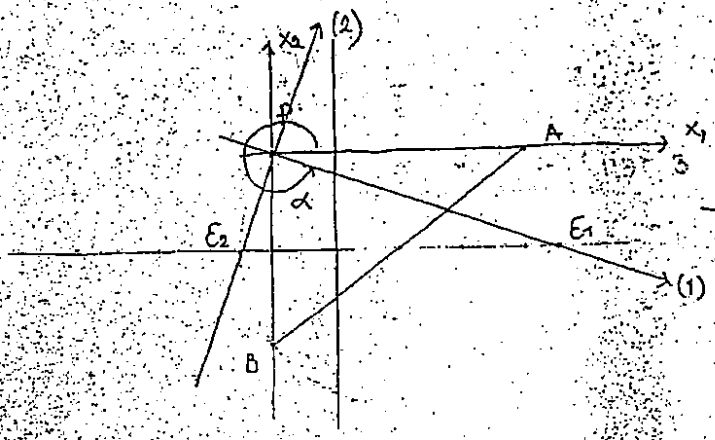
$$\varepsilon_{1,2} = \frac{30-10}{2} \pm \sqrt{\left(\frac{30+10}{2}\right)^2 + 15^2}$$

$\varepsilon_1 = 35\cdot10^{-6}$

$\varepsilon_2 = -15\cdot10^{-6}$

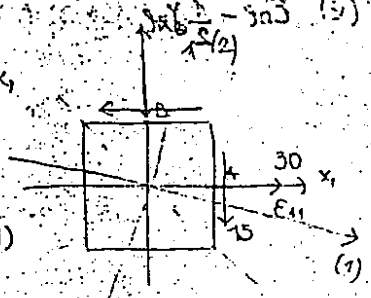
$$\operatorname{tg} 2\alpha = \frac{\gamma_{12}}{\varepsilon_{11}-\varepsilon_{22}} = \frac{-30}{30+10} > 0$$

$\alpha = 360° + \frac{1}{2}\operatorname{arctg}(\,) = 341{,}56°$



A(ε11, −ε12)

B(ε22, ε12)



(172) У ПОЉЕ ДЕФОРМАЦИЈЕ У НАПРЕГНУТОМ ТЕЛУ ЈЕ ДАТО ИЗРАЗОМ

$\varepsilon_{11} = 16Kx_3 \quad \varepsilon_{22} = \varepsilon_{33} = -4Kx_3 \quad$ ГДЕ ЈЕ K КОНСТАНТА

$\varepsilon_{12} = -Kx_3 \quad \varepsilon_{23} = 0 \quad \varepsilon_{31} = 3Kx_2$

(a) ПОКАЗАТИ ДА СУ УСЛОВИ КОМПАТИБИЛНОСТИ ДЕФОРМАЦИЈА ЗАДОВОЉЕНИ ДАТО ПОЉЕ ДЕФОРМАЦИЈЕ ЗАДОВОЉЕНО

(b) НАПИСАТИ МАТРИЦУ ТЕНЗОРА ДЕФ. ЗА ПРОИЗВОЉНУ ТАЧКУ ТЕЛА

(c) ОДРЕДИТИ МАТРИЦУ ТЕНЗОРА ДЕФ И КУБНУ ДИЛАТАЦИЈУ У ТАЧКИ $M(x_1, 1, 1)$

(d) У ТАЧКИ $M(x_1, 1, 1)$ ОДРЕДИТИ ДИЛАТАЦИЈУ У ПРАВЦУ

$$\vec{n} = \sqrt{2}/6\,(4\vec{e}_1 + \vec{e}_2 - \vec{e}_3)$$

(e) У ТАЧКИ $M(x_1, 1, 1)$ ОДРЕДИТИ КЛИЗАЊЕ ИЗМЕЂУ ПРАВАЦА

$$\vec{n} = \sqrt{2}/6\,(4\vec{e}_1 + \vec{e}_2 - \vec{e}_3) \qquad \vec{\ell} = \frac{\sqrt{2}}{2}(\vec{e}_2 + \vec{e}_3)$$

(b)
$$D(x_1, x_2, x_3) = [\varepsilon_{ij}] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} = K \begin{bmatrix} 16x_3 & -x_3 & 3x_2 \\ -x_3 & -4x_3 & 0 \\ 3x_2 & 0 & -4x_3 \end{bmatrix}$$

(c) $M(x_1, 1, 1)$

$$D(x_1, 1, 1) = \begin{bmatrix} 16 & -1 & 3 \\ -1 & -4 & 0 \\ 3 & 0 & -4 \end{bmatrix} K$$

(d) $M(x_1, 1, 1) \Rightarrow \Theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = (16 - 4 - 4)K = 8K$

$$\varepsilon_{nn} = n^T D\, n = \left(\frac{\sqrt{2}}{6}\right)^2 K \begin{bmatrix} 4 & 1 & -1 \end{bmatrix} \begin{bmatrix} 16 & -1 & 3 \\ -1 & -4 & 0 \\ 3 & 0 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{2}{36} K \begin{bmatrix} 60 & -8 & 16 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} = \underline{\underline{12K}}$$

(e) $$\varepsilon_{n\ell} = \frac{1}{2}\gamma_{n\ell} = n^T D\, \ell = \frac{\sqrt{2}}{6}\frac{\sqrt{2}}{2} K \begin{bmatrix} 4 & 1 & -1 \end{bmatrix} \begin{bmatrix} 16 & -1 & 3 \\ -1 & -4 & 0 \\ 3 & 0 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{6} K \begin{bmatrix} 60 & -8 & 16 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 1{,}333\,K$$

$$\underline{\underline{\gamma_{n\ell} = 2{,}667\,K}}$$

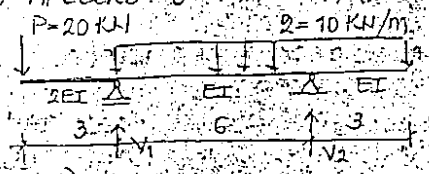
ГРЕДНИ НОСАЧ

НАПИСАТИ ИЗРАЗ ЗА МОМЕНТ САВИЈАЊА M_y И ТРАНСФЕРЗАЛНУ СИЛУ T_z
И НАЦРТАТИ ОДГОВАРАЈУЋЕ ДИЈАГРАМЕ ЗА ГРЕДУ

179

27. 06. 2003.

Mohr-Maxwell-ovom analogijom odrediti ugib i nagib u preseku u tački A ($EI = 1,5\ MN\,m^2$)

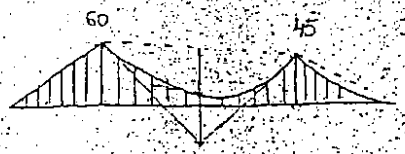


$\Sigma M_A^{+\circlearrowleft} = 0 \quad 90 \cdot 7,5 - 3V_1 - 9V_2 = 0 \qquad V_1 = 110 - V_2 \qquad \underline{V_1 = 52,5\ kN}$

$V_1 + V_2 - 20 - 90 = 0$

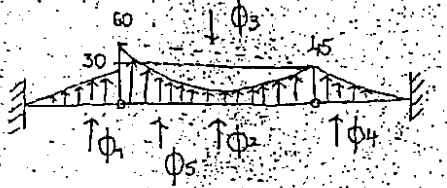
$675 - 330 + 3V_2 - 9V_2 = 0 \quad 6V_2 = 345 \quad \underline{V_2 = 57,5\ kN}$

(M)



$f_1 = \frac{10 \cdot 9}{8} = 11,25$

$f_2 = \frac{10 \cdot 36}{8} = 45$

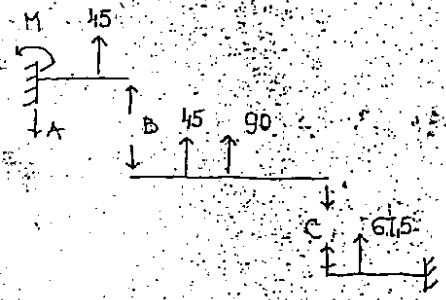


$\phi_1 = \frac{1}{2} 30 \cdot 3 = 45$

$\phi_2 = 45 \cdot 6 = 270 \qquad \phi_5 = \frac{1}{2} 15 \cdot 6 = 45$

$\phi_3 = \frac{2}{3} 45 \cdot 6 = 180$

$\phi_4 = \frac{1}{2} 45 \cdot 3 = 67,5$ ← NIJE BITNO



$\Sigma M_B^{+\circlearrowleft} = 0 \quad 45 \cdot 2 + 90 \cdot 3 + C \cdot 9 = 0 \qquad C = -40$

