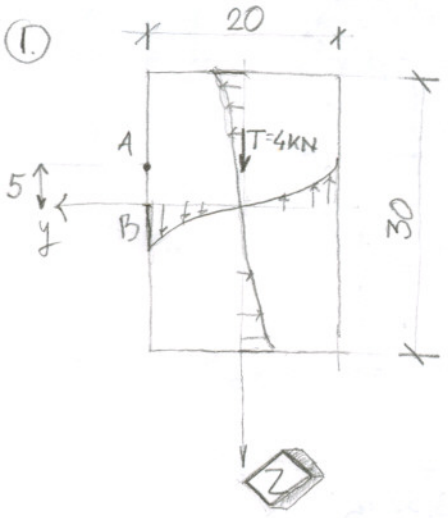
III KOLOKVIJUM

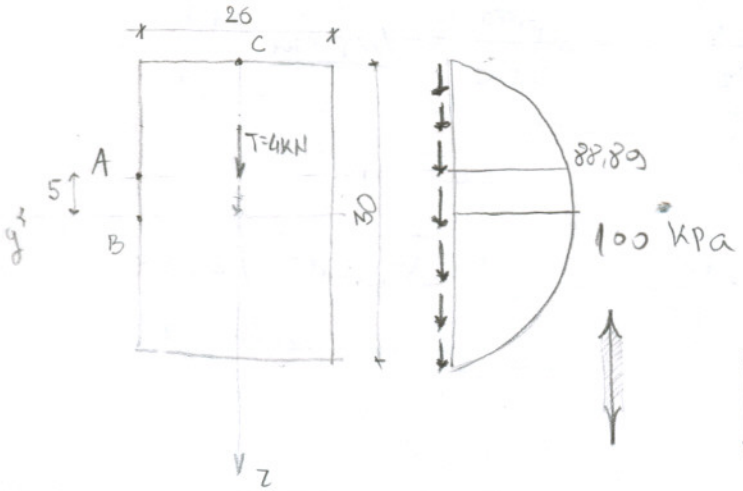
TORZIJA

① 

$\frac{c}{b} = \frac{3}{2} = 1,5$

$\alpha = 0,195 \Rightarrow I_t = \alpha b^3 c = 0,195 \cdot 30^3 \cdot 20 = 105300\ cm^4$

$$\varphi = \int_0^l \frac{M_t}{G I_t} dx = \frac{M_t \cdot l}{G I_t}$$



$\tau^B = \frac{M_T}{W_{T,B}}$, $W_{T,B} = \beta b^2 c$

$\tau^C = \frac{M_T}{W_{T,C}}$, $W_{T,C} = \gamma b^2 c$

$$\tau_{xz}^{T_z} = \frac{T_z \cdot S_y^*}{I_y \cdot b(z)}$$

$S_y^A = 10 \cdot 20 \cdot 10 = 2000\ cm^3 \Rightarrow \tau_{xz}^A = \frac{4000\,N \cdot 2000 \cdot 10^{-6}\,m^3}{45000 \cdot 10^{-8}\,m^4 \cdot 20 \cdot 10^{-2}\,m} = 88,89\ kPa$

$S_y^T = 20 \cdot 15 \cdot 7,5 = 2250\ cm^3 \Rightarrow \tau_{xz}^T = \frac{-11- \ 2250}{-11- \ -11-} = 100\ kPa$ 100000

$I_y = \frac{1}{12} \cdot 30^3 \cdot 20 = 45000\ cm^4$

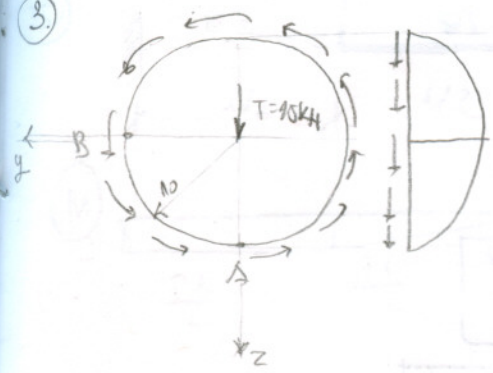
$\frac{3}{2} \frac{4000\ N}{300 \cdot 10^{-4}\,m^2} = 100000 \frac{N}{m^2}$

$$S = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

$\sigma_x = -\frac{M_z}{I_z} y_A$

$$S = \begin{bmatrix} \sigma_x & 0 & -88,89 \\ 0 & 0 & 0 \\ -88,89 & 0 & 0 \end{bmatrix}$$

3.



$\frac{4}{3}\frac{15000\ N}{10^2\pi \cdot 10^{-4}\ m^2} = 636{,}62\ kPa$

$I_t = I_o = \frac{R^4\pi}{2} = \frac{10^4\pi}{2} = 15707{,}96\ cm^4$

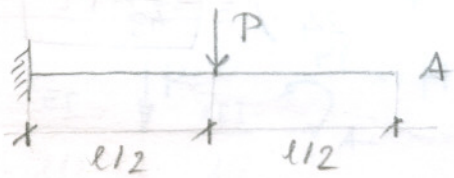
$S_A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\varphi = \int_0^l \frac{M_t}{GI_t}\,dx = \frac{M_t \cdot l}{GI_t}\ [rad]$

$\tau_{max} = \frac{M_t}{I_t} \cdot R = \frac{M_t}{W_t}\ ,\ W_t = \frac{R^3\pi}{2}$

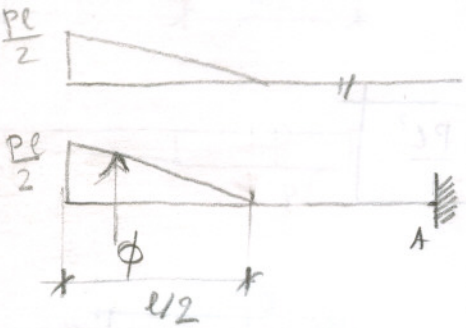
УГИБ И НАГИБ

1.



$\phi = \frac{1}{2}\frac{Pl}{2}\frac{l}{2} = \boxed{\frac{Pl^2}{8}}$

M



$T_A = \frac{Pl^2}{8}$

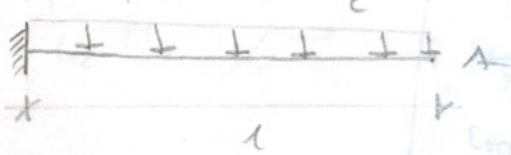
$M_A = \frac{Pl^2}{8}\left(\frac{2}{3}\frac{l}{2} + \frac{l}{2}\right) =$

$= \frac{Pl^2}{8}\frac{5}{6}l = \boxed{\frac{5}{48}Pl^3}$

$\boxed{\varphi = \frac{Pl^2}{8EI}}$ $\boxed{W = \frac{5Pl^3}{48EI}}$

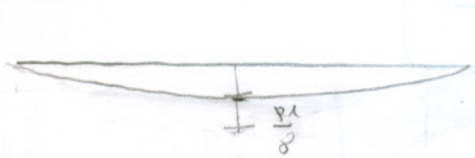
$q = \frac{P}{l}$

2.



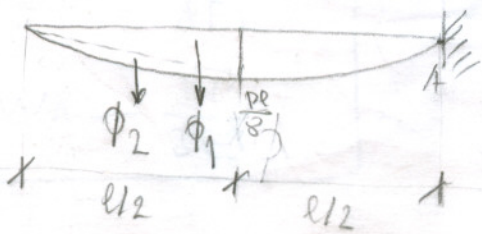
$f = \frac{ql^2}{8} = \frac{\frac{P}{l}\cdot l^2}{8} = \boxed{\frac{Pl}{8}}$

M



$\phi_1 = \frac{1}{2}\frac{l}{2}\frac{Pl}{8} = \boxed{\frac{Pl^2}{32}}$ $f_2 = \frac{ql^2}{8} = \frac{\frac{P}{l}\cdot l^2}{8} = \frac{Pl}{8} = \frac{Pl}{32}$

$\phi_2 = \frac{2}{3}f_2 l = \frac{2}{3}\frac{Pl}{32}\cdot\frac{1}{2} = \boxed{\frac{Pl^2}{96}}$

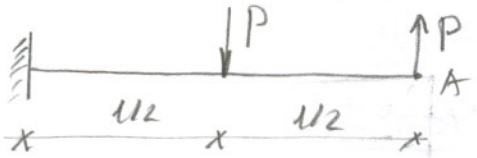


$\boxed{T_A = -\frac{Pl^2}{12}} = (\phi_1 + \phi_2)\cdot 2$

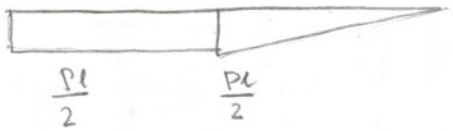
$M_A = \left(\phi_1\frac{2}{3}l + \phi_1\frac{l}{3} + \phi_2\frac{3}{4}l + \phi_2\frac{l}{4}\right) = \phi_1 l + \phi_2 l =$

$= \boxed{-\frac{Pl^3}{24}}$ $\boxed{\varphi_A = -\frac{Pl^2}{12EI}}$ $\boxed{W_A = -\frac{Pl^3}{24EI}}$

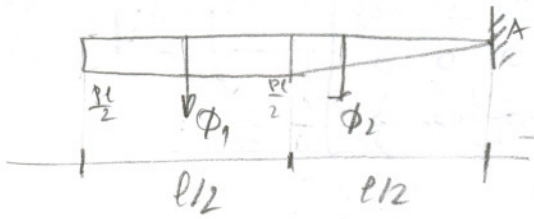
3.



$+ \curvearrowleft Pl \downarrow \frac{3l}{2}$



$$\phi_1 = \frac{Pl}{2}\,\frac{1}{2} = \boxed{\frac{Pl^2}{4}}$$



$$\phi_2 = \frac{1}{2}\,\frac{Pl}{2}\,\frac{l}{2} = \boxed{\frac{Pl^2}{8}}$$

$$\boxed{T_A = -\frac{3}{8}\,Pl^2}$$

$$M_A = -\left(\phi_1 \cdot \frac{3}{4}\,l + \phi_2\,\frac{l}{3}\,\frac{1}{2}\right) =$$

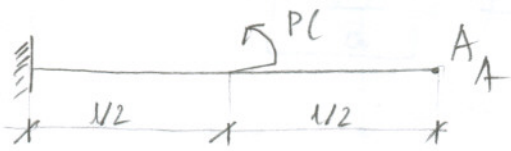
$$= -\left(\frac{Pl^2}{4}\cdot\frac{3}{4}\,l + \frac{Pl^2}{8}\,\frac{1}{3}\right) =$$

$$= -\left(\frac{3Pl^3}{16} + \frac{Pl^3}{24}\right) = \boxed{-\frac{11}{48}\,Pl^3}$$

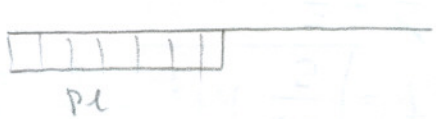
$$\boxed{\varphi_A = -\frac{3Pl^2}{8EI}}$$

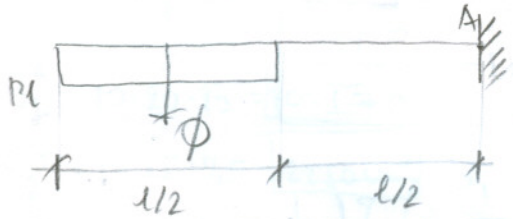
$$\boxed{W_A = -\frac{11\,Pl^3}{48EI}}$$

4



M





$$\phi = Pl \cdot \frac{1}{2} = \boxed{\frac{Pl^2}{2}}$$

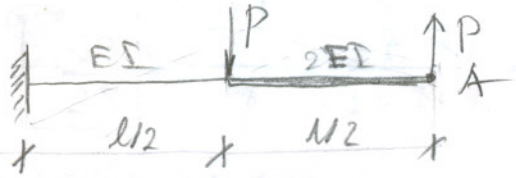
$$\boxed{T_A = -\frac{Pl^2}{2}}$$

$$M_A = -\frac{Pl^2}{2}\cdot\frac{3}{4}\,l = \boxed{-\frac{3Pl^3}{8}}$$

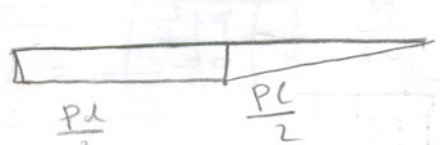
$$\boxed{\varphi_A = -\frac{Pl^2}{2EI}}$$

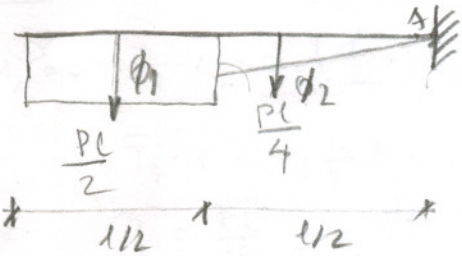
$$\boxed{W_A = -\frac{3Pl^3}{8EI}}$$

5.



M





$$\varphi_1 = \frac{Pl}{2}\,\frac{1}{2} = \boxed{\frac{Pl^2}{4}}$$

$$\varphi_2 = \frac{1}{2}\,\frac{Pl}{4}\,\frac{1}{2} = \boxed{\frac{Pl^2}{16}}$$

$$T_A = -\left(\frac{4Pl^2}{16} + \frac{Pl^2}{16}\right) = \boxed{-\frac{5}{16}Pl^2}$$

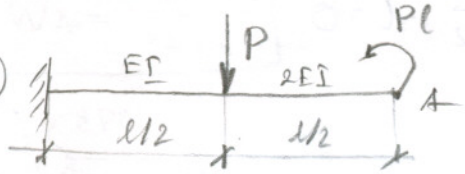
$$M_A = -\left(\frac{Pl^2}{4}\cdot\frac{3l}{4} + \frac{Pl^2}{16}\,\frac{2}{3}\,\frac{l}{2}\right) =$$

$$= -\left(\frac{3Pl^3}{16} + \frac{Pl^3}{48}\right) = \boxed{-\frac{5}{24}Pl^3}$$

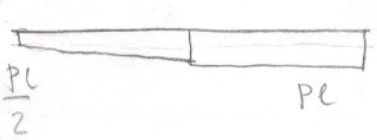
$$\boxed{\varphi_A = -\frac{5Pl^2}{16EI}}$$

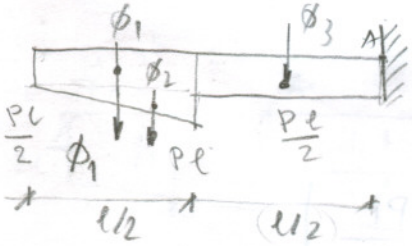
$$\boxed{w_A = -\frac{5Pl^3}{24EI}}$$

6.



M





$$\varphi_1 = \frac{Pl}{2}\,\frac{l}{2} = \boxed{\frac{Pl^2}{4}}$$

$$\varphi_2 = \frac{1}{2}\,\frac{Pl}{2}\,\frac{l}{2} = \boxed{\frac{Pl^2}{8}}$$

$$\varphi_3 = \frac{Pl}{2}\,\frac{l}{2} = \boxed{\frac{Pl^2}{4}}$$

$$\boxed{T_A = -\frac{5}{8}Pl^2}$$

$$M_A = -\left(\frac{Pl^2}{4}\,\frac{3}{4}\,l + \frac{Pl^2}{8}\left(\frac{l}{2} + \frac{1}{3}\,\frac{l}{2}\right) + \frac{Pl^2}{4}\,\frac{l}{4}\right) =$$

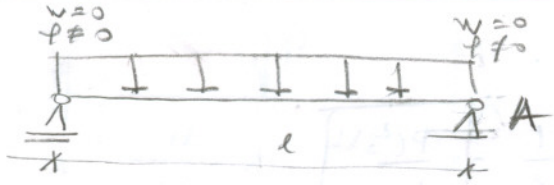
$$= -\left(\frac{3Pl^3}{16} + \frac{Pl^2}{12} + \frac{Pl^2}{16}\right)\frac{4}{3} \qquad \left(\frac{l}{2} + \frac{1}{3}\,\frac{l}{2} = \frac{4}{3}\right)$$

$$= \boxed{-\frac{1}{3}Pl^3}$$

$$\boxed{\varphi_A = -\frac{5Pl^2}{8EI}}$$

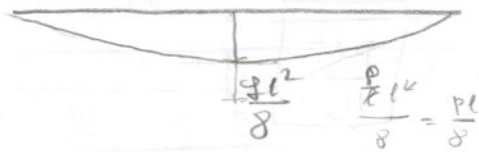
$$\boxed{w_A = -\frac{Pl^3}{3EI}}$$

7.



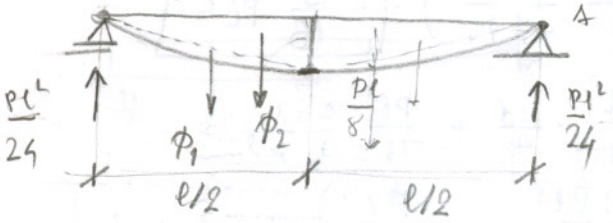
$$f = \frac{\frac{p}{k} l^2}{8} = \frac{\frac{p}{k}\frac{l^k}{4}}{8} = \frac{\frac{pl}{4}}{\frac{8}{1}} = \boxed{\frac{pl}{32}}$$

M



$$\phi_1 = \frac{2}{3} \bar{f} \cdot \bar{l} = \frac{2}{3} \frac{pl}{32} \cdot \frac{1}{k} = \boxed{\frac{pl^2}{96}}$$

$$\phi_2 = \frac{1}{2} \frac{1}{2} \frac{pl}{8} = \boxed{\frac{pl^2}{32}}$$



$$\boxed{T_A = -\frac{pl^2}{24}}$$

$$M_A = \frac{pl^2}{24} \cdot l - \phi_1 \frac{3}{4} l + \phi_1 \cdot \frac{l}{4} - \phi_2 \frac{2}{3} l - \phi_2 \frac{l}{3}$$

$$\overset{\oplus}{M_A} = \frac{pl^3}{24} - \phi_1 \left(\frac{3}{4} l + \frac{1}{4} l\right) - \phi_2 \left(\frac{2}{3} l + \frac{1}{3} l\right) =$$

$$= \frac{pl^3}{24} - \phi_1 l - \phi_2 l = \frac{pl^3}{24} - \frac{pl^2}{96} \cdot l - \frac{pl^2}{32} \cdot l = 0$$

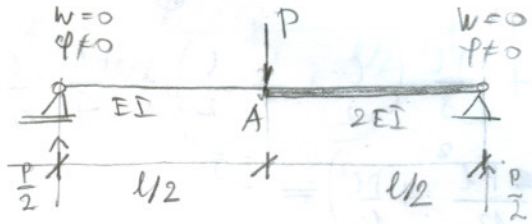
$$\boxed{\varphi_A = -\frac{pl^2}{24EI}} \qquad \boxed{w_A = 0}$$

$$\overset{\oplus}{M_{max}} = \frac{pl^2}{24} \cdot \frac{l}{2} - \phi_1 \frac{l}{4} - \phi_2 \frac{1}{3} \frac{l}{2} =$$

$$M\left(\frac{l}{2}\right) = \frac{pl^3}{48} - \frac{pl^2}{96} \frac{l}{4} - \frac{pl^2}{32} \frac{l}{6} = \boxed{\frac{5}{384} pl^3}$$

$$\boxed{\varphi_{max} = \frac{5\, pl^3}{384\, EI}}$$

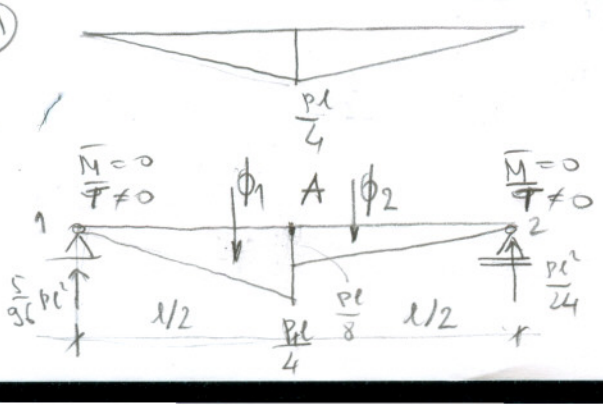
8.