$-B + 45 + 90 - C = 0 \qquad B = 95$

$\underline{M} = 45 \cdot 2 + 95 \cdot 3 = \underline{375}\ kN\,m$

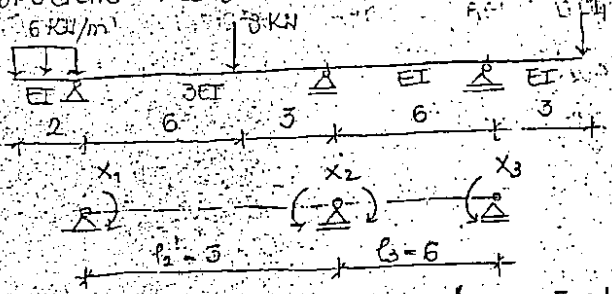
$-A + 45 + 95 = 0 \qquad A = 140\ kN$

$$W_A = \frac{M_A}{EI} = \frac{375 \cdot 10^6}{1,5 \cdot 10^6} = 0,25\ m$$

$$\gamma_A = \frac{T_A}{EI} = \frac{140 \cdot 10^3}{1,5 \cdot 10^6} = 0,09\ rad$$

30.01.2004. IV

Користећи тромоментну једначину, нацртати дијаграме момента у пресецима



$X_1 = -12$

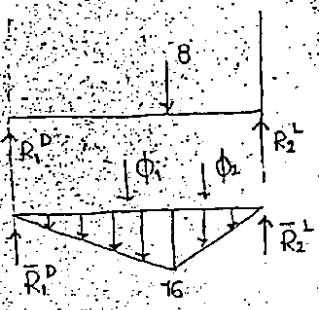
$X_3 = -12$

$$x_{i-1}\,l_i' + 2x_i(l_i' + l_{i+1}') + x_{i+1}\,l_{i+1}' = -6\left[\bar{R}_i^L\frac{I_c}{I_i} + \bar{R}_i^D\frac{I_c}{I_{i+1}}\right]$$

$$i=2 \quad x_1 l_2' + 2x_2(l_2' + l_3') + x_3 l_3' = -6\left[\bar{R}_2^L\frac{I_c}{I_2} + \bar{R}_2^D\frac{I_c}{I_3}\right]$$

$$-12\cdot 3 + 2x_2(3+6) - 12\cdot 6 = -6\left[\bar{R}_2^L\frac{1}{3} + \bar{R}_2^D\right]$$

$$18x_2 - 108 = -6\left[\bar{R}_2^L\frac{1}{3} + \bar{R}_2^D\right]$$



$\Sigma M^{+\circlearrowright} = 0 \quad 8\cdot 6 - R_2^L\cdot 9 = 0 \quad R_2^L = 5{,}33$

$R_1^D = 2{,}67$

$\Phi_1 = \frac{1}{2}\,16\cdot 6 = 48$

$\Phi_2 = \frac{1}{2}\,16\cdot 3 = 24$

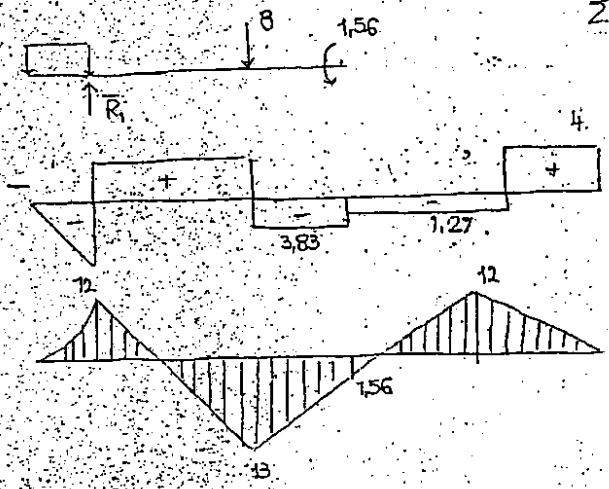
$\Sigma M^{+\circlearrowright} = 0 \quad 48\cdot 4 + 24\cdot 7 - \bar{R}_2^L\cdot 9 = 0$

$\bar{R}_2^L = 40$

$\bar{R}_1^D + 40 - 48 - 24 = 0 \quad \bar{R}_1^D = 32$

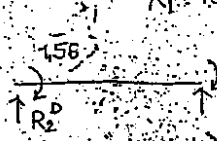
$\bar{R}_2^D = 0$

$18x_2 - 108 = -6\left(40\cdot\frac{1}{3}\right) \quad x_2 = 1{,}56$



$\Sigma M^{+\circlearrowright} = 0 \quad -1{,}56 + 8\cdot 3 + R_1\cdot 9 - 12\cdot 10 = 0$

$R_1 = 16{,}17$



$\Sigma M^{+\circlearrowright} = 0 \quad R_2^D\cdot 6 + 1{,}56 + 12 = 0$

$R_2^D = -2{,}26$

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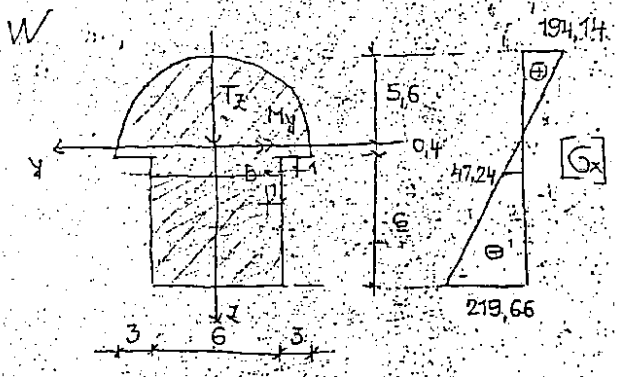
24.09.2004.

За дати попречни пресек и пресечне силе $|M_y| = 32\,kNm$ и $|T_z| = 12{,}8\,kN$ и $N = 9{,}6\,kN$ одредити правце главних напона у тачки B

$I_y = 928\,cm^4$ $I_z = 617\,cm^4$ $F = 93\,cm^2$



$S_y^B = 5{,}6 \cdot 3{,}9 = 117\,cm^3$

$$\sigma_x = \frac{N}{F} + \frac{M_y}{I_y} z = \frac{9{,}6 \cdot 10^3}{93 \cdot 10^{-4}} + \frac{-32 \cdot 10^3}{928 \cdot 10^{-8}} \cdot 6{,}4 \cdot 10^{-2} = -219{,}65\,MPa$$

$$= 194{,}14\,MPa$$

$$\sigma_x^B = \frac{9{,}6 \cdot 10^3}{93 \cdot 10^{-4}} + \frac{-32 \cdot 10^3}{928 \cdot 10^{-8}} \cdot 1{,}4 \cdot 10^{-2} = -47{,}24\,MPa$$

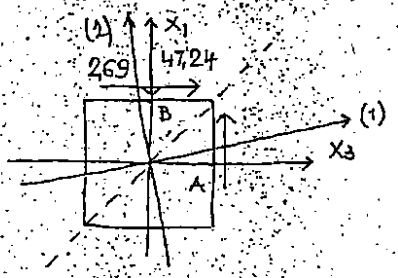
$$\tau_x^{B'} = \frac{T_z \cdot S_y^+}{I_y \cdot b(z)} = \frac{12{,}8 \cdot 10^3 \cdot 117 \cdot 10^{-6}}{928 \cdot 10^{-8} \cdot 6 \cdot 10^{-2}} = 2{,}69\,MPa$$

$$S_B = \begin{bmatrix} -47{,}24 & 0 & 2{,}69 \\ 0 & 0 & 0 \\ 2{,}69 & 0 & 0 \end{bmatrix}$$

$$tg\,2\alpha = \frac{2\sigma_{31}}{\sigma_{33} - \sigma_{11}} = \frac{2 \cdot 2{,}69}{47{,}24}$$