$$\phi_1 = \frac{1}{2} \frac{l}{2} \frac{pl}{4} = \boxed{\frac{pl^2}{16}}$$

$$\phi_2 = \frac{1}{2} \frac{l}{2} \frac{pl}{8} = \boxed{\frac{pl^2}{32}}$$

M



$$\bar{T}_A = 0$$

$$\overset{\oplus}{\Sigma M_1} = 0 : \phi_1 \cdot \frac{2}{3} \frac{l}{2} + \phi_2 \left(\frac{l}{2} + \frac{l}{6}\right) - T_2 l = 0$$

$$T_2 = \phi_1 \frac{1}{3} + \phi_2 \frac{2}{3}$$

$$T_2 = \frac{pl^2}{16} \frac{1}{3} + \frac{pl^2}{32} \frac{2}{3} = \boxed{\frac{pl^2}{24}}$$

$$\bar{M}_A = 0: \quad \phi_2 \frac{l}{6} - \frac{Pl^2}{24} \cdot \frac{1}{2} = \frac{Pl^2}{32} \frac{l}{6} - \frac{Pl^3}{48} = -\frac{Pl^3}{64}$$

По конгруэнции

$$\bar{M}_A = \frac{Pl^3}{64}$$

(T) $\frac{5}{96}$ $\frac{1}{24}$ $\frac{1}{96}$ ⊕ $\Rightarrow T_A = -\frac{1}{96} Pl^2$

II начин

$$\sum M_2 = 0: \quad \bar{T}_1 \cdot l - \phi_1 \left(\frac{l}{2} + \frac{l}{6}\right) - \phi_2 \left(\frac{2}{3} \frac{1}{2}\right) = 0$$

$$\varphi_A = -\frac{Pl^2}{96 EI}$$

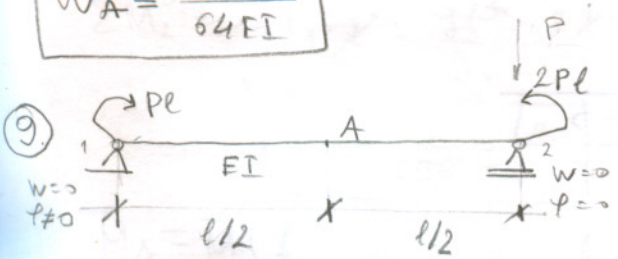
$$\bar{T}_1 \cdot l - \phi_1 \frac{2}{3} l - \phi_2 \frac{l}{3} = 0$$

$$T_1 = \frac{Pl^2}{16} \frac{2}{3} + \frac{Pl^2}{32} \frac{1}{3} = \frac{5}{96} Pl^2$$

$$\bar{M}_A = M_A\left(\frac{l}{2}\right) = \frac{5}{96} Pl^2 \frac{1}{2} - \frac{Pl^2}{16} \cdot \frac{l}{6} = \frac{1}{64} Pl^3$$

У по конгруэнции

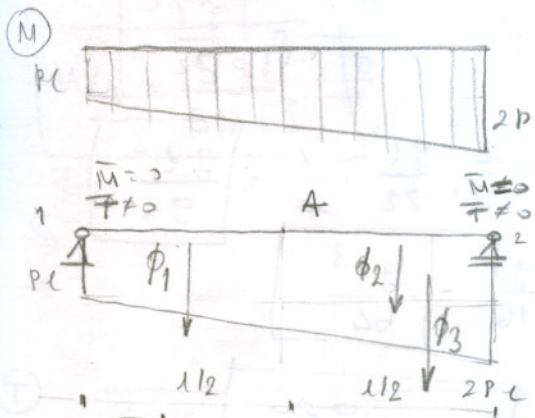
$$w_A = \frac{Pl^3}{64 EI}$$



$$\sum M_1 = 0: \quad -2Pl - T_2 \cdot l + Pl = 0 \Rightarrow T_2 \cdot l = -Pl$$

$$T_2 = -P$$

$$\sum M_2 = 0: \quad P(-2l + l) = 0$$



$$\phi_1 = Pl \cdot \frac{l}{2} = \frac{Pl^2}{2} = \phi_2$$

$$\phi_3 = \frac{1}{2} Pl \frac{l}{2} = \frac{Pl^2}{4}$$

$$\sum M_2 = 0: \quad T_1 \cdot l - Pl^2 \frac{l}{2} - \frac{Pl^2}{4} \cdot \frac{l}{6} = 0$$

$$T_1 = \frac{Pl^2}{2} + \frac{Pl^2}{24} = \frac{13}{24} Pl^2$$

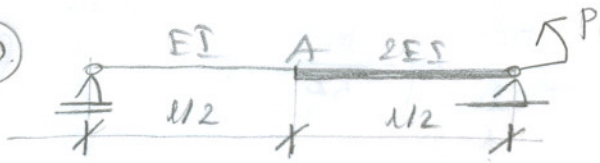
$$\bar{M}_A = M\left(\frac{l}{2}\right) = \frac{13}{24} Pl^2 \frac{l}{2} - \frac{Pl^2}{2} \cdot \frac{l}{4} = \frac{7}{48} Pl^3$$

$$\sum M_1 = 0: \quad Pl^2 \frac{l}{2} + \frac{Pl^2}{4} \left(\frac{1}{2} + \frac{2}{3} \frac{1}{2}\right) = \frac{Pl^2}{2} + \frac{Pl^2}{4} \frac{5}{6} = \frac{17}{24} Pl^2 = T_2$$

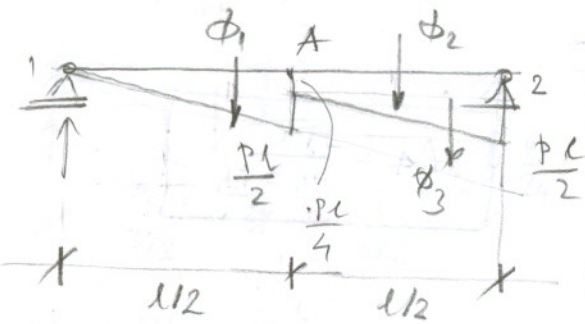
$$-T_2 l =$$

$$\bar{M}_A = M\left(\frac{l}{2}\right) = \frac{17}{24} Pl^2 \cdot \frac{l}{2} - \frac{Pl^2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} - \frac{Pl^2}{2} \frac{l}{4} = \frac{7}{48} Pl^3 \Rightarrow w_A = \frac{7Pl^3}{48 EI}$$

10.



M



$$\Phi_1 = \frac{1}{2}\frac{l}{2}\frac{Pl}{2} = \boxed{\frac{Pl^2}{8}}$$

$$\Phi_2 = \frac{Pl}{4}\frac{l}{2} = \boxed{\frac{Pl^2}{8}}$$

$$\Phi_3 = \frac{Pl}{4}\cdot\frac{l}{2}\frac{1}{2} = \boxed{\frac{Pl^2}{16}}$$

$$\Sigma M_2 \overset{\oplus}{=} 0: \quad T_1 l - \frac{Pl^2}{8}\frac{2}{3}l - \frac{Pl^2}{8}\frac{l}{4} - \frac{Pl^2}{16}\frac{l}{6} = 0$$

$$\boxed{T_1 = \frac{1}{8}Pl^2}$$

$$\bar{M}_A \equiv \bar{M}\left(\frac{l}{2}\right) \overset{\oplus}{=} \frac{1}{8}Pl^2\cdot\frac{l}{2} - \frac{Pl^2}{8}\frac{l}{6} = \boxed{\frac{Pl^3}{24}}$$

$$\boxed{w_A = \frac{Pl^3}{24EI}}$$

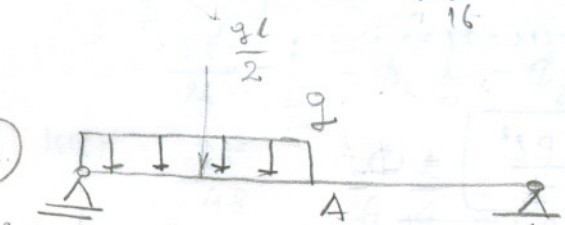
$$\Sigma y = 0: \quad \frac{1}{8}Pl^2 + T_2 - \frac{Pl^2}{8} - \frac{Pl^2}{8} - \frac{Pl^2}{16} = 0$$

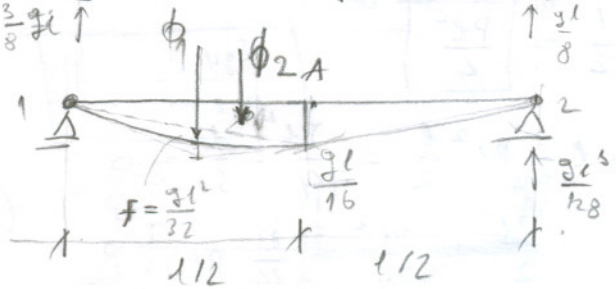
$$\boxed{T_2 = \frac{3}{16}Pl^2}$$

T

$$T_A = 0 \Rightarrow \boxed{\varphi_A = 0}$$

11.





$$F = \frac{ql^2}{8} = \frac{q\frac{l^2}{4}}{8} = \frac{\frac{ql^2}{4}}{8} = \boxed{\frac{ql^2}{32}}$$

$$\Phi_1 = \frac{2}{3}\bar{F}\bar{l} = \frac{2}{3}\frac{ql^2}{32}\cdot\frac{1}{2} = \boxed{\frac{ql^3}{96}}$$

$$\Phi_2 = \frac{1}{2}\frac{l}{2}\frac{ql}{16} = \boxed{\frac{ql^3}{64}}$$

$\frac{2}{3}\ \frac{1}{2}$

$$\Sigma M_1 = 0: \quad \frac{ql^3}{96}\cdot\frac{l}{4} + \frac{ql^3}{64}\frac{l}{3} - T_2 l = 0 \Rightarrow \boxed{T_2 = \frac{ql^3}{128}}$$

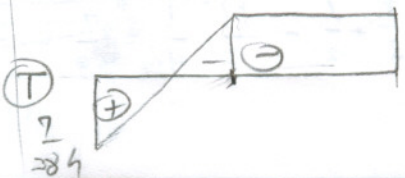
$$\bar{M}_A = M\left(\frac{l}{2}\right) = \frac{ql^3}{128}\cdot\frac{l}{2} = \boxed{\frac{ql^4}{256}}$$

$$\boxed{w_A = \frac{ql^4}{256EI}}$$

$$\Sigma y = 0: \quad \frac{ql^3}{128} + T_1 - \frac{ql^3}{96} - \frac{ql^3}{64} = 0$$

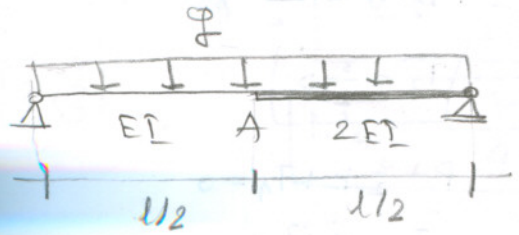
$$\boxed{T_1 = \frac{7}{384}ql^3}$$

T



$$T_A = -\frac{1}{128}ql^3 \Rightarrow \boxed{\varphi_A = -\frac{ql^3}{128EI}}$$

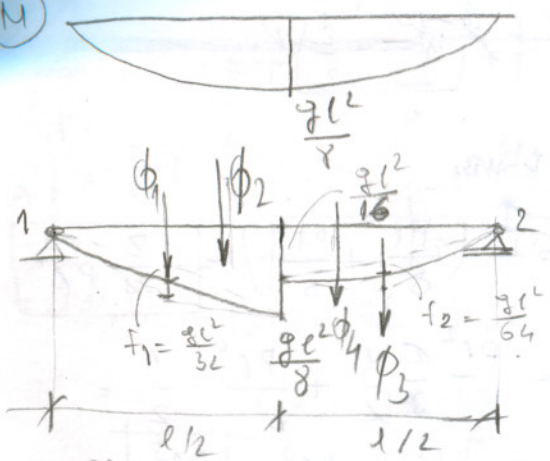
(12)



$$f_1 = \frac{q l^2}{8} = \frac{q \frac{l^2}{4}}{8} = \frac{\frac{q l^2}{4}}{8} = \boxed{\frac{q l^2}{32}}$$

$$f_2 = \frac{q l^2}{64}$$

(M)



$$\Phi_1 = \frac{2}{3} f \cdot \frac{l}{2} = \frac{1}{3} \frac{q l^2}{32} \frac{l}{2} = \boxed{\frac{q l^3}{96}}$$

$$\Phi_2 = \frac{1}{2} \frac{l}{2} \frac{q l^2}{8} = \boxed{\frac{q l^3}{32}}$$

$$\Phi_3 = \frac{2}{3} \frac{q l^2}{64} \frac{l}{2} = \boxed{\frac{q l^3}{192}}$$

$$\Phi_4 = \frac{1}{2} \frac{l}{2} \frac{q l^2}{16} = \boxed{\frac{q l^2}{64}}$$

$$\Sigma \overset{\curvearrowright}{M_2} = 0: \quad T_1 \cdot l - \frac{q l^3}{96} \cdot \frac{3}{4} l - \frac{q l^3}{32} \frac{2}{3} l - \frac{q l^2}{64} \frac{l}{3} - \frac{q l^3}{192} \frac{l}{4} = 0$$

$$\boxed{T_1 = \frac{9}{256} q l^3}$$

$$M_A = M\left(\frac{l}{2}\right) = \frac{9}{256} q l^3 \frac{l}{2} - \frac{q l^3}{96} \frac{l}{4} - \frac{q l^3}{32} \frac{l}{6} = \boxed{\frac{5}{512} q l^4}$$

$$\boxed{w_A = \frac{5 q l^4}{512 EI}}$$

$$\Sigma y = 0: \quad \frac{9}{256} - \frac{1}{96} - \frac{1}{32} - \frac{1}{192} - \frac{1}{64} + T_2 = 0$$

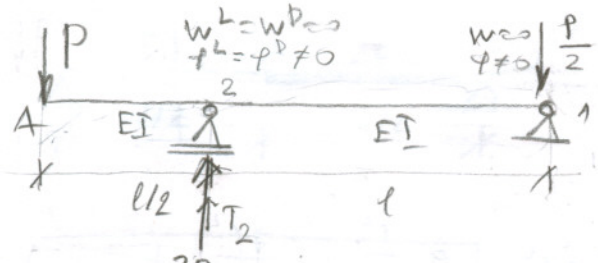
$$\boxed{T_2 = \frac{7}{256} q l^3}$$

(T)

$$T_A = \frac{-5}{768} q l^3$$

$$\boxed{\varphi_A = \frac{-5 q l^3}{768 EI}}$$

15.



$$\sum M_1 = 0: \quad -P\frac{3}{2}l + T_2 \cdot l = 0$$

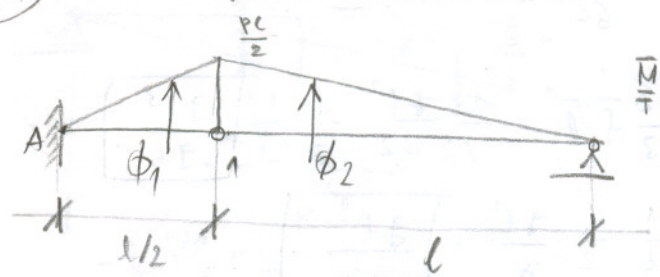
$$T_2 \cdot l = \frac{3P}{2}l$$

$$\boxed{T_2 = \frac{3}{2}P}$$

$$\sum y = 0: \quad -P + \frac{3}{2}P + T_1 = 0$$

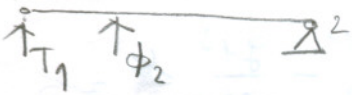
$$T_1 = P - \frac{3}{2}P$$

$$\boxed{T_1 = -\frac{P}{2}}$$



$$\phi_1 = \frac{1}{2}\frac{l}{2}\frac{Pl}{2} = \boxed{\frac{Pl^2}{8}}$$

$$\phi_2 = \frac{1}{2}l \cdot \frac{Pl}{2} = \boxed{\frac{Pl^2}{4}}$$



$$\sum M_2 = 0 \;\div\; T_1 \cdot l + \frac{Pl^2}{24} \cdot \frac{2}{3}l = 0$$

$$\boxed{T_1 = -\frac{Pl^2}{6}}$$

кинв.

$$T_A = \uparrow\left(\frac{Pl^2}{8} + \frac{Pl^2}{6}\right) = \boxed{-\frac{7}{24}Pl^2}$$

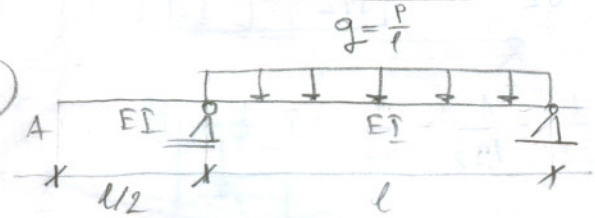
$$M_A = \frac{Pl^2}{8}\frac{2}{3}\frac{l}{2} + \frac{Pl^2}{6}\frac{l}{2} =$$

$$= \frac{Pl^3}{24} + \frac{Pl^3}{12} = \boxed{\frac{Pl^3}{8}}$$

$$\boxed{\varphi_A = -\frac{7Pl^2}{24EI}}$$

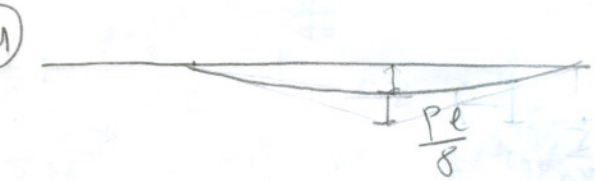
$$\boxed{w_A = \frac{Pl^3}{8EI}}$$

16.

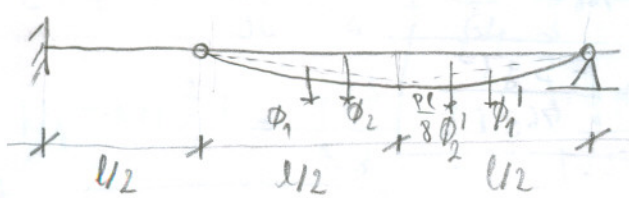


$$f = \frac{5}{8}\frac{ql^2}{8} = \frac{5}{8}\frac{Pl}{8}$$

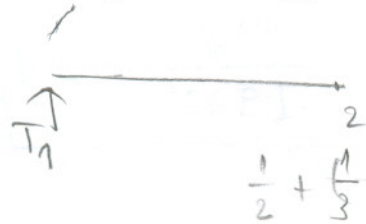
$$f_1 = \frac{\frac{P}{2}\frac{l^2}{4}}{8} = \frac{\frac{Pl}{4}}{\frac{8}{1}} = \boxed{\frac{Pl}{32}}$$



$$\phi_1 = \frac{2}{3}f \cdot \bar{l} = \frac{2}{3}\frac{Pl}{32}\frac{l}{2} = \boxed{\frac{Pl^2}{96}}$$



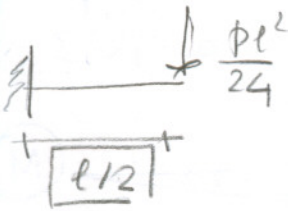
$$\phi_2 = \frac{1}{2}\frac{l}{2}\frac{Pl}{8} = \boxed{\frac{Pl^2}{32}}$$



$$\sum M_2 = 0 \quad T_1 \cdot l - \phi_1\frac{3}{4}l - \phi_2\frac{2}{3}l - \phi_2\frac{2}{3}\frac{l}{2} - \phi_1\frac{l}{4} = 0$$

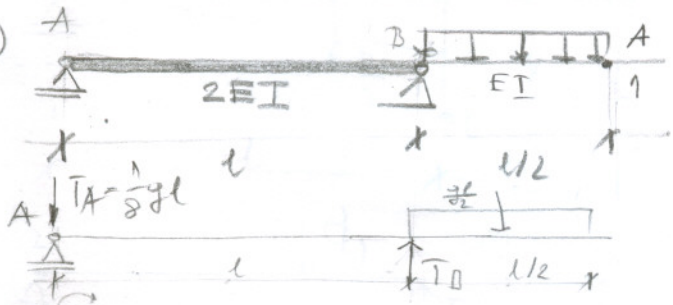
$$T_1 = \phi_1\left(\frac{3}{4} + \frac{1}{4}\right) + \phi_2\left(\frac{2}{3} + \frac{1}{3}\right)$$

$$T_1 = \Phi_1 + \Phi_2 = \frac{pl^2}{96} + \frac{pl^2}{32} = \boxed{\frac{pl^2}{24}}$$



$$\boxed{T_A = \frac{pl^2}{24}} \qquad \boxed{M_A = -\frac{pl^3}{48}} \Rightarrow \boxed{\varphi_A = \frac{pl^2}{24\,EI}} \qquad \boxed{w_A = -\frac{pl^3}{48\,EI}}$$

17.