$\alpha_1 = 3{,}25°$

$\alpha_2 = 93{,}25°$



$A(\sigma_{33}, -\sigma_{31})$ $A(0, -2{,}69)$

$B(\sigma_{11}, \sigma_{31})$ $B(-47{,}24, 2{,}69)$

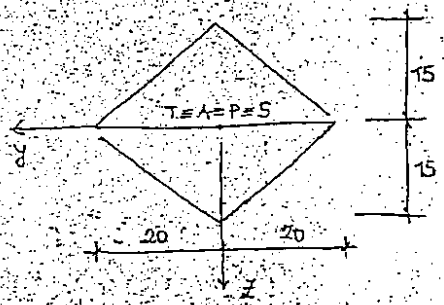
18.06.2004. W

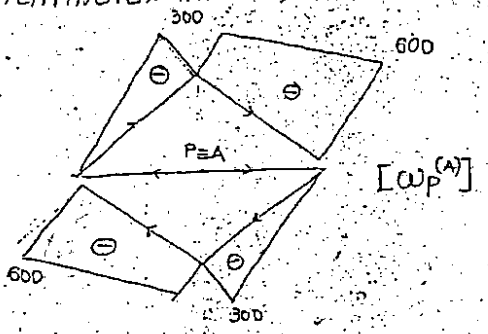
У ПРЕСЕКУ ДАТОМ НА СЛИЦИ ДЕЛУЈУ ПРЕСЕЧНЕ СИЛЕ

$M_\omega = 3\ kNcm^2 \quad M_{t\omega} = -9\ kNcm$

а) ОДРЕДИТИ ГЕОМЕТРИЈСКЕ КАРАКТЕРИСТИКЕ I_ω и S_ω

б) НАЦРТАТИ ДИЈАГРАМЕ КОМПОНЕНТАЛНИХ НАПОНА $t=1\ cm$



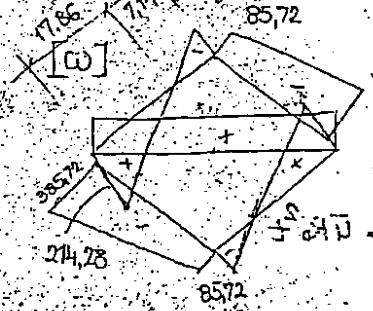


$F = 4 \cdot 25 \cdot 1 + 40 \cdot 1 = 140\ cm^2$

$S_\omega^{(n)} = 2 \cdot 1 \cdot \frac{1}{2} 300 \cdot 25 + 2 \cdot 1 \cdot \frac{300+600}{2} \cdot 25 = 30\,000\ cm^4$

$\omega_0 = -\frac{S\omega_P^{(B)}}{F} = -\frac{30\,000}{140} = \boxed{-214{,}28\ cm^2}$

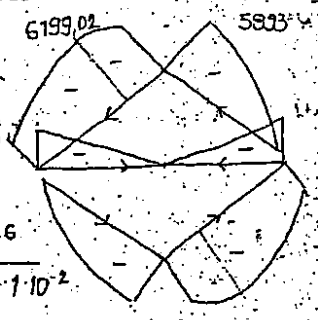
$\omega = \omega_P^{(B)} + \omega_0$



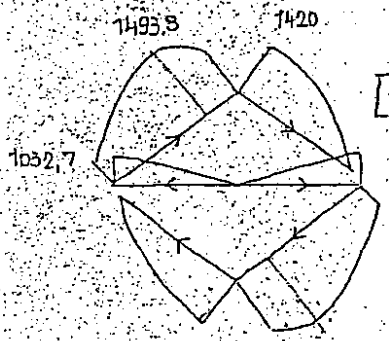
$I_\omega = \frac{25}{3}\left(385{,}72^2 + 85{,}72\,385{,}72 + 85{,}72^2\right) \cdot 2$

$+ 2 \cdot \frac{25}{3}\left(85{,}72^2 - 85{,}72 \cdot 214{,}28 + 214{,}28^2\right)$

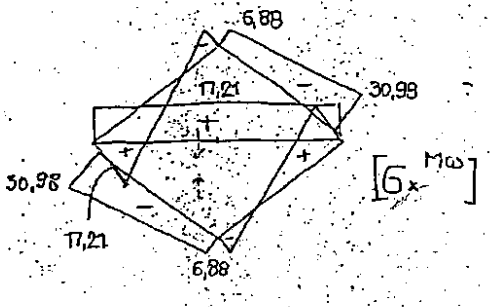
$\therefore I_\omega = 3\,734\,791{,}84\ cm^6$



$\tau_{xz}^{M_\omega} = -\frac{M_{t\omega} S_\omega^*}{I_\omega t} = -\frac{(-9 \cdot 10^3) \cdot S_\omega^* \cdot 10^{-6}}{3\,734\,791{,}84\ 10^{-12} \cdot 1 \cdot 10^{-2}}$



$\sigma_x^{M_\omega} = \frac{M_\omega}{I_\omega} \cdot \omega = \frac{3 \cdot 10^3 \cdot \omega \cdot 10^{-4}}{3\,734\,791{,}84\ 10^{-12}}$



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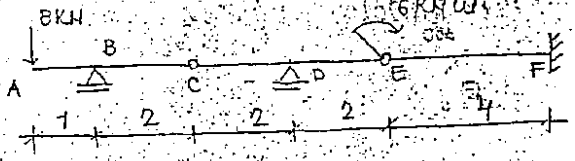
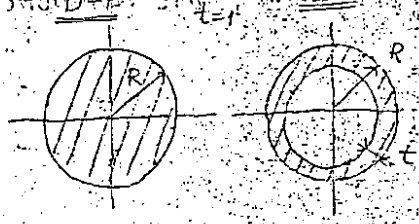
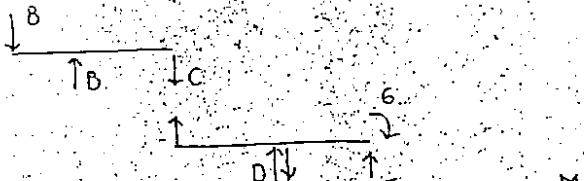
18.06.2004.

(а) Димензионисати дати носач тако да $\sigma_{max} < \sigma_{dop} = 12\,MPa$

(б) Одредити угиб тачке C применом II Castigliano-вог става. $E = 20\,GPa$







$\Sigma M_C = 0 \quad B \cdot 2 - 8 \cdot 3 = 0 \quad B = 12$

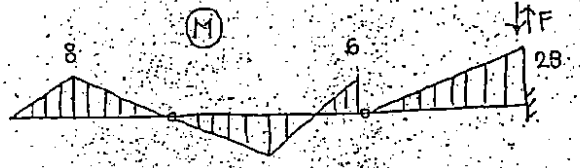
$-8 + 12 - C = 0 \quad C = 4$

$\Sigma M_E = 0 \quad 6 + D \cdot 2 + 4 \cdot 4 = 0$

$D = -11$

$4 - 11 + E = 0 \quad E = 7$

$E + F = 0 \quad F = -7$



$M_{max}^{D-F} = 28\,kNm \qquad \sigma_{max}^{D-E} = \frac{M_{max}}{W_y}$

$\sigma_{max}^{D-E} = \frac{28 \cdot 10^3}{\frac{1}{4}\bar{u} \cdot R^3} < 12 \cdot 10^6 \qquad R < 0{,}1438\,m$

$R > 14{,}38$

$\boxed{R < 14{,}38\,cm} \qquad R_{usv} = 14\,cm$

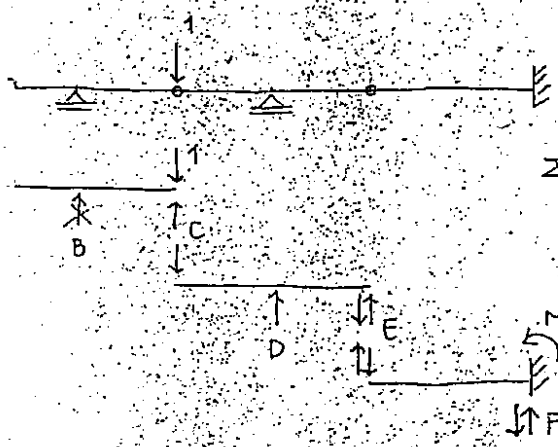
$M_{max}^{A-D} = 8\,kNm \qquad \sigma_{max}^{D-D} = \frac{M_{max}}{W_y}$

$W_y = \bar{u} R_0^2 t$

$\sigma_{max}^{A-D} = \frac{8 \cdot 10^3}{\bar{u} \cdot R_0^2 \cdot 1 \cdot 10^{-2}} < 12 \cdot 10^6 \qquad R_0 < 0{,}1457$

$\boxed{R_0 < 14{,}57\,cm} \qquad R_{0usv} = 14{,}5\,cm$

$\Rightarrow \underline{R = 15\,cm}$



$\Sigma M_C = 0 \quad 2B = 0 \quad B = 0 \quad C = 1$

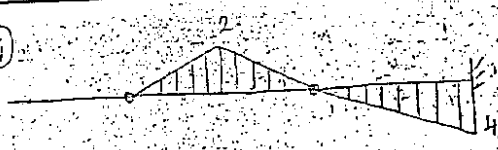
$\Sigma M_E = 0 \quad D \cdot 2 - 1 \cdot 4 = 0 \quad D = 2$

$-1 + 2 + E = 0 \quad E = -1$

$F = 1$

$M = 1 \cdot 4 = 4$

(1)



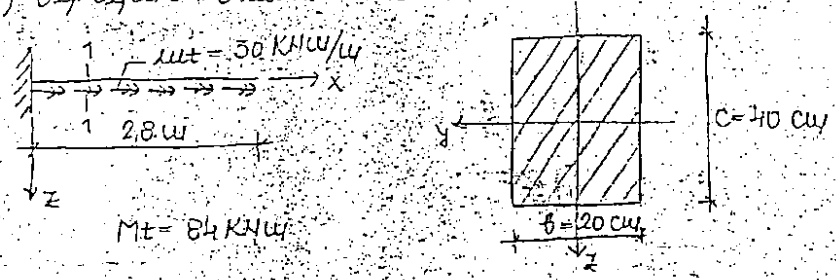
$$V_c = \frac{1}{EI}\int m\bar{M}\,ds =$$

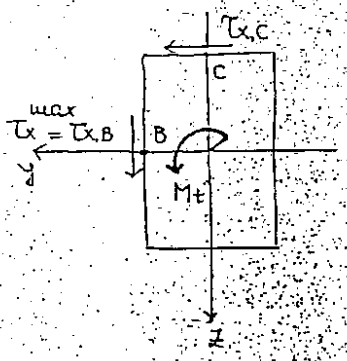
$$= \frac{1}{EI}\left[-\frac{2}{3}2\cdot 8 + \frac{2}{6}\left[2(-2\cdot 8+6)\right] - \frac{4}{3}2\cdot 4\right] = -\frac{166{,}67}{EI}$$

$$V_c = -\frac{166{,}67\cdot 10^3}{20\cdot 10^9 \cdot 9577{,}54\cdot 10^{-8}} = -0{,}087\ m$$

$$I_g = \pi R_e^3 \cdot t = \pi \cdot 14{,}5^3 \cdot 1 = 9577{,}54\ cm^4$$

(18c) За конзолни носач оптерећен једнакоподељеним моментом торзије
а) одредити σ_{max} и τ_{max} у конзоли ако је поп. пресек 1-1
б) одредити ϑ_{max} и γ_{max} ако је $G = 80\ GPa$





$c/b = 40/20 = 2$

$\alpha = 0{,}229 \quad \beta = 0{,}246 \quad \gamma = 0{,}309$

$\pm \sigma_{max} = \frac{M}{W}$

$$\tau_{max} = \tau_{x,B} = \frac{Mt}{\beta b^2 c} = \frac{84 \cdot 10^3}{0{,}246 \cdot 0{,}2^2 \cdot 0{,}4} = 21{,}3\ MPa$$

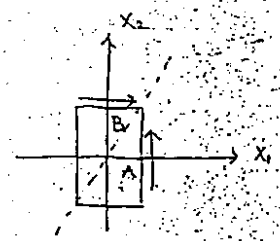
$$\tau_{x,C} = \frac{Mt}{\gamma b^2 c} = \frac{84 \cdot 10^3}{0{,}309 \cdot 0{,}2^2 \cdot 0{,}4} = 17\ MPa$$

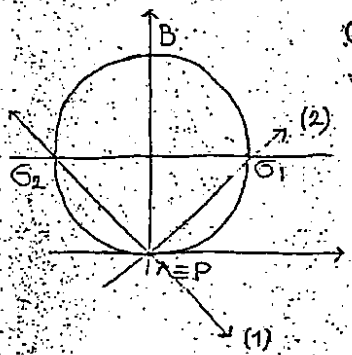
$$\vartheta_{max} = \frac{Mt}{G \cdot It} = \frac{Mt}{G \alpha b^3 c} = \frac{84 \cdot 10^3}{80 \cdot 10^9 \cdot 0{,}229 \cdot 0{,}2^3 \cdot 0{,}4} = 0{,}0014$$

За тачку B

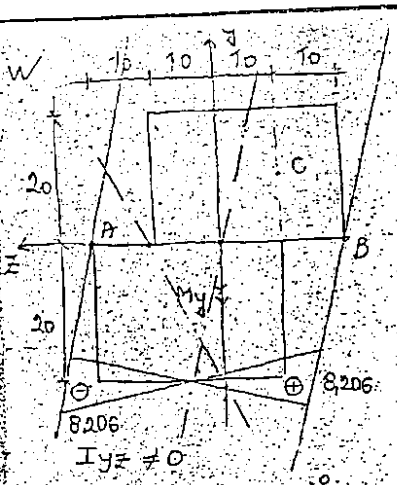
$$\begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$A(0; -\tau)$
$B(0; \tau)$





$\sigma_{max} = \tau_{max} = 21{,}3\ MPa$



$I_y = 120\,000\ cm^4$

$I_z = 160\,000\ cm^4$

$I_{yz} = -60\,000\ cm^4$

$M_y = -40\ kNm$

$$\sigma_x = \frac{M_y I_z + M_z I_{yz}}{J_2} z - \frac{M_z I_y + M_y I_{yz}}{J_2} y \qquad J_2 = I_y I_z - I_{yz}^2 = 156 \cdot 10^8\ cm^8$$

$$\sigma_x = \frac{-40 \cdot 10^3 \cdot 160\,000 \cdot 10^{-8}}{156 \cdot 10^8 \cdot 10^{-16}} z - \frac{-40 \cdot 10^3 \cdot (-60\,000 \cdot 10^{-8})}{156 \cdot 10^8 \cdot 10^{-16}} y$$

$$\sigma_x = -41{,}03\, z - 15{,}38\, y \quad [MPa]$$

* ГЕОМЕТРИЈСКО МЕСТО ТАЧАКА НА КОЈИМА ЈЕ НОРМАЛНИ НАПОН ЈЕДНАК 0 (НЕУТРАЛНА ОСА)

$$\sigma_x = 0 \qquad 41{,}03 z + 15{,}38 y = 0 \qquad z + 0{,}375 y = 0 \qquad z = -0{,}375 y$$

y	0	10
z	0	-3,75

$A(0, 20) \quad B(0, -20)$ $\qquad \sigma_x^A = -41{,}03 \cdot 0{,}2 = -8{,}206\ MPa$

$\sigma_x^B = -41{,}03 \cdot (-0{,}2) = 8{,}206\ MPa$

* ПРОВЕРИТИ ДА ЛИ ТАЧКА С ПРИЛАЗИ ЈЕЗГРУ ПРЕСЕКА

$C(10, -10)$ $\qquad e_y \quad e_z$ $\qquad F = 20 \cdot 30 \cdot 2 = 1200\ cm^2$