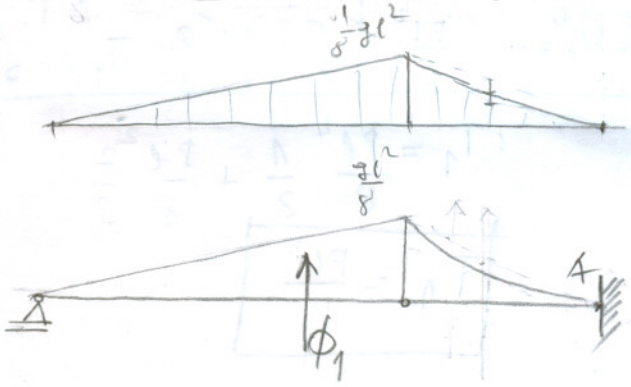
$$\Sigma M_A = 0: \quad -T_B \cdot l + \frac{gl}{2}\left(l + \frac{l}{4}\right) = 0$$

$$T_B\, l = \frac{gl}{2}\,\frac{5l}{4} \Rightarrow \boxed{T_B = \frac{5}{8}gl}$$

$$T_A = +\frac{gl}{2} - \frac{5}{8}gl = 0 \Rightarrow \boxed{T_A = \frac{1}{8}gl}$$

$$\Sigma M_1 = 0: \quad -\frac{1}{8}gl \cdot \frac{3}{2}l + \frac{5}{8}gl \cdot \frac{l}{2} - \frac{gl}{2}\,\frac{l}{4} = M_1$$

M



$$f = \frac{gl^2}{8} - \frac{g\frac{l^2}{4}}{8} - \frac{\frac{gl^2}{4}}{\frac{8}{1}} = \boxed{\frac{gl^2}{32}}$$

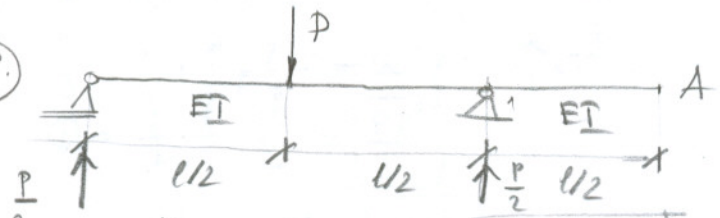
$$P_{PARAB} = \frac{2}{3}\,f\,l = \frac{2}{3}\,\frac{gl^2}{32}\cdot\frac{l}{2} = \boxed{\frac{gl^3}{96}} = \frac{1}{96}$$

$$P_\Delta = \frac{1}{2}\,\frac{gl^2}{8}\cdot\frac{l}{2} = \boxed{\frac{gl^3}{32}} = \frac{1}{32}$$

$$\boxed{P_{UK} = \frac{1}{48}} = \Phi_2$$

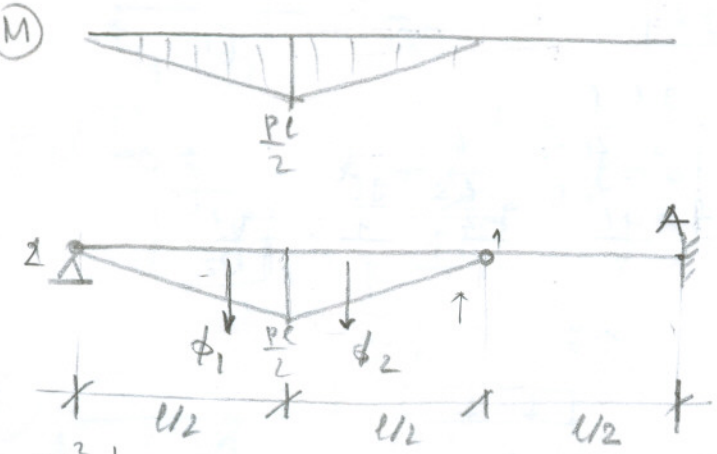
$$X_T = \frac{\frac{1}{6}\cdot\frac{1}{32} - \frac{1}{4}\,\frac{1}{96}}{\frac{1}{32} - \frac{1}{96}} = \frac{1}{8}$$

(18.)



$$P\cdot\frac{l}{2} - T_1\, l = 0 \Rightarrow \boxed{T_1 = \frac{P}{2}}$$

(M)

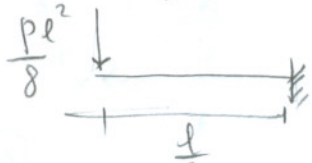


$$\Phi_1 = \frac{1}{2}\,\frac{l}{2}\,\frac{Pl}{2} = \boxed{\frac{Pl^2}{8}} = \Phi_2$$

$$\sum M_2 = 0: \quad \frac{Pl^2}{8}\cdot\frac{2}{3}\cdot\frac{l}{2} + \frac{Pl^2}{8}\cdot\frac{2}{3}l - T_1\, l = 0$$

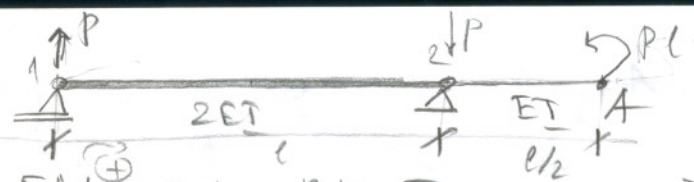
$$T_1 = \frac{Pl^2}{8}\,\frac{1}{3} + \frac{Pl^2}{8}\,\frac{2}{3}$$

$$\boxed{T_1 = \frac{Pl^2}{8}}$$



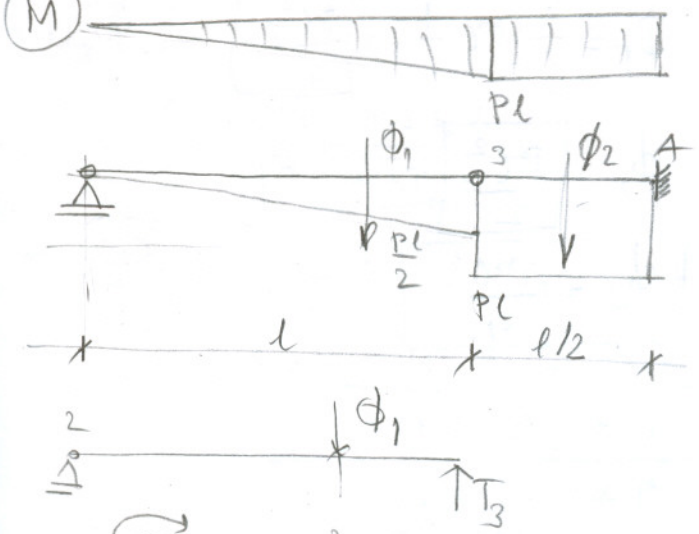
$$\left.\begin{array}{l}\boxed{T_A = -\frac{Pl^2}{8}}\\ \boxed{M_A = -\frac{Pl^2}{16}}\end{array}\right\} \Rightarrow \boxed{\begin{array}{l}\varphi_A = -\frac{Pl^2}{8EI}\\ w_A = \frac{-Pl^2}{16EI}\end{array}}$$

(19)



$\Sigma M_1 \overset{\oplus}{=} 0 : -Pl - T_2 \cdot l = 0 \Rightarrow \boxed{T_2 = -P}$

(M)

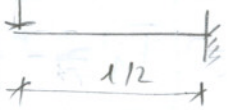


$\Phi_1 = \frac{1}{2} \cdot l \cdot \frac{Pl}{2} = \boxed{\frac{Pl^2}{4}}$

$\Phi_2 = Pl \cdot \frac{l}{2} = \boxed{\frac{Pl^2}{2}}$

$\Sigma M_2 \overset{\oplus}{=} 0 : \frac{Pl^2}{4} \cdot \frac{2}{3} l = T_3 \cdot l$

$\boxed{T_3 = \frac{Pl^2}{6}}$

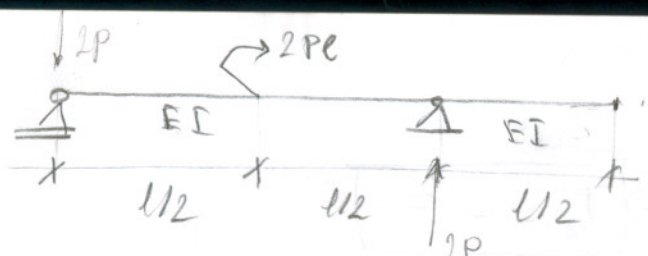


$\boxed{T_A = -\frac{Pl^2}{6}}$

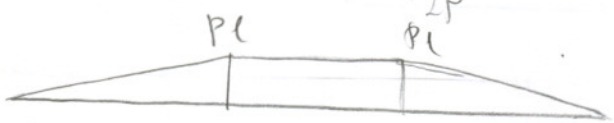
$\boxed{M_A = -\frac{Pl^2}{12}}$

$\boxed{\varphi_A = -\frac{Pl^2}{6EI} \; ; \; w_A = -\frac{Pl^2}{12EI}}$

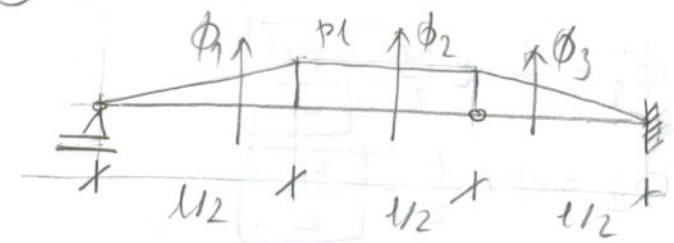
(20.)



(M)



$$\Phi_1 = \frac{1}{2} P\ell \frac{1}{2} = \boxed{\frac{P\ell^2}{4}}$$

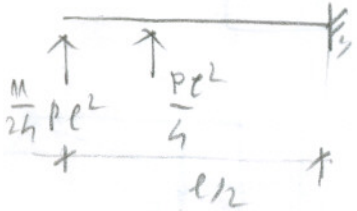


$$\Phi_2 = \boxed{-\frac{P\ell^2}{2}}$$

$$\boxed{\Phi_3 = \frac{P\ell^2}{4}}$$

$$\Sigma M_2^{(+)} = 0 : -\frac{P\ell^2}{4}\frac{2}{3}\frac{\ell}{\ell} - \frac{P\ell^2}{2}\frac{3}{4}\ell - T_1 \cdot \ell = 0$$

$$T_1 = -\frac{P\ell^2}{12} - \frac{3P\ell^2}{8} \Rightarrow \boxed{T_1 = -\frac{11}{24} P\ell^2}$$



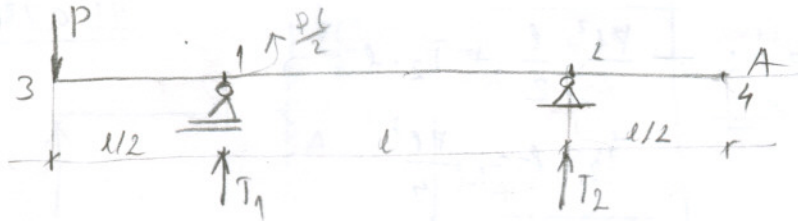
$$\boxed{T_A = \frac{11}{24} P\ell^2}$$

$$M_A = \frac{11}{24} P\ell^2 \frac{1}{2} + \frac{P\ell^2}{4} \cdot \frac{2}{3} \frac{\ell}{\ell} = \boxed{\frac{5}{15} P\ell^3}$$

$$\boxed{\varphi_A = \frac{17\, P\ell^2}{24\, EI}}$$

$$\boxed{w_A = \frac{5\, P\ell^3}{24\, EI}}$$

13.



$\Sigma M_3 \overset{(+)}{=} 0: \quad -T_1 \frac{l}{2} - T_2 \frac{3}{2} l = 0 \quad / \cdot 2$

(1) $\boxed{T_1 l + 3 T_2 l = 0}$

$\boxed{T_1 + 3T_2 = 0}$

$\Sigma M_4 \overset{(+)}{=} 0: \quad -P \cdot 2l + T_1 \cdot \frac{3}{2} l + T_2 \frac{1}{2} = 0 \quad / \cdot \frac{2}{l}$

(2) $\boxed{3T_1 + T_2 = 4P}$

$T_1 + 3T_2 = 0 \quad / (-3)$

$3T_1 + T_2 = 4P$

$-3T_1 - 9T_2 = 0$

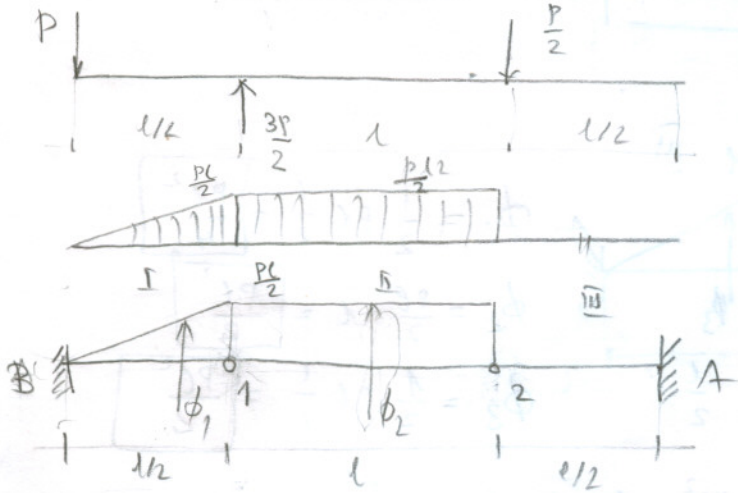
$-8T_2 = 4P \Rightarrow \boxed{T_2 = -\frac{P}{2}}$

$T_1 = \left(4P + \frac{P}{2}\right) \cdot \frac{1}{3}$

$T_1 = \frac{9P}{2} \cdot \frac{1}{3} \Rightarrow \boxed{T_1 = \frac{3P}{2}}$

II НАЧИН

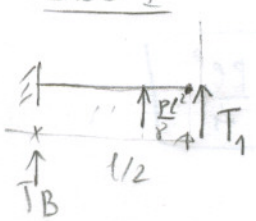
$\Sigma M_1 \overset{(+)}{=} \frac{Pl}{2}: \Rightarrow \frac{Pl}{2} + T_2 \cdot l = 0 \Rightarrow \boxed{T_2 = -\frac{P}{2}}$



$\Phi_1 = \frac{1}{2} \frac{Pl}{2} \frac{l}{2} = \boxed{\frac{Pl^2}{8}}$

$\Phi_2 = \frac{Pl}{2} \cdot l = \boxed{\frac{Pl^2}{2}}$

II ЕО I

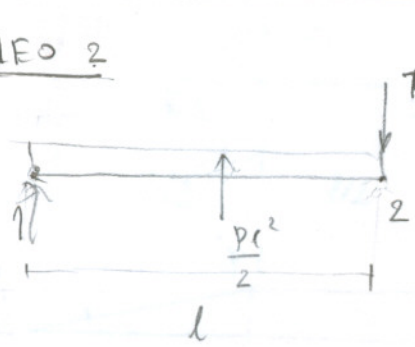


$\Sigma M_1 \overset{(+)}{=} 0: \quad T_B \cdot \frac{l}{2} + \frac{Pl^2}{8} \cdot \frac{l}{6} = 0$

$T_B \frac{l}{2} = -\frac{Pl^2}{48} \Rightarrow \boxed{T_B = -\frac{1}{24} Pl^2}$

$\Sigma y = 0: \quad -\frac{Pl^2}{24} + \frac{Pl^2}{8} + T_1 = 0 \Rightarrow T_1 = -\frac{Pl^2}{12}$

ДЕО 2

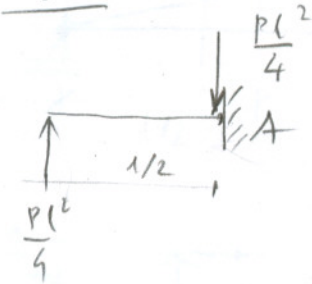


$\sum M_1 = 0 : \quad -\frac{pl^2}{2}\cdot\frac{l}{2} + T_2 \cdot l = 0$

$T_2 \cdot l = \frac{pl^3}{4}$

$$\boxed{T_2 = \frac{pl^2}{4}}$$

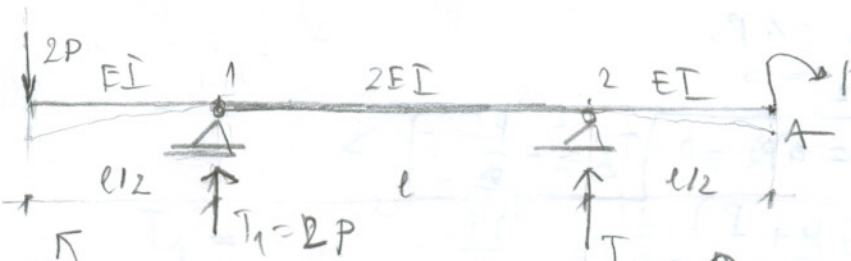
ДЕО 3



$$\boxed{T_A = \frac{pl^2}{4}} \qquad \boxed{M_A = \frac{pl^3}{8}}$$

$$\Rightarrow \boxed{\varphi_A = \frac{pl^2}{4EI} \qquad w_A = \frac{pl^3}{8EI}}$$

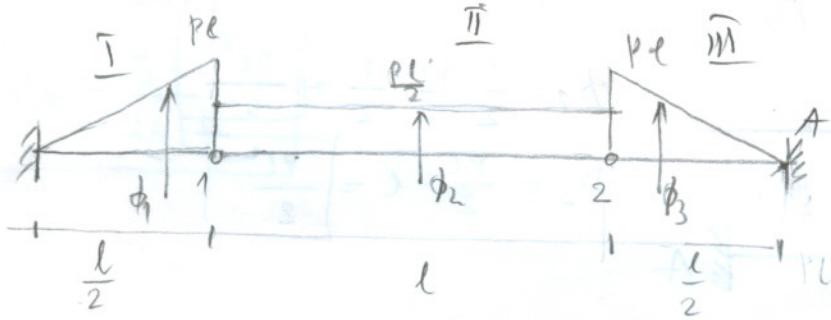
14.



$\sum M_1 = 2P \cdot \frac{l}{2} = Pl$

$\Rightarrow \quad T_2 \cdot l - Pl = Pl \Rightarrow \boxed{T_2 = 0}$

M

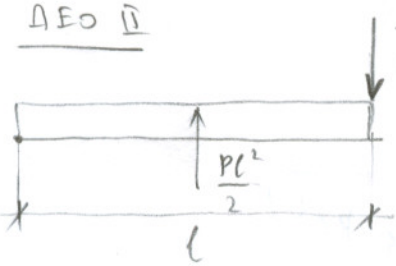


$\phi_1 = \frac{1}{2} Pl \frac{l}{2} = \boxed{\frac{Pl^2}{4}}$

$\phi_2 = \frac{Pl}{2} \cdot l = \boxed{\frac{Pl^2}{2}}$

$\phi_3 = \frac{1}{2} Pl \frac{l}{2} = \boxed{\frac{Pl^2}{4}}$

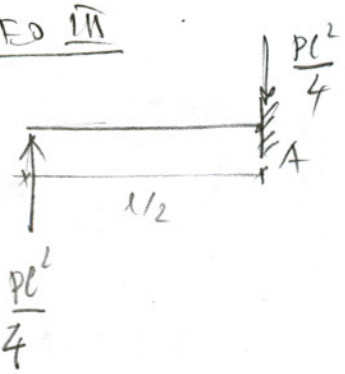
ДЕО II



$\sum M_1 = 0 : -\frac{Pl^2}{2} \cdot \frac{l}{2} + T_2 \cdot l = 0$

$T_2 \cdot l = \frac{Pl^2}{4} \Rightarrow \boxed{T_2 = \frac{Pl^2}{4}}$

1 EJ 1 II



$$T_A = \frac{pl^2}{4}$$

$$M_A = \frac{pl^3}{8}$$

$$\Rightarrow$$

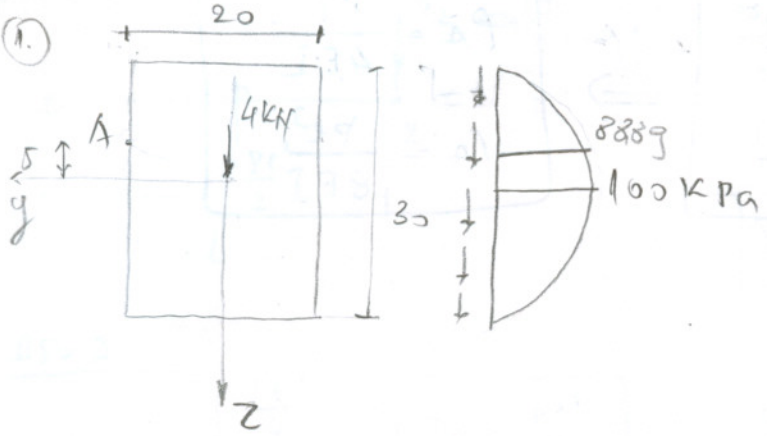
$$\varphi_A = \frac{pl^2}{4EI}$$

$$=$$

$$w_A = \frac{pl^3}{8EI}$$

2.

1.



$$\tau_{xz} = \frac{T_z \cdot S_y^*}{I_y \, b(z)}$$

$$I_y = \frac{1}{12} \cdot 30^3 \cdot 20 = 45000\,cm^4$$

$$S_y^A = 20 \cdot 10 \cdot 10 = 2000\,cm^3$$

$$S_y^T = 20 \cdot 15 \cdot 7{,}5 = 2250\,cm^3$$

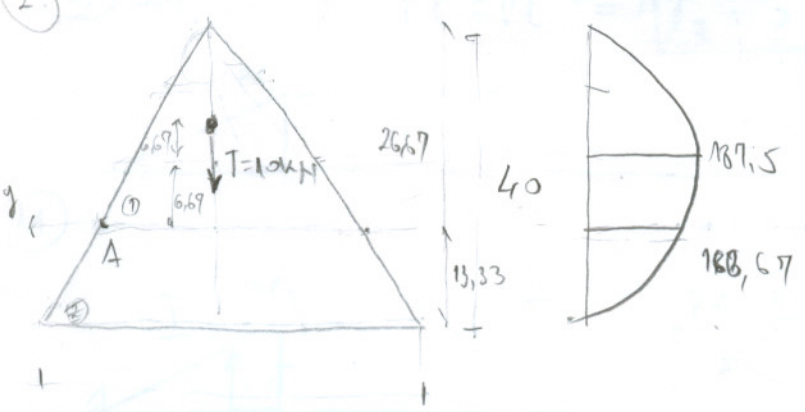
$$\tau_{xz}^T = \frac{4 \cdot 10^3 N \cdot 2250 \cdot 10^{-6} m^3}{45000 \cdot 10^{-8} m^4 \cdot 20 \cdot 10^{-2} m} = 100\,kPa$$

$$\tau_{xz}^T = \frac{3}{2}\frac{T}{A} = \frac{3}{2}\,\frac{4000}{20 \cdot 30 \cdot 10^{-4}} = 100\,kPa$$

$$\tau_{xz}^A = -11-$$

$$S = \begin{bmatrix} 0 & 0 & 88{,}9 \\ 0 & 0 & 0 \\ 88{,}9 & 0 & 0 \end{bmatrix}$$

2.



40

$$I_y = \frac{1}{36} \cdot 40^4 = 71111{,}11\,cm^4$$

$$p^{(1)} = \frac{1}{2}\, b(z) \cdot 26{,}67$$

$$\tau_{xz}^{T_z} = \frac{T_z \cdot S_y^*}{I_y \, b(z)}$$

$$S_y^T = \frac{1}{2}\, b(z)\, 26{,}67 \cdot 8{,}89 = 118{,}55\, b(z)\,cm^3$$

$$S_y^{SR} = \frac{1}{2}\, b(z) \cdot 20 \cdot 13{,}34 = 133{,}4\, b(z)\,cm^3$$

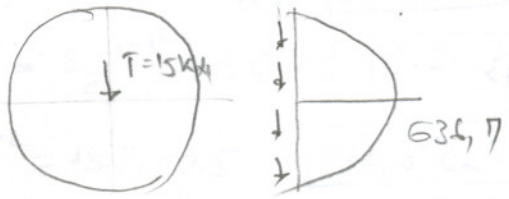
$$\tau_{xz}^T = \frac{10 \cdot 10^3 \cdot 118{,}55\, b(z) \cdot 10^{-4}\, b(z)}{71111{,}11 \cdot 10^{-8}\, b(z)} = 166{,}71\,kPa$$

$$\tau_{xz}^{T_z} = \frac{10 \cdot 10^3 \cdot 133{,}4 \cdot 10^{-6}\, b(z)}{71111{,}11 \cdot 10^{-8}\, b(z)} = 187{,}5\,kPa$$

$$\tau_{xz}^T = \frac{4}{3}\frac{T_z}{A} = \frac{4}{3}\,\frac{10 \cdot 10^3 N}{800 \cdot 10^{-4} m^2} = 166{,}67\,kPa$$

$$\tau_{xz}^{SR} = \frac{3}{2}\frac{T_z}{A} = \frac{3}{2}\,\frac{10 \cdot 10^3 N}{800 \cdot 10^{-4}} = 187{,}5\,kPa$$

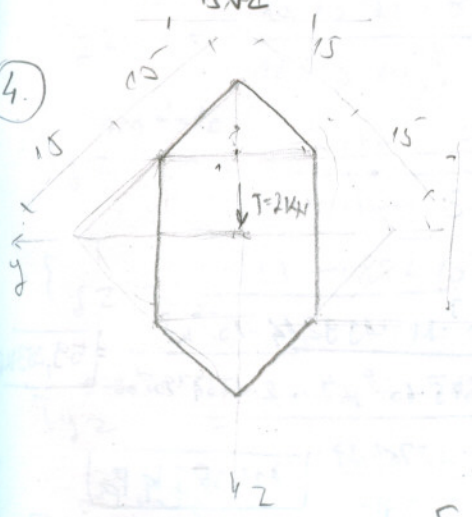
3.

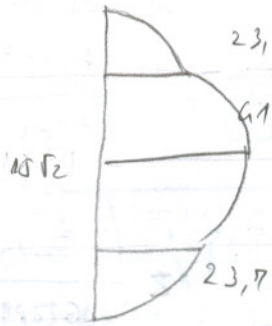


$$\tau = \frac{4}{3}\frac{T_z}{A} = \frac{4}{3}\frac{15\cdot 10^3}{20{,}1^2\,\pi} = \boxed{636{,}7\ kPa}$$

$$A = 10{,}1^2\,\pi$$

4.





$$\frac{7}{5}\frac{T_z}{A} = \frac{7}{5}\frac{2000\,N}{675\cdot 10^{-4}\,m^2} = \boxed{41{,}481\ kPa}$$

$$\frac{4}{5}\frac{T_z}{A} = \frac{4}{5}\ \text{--11--} = \underline{23{,}703\ kPa}$$

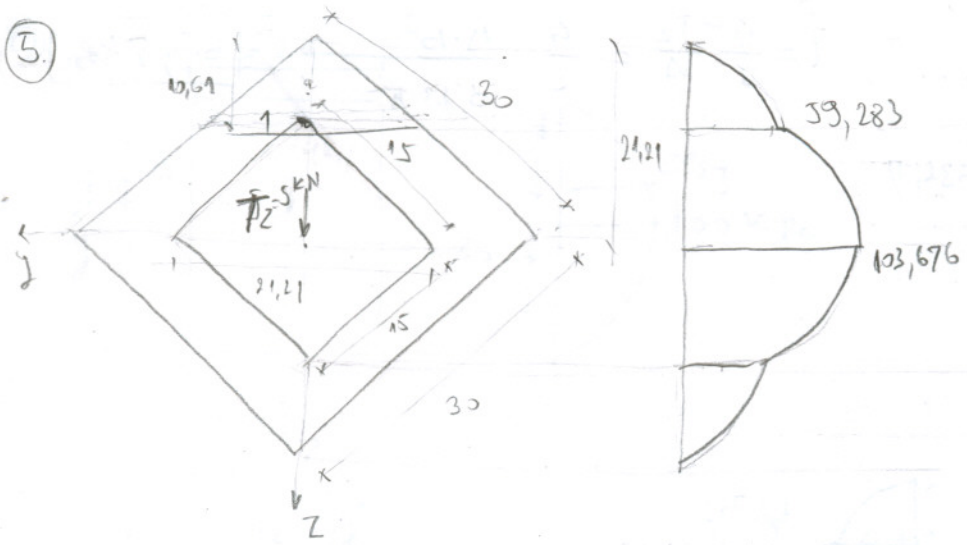
$$I_y = \frac{1}{12}\cdot(15\sqrt{2})^4 + 2\left[\frac{1}{36}\cdot\left(\frac{15\sqrt{2}}{2}\right)^3\cdot 15\sqrt{2} + (10\sqrt{2})^2\cdot\frac{1}{2}\,15\sqrt{2}\,\frac{15\sqrt{2}}{2}\right] = 63289{,}25$$

$$\tau_{xz} = \frac{T_z\cdot S_y^*}{I_y\ B(z)}$$

$$S_y^* = \frac{1}{2}\cdot 10{,}61^2\cdot$$

$$A = 2\,\frac{1}{2}\,15\frac{\sqrt{2}}{2}\cdot 15\sqrt{2} + (15\sqrt{2})^2 = \boxed{675\ cm^2}$$

5.



$$I_y = \frac{1}{12}(30^4 - 15^4) = 63281{,}25\ cm^4$$

$$\tau_{xz}^T = \frac{T_z \cdot S_y^*}{I_y \cdot b(z)}$$

$$S_y^1 = \frac{1}{2} \cdot 10{,}61 \cdot 21{,}21 \cdot 14{,}15 = 1592{,}14\ cm^3$$

$$S_y^T = \frac{1}{2}(30^2 - 15^2) \cdot 8{,}25 = 2784{,}38\ cm^4$$

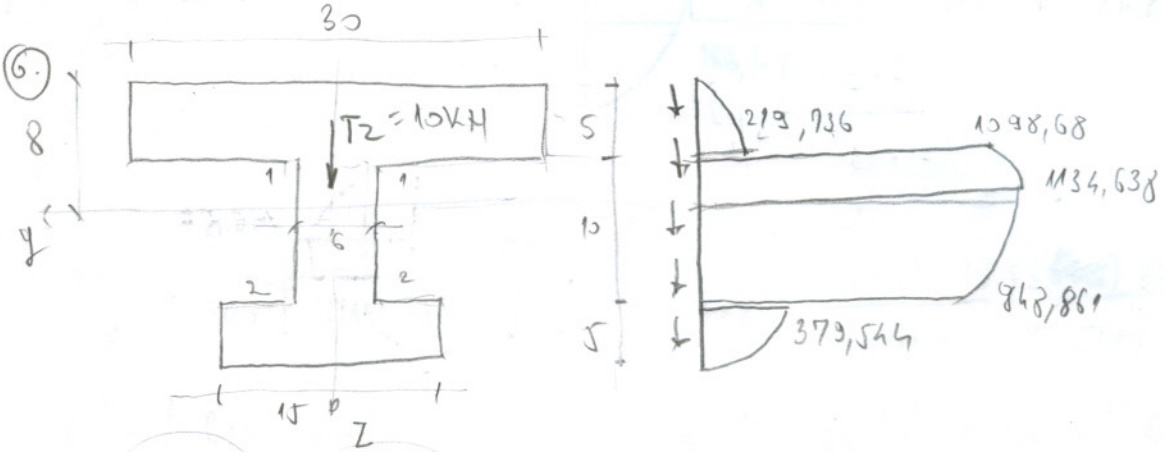
$$\tau_{xz}^1 = \frac{5 \cdot 10^3 \cdot N \cdot 1592{,}14 \cdot 10^{-6}\ m^3}{63289{,}25 \cdot 10^{-8}\ m^4 \cdot 2 \cdot 10{,}61 \cdot 10^{-2}\ m} = 59{,}283\ kPa$$

$$\tau_{xz}^T = \frac{-11- \cdot 2784{,}38}{-11- \cdot -11-} = 103{,}676\ kPa$$

$$z_T = \frac{(3{,}53 \cdot 56{,}29) \cdot 2 + (9{,}07 \cdot 56{,}29) \cdot 2 + 14{,}15 \cdot 112{,}58}{337{,}69} = 8{,}249\ cm$$

$$z_T = \frac{5{,}305 \cdot 112{,}58 \cdot 2 + 14{,}15 \cdot 112{,}58}{337{,}74} = 8{,}25\ cm$$

6.



$$I_y = \frac{1}{12}10^3 \cdot 6 + \frac{1}{12} \cdot 5^3 \cdot 30 + 5{,}5^2 \cdot 150 + \frac{1}{12}5^3 \cdot 15 + 9{,}5^2 \cdot 15 \cdot 5 + 2^2 \cdot 60 = 12515\ cm^4$$

12275

$S_y^{(1-1)} = 5 \cdot 30 \cdot 5{,}5 = \boxed{825\ cm^3}$

$S_y^T = S_y^{(1-1)} + 3 \cdot 6 \cdot 1{,}5 = \boxed{852\ cm^3}$

$S_y^{2-2} = 15 \cdot 5 \cdot 9{,}5 = \boxed{712{,}5\ cm^3}$

$\tau_{yz} = \frac{T_z \cdot S_y^*}{I_y \, b(z)}$

$\tau_{yz}^{1-1\ gore} = \frac{10 \cdot 10^3 N \cdot 825 \cdot 10^{-6}\ m^3}{12515 \cdot 10^{-8}\ m^4 \cdot 30 \cdot 10^{-2}\ m} = \boxed{219{,}736\ kPa}$

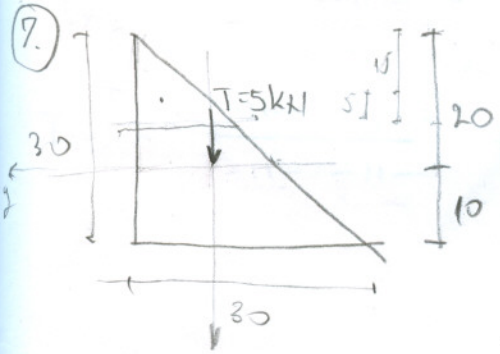
$\tau_{yz}^{1-1\ dole} = \frac{-11- \quad -11-}{-11- \quad 6 \cdot 10^{-2}} = \boxed{1098{,}68\ kPa}$

$\tau_{yz}^{T} = \frac{-11- \cdot 852 \cdot 10^{-6}}{-11- \quad -11-} = \boxed{1134{,}638\ kPa}$

$\tau_{yz}^{2-2\ gore} = \frac{-11- \quad 7125 \cdot 10^{-6}}{-11- \quad -11-} = \boxed{948{,}861\ kPa}$

$\tau_{yz}^{2-2\ dole} = \frac{-11- \quad -11-}{-11- \quad 15 \cdot 10^{-2}} = \boxed{379{,}544\ kPa}$

7.

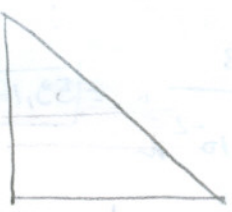


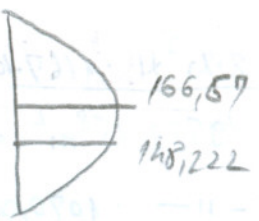
$\tau_{xz}^{T_z} = \frac{5 \cdot 10^3 N \cdot 66{,}7\, b(z) \cdot 10^{-4}\ m^2}{22500 \cdot 10^{-8}\ m^4 \cdot b(z)} = \boxed{148{,}222\ kPa}$

$\tau_{xz}^{(15)} = \frac{-11- \cdot 75\, b(z) \cdot 10^{-4}}{-11-} = \boxed{166{,}67\ kPa}$

$\tau_{yz}^{T_z} = \frac{T_z \cdot S_y^*}{I_y \, b(z)}$

$I_y = \frac{1}{36} 30^4 = \boxed{22500\ cm^4}$

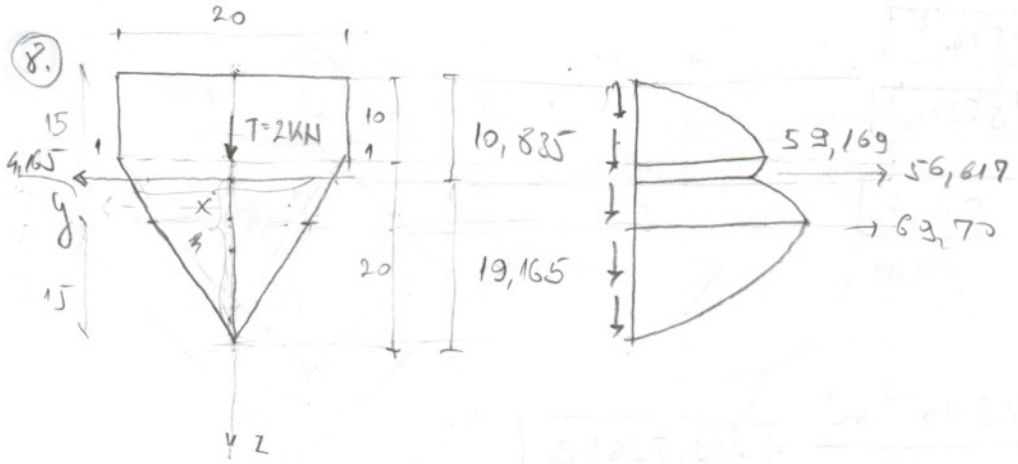




$S_y^T = \frac{1}{2} 20 \cdot b(z) \cdot 6{,}67 = 66{,}7\, b(z)$

$S_y^{(15)} = \frac{1}{2} \cdot 15 \cdot b(z) \cdot 10 = 75\, b(z)$

8.