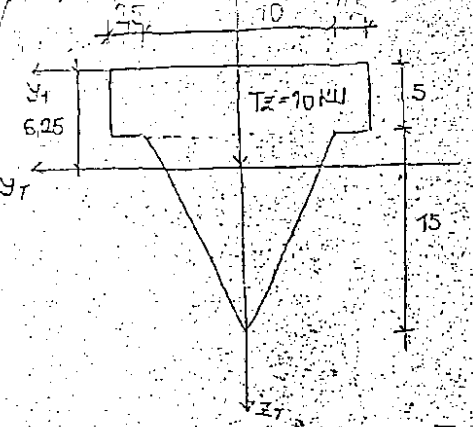
$I_{yz} \neq 0$

$$p_y = -\frac{J_2}{F(e_y I_y - e_z I_{yz})} = \frac{156 \cdot 10^8 \cdot 10^{-16}}{1200 \cdot 10^{-4} \cdot 10^{-8} (0{,}1 \cdot 120\,000 + 0{,}1 \cdot 60\,000)} = +0{,}217\ m$$

$$p_z = -\frac{J_2}{F(e_z I_z - e_y I_{yz})} = -\frac{156 \cdot 10^8 \cdot 10^{-16}}{1200 \cdot 10^{-4} \cdot 10^{-8} (-0{,}1 \cdot 160\,000 + 0{,}1 \cdot 60\,000)} = -0{,}186\ m$$

⇒ НЕУТРАЛНА ОСА СЕЧЕ ПРЕСЕК → ТАЧКА С НЕ ПРИЛАЗИ ЈЕЗГРУ ПРЕСЕКА

(182) Одредити највећу компоненталну смичућу напон τ_{xz}

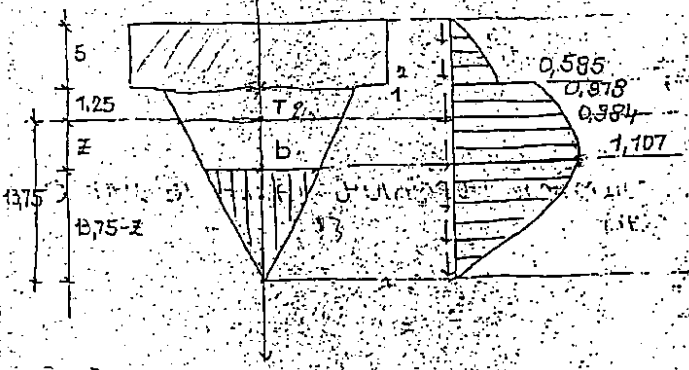


$$F_1 = 15\cdot 5 + \frac{1}{2}\,15\cdot 10 = 150\ cm^2$$

$$z_T = \frac{15\cdot 5\cdot 2{,}5 + 75\cdot 10}{150} = 6{,}25\ cm$$

$$\tau^{T_z} = \frac{T_z\, S_y^*}{I_y\cdot b(z)}$$

$$I_y = \frac{1}{12}15\cdot 5^3 + 3{,}75^2\cdot 75 + \frac{1}{36}10\cdot 15^3 + 3{,}75^2\cdot 75 = 3205{,}12\ cm^4$$



$$S_y^* = F^*\cdot z_T^* = \frac{1}{2}b(13{,}75-z)\left(z+\frac{13{,}75-z}{3}\right) = \frac{1}{6}b(13{,}75-z)(2z+13{,}75)\ cm^3$$

$$\tau_{xz}^{T_z} = \frac{T_z\cdot S_y^*}{I_y\cdot b} = \frac{10\cdot 10^3\cdot\frac{1}{6}b(13{,}75-z)(2z+13{,}75)\cdot 10^{-6}}{3205{,}12\cdot 10^{-8}\cdot b\cdot 10^{-2}} = 0{,}005203\,(13{,}75-z)(2z+13{,}75)\ MPa$$

$$\tau_{xz}^{T_z\,\prime} = 0 \qquad -1(2z+13{,}75)+2(13{,}75-z)=0$$

$$-2z-13{,}75+27{,}5-2z=0 \qquad z = 3{,}44$$

$$\tau_{xz,max}^{T_z} = 1{,}107\ MPa \quad (z=3{,}44)$$

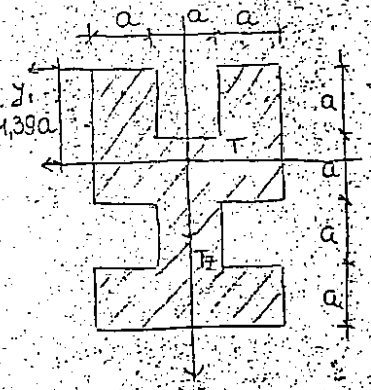
$$\tau_{xz,T}^{T_z} = 0{,}984\ MPa \quad (z=0)$$

$$\tau_{xz}^{T_z}(-1{,}25) = 0{,}878\ MPa$$

$$\tau_{xz,2} = \tau_{xz_1}^{(1)}\frac{b_1}{b_2} = 0{,}878\cdot\frac{10}{15} = 0{,}585\ MPa$$

03.09.2004

Услед дејства трансферзалне силе T_z одредити вредност компоненталног напона τ_{xz} у тежишту пресека и наскицирати дијаграм компоненталног напона τ_{xz} (без рачунања вредности)



$$F = 3a \cdot 4a - 3a^2 = 9a^2$$

$$z_T = \frac{9a^2 \cdot 2a - a^2 \cdot a/2 - 2a^2 \cdot 5/2a}{9a^2} = 1{,}39a$$

$$\tau_{xz,T} = \frac{T_z \cdot S_y^*}{I_y \cdot b(z)}$$

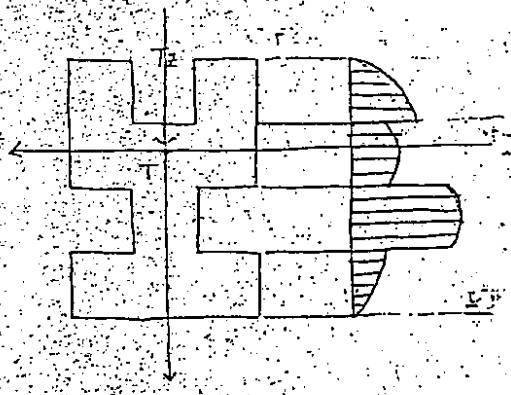
$$I_y = \frac{1}{12} 3a (4a)^3 + (0{,}61a)^2 \cdot 12a^2 - \frac{1}{12} a \cdot a^3 - (0{,}89a)^2 \cdot a^2$$

$$- 2 \frac{1}{12} a \cdot a^3 - (1{,}11a)^2 a^2$$

$$I_y = 3{,}191\, a^4$$

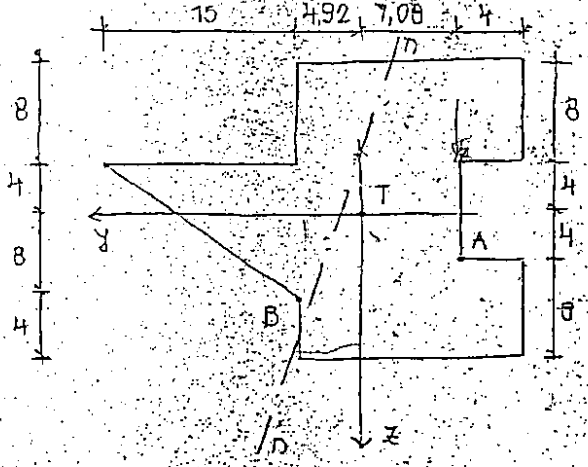
$$S_y^* = 2a^2 \cdot 0{,}89a + 0{,}39a \cdot 3a \cdot 0{,}195a = 2{,}01 a^3$$

$$\tau_{xz,T} = \frac{T_z \cdot S_y^*}{I_y \cdot b(z)} = \frac{T_z \cdot 2{,}01 a^3}{3{,}191 a^4 \cdot 3a} = 0{,}21 \frac{T_z}{a^2}$$



05.09.2004.

W Управно на задати попречни пресек (у правцу осе x) дејству у тачки A сила притиска 300 kN, а у тачки B сила затезања 300 kN. Одредити положај неутралне осе.