$$z_T = \frac{10 \cdot 20 \cdot 5 + \frac{20 \cdot 20}{2} \cdot 16{,}67}{400} = 10{,}835\,cm$$

$$\tau_{xz} = \frac{T_z \cdot S_y^*}{I_y \cdot b(z)}$$

$$I_y = \frac{1}{12} 10^3 \cdot 20 + 5{,}835^2 \cdot 200 + \frac{1}{36} 20^4 + 5{,}832^2 \cdot 200 = 19923\,cm^4$$

$$S_y^{1-1} = 20 \cdot 10 \cdot 5{,}835 = 1167\,cm^3$$

$$19{,}165 : x = 1$$

$$\frac{19{,}165}{x} = 1 \Rightarrow x = 19{,}165$$

$$S_y^T = \frac{19{,}165^2}{2} \cdot 5{,}832 = 1070{,}04\,cm^3$$

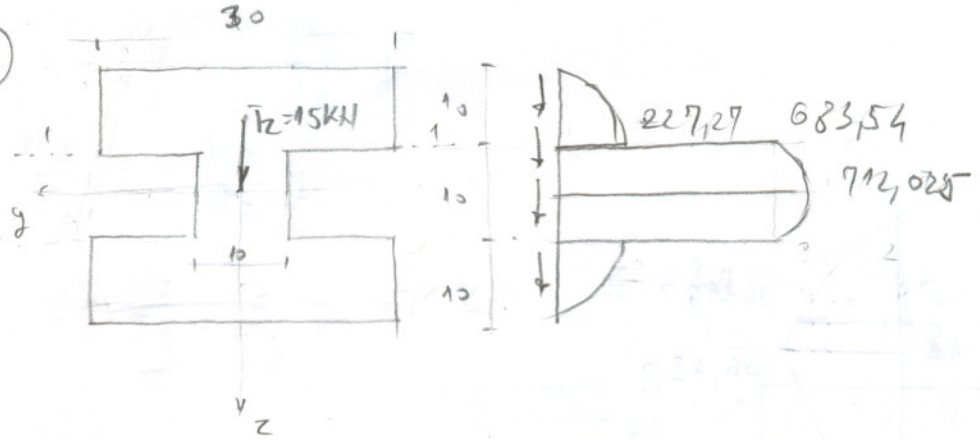
$$S_y^{(15)} = \frac{15^2}{2} \cdot 9{,}165 = 1031{,}0625\,cm^3$$

$$\tau_{xz}^{(1-1)} = \frac{2 \cdot 10^3 \cdot 1 \cdot 1167 \cdot 10^{-6}\,m^3}{19923 \cdot 10^{-8}\,m^4 \cdot 20 \cdot 10^{-2}\,m} = 59{,}169\,kPa$$

$$\tau_{xz}^{T} = \frac{-11- \cdot 1070{,}04 \cdot 10^{-6}}{-11- \cdot 19{,}165 \cdot 10^{-2}} = 56{,}617\,kPa$$

$$\tau_{xz}^{(15)} = \frac{-11- \cdot 1031{,}0625}{-11- \cdot 15 \cdot 10^{-2}} = 69{,}70\,kPa$$

9.



$$\tau_{xz}^{T_z} = \frac{T_z \cdot S_y^*}{J_y \, b(z)}$$

$$I_y = \frac{1}{12} 30^4 - 2\left(\frac{1}{12} 10^4 + 10^2 \cdot 150\right) = 65833{,}33\ cm^4$$

$$S_y^{(1-1)} = 10 \cdot 30 \cdot 10 = 3000\ cm^3$$

$$S_y^{T} = S_y^{(1-1)} + 10 \cdot 5 \cdot 2{,}5 = 3125\ cm^3$$

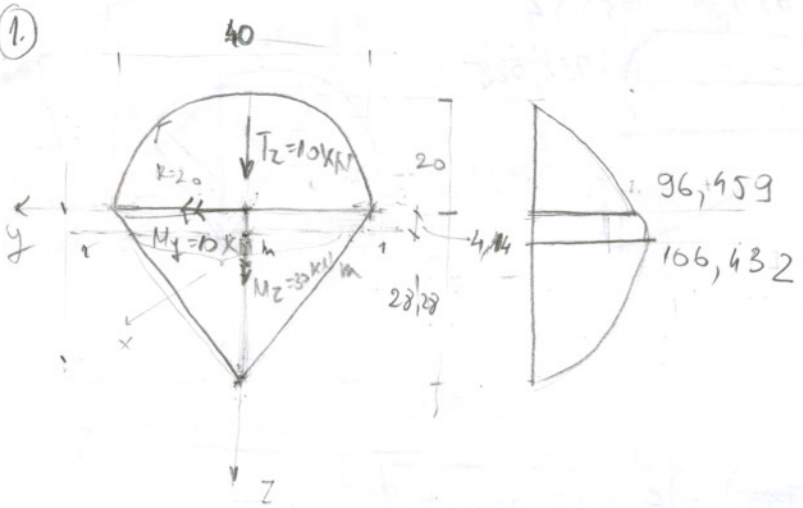
$$\tau_{xz}^{(1-1)\,gore} = \frac{15 \cdot 10^3 \cdot 3000 \cdot 10^{-6}\ m^3}{65833{,}33 \cdot 10^{-8}\ m^4 \cdot 30 \cdot 10^{-2}\ m} = 227{,}27\ kPa$$

$$\tau_{xz}^{(1-1)\,dole} = \frac{-\!\!-\!\!-}{-\!\!-\!\!-} = 683{,}54\ kPa$$

$$\tau_{xz}^{T} = \frac{-\!\!-\!\!-}{-\!\!-\!\!-} = 712{,}025\ kPa$$

СЛОЖЕНО НАПРЕЗАНЕ

1.



$$z_T = \frac{\frac{1}{2}\cdot 40\cdot h\cdot\frac{2}{3}h + 200\pi(h+8{,}488)}{20h+200\pi} \quad /\cdot(20h+200\pi)$$

z_T = h

$$20h^2 + 200h\pi = \frac{40}{3}h^2 + 200h\pi + 200\pi\cdot 8{,}488$$

$$\frac{20}{3}h^2 - 5333{,}168 = 0 \Rightarrow \boxed{h = 28{,}28\ cm}$$

$$\tau_{xz}^{T_z} = \frac{T_z\cdot S_y^*}{I_y\, b(z)}$$

$$I_y = \frac{1}{36}\cdot 28{,}28^3\cdot 40 + 9{,}427^2\cdot 565{,}6 + 2\cdot 0{,}95488\cdot 20^4 + 8{,}488^2\cdot 200\pi = \boxed{138223{,}64\ cm^4}$$

$$I_z = \frac{1}{48}\cdot 40^3\cdot 28{,}28 + \frac{25\pi}{8} = \boxed{100538{,}52\ cm^4}$$

$$S_y^T = 200\pi\cdot 8{,}488 = \boxed{5333{,}168\ cm^3}$$

$$\frac{28{,}28}{40} = \frac{24{,}14}{x} \qquad x = \frac{40\cdot 24{,}14}{28{,}28} = 34{,}144$$

$$S_y^{24,14} = \frac{1}{2}\cdot 24{,}14\cdot 34{,}144\cdot 12{,}187 = \boxed{5022{,}48\ cm^3}$$

$$\frac{28{,}28}{40} = \frac{14{,}14}{x_1} \qquad \boxed{x_1 = \frac{14{,}14\cdot 40}{28{,}28} = 20}$$

$$\tau_{xz}^T = \frac{10\cdot 10^6\cdot 5333{,}168\cdot 10^{-6}\ m^3}{138223{,}64\cdot 10^{-8}\ m^4\cdot 40\cdot 10^{-2}\ m} = \boxed{96{,}459\ kPa}$$

$$S_y^{1-1} = \frac{1}{2}\cdot 20\cdot 14{,}14\cdot 18{,}85 = \boxed{2665{,}39\ cm^3}$$

$$\tau_{xz}^{(24,14)} = \frac{-11-\ 5022{,}48\cdot 10^{-6}}{=11-\ 34{,}14\cdot 10^{-2}} = \boxed{106{,}432\ kPa}$$

$$\tau_{xz}^{1+1} = \frac{10\cdot 10^3\cdot 2665{,}39\cdot 10^{-6}}{138223{,}64\cdot 10^{-8}\cdot 20\cdot 10^{-2}} = \boxed{96{,}415\ kPa}$$

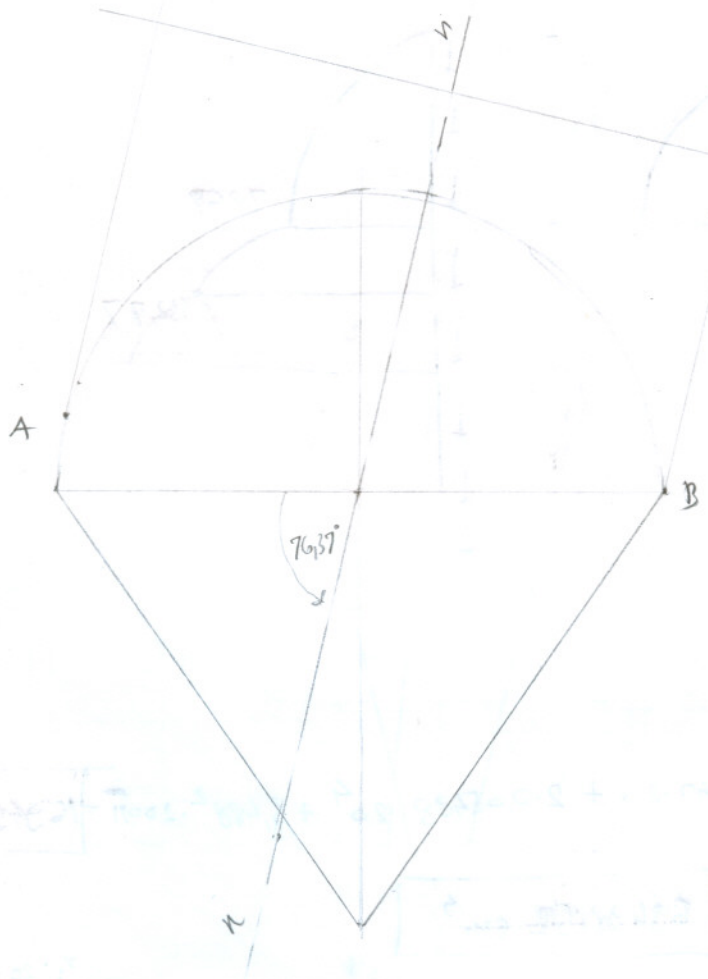
$$\sigma_x = + \frac{M_y}{I_y} z - \frac{M_z}{I_z} y$$

$$\frac{M_y}{I_y} z = \frac{M_z}{I_z} y$$

$$z = \frac{M_z}{I_z} \frac{I_y}{M_y} y \Rightarrow z = \frac{M_z}{M_y} \frac{I_y}{I_z} y = \frac{30}{10} \frac{138223,64}{100538,52} = 4,124\, y$$

$$\boxed{z = 4,124\, y} \Rightarrow \boxed{\alpha = 76,37°}$$

x: 486,2442
y: 552,5815



A (13,41; -4,83)

$$\sigma_{x,A} = \frac{10000\,Nm}{138223,64 \cdot 10^{-8} m^4} \cdot (-4,83) \cdot 10^{-2} m - \frac{30000\,Nm}{100538,52 \cdot 10^{-8} m^4} \cdot 13,41 \cdot 10^{-2} m = \boxed{-6,441\,MPa}$$

$$\sigma_{x,B} = 5,197\,MPa$$

2.) $$T_T = \frac{\frac{1}{2} 20 \cdot h \cdot \frac{2}{3} h + 200\pi \cdot (h + 8{,}488)}{10h + 200\pi} \quad / (10h + 200\pi)$$

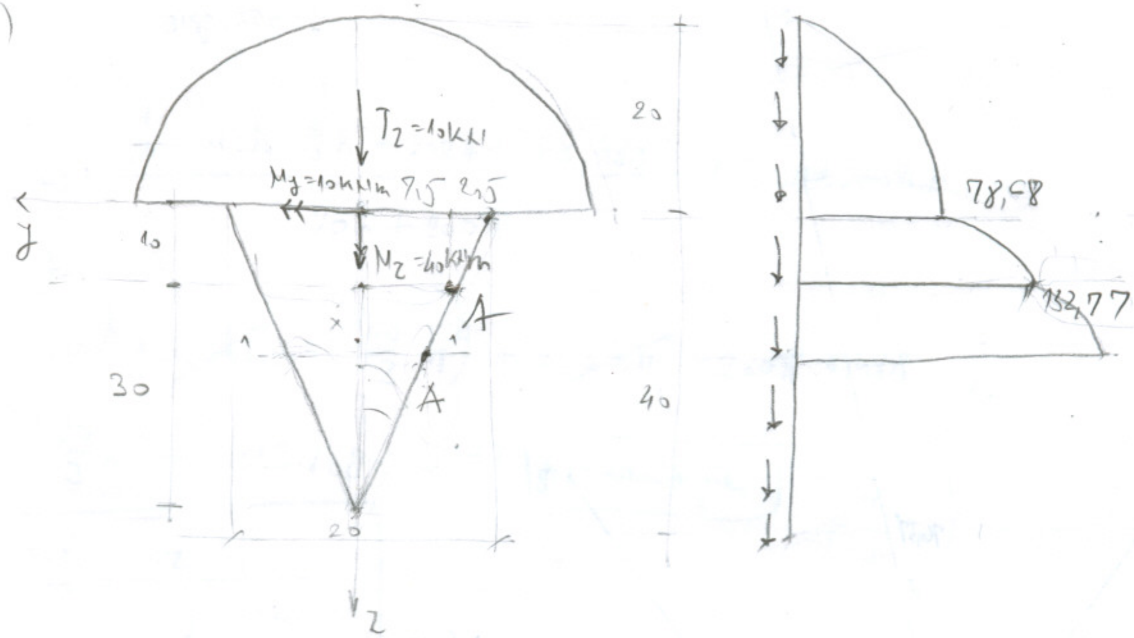
$\| $

h

3) $$10h^2 + \cancel{200h\pi} = \frac{20}{3} h^2 + \cancel{200h\pi} + 200\pi \cdot 8{,}488$$

$$\frac{10}{3} h^2 - 200\pi \cdot 8{,}488 = 0 \Rightarrow \boxed{h = 40\,cm}$$

b)



$$I_y = \frac{1}{36} \cdot 40^3 \cdot 20 + 13{,}33^2 \cdot \frac{1}{2} 40 \cdot 20 + 2 \cdot 0{,}05488 \cdot 20^4 + 8{,}488^2 \cdot 200\pi = \boxed{169399{,}24\,cm^4}$$

$$I_z = \frac{1}{36} \cdot 20^3 \cdot 40 + \frac{20^4 \pi}{8} = \boxed{69438{,}52\,cm^4}$$

$x_1 : 20 = 20 : 40$

$40 x_1 = 400$

$x_1 = 10$

c) $$\boxed{\tau_{xz}^{T_z} = \frac{T_z \cdot S_y^*}{I_y \cdot b(z)}}$$

$x : 30 = 20 : 40$

$40x = 600 \Rightarrow x = \frac{600}{40} = 15$

$$S_y^T = 200\pi \cdot 8{,}488 = \boxed{5333{,}17\,cm^3}$$

$$S_y^{1-1} = 10 \cdot 20 \frac{1}{2} \cdot 26{,}67 = \boxed{2667\,cm^3}$$

$$S_y^{POLA} = 15 \cdot 30 \frac{1}{2} \cdot (10 + 10) = \boxed{4500\,cm^3}$$

$$\tau_{xz}^{1-1} = \frac{-\|- \; 2667 \cdot 10^{-6}}{-\|- \; 10 \cdot 10^{-2}} = 157{,}38$$

$$\tau_{xz}^T = \frac{10 \cdot 10^3 N \cdot 5333{,}17 \cdot 10^{-6} m^3}{169460{,}64 \cdot 10^{-8} m^4 \cdot 40 \cdot 10^{-2} m} = \boxed{78{,}68\,kPa}$$

0,02667

0,0225

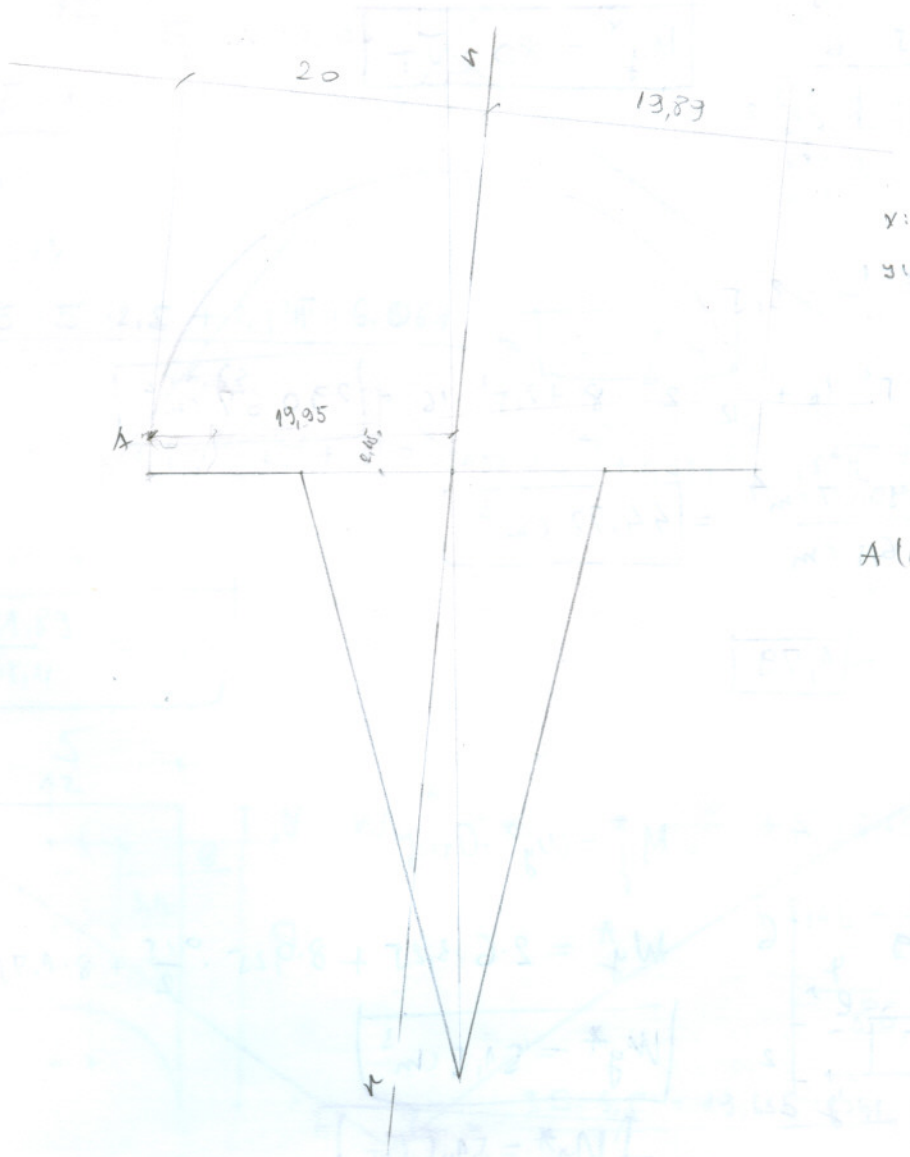
$$\tau_{xz}^{POLA} = \frac{-\|- \; \cdot 4500 \cdot 10^{-6}}{-\|- \; \cdot 20 \cdot 10^{-2}} = \boxed{132{,}77\,kPa}$$

d) $\sigma_x = -\frac{M_z}{I_z} y + \frac{M_y}{I_y} z$

$\frac{M_y}{I_y} z = \frac{M_z}{I_z} y$

$z = \frac{M_z}{I_z} \frac{I_y}{M_y} y = \frac{M_z}{M_y} \frac{I_y}{I_z} y = \frac{40}{10} \frac{16946,64}{71720,74} = 9,45\, y$

$\alpha = 83,96°$



x: 678,5930

y: 421,5845

A (19,95 ; -2,45)

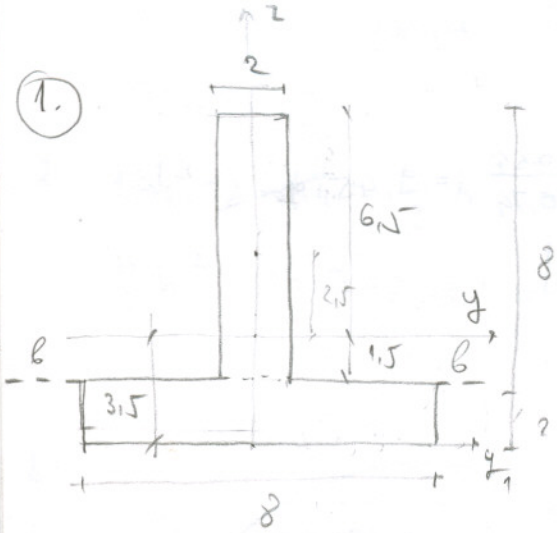
$\sigma_{x,A} = -\frac{40 \cdot 10^3 kNm}{71720,74 \cdot 10^{-8} m^4} \cdot 19,95 \cdot 10^{-2} m + \frac{10 \cdot 10^3 kNm}{16946 \cdot 10^{-8} m^4} (-2,45 \cdot 10^{-2}) = -11,27 MPa$

I_z I_y

A (13,86; -2,37) → $\sigma_{x,A} = -11,22 MPa$

ЕЛАСТО - ПЛАСТИЧНА ЗОНА ГРЕДНИ НОСАЧА

1.



$M_y^* = W_y^* \cdot \sigma_T$

$W_y^* = 2 \cdot 8 \cdot 4 + 2 \cdot 8 \cdot 1 = 80\ cm^3$

$M_y^* = 80 \cdot \sigma_T$

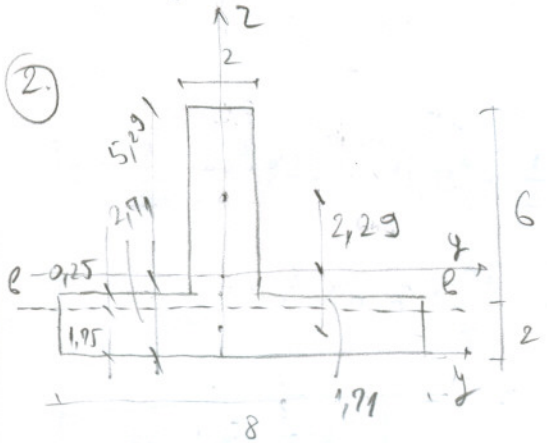
$z_T = \frac{2 \cdot 8 \cdot 6 + 2 \cdot 8 \cdot 1}{32} = 3,5$

$I_y = \frac{1}{12} \cdot 8^3 \cdot 2 + 2,5^2 \cdot 16 + \frac{1}{12} 2^3 \cdot 8 + 2,5^2 \cdot 16 = 290,67\ cm^4$

$W_y = \frac{I_y}{z_{max}} = \frac{290,67\ cm^4}{6,5\ cm} = 44,72\ cm^3$

$f = \frac{W_y^*}{W_y} = \frac{80}{44,72} = 1,79$

2.



$M_y^* = W_y^* \cdot \sigma_T$

$W_y^* = 2 \cdot 6 \cdot 3,25 + 8 \cdot 0,25 \cdot \frac{0,25}{2} + 8 \cdot 1,75 \cdot \frac{1,75}{2} =$

$W_y^* = 51,5\ cm^3$

$M_y^* = 51,5\ \sigma_T$

$A_1 = 12$

$A_2 = 16$

$A = 28$

$\frac{A}{2} = 14$

$8 \cdot x = 2 \Rightarrow x = 0,25$

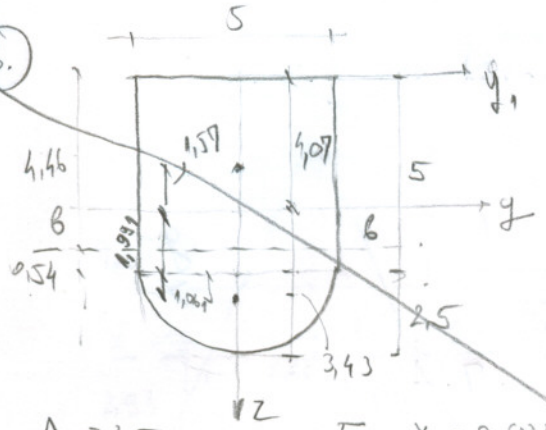
$z_T = \frac{2 \cdot 6 \cdot 5 + 2 \cdot 8 \cdot 1}{28} = 2,71\ cm$

$I_y = \frac{1}{12} \cdot 6^3 \cdot 2 + 2,29^2 \cdot 12 + \frac{1}{12} \cdot 2^3 \cdot 8 + 1,71^2 \cdot 16 = 151,048\ cm^4$

$W_y = \frac{I_y}{z_{max}} = \frac{151,048}{5,29} = 28,55\ cm^3$

$f = \frac{51,5}{28,55} = 1,8$

3.



$A_1 = 25$

$A_2 = 2{,}5^2\pi = 19{,}634$

$A = 44{,}63$

$\frac{A}{2} = 22{,}317$

$5 \cdot X = 2{,}683$

$X = 0{,}5366$

$A_1(0 ; 2{,}5)$

$A_2(0 ; 6{,}061)$

$z_T = \frac{5 \cdot 5 \cdot 2{,}5 + 2{,}5^2\pi \cdot 6{,}061}{44{,}63} = 4{,}07\ cm$

$I_y = \frac{1}{12}5^4 + 1{,}57^2 \cdot 25 + 2 \cdot 0{,}05488 \cdot 2{,}5^4 + 1{,}391^2 \cdot 2{,}5^2\pi = 135{,}83\ cm^4$

$W_y = 48{,}11\ cm^3$

$M_y = W_y^* \cdot \sigma_T$

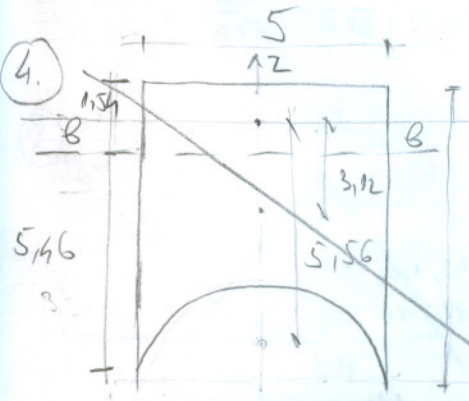
$W_y^* = 5 \cdot 4{,}46 \cdot \frac{4{,}46}{2} + 5 \cdot 0{,}54 \cdot \frac{0{,}54}{2} + 2{,}5^2\pi \cdot \left(\frac{4}{3}\frac{2{,}5}{\pi} + 0{,}54\right)$

$W_y^* = 81{,}89\ cm^3$

$f = \frac{81{,}89}{48{,}11} = 1{,}70$

80,458

4.



$A_1 = 35$

$A_2 = 2{,}5^2\pi = 19{,}635$

$A = A_1 - A_2 = 15{,}365\ cm^2$

$\frac{A}{2} = 7{,}6825$

$5 \cdot X = 7{,}6825$

$X = 1{,}54$

$W_y^* = 5 \cdot 1{,}54 \cdot \frac{1{,}54}{2} + 5 \cdot 5{,}46 \frac{5{,}46}{2} - 2{,}5^2\pi \cdot \left(5{,}46 - \frac{4}{3}\frac{2{,}5}{\pi}\right) = -5{,}916\ cm^3$

86,37

$z_T = \frac{35 \cdot 3{,}5 - 19{,}635 \cdot 1{,}061}{15{,}365} = 6{,}62\ cm$

$I_y = \frac{1}{12} \cdot 7^3 \cdot 5 + 3{,}12^2 \cdot 35 - \left(2 \cdot 0{,}05488 \cdot 2{,}5^4 + 5{,}56^2 \cdot 19{,}635\right) =$

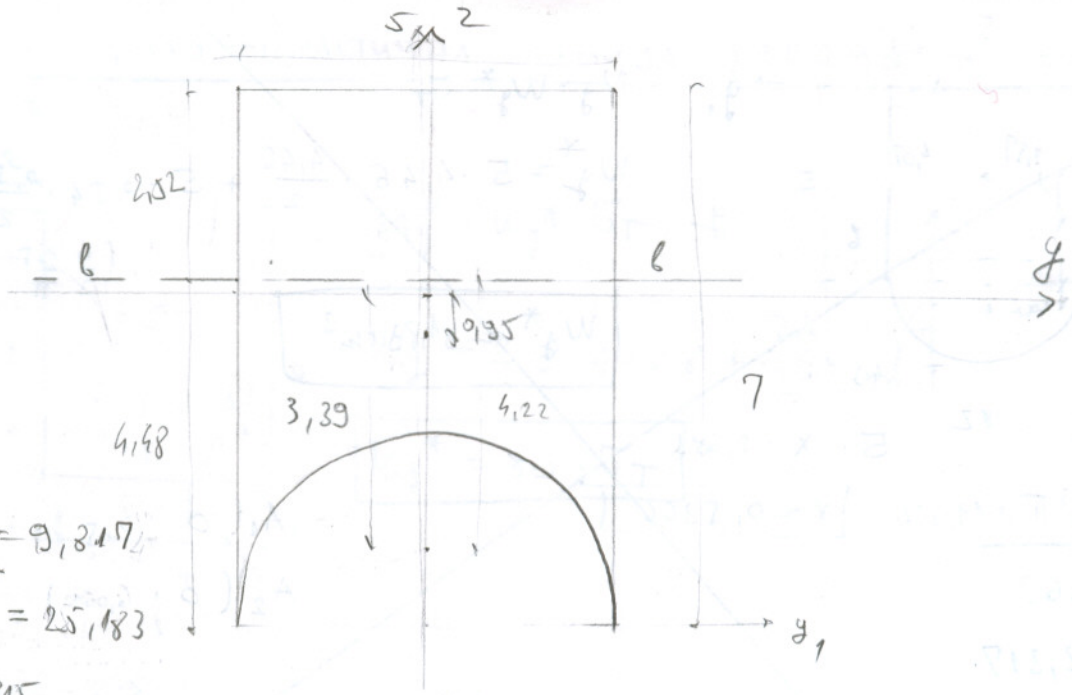
$I_y = -127{,}66$

3,08

x: 573,9953

y: 585,0862

4.



$A_1 = 35$

$A_2 = 2,5^2 \pi = 9,817$

$A = A_1 - A_2 = 25,183$

$\frac{A}{2} = 12,5915$

$5 \cdot x = 12,5915$

$x = 2,52$

$W_y^* = 5 \cdot 2,52 \cdot \frac{2,52}{2} + 5 \cdot 4,48 \cdot \frac{4,48}{2} - 9,817 \cdot 4,22 = 14,59\ cm^3$

$y = \frac{35 \cdot 3,5 - 9,817 \cdot 1,061}{25,183} = 4,45\ cm$

$I_y = \frac{1}{12} 7^3 \cdot 5 + 0,95^2 \cdot 35 - (2 \cdot 0,05488 + 3,39^2 \cdot 9,817) = 57,40\ cm^4$

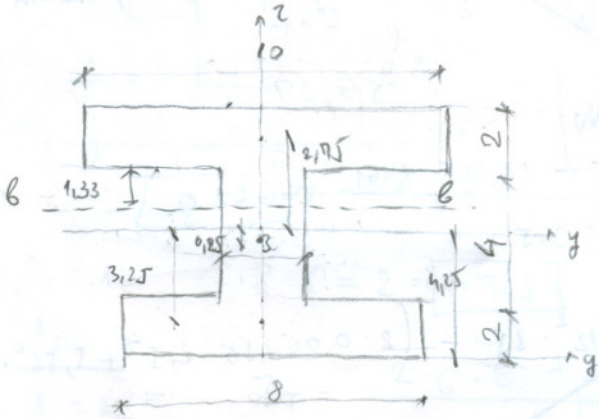
$W_y = \frac{57,40}{4,48} = 12,81\ cm^3$

x: 620,1581

y: 745,6020

$f = \frac{14,59}{12,81} = 1,139$

5.



x: 545,1046

y: 687,2058

$W_y^* = W_y^* \cdot G_T$

$W_y^* = 10 \cdot 2 + 2,33 + 1,33 \cdot 3 \cdot \frac{1,33}{2} + 3 \cdot 2,67 \frac{2,67}{2} + 8 \cdot 2 \cdot 3,67 \Rightarrow W_y^* = 118,67\ cm^3$

$z_T = \frac{16 \cdot 1 + 12 \cdot 4 + 20 \cdot 7}{48} = 4,25\ cm$

$A_1 = 10 \cdot 2 = 20\ cm^2$

$A_2 = 3 \cdot 4 = 12\ cm^2$

$A_3 = 8 \cdot 2 = 16\ cm^2$

$A = 48\ cm^2$

$\frac{A}{2} = 24\ cm^2$

$x \cdot 3 = 4$

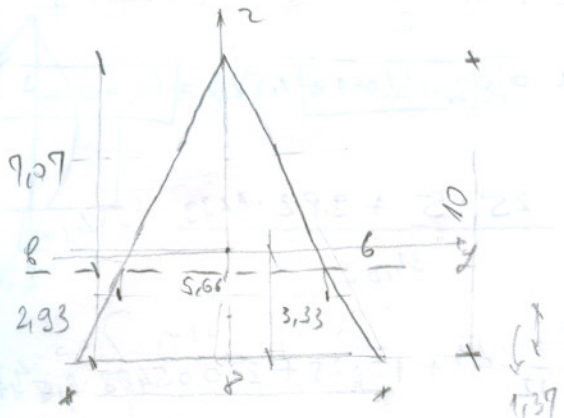
$x = 1,33$

$I_y = \frac{1}{12} 2^3 \cdot 10 + 2,75^2 \cdot 20 + \frac{1}{12} \cdot 4^3 \cdot 3 + 0,25^2 \cdot 12 + \frac{1}{12} \cdot 2^3 \cdot 8 + 3,25^2 \cdot 16 = 349\ cm^4$

$W_y = \frac{349}{4,25} = 82,12\ cm^3 \Rightarrow f = \frac{118,67}{82,12} = 1,445$

Težišče trikotnika

6.



$A = \frac{8 \cdot 10}{2} = 40\,cm^2$

$\frac{A}{2} = 20\,cm^2$

$\frac{a \cdot h}{2} = 20$

$a \cdot h = 40$

$a \cdot \frac{10}{8} a = 40 / \cdot 8$

$10a^2 = 320$

$a^2 = 32 \Rightarrow \boxed{a = 4\sqrt{2} = 5,66}$

$h = \frac{40}{4\sqrt{2}} = 5\sqrt{2} = \boxed{7,07\,cm}$

$8 : 10 = a : h$

$8h = 10a$

$h = \frac{10}{8} a$

$W_y^* = W_y^* \cdot \sigma_T$

$W_y^* = \frac{7,07 \cdot 5,66}{2} \cdot \frac{7,07}{3} + \frac{8 + 5,66}{2} \cdot 2,93$

$\boxed{W_y^* = 74,77\,cm^3}$

$2,93 \cdot a = 1,38 \Rightarrow a =$

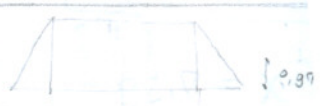
$2,62 \cdot a = 1,22 \Rightarrow a =$

$I_y = \frac{1}{36} \cdot 10^3 \cdot 8 = \underline{222,22\ cm^4}$

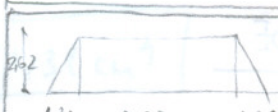
$W_y = \frac{222,22}{6,67} = \boxed{33,32\,cm^3}$

$\boxed{f = \frac{W_y^*}{W_y} = \frac{74,77}{33,32} = 2,24}$

Težišče trikotnika



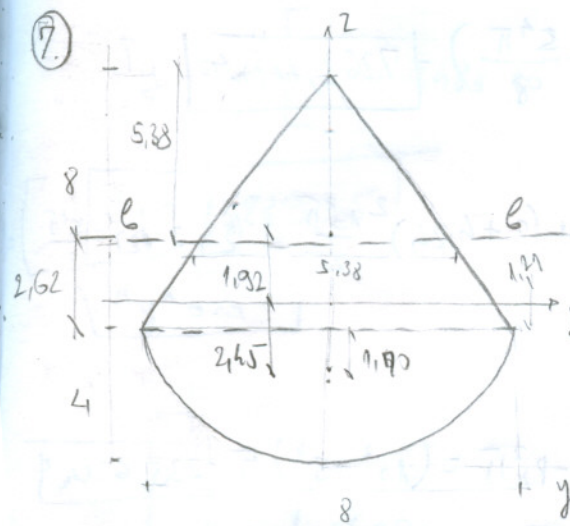
$z_T = \frac{5,66 \cdot 2,93 \cdot \frac{2,93}{2} + 2\,\frac{2,93 \cdot 1,17}{2} \cdot 0,39}{20,0119} = \boxed{1,38}$



$z_T = \frac{5,38 \cdot 2,62 \cdot \frac{2,62}{2} + 2\,\frac{2,62 \cdot 1,31}{2} \cdot 0,8}{17,5278}$

$\boxed{z_T = 1,22\,cm}$

7.