ТАЧКА A: $N = -300$ kN

$\Rightarrow M_z = -300 \cdot 7{,}08 \cdot 10^{-2} = -21{,}24$ kNm

$M_y = -300 \cdot 4 \cdot 10^{-2} = -12$ kNm

ТАЧКА B: $N = 300$ kN

$M_z = -300 \cdot 4{,}92 \cdot 10^{-2} = -14{,}76$ kNm

$M_y = 300 \cdot 8 \cdot 10^{-2} = 24$ kNm

ТЕЖИШТЕ: $M_z = -36$ kNm $M_y = 12$ kNm $N = 0$ kN

$I_y = 18981$ cm^4 $I_z = 19135$ cm^4 $I_{yz} = -450$ cm^4

$I_{yz} \neq 0$

$$\sigma_x = \frac{M_y I_z + M_z I_{yz}}{J_z} z - \frac{M_z I_y + M_y I_{yz}}{J_z} y \qquad \boxed{J_z = I_y I_z - I_{yz}^2}$$

$$\sigma_x = \frac{1200 \cdot 19135 + 3600 \cdot 450}{J_z} z - \frac{-3600 \cdot 18981 - 1200 \cdot 450}{J_z} y$$

$\sigma_x = 0{,}677\, z + 1{,}897\, y$ [MPa]

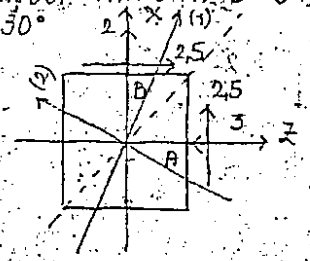
$\sigma_x = 0 \Rightarrow \boxed{z + 2{,}802\, y = 0}$ $z = -2{,}802\, y$

y	0	2
z	0	-5,60

03.09.2004. W

Дат је тензор-напона S у коорд. систему xyz. Одредити помоћу Мохр-овог круга тензор напона S' у ротираном коорд. систему x'y'z' око y за +30°

$$S_{xyz} = \begin{bmatrix} 2 & 0 & 2,5 \\ 0 & 0 & 0 \\ 2,5 & 0 & -3 \end{bmatrix} \text{ MPa}$$



$A(\sigma_{zz}, -\sigma_{zx})$

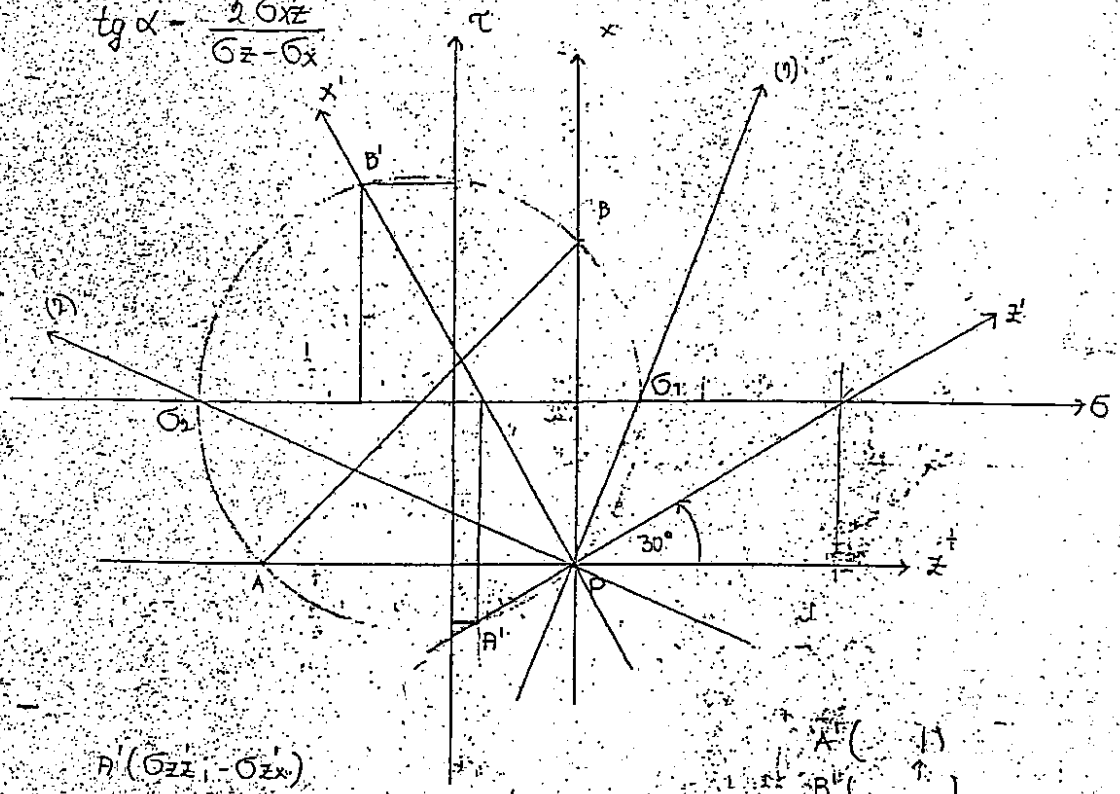
$B(\sigma_{xx}, \sigma_{zx})$

$$\sigma_{1,2} = \frac{\sigma_{zz}+\sigma_{xx}}{2} \pm \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \sigma_{zx}^2}$$

$\sigma_1 = 3,04$ MPa

$\sigma_2 = -4,53$ MPa

$$tg\alpha = \frac{2\sigma_{xz}}{\sigma_z - \sigma_x}$$



$A'(\sigma_{z'z'}, -\sigma_{z'x'})$

$B'(\sigma_{x'x'}, \sigma_{z'x'})$

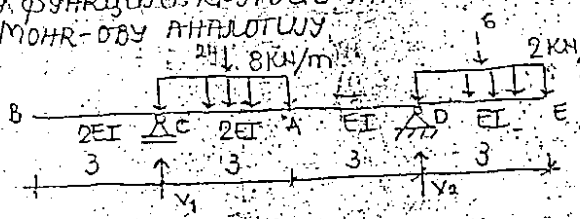
$A'(\quad\uparrow)$

$B'(\quad\uparrow)$

$$S' = \begin{bmatrix} -1,5 & 0 & 3,5 \\ 0 & 0 & 0 \\ 3,5 & 0 & 0,5 \end{bmatrix}$$

Пр. по 2 пат.

Одредити интензитете вертикалног померања и ротације тачке А користећи функцију крутости на савијање EI) користећи Мохр-ову аналогију



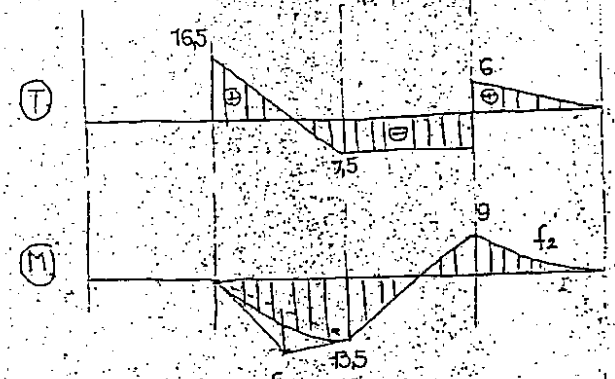
$\Sigma M_L^{\curvearrowleft} = 0 \quad V_1 3 - 24\,4{,}5 + V_2 9 - 6\,10{,}5 = 0 \quad 90 - 3V_2 + 9V_2 - 171 = 0$

$V_1 + V_2 = 30 \quad V_1 = 30 - V_2$

$6V_2 = 81$

$V_2 = 13{,}5$ kN

$V_1 = 16{,}5$ kN



$f_1 = \frac{8 \cdot 9}{8} = 9$

$f_2 = \frac{2 \cdot 9}{8} = 2{,}25$

$\phi_1 = \frac{2}{3} 4{,}5 \cdot 3 = 9$

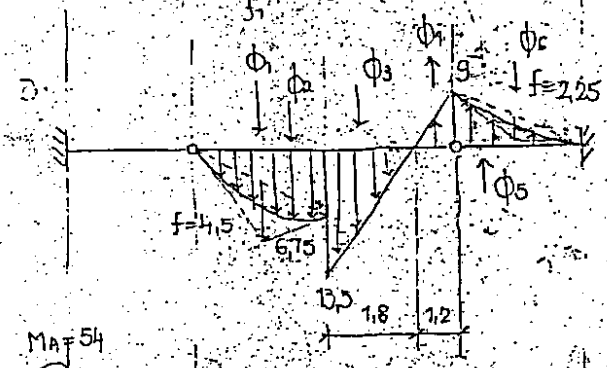
$\phi_2 = \frac{1}{2} 6{,}75 \cdot 3 = 10{,}12$