$A_1 = \frac{8 \cdot 8}{2} = 32\,cm^2$

$A_2 = \frac{4^2 \pi}{2} = 25,13\,cm^2$

$A = 57,13$

$\boxed{\frac{A}{2} = 28,57\,cm^2}$

$8 : 8 = a : h$

$\frac{a}{h} = 1$

$\boxed{a = h}$

$\frac{a \cdot h}{2} = 3,43$

$\frac{a^2}{2} = 3,43$

$\boxed{a = 2,62}$

$W_y^* = \frac{5,38 \cdot 5,38}{2} \cdot \frac{5,38}{3} + \frac{8 + 5,38}{2} \cdot 2,62 \cdot 1,$

$+ 25,13 \cdot 4,32$

$\boxed{W_y^* = 155,89\,cm^3}$

$z_T = \frac{32 \cdot 6,67 + 25,13 \cdot 2,\overline{3}}{57,13} = \boxed{4,75\,cm}$

$I_y = \frac{1}{36} \cdot 8^3 \cdot 8 + 1,92^2 \cdot 32 + 2 \cdot 0,05488 \cdot 4^4 + 2,45^2 \cdot 25,13 = \boxed{410,68\,cm^4}$

$W_y = \frac{410,68}{7,25} = 56,64$

$\boxed{f = \frac{155,89}{56,64} = 2,75}$

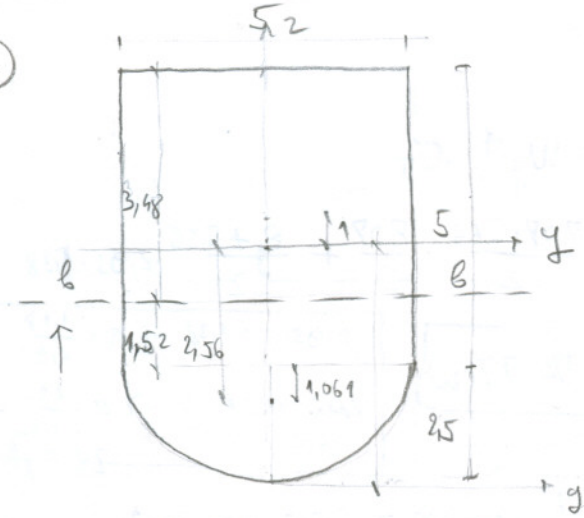
x: 1279,5957 3398070932,2316

y: 2415,7548 899465514,2506

x: 582,1663

y: 436,6629

3.



$$W_y^* = 5 \cdot 3{,}48 \frac{3{,}48}{2} + 5 \cdot 1{,}52 + \frac{1{,}52^2}{2} + $$

$$+ 9{,}82 \cdot (1{,}061 + 1{,}52) = \boxed{61{,}40\,cm^3}$$

$$z_T = \frac{25 \cdot 5 + 9{,}82 \cdot 1{,}439}{34{,}82} = \boxed{4\,cm}$$

$$I_y = \frac{1}{12} \cdot 5^4 + 1^2 \cdot 25 + 2 \cdot 0{,}05488 \cdot 2{,}5^4 + 2{,}56^2 \cdot 9{,}82$$

$$\boxed{I_y = 145{,}73\,cm^4}$$

$A_1 = 25\,cm^2$

$A_2 = \frac{2{,}5^2 \pi}{2} = 9{,}82\,cm^2$

$\boxed{A = 34{,}82\,cm^2}$

$\boxed{\frac{A}{2} = 17{,}41\,cm^2}$

$$W_y = \frac{145{,}73}{4} = \boxed{36{,}43\,cm^3}$$

$$\boxed{f = \frac{61{,}40}{36{,}43} = 1{,}685}$$

$5 \cdot x = 7{,}59$

$x = 1{,}518$

ИЗВИЈАЊЕ

1.

a) $I_y = 2\left(\frac{R_1^4 \pi}{8} - \frac{R_2^4 \pi}{8}\right) = 2\left(\frac{10^4 \pi}{8} - \frac{5^4 \pi}{8}\right) = \boxed{7363{,}11\,cm^4}$

$$I_z = 2\left[2 \cdot 0{,}05488 R_1^4 - 2 \cdot 0{,}05488 \cdot R_2^4 + (2+4{,}24)^2 \frac{10^2 \pi}{2} - (2+2{,}12)^2 \frac{5^2 \pi}{2}\right]$$

(R_1^4: 10^4; R_2^4: 5^4)

$\boxed{I_z = 12957{,}44\,cm^4}$

"y" osa

$A = R_1^2 \pi - R_2^2 \pi = (10^2 - 5^2)\pi = \boxed{235{,}62\,cm^4}$

$l_{yi} = 300\,cm$

$i_y = \sqrt{\frac{7363{,}11}{235{,}62}} = \boxed{5{,}59\,cm}$

$i_z = \sqrt{\frac{12957{,}44}{235{,}62}} = \boxed{7{,}42\,cm}$

$\lambda_y = \frac{l_{yi}}{i_y}$

$\boxed{\lambda_y = \frac{300}{5{,}59} = 53{,}67}$

"z" oca

$0,7 \cdot 7 l = \boxed{424,2\ cm}$ $\quad \lambda_z = \frac{l_{zi}}{i_z} = \frac{424,2}{7,42} = \boxed{57,17}$ = λ_{max}

$$\sigma_{kR\,min} = \begin{cases} \sigma^{EL} = \Pi^2 \frac{E}{\lambda^2} = \Pi^2 \frac{210 \cdot 10^9}{57,17^2} = \boxed{634,14\ MPa} \\ \sigma^T = 310 - 1,12\,\lambda\ [MPa] = 310 - 1,12 \cdot 57,17 = 200,24\ MPa \\ \sigma^T = 240\ MPa \end{cases}$$

$P_{kR} = \sigma_{kR} \cdot A = 634,14 \cdot 235,62 = \underline{14941,61\ kN}$

$$I_z = \frac{R_1^4 \Pi}{4} - \frac{R_2^4 \Pi}{4} + 2(2+4,24)^2 \frac{10^2 \Pi}{2} - 2\left((2+2,12)^2 \frac{5^2 \Pi}{2}\right)$$

2.

$$I_z = \frac{10^4 \Pi}{4} - \frac{1}{12} 4^3 \cdot 8 = \boxed{7811,31\ cm^4}$$

$$I_y = 2\left[2 \cdot 0,05488\, 10^4 + \left(2 + \frac{4}{3}\frac{10}{\Pi}\right)^2 \frac{16^2 \Pi}{2} - \left(\frac{1}{12} 4^4 + 4^2 \cdot 16\right)\right]$$

$\boxed{I_y = 13889,35}$ $\qquad A = 10^2 \Pi - 2 \cdot 16 = \boxed{282,16\ cm^2}$

"y" oca

$$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{13889,35}{282,16}} = \boxed{7,016\ cm}$$

$l = 600\ cm$ $\qquad i_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{7811,31}{282,16}} = \boxed{5,26\ cm}$

$$\lambda_y = \frac{l_y}{i_y} = \frac{600}{7,016} = \boxed{85,52}$$

"z" oz

$\lambda_z = \frac{l_{iz}}{i_z} = \frac{424,2}{5,26} = 80,64$

$0,707 l = 0,707 \cdot 600 = 424,2\,cm$

$\lambda_{max} = \lambda_y = 85,52$

$$\sigma_{kr,min} = \begin{cases} \pi^2 \frac{E}{\lambda^2} = \pi^2 \frac{210 \cdot 10^9}{85,52^2} = 283,39\,MPa \\ \sigma^{TET} = 310 - 1,92\lambda = 145,80\,MPa \\ \sigma^T = 240 \end{cases}$$

$\sigma_{min} = 145,80\,MPa$

$P_{KR} = 145,80 \cdot 10^6 \cdot 282,16 \cdot 10^{-4} = 4113,89\,kN$

(2.)(1.)d) $I_y = \frac{1}{12} b h^3$

$I_z = \frac{1}{12} b^3 h$

$i_y = \sqrt{\frac{\frac{1}{12} b h^3}{bh}} = \frac{1}{\sqrt{12}} h = 0,289\,h$

$l_i = \frac{d}{2}$

$\lambda = \frac{\frac{d}{2}}{0,289\,h} = 1,73 \frac{d}{h}$

$$\sigma_{KR} = \begin{cases} \pi^2 \frac{E}{\lambda^2} = \end{cases}$$

$I_y = \frac{1}{12} \cdot 10^3 \cdot 5 = \boxed{416{,}67\ cm^4}$ $\quad i_y =$

$I_z = \frac{1}{12} \cdot 5^3 \cdot 10 = \boxed{104{,}17\ cm^4}$

$i_z = \sqrt{\frac{104{,}17}{50}} = \boxed{1{,}44\ cm}$

$l_{iz} = 300\ cm, \quad \lambda = \frac{300}{1{,}44} = 208{,}33$

$\sigma_{kr} = \begin{cases} \pi^2 \frac{E}{\lambda^2} = \pi^2 \frac{32 \cdot 10^9}{208{,}33^2} = 7{,}28\ MPa \\ 37 - 0{,}12\lambda = 37 - 0{,}12 \cdot 208{,}33 = 12\ MPa \\ \sigma^T = 32\ MPa \end{cases}$

$P_{kr} = 7{,}28 \cdot 10^6 \cdot 50 \cdot 10^{-4} = \boxed{3640\ kN}$

2. a) $I_y = \frac{1}{36} 17^3 \cdot 12 = \boxed{1639{,}67\ cm^4}$ $\quad \boxed{A = \frac{17 \cdot 12}{2} = 102\ cm^2}$

$I_z = \frac{1}{48} \cdot 12^3 \cdot 17 = \boxed{612\ cm^4}$

$I_{zz} = I_{min}$

$i_z = \sqrt{\frac{612}{102}} = \boxed{2{,}45\ cm}$ $\quad l_{iz} = 0{,}707 \cdot 600 = 424{,}2\ cm$

$\lambda = \frac{424{,}2}{2{,}45} = 173{,}14$

$\sigma_{kr,min} = \begin{cases} \pi^2 \frac{32 \cdot 10^9}{173{,}14^2} = \boxed{10{,}54\ MPa} \\ 37 - 0{,}12 \cdot 173{,}14 = 16{,}22\ MPa \\ 32\ MPa \end{cases}$

$P_{kr} = 10{,}54 \cdot 10^6 \cdot 102 \cdot 10^{-4} = \boxed{107{,}508\ kN}$

δ) $P = 700\,kN$

$n = 2$

$\sigma_{kr} = \frac{P_{kr}}{A} = \frac{1400\,kN}{102\,cm^2} \cdot 10 = \boxed{137,25\,MPa}$

$P_{dopus} = \frac{1}{2} P_{kr}$

$\sigma^{EL} = \pi^2 \frac{E}{\lambda^2} \Rightarrow \lambda^2 = \frac{\pi^2 E}{\sigma^{EL}}$

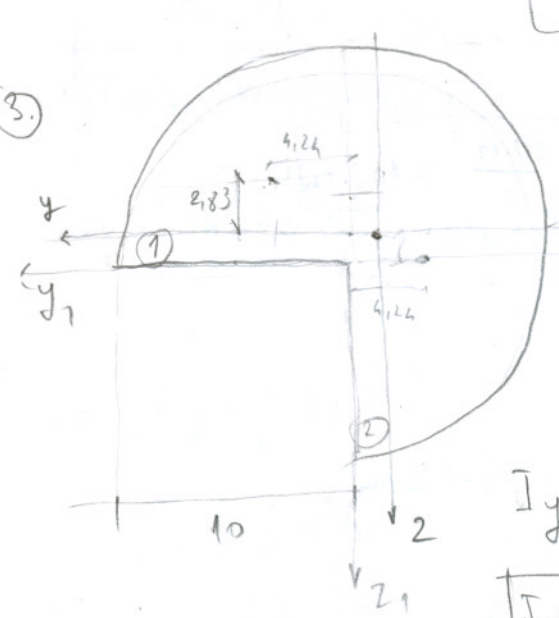
$\boxed{P_{kr} = 1400\,kN}$

$\lambda = \sqrt{\frac{\pi^2 E}{\sigma^{EL}}} = \sqrt{\frac{\pi^2 \cdot 32 \cdot 10^9 \frac{N}{m^2}}{137,25 \cdot 10^6 \frac{N}{m^2}}} = \boxed{47,97}$

$\lambda = \frac{l_i}{i_{min}} \Rightarrow \boxed{l_i = 47,97 \cdot 2,15 = 117,52\,cm}$

$0,707 \cdot l = 117,52 \Rightarrow \boxed{l = \frac{117,52}{0,707} = 166,23\,cm}$

3.



$z_T = \frac{-4,24 \cdot 78,54}{235,62} = \boxed{-1,41\,cm}$

$y_T = \frac{4,24 \cdot 78,54 - 4,24 \cdot 157,08}{235,62} = \boxed{-1,41}$

$I_y = 0,05488 \cdot 10^4 + 2,83^2 \cdot 78,54 + \frac{10^4 \pi}{8} + 1,41^2 \cdot 157,08$

$\boxed{I_y = 5417,10\,cm^4}$

$A_1 = \frac{10^2 \pi}{4} = 78,54$

$A_2 = \frac{10^2 \pi}{2} = 157,08$

$A_1 (4,24 ; -4,24)$ (y_1 ; z_1)

$A_2 (-4,24 ; 0)$

$A_1 (5,65 ; -2,83)$ (y ; z)

$A_2 (-2,83 ; 1,41)$

$I_z = 0,05488 \cdot 10^4 + 5,65^2 \cdot 78,54 + 2 \cdot 0,05488 \cdot 10^4 + 1,41^2 \cdot 157,08$

$\boxed{I_z = 5411,63\,cm^4}$

$I_{yz} = 0,01647 \cdot 10^4 + (-2,83) \cdot 5,65 \cdot 78,54 + 1,41 \cdot (-2,83) \cdot 157,08$

$\boxed{I_{yz} = -1717,91\,cm^4}$

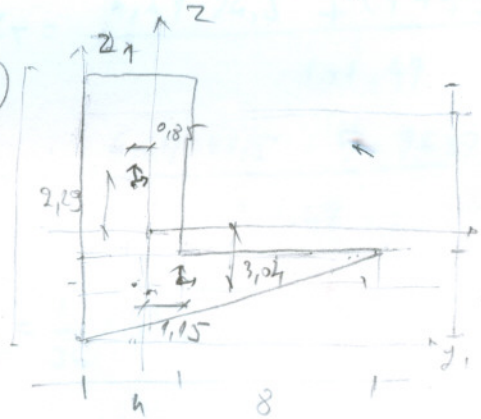
$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} \Rightarrow \boxed{I_2 = I_{min} = 3696,45}$

$$i_{min} = \sqrt{\frac{I_{min}}{A}} = \boxed{3,96\ cm}$$

$$\lambda = \frac{l_i}{i_{min}} = \frac{600}{3,96} = \boxed{151,52}$$

$$\sigma_{kR,min} = \begin{cases} \sigma_{el} = \dfrac{\pi^2 E}{\lambda^2} \\ 37 - 0,12\lambda \\ 32\ MPa \end{cases}$$

4.



$A_1 = \frac{12 \cdot 4}{2} = 24\ cm^2$ $\quad A_1(4 ; 2,67)$

$A_2 = 32\ cm^2$ $\quad A_2(2 ; 6)$

$\boxed{A = 56\ cm^2}$

$$y_T = \frac{4 \cdot 24 + 2 \cdot 32}{56} = \boxed{2,85\ cm}$$

$$z_T = \frac{2,67 \cdot 24 + 8 \cdot 32}{56} = \boxed{5,71\ cm}$$

$$I_1 = \frac{1}{36} 4^3 \cdot 12 + 3,04^2 \cdot 24 + \frac{1}{12} \cdot 8^3 \cdot 4 + 2,29^2 \cdot 32 = \boxed{581,61\ cm^4}$$

$$I_2 = \frac{1}{36} \cdot 12^3 \cdot 4 + 1,15^2 \cdot 24 + \frac{1}{12} \cdot 4^3 \cdot 8 + 0,85^2 \cdot 32 = \boxed{289,35\ cm^4}$$

$$I_y = \frac{1}{72} \cdot 12^2 \cdot 4^2 + 1,15(-3,04) \cdot 24 + 2,29 \cdot (-0,85) \cdot 32 = \boxed{-114,172\ cm^4}$$

$\boxed{I_2 = I_{min} = 250,02\ cm^4}$

$$i = \sqrt{\frac{250,02}{56}} = 2,11\ cm$$

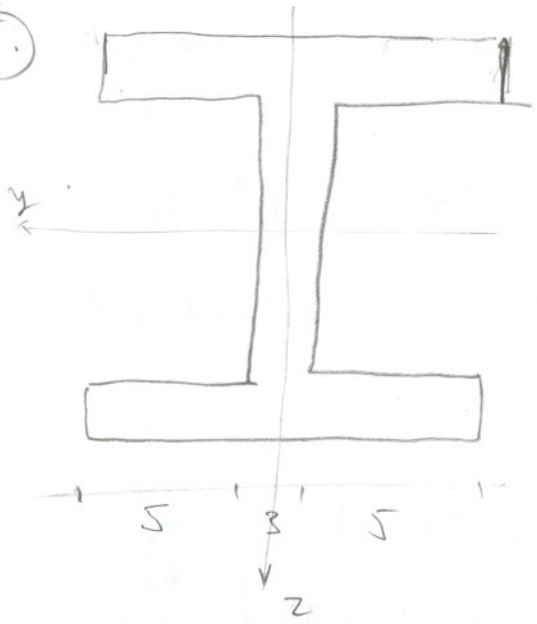
$l_i = 1200\ cm$

$$\lambda = \frac{1200}{2,11} = \boxed{568,72}$$

$$\sigma_{k,min} = \begin{cases} \dfrac{\pi^2 \cdot 32 \cdot 10^9}{568,72^2} = \sigma^{EL} = 0,996\ MPa \\ 37 - 0,12 \cdot 568,72 = \ominus \end{cases}$$

$32\ MPa$

5.



$I_y = \frac{1}{12}(21^3 \cdot 13) - 2\left(\frac{1}{12} 15^3 \cdot 5\right)$

$\boxed{I_y = 7220{,}25\ cm^4}$

$I_z = \frac{1}{12} \cdot 3^3 \cdot 15 + 2\left(\frac{1}{12} \cdot 5 \cdot 13^3 \cdot 3\right) =$

$\boxed{I_z = 1132{,}25\ cm^4}$

$l = 300\ cm$

$i_z = \sqrt{\frac{1132{,}25}{123}} = \boxed{3{,}03\ cm}$

$\lambda = \frac{300}{3{,}03} = 99$

$\sigma_{kr,min} = \begin{cases} \sigma^{El} = \pi^2 \frac{32 \cdot 10^9}{99^2} = 32{,}22\ MPa \\ 37 - 0{,}12 \cdot \lambda = 37 - 0{,}12 \cdot 99 = \boxed{25{,}12\ MPa} \\ 32\ MPa \end{cases}$

$P_{kr} = 25{,}12 \cdot 10^6 \cdot 123 \cdot 10^{-4} = \boxed{309{,}345\ kN}$

6.

$I_y = I_z = \frac{15^4 \pi}{2} - \frac{5^4 \pi}{2} = \boxed{14726{,}21\ cm^4}$

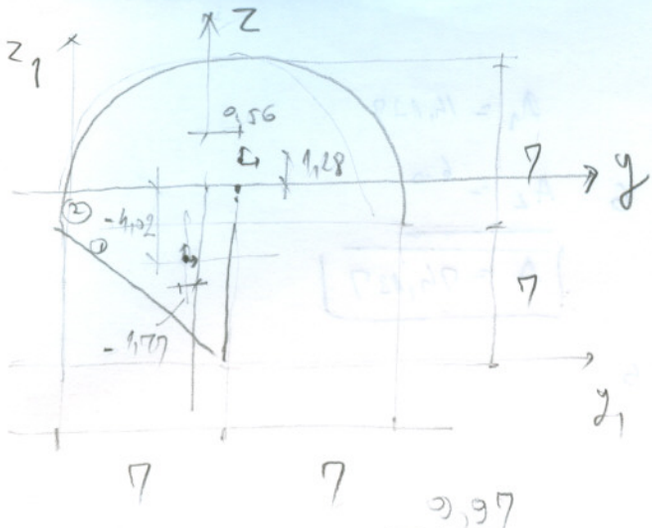
$\boxed{A = 235{,}62\ cm^2}$

$i_{min} = \sqrt{\frac{14726{,}21}{235{,}62}} = 7{,}91\ cm$

$\lambda = \frac{0{,}7 \cdot 600}{7{,}91} = 53{,}1$

$\sigma_{kr,min} = \begin{cases} \sigma^{El} = \pi^2 \frac{32 \cdot 10^9}{53{,}1^2} = 112{,}01\ MPa \\ \sigma^{T} = 37 - 0{,}12 \cdot 53{,}1 = \boxed{30{,}63\ MPa} \\ 32\ MPa = \end{cases}$

$P_{kr} = 32{,}63 \cdot 10^6 \cdot 235{,}62 \cdot 10^{-4} = \boxed{721{,}90\ kN}$

9. 