$\phi_3 = \frac{1}{2} 1{,}8 \cdot 13{,}5 = 12{,}15$

$\phi_4 = \frac{1}{2} 1{,}2 \cdot 9 = 5{,}4$

$\phi_5 = \frac{1}{2} 9 \cdot 3 = 13{,}5$

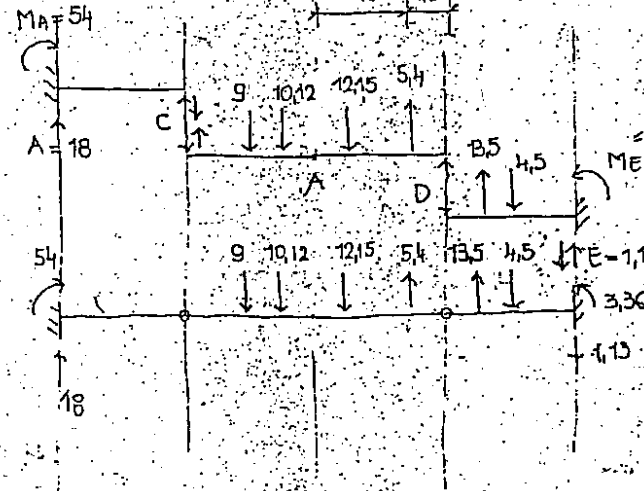
$\phi_6 = \frac{2}{3} 2{,}25 \cdot 3 = 4{,}5$



$\Sigma M_C^{\curvearrowright} = 0 \quad 9 \cdot 1{,}5 + 10{,}12 \cdot 2 + 12{,}15 \cdot 3{,}6 - 54{,}5{,}6 - 6D$

$D = 7{,}87$

$C = -18$



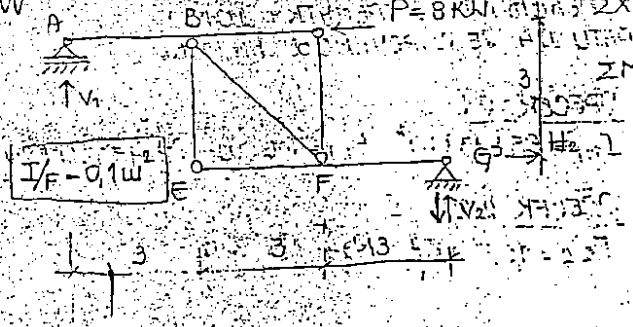
$M_E = 3{,}36 \quad 10{,}12 \cdot 1 + 9 \cdot 1{,}5 - 18 \cdot 3 = M_A$

$M_A^{\curvearrowleft} = -30{,}38$ kN m

$\varphi_A = \frac{T_A}{EI} = \frac{-1{,}12}{EI}$

$w_A = \frac{M_A}{EI} = \frac{30{,}38}{EI}$

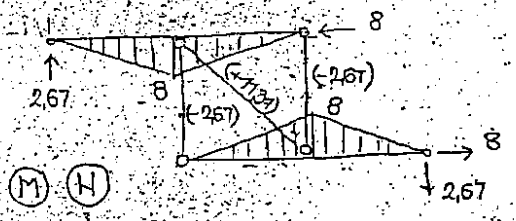
03.09.2004. Одредити вертикално померање тачке E у функцији EI

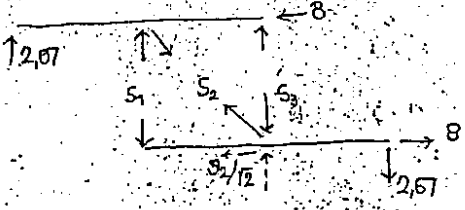


$\Sigma X = 0 \quad -H_2 + 8 = 0 \quad H_2 = 8$ kN

$\Sigma M_A^{+\circlearrowleft} = 0 \quad 8 \cdot 3 + V_2 \cdot 9 = 0 \quad V_2 = -2{,}67$ kN

$V_1 = 2{,}67$ kN

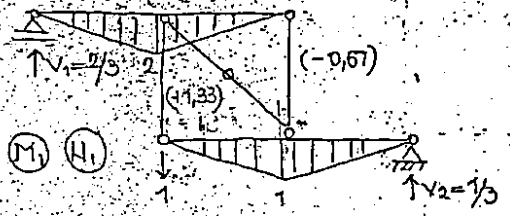




$\Sigma X = 0 \quad -\frac{S_2}{\sqrt{2}} + 8 = 0 \quad S_2 = 8\sqrt{2}$

$\Sigma M_E^{+\circlearrowleft} = 0 \quad -\frac{S_2}{\sqrt{2}} \cdot 3 + S_3 \cdot 3 + 2{,}67 \cdot 6 = 0 \quad S_3 = 2{,}67$

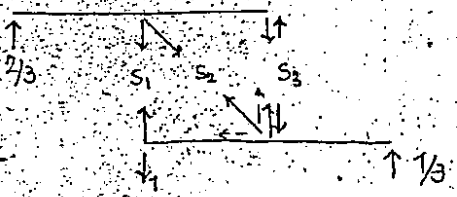
$\Sigma Y = 0 \quad -S_1 + \frac{S_2}{\sqrt{2}} - 2{,}67 - 2{,}67 = 0 \quad S_1 = 2{,}67$



$\Sigma M_A^{+\circlearrowleft} = 0 \quad 1 \cdot 3 - V_2 \cdot 9 = 0 \quad V_2 = 1/3$

$V_1 - 1 + 1/3 = 0 \quad V_1 = 2/3$

3



$\Sigma X = 0 \quad -\frac{S_2}{\sqrt{2}} = 0 \quad S_2 = 0$

$\Sigma M_E^{+\circlearrowleft} = 0 \quad -S_3 \cdot 3 - \frac{2}{3} \cdot 6 = 0$

$S_3 = -2/3$

$\Sigma Y = 0 \quad -1 + S_1 - 2/3 + 1/3 = 0$

$S_1 = 4/3$

$$V_e = \frac{1}{EI}\int M M_1 \, ds = \frac{1}{EI}\left[2 \cdot \frac{3}{3} \cdot 8 \cdot 2 - 2 \cdot \frac{3}{3} \cdot 8 \cdot 1\right] + \frac{0{,}1}{EI}\left[-3 \cdot 2{,}67 \cdot 1{,}33 + 3 \cdot 2{,}67 \cdot 0{,}67\right] = \frac{15{,}47}{EI}$$

03.09.2004.

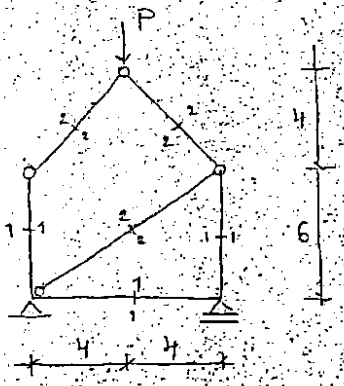
Одредити вредност граничног оптерећења P^+ у тренутку лома методом корак по корак (сматрати да је спречено извијање штапова).

ПРЕСЕК 1-1

$I_{1-1} = 65\,833\ cm^4$ $F_{1-1} = 700\ cm^2$ $M^+_{1-1} = 1500\ kNm$

ПРЕСЕК 2-2

$F_{2-2} = 15{,}7\ cm^2$ $N^+_{2-2} = 377\ kN$



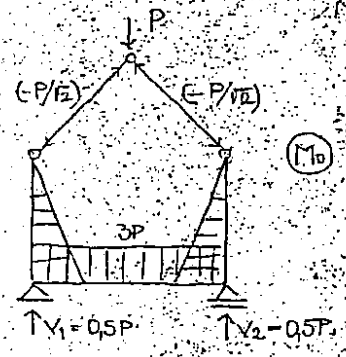
$z_{S1} = 3$ $z_{Dx} = 1$ $K_{0x} = 6$

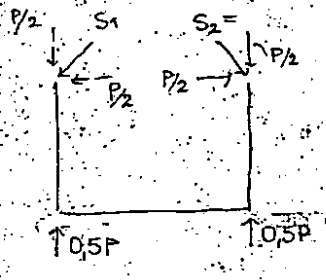
$z_{S2} = 2$ $z_{Dy} = 2$ $K_{0y} = 6$

$z_{13} = 1$ $z_u = 0$ $n = 2$

$$\frac{I_{1-1}}{F_{2-2}} = \frac{65833 \cdot 10^{-8}}{15{,}7 \cdot 10^{-4}} = 0{,}419\ m^2$$

$14 - 3 \cdot 1 - 2 \cdot 2 - 3 \cdot 3 = 1$



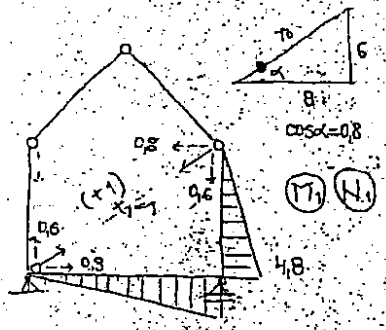


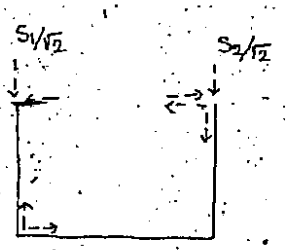
$\Sigma M_1 = 0 \quad \frac{S_2}{\sqrt{2}} \cdot 8 - 0{,}5P \cdot 8 = 0$

$S_2 = 0{,}707P$

$\frac{S_1}{\sqrt{2}} - 0{,}5P + P = 0$

$S_1 = 0{,}707P$





$\Sigma M_1 = 0$

$-0{,}8 \cdot 6 + 0{,}6 \cdot 8 + \frac{S_2}{\sqrt{2}} \cdot 8 = 0$

$S_2 = 0$

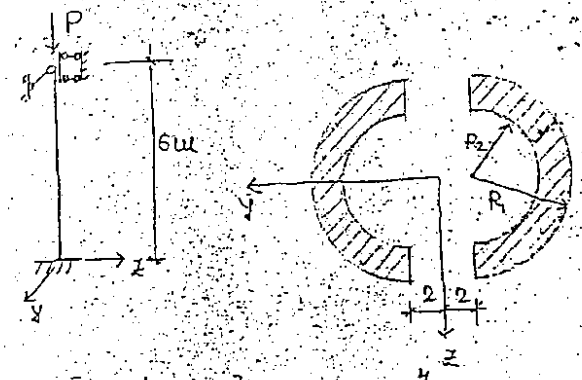
$S_1 = 0$

$$\delta_{11} = \frac{1}{EI}\left(\frac{8}{3} 4{,}8^2 + \frac{6}{3} 4{,}8^2\right) + \frac{0{,}419}{EI} \cdot 10^2 = \frac{111{,}71}{EI}$$

$$X_1 = \frac{86{,}4P}{111{,}71} = 0{,}773P$$

$$\delta_{10} = \frac{1}{EI}\left(-\frac{8}{2} 4{,}8 \cdot 3P - \frac{6}{3} 3P \cdot 4{,}8\right) = -\frac{86{,}4P}{EI}$$

06.09.2004. Одредити вредност критичне силе извијања P_{KR}.



$R_1 = 10$ cm $\quad R_2 = 5$ cm $\quad E = 210$ GPa

$\sigma_T = 240$ MPa

$\sigma_{KR}^T = 310 - 1{,}92\lambda$ (MPa)

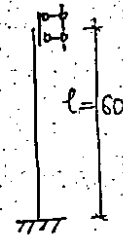
$\frac{4}{3}\frac{R_1}{U} = 4{,}244 \quad \frac{4}{3}\frac{R_2}{U} = 2{,}122$

$$I_y = 2\left[\frac{U \cdot 10^4}{8} - \frac{U \cdot 5^4}{8}\right] = 7363{,}11 \text{ cm}^4$$

$$I_z = 2\cdot\left[10^4\, 2\, 0{,}05488 + 6{,}244^2 \cdot 157{,}08 - 5^4\, 2\, 0{,}05488 - 4{,}122^2 \cdot 39{,}27\right] = 12971{,}86 \text{ cm}^4$$

$$F = R_1^2 U - R_2^2 U = 235{,}62 \text{ cm}^2$$

* ИЗВИЈАЊЕ ОКО y ОСЕ (ГЛЕДАМ ХОД РАВАН)

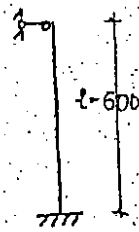


$l_i = \mu l = 0{,}5 \cdot 600 = 300$ cm

$i_y = \sqrt{\frac{I_y}{F}} = \sqrt{\frac{7363{,}11}{235{,}62}} = 5{,}59$ cm

$\lambda_y = \frac{l_i}{i_y} = \frac{300}{5{,}59} = 53{,}67$

* ИЗВИЈАЊЕ ОКО z ОСЕ (ГЛЕДАМ ХОД y РАВАН)



$l_i = \mu l = 0{,}7 \cdot 600 = 420$ cm

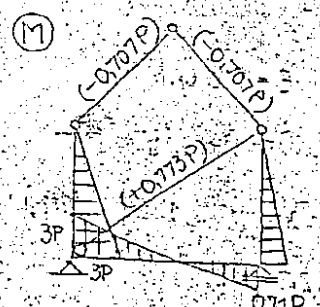
$i_z = \sqrt{\frac{I_z}{F}} = \sqrt{\frac{12971{,}86}{235{,}62}} = 7{,}42$

$\lambda_z = \frac{l_i}{i_z} = \frac{420}{7{,}42} = 56{,}6$

$\Rightarrow \lambda = \max\{\lambda_y, \lambda_z\} = 56{,}6$

$$\sigma_{KR} = \min\begin{cases} \sigma_{KR}^E = \left(\frac{U}{\lambda}\right)^2 E = \left(\frac{U}{56{,}6}\right)^2 \cdot 210 \cdot 10^3 = 646{,}97 \text{ MPa} \\ \sigma_T = 240 \text{ MPa} \\ \sigma_{KR}^T = 310 - 1{,}92 \cdot 56{,}6 = 201{,}33 \text{ MPa} \end{cases}$$

$$P_{KR} = \sigma_{KR,\min} \cdot F = 201{,}33 \cdot 235{,}62 \frac{1}{10} = 4743{,}74 \text{ kN}$$



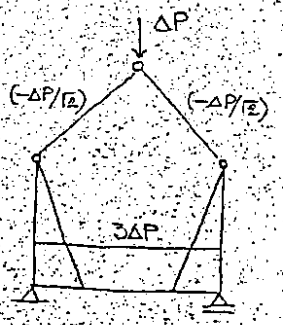
$M_{max} = M^*$

$3P_1' = 1500 \quad P_1' = 500\ KN$

$N_{max} = N^*$

$0{,}773\,P_1'' = N^* \quad P_1'' = 487{,}71\ KN$

$\underline{P_1 = \min\{500;\ 487{,}71\} = 487{,}71\ KN}$



$M^* = 3P_1 + 3\Delta P'$

$\Delta P' = \frac{M^* - 3P_1}{3} = 12{,}29\ KN$

$N^* = \frac{P}{\sqrt{2}} + \frac{\Delta P''}{\sqrt{2}}$

$\Delta P'' = N^*\sqrt{2} - P_1 = 45{,}45\ KN$

$\underline{\Delta P = \min\{12{,}29;\ 45{,}45\} = 12{,}29\ KN}$

$\underline{\underline{P^* = P_1 + \Delta P = 487{,}71 + 12{,}29 = 500\ KN}}$