$A_1 = 24,5\ cm^2$

$A_2 = 76,97\ cm^2$

$A = 101,47\ cm^2$

$$z_T = \frac{4,67 \cdot 24,5 + (7 + \underbrace{2,97}) \cdot 76,97}{101,47} = 8,69\ cm$$

$$y_T = \frac{4,67 \cdot 24,5 + 7 \cdot 76,97}{101,47} = 6,44\ cm$$

$$I_y = \frac{1}{36} \cdot 7^4 + 4,02^2 \cdot 24,5 + 2 \cdot 0,05488 \cdot 7^4 + 1,28^2 \cdot 76,97 = 852,27\ cm^4$$

$$I_z = \frac{1}{36} 7^4 + 1,77^2 \cdot 24,5 + \frac{7^4 \pi}{8} + 0,56^2 \cdot 76,97 = 1110,46\ cm^4$$

$$I_{yz} = -\frac{1}{72} 7^4 + (-4,02)(-1,77) \cdot 24,5 + 0,56 \cdot 1,28 \cdot 76,97 = 199,10\ cm^4$$

$I_{min} = 718,66\ cm^4$

$$i_{min} = \sqrt{\frac{718,66}{101,47}} = 2,66\ cm$$

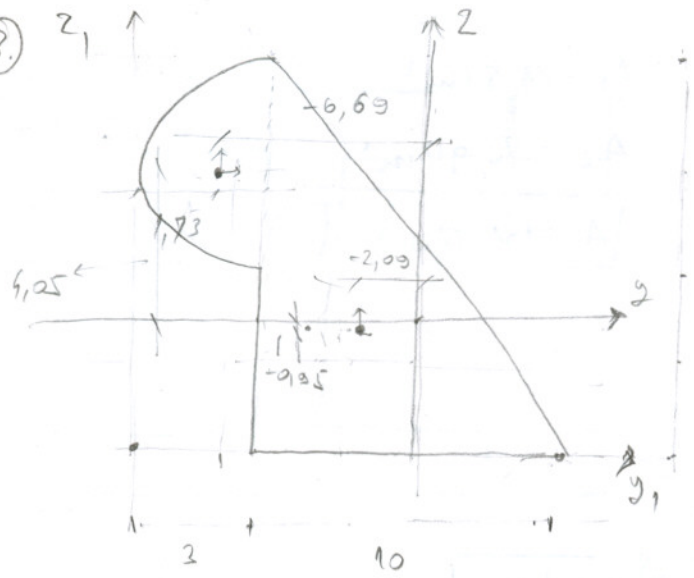
$$\lambda = \frac{600}{2,66} = 225,56$$

$$\sigma_{kR,min} = \begin{cases} \sigma^{EL} = \pi^2 \frac{32 \cdot 10^3}{225,56^2} = 6,20\ MPa \\ \sigma^T = 37 - 0,2 \cdot 225,56 = 3,93\ MPa \end{cases}$$

$32\ MPa$

$$P_{kR} = 6,20 \cdot 10^6 \cdot 101,47 \cdot 10^{-4} = 62,911\ kN$$

(8.)



$A_1 = 14,137$

$A_2 = 60$

$A = 74,137$

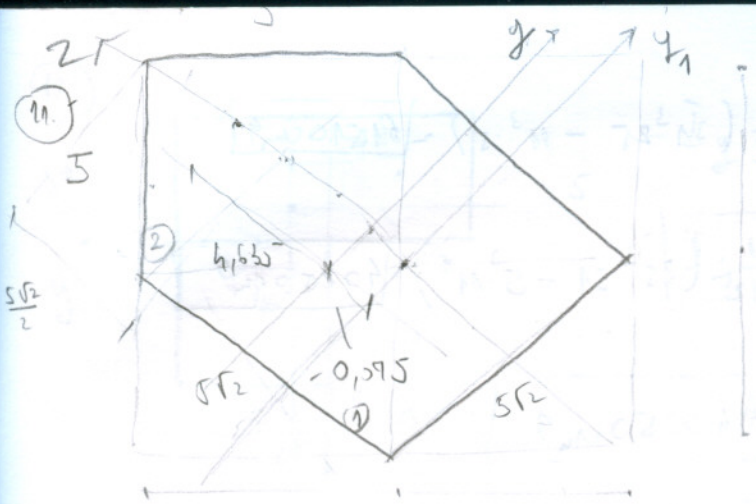
$$z_T = \frac{9 \cdot 14,137 + 4 \cdot 60}{74,137} = 4,95\ cm$$

$$y_T = \frac{1,73 \cdot 14,137 + \underbrace{(3 + \frac{10}{3})}_{6,33} \cdot 60}{74,137} = 5,43\ cm$$

$$I_y = \frac{3^4 \pi}{8} + 4,05^2 \cdot 14,137 + \frac{1}{36} \cdot 12^3 \cdot 10 + 0,95^2 \cdot 60 = 797,84\ cm^4$$

$$I_z = 2 \cdot 0,05488 \cdot 3^4 + 6,69^2 \cdot 14,137 + \frac{1}{36} \cdot 10^3 \cdot 12 + 2,09^2 \cdot 60 = 1237,03\ cm^4$$

$$I_{yz} = \frac{4,05}{1,73} \cdot (-6,69) \cdot 14,137 - \frac{1}{72} \cdot 10^2 \cdot 12^2 + (-0,95)(-2,09) \cdot 60 = -635,38\ cm^4$$



$A_1 = 5\sqrt{2} \cdot 5\sqrt{2} = \boxed{50\,cm^2}$

$\boxed{A_2 = 12,5\,cm^2}$

$\boxed{A = 62,5\,cm^2}$

$$z_T = \frac{\frac{5\sqrt{2}}{2} \cdot \frac{1}{3} + \frac{5\sqrt{2}}{2}}{62,5} = 4,871 = \boxed{0,075\,cm}$$

$$I_z = \frac{1}{12} \cdot (5\sqrt{2})^4 + \frac{1}{48} \cdot (5\sqrt{2})^3 \cdot \frac{5\sqrt{2}}{2} = \boxed{234,375\,cm^4} \quad \boxed{I_{min}}$$

$$I_y = \frac{1}{12} (5\sqrt{2})^4 + 0,375^2 \cdot 50 + \frac{1}{36} \cdot \left(\frac{5\sqrt{2}}{2}\right)^3 \cdot 5\sqrt{2} + 4,635^2 \cdot 12,5 = \boxed{483,835\,cm^4}$$

$$i_{min} = \sqrt{\frac{234,375}{62,5}} = \boxed{1,94\,cm}$$

$$\lambda = \frac{l_i}{i_{min}} = \frac{300}{1,94} = \boxed{154,64}$$

$$\sigma_{KR,min} = \begin{cases} \sigma_{EL} = \pi^2 \frac{E}{\lambda^2} = \pi^2 \frac{32 \cdot 10^3}{154,64^2} = \end{cases}$$

x: 426,5790

y: 428,2496

I_x: 336,8056

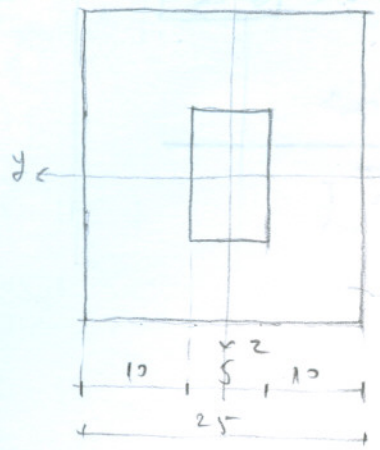
I_y: 336,8051

I_{xy} = -132,4306

$\underline{I}$: 234,3750

J: 439,2361

9.



$I_y = \frac{1}{12}(31^3 25 - 11^3 \cdot 5) = 61510\,cm^4$

$I_z = \frac{1}{12}(25^3 31 - 5^3 \cdot 11^3) = 40250\,cm^4$

$I_{min} = 40250\,cm^4$

$A = 31 \cdot 25 - 11 \cdot 5 = 720\,cm^2$

$i_{min} = \sqrt{\frac{40250}{720}} = 7{,}48\,cm$

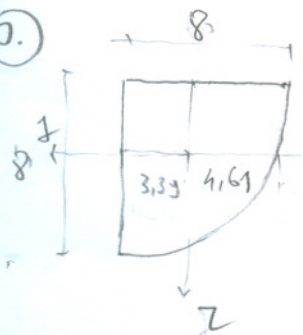
$l_i = 0{,}7 \cdot 600 = 420\,cm$

$\lambda = \frac{420}{7{,}48} = 56{,}15$

$\sigma_{KR,min} = \begin{cases} \sigma^{EL} = \pi^2 \frac{32 \cdot 10^3}{56{,}15^2} = 100{,}17\,MPa \\ \sigma^{T} = 37 - 0{,}12 \cdot 56{,}15 = 30{,}262\,MPa \\ 32\,MPa \end{cases}$

$P_{KR} = 32 \cdot 10^6 \cdot 720 \cdot 10^{-4} = 2{,}304\,kN$

10.



$I_y = I_z = 0{,}05488 \cdot 8^4 = 224{,}788\,cm^4$

$I_{yz} = 0{,}01649 \cdot 8^4 = 67{,}46\,cm^4$

$A = \frac{8^2 \pi}{4} = 50{,}265\,cm^4$

$I_2 = 157{,}328 = I_{min}$

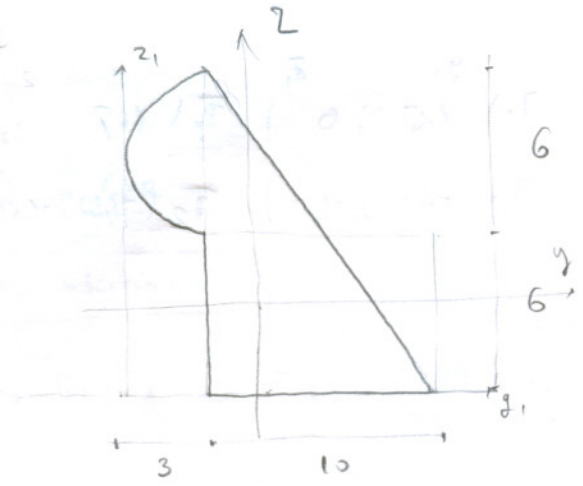
$i_{min} = \sqrt{\frac{157{,}328}{50{,}265}} = 1{,}77\,cm$

$\lambda = \frac{600}{1{,}77} = 338{,}98$

$\sigma_{kr} = \begin{cases} 2{,}74\,MPa \\ - \\ 32\,MPa \end{cases}$

$P_{KR} = 13{,}772\,kN$

8.



$A_1 = \frac{3^2\pi}{2} = 14,137\,cm^2$

$A_2 = \frac{10 \cdot 12}{2} = 60\,cm^2$

$A = 74,137\,cm^2$

	y_1	z_1		y	z
T_1	1,73	9	T_1	−3,72	4,05
T_2	6,33	4	T_2	0,88	−0,95

$$y_T = \frac{1,73 \cdot 14,137 + 6,33 \cdot 60}{74,137} = 5,45\,cm$$

$$z_T = \frac{9 \cdot 14,137 + 4 \cdot 60}{74,137} = 4,95$$

$$I_y = \frac{3^4\pi}{8} + 4,05^2 \cdot 14,137 + \frac{1}{36} \cdot 12^3 \cdot 10 + 0,95^2 \cdot 60 = 237,84$$

$$I_z = 2 \cdot 0,05488 \cdot 3^4 + 3,72^2 \cdot 14,137 + \frac{1}{36} \cdot 10^3 \cdot 12 + 0,88^2 \cdot 60 = 584,32\,cm^4$$

$$I_{yz} = (-3,72)\,4,05 \cdot 14,137 + \frac{1}{72} \cdot 10^2 \cdot 12^2 + 0,88(-0,95) \cdot 60 = -463,15\,cm^4$$

$I_{min} = 215,79\,cm^4$

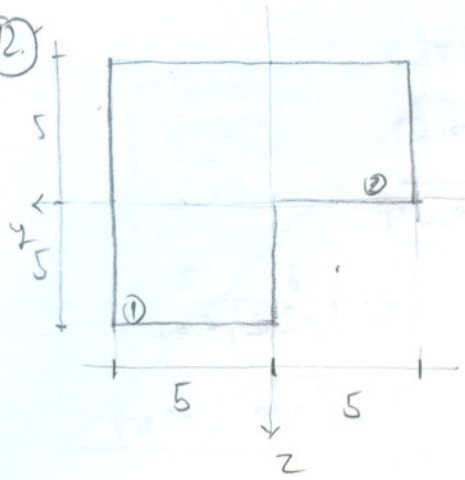
$$i_{min} = \sqrt{\frac{215,79}{74,137}} = 1,71\,cm$$

$$\lambda = \frac{600}{1,71} = 350,87$$

$$\sigma_{kr,min} = \begin{cases} \sigma^{EL} = \pi^2 \frac{32 \cdot 10^9}{350,87^2} = 2,56\,MPa \\ \sigma^T = 37 - 0,12 \cdot 350,87 = \ominus \\ 32\,MPa \end{cases}$$

$$P_{kr} = 2,56 \cdot 10^6 \cdot 74,137 \cdot 10^{-4} = 18,979\,kN$$

12.



$A_1 = 50$

$A_2 = 25$

$A = 75\,cm^2$

$T_1(2,5;0)$ $T_1(1,67;-0,83)$

$T_2(-2,5;-2,5)$ $T_2(-3,33;-1,67)$

$$I_y = \frac{1}{12}\cdot 10^4 - \left(\frac{1}{12}5^4 + 2,5^2 \cdot 25\right) = 625\,cm^4 = J_2$$

$$y_T = \frac{2,5\cdot 50 - 2,5\cdot 25}{75} = 0,83\,cm$$

$$z_T = \frac{-2,5\cdot 25}{75} = -0,83\,cm$$

$$I_{yz} = 1,67(-0,83)\cdot 50 + (-3,33)(-1,67)\cdot 25 = 69,72\,cm^4$$

$$I_{min} = 555,28\,cm^4$$

$$i_{min} = \sqrt{\frac{555,28}{75}} = 2,72\,cm$$

$$\lambda = \frac{0,909\cdot 600}{2,72} = 155,96$$

$$\sigma_{kr,min} = \begin{cases} \pi^2 \dfrac{32\cdot 10^9}{155,96^2} = 12,98\,MPa \\ 37 - 0,12\cdot 155,96 = 18,28\,MPa \\ 32\,MPa \end{cases}$$

$$P_{kr} = 12,98\cdot 10^6 \cdot 75\cdot 10^{-4} = 97,35\,kN$$

$P_{dop} = 700 kN$

$P_{dop} = \frac{1}{n} P_{kR}$

$\boxed{P_{kR} = 1400 kN}$

$P_{kR} = \sigma_{kR} \cdot A \Rightarrow \sigma_{kR} = \frac{1400}{942,48} \cdot 10 = \boxed{14,85 MPa}$

$14,85 \cdot 10^6 = \pi^2 \frac{32 \cdot 10^9}{\lambda^2}$

$\lambda^2 = \frac{32 \cdot 10^9 \pi^2}{14,85 \cdot 10^6} \Rightarrow \boxed{\lambda = 145,83}$

$14,85 = 37 - \lambda \cdot 0,12$

$0,12 \lambda = 37 - 14,85$

$\lambda = \frac{37 - 14,85}{0,12} = \boxed{184,58}$

$\lambda = \frac{l_{i1}}{l_{min}}$

$l_{i1} = 145,83 \cdot 11,18$ $\quad l_{i2} = 184,58 \cdot 11,8$

$l_{i1} = 1630,39 cm$ $\quad l_{i2} = 2063,60$

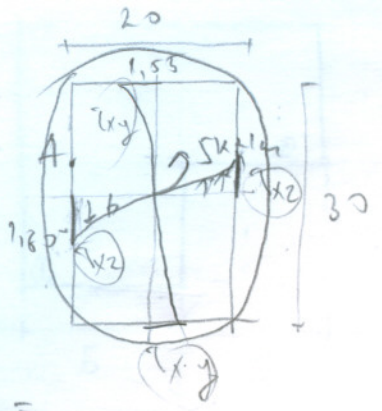
$l_i = 0,707 \cdot l_k$

$l_{k1} = \frac{1630,39}{0,707} = 2306,04 cm$ $\quad l_2 = \frac{2063,60}{0,707}$ ~~$= 3093,34 cm$~~

$\boxed{29,18 m}$

ОБЯВЛЯВАНЕ

1.



$I_t = \alpha \cdot b^3 c = 0{,}135 \cdot 20^3 \cdot 30 = 46800\,cm^4$

$W_{t,B} = \beta b^2 c = 0{,}231 \cdot 20^2 \cdot 30 = 2772\,cm^3$

$W_{t,C} = \gamma b^2 c = 0{,}270 \cdot 20^2 \cdot 30 = 3240\,cm^3$

$\frac{c}{b} =$

$\tau_B = \frac{M_t}{W_{t,B}} = \frac{5 \cdot 1000 \cdot 1\,kg}{2772 \cdot 10^{-6}} = 1{,}80\,MPa$

$\tau_C = \frac{5 \cdot 1000}{3240 \cdot 10^{-6}} = 1{,}54\,MPa$

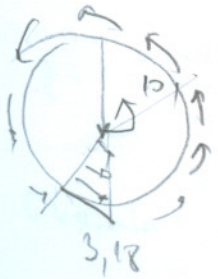
$\varphi = \frac{M_t}{G I_t} l = \frac{5000 \cdot 3}{G \cdot 46800} = \frac{320512{,}8}{G}$ [rad]

$G[Pa]$

τ_x

$\tau = \frac{T_z \cdot S_y}{I_y \, b(z)} =$ ———

$S_y^A =$



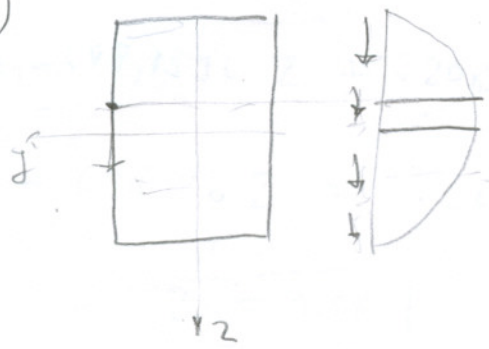
$I_t = \frac{R^4 \pi}{2} = 15707{,}96\,cm^4$

$\tau = \frac{M_t}{I_t} \cdot R = \frac{5000\,kg}{15707{,}96 \cdot 10^{-8}\,m^4} \cdot 10 \cdot 10^{-2}\,m = 3{,}18\,MPa$

$\varphi = \frac{M_t \cdot l}{G I_t} = \frac{5000 \cdot 3}{G \cdot 15707{,}96 \cdot 10^{-8}} = \frac{95{,}49 \cdot 10^6}{G}$ [rad]

$G[Pa]$

2.



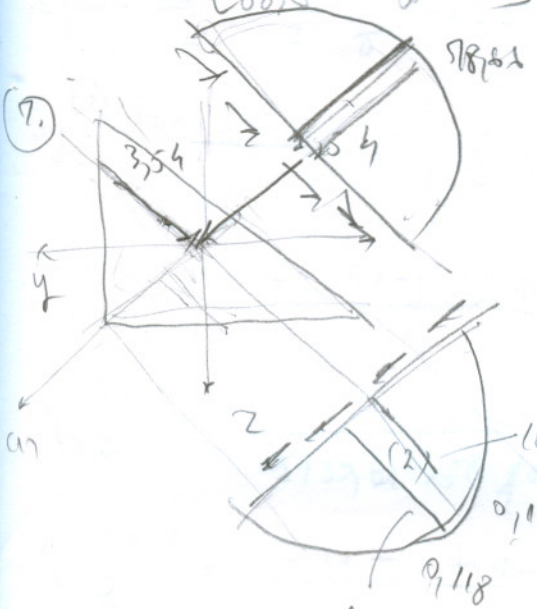
$I_y = \frac{1}{12} \cdot 30^3 \cdot 20 = \boxed{45000\ cm^4}$

$S_y^A = 20 \cdot 10 \cdot 10 = 2000\ cm^3$

$\frac{3}{2} \frac{4000}{20 \cdot 30 \cdot 10^{-4}} = 100\ kPa$ $\quad \tau_{xz}^A = \frac{4000 \cdot 2000 \cdot 10^{-6}}{45000 \cdot 10^{-8} \cdot 20 \cdot 10^{-2}} = 88,89\ kPa$

$\tau_{xz}^A = \frac{T_z \cdot S_y^A}{I_y \cdot b(z)}$

$S = \begin{bmatrix} 0 & 0 & 88,89 \\ 0 & 0 & 0 \\ 88,89 & 0 & 0 \end{bmatrix} kPa$



1.

$I_y = \frac{1}{36} \cdot 30^4 = I_z = \boxed{22500\ cm^4}$

$I_{yz} = -\frac{1}{12} \cdot 30^4 = \boxed{-11250\ cm^4}$

$tg\,2\alpha = \frac{-2 I_{yz}}{I_y - I_z} = \infty$

$\alpha = 45$

$I_1 = 33750\ cm^4$

$I_2 = 11250\ cm^4$

$\frac{3}{2} \frac{3,54 \cdot 10^3}{150 \cdot 10^{-4}} = 0,118\ MPa$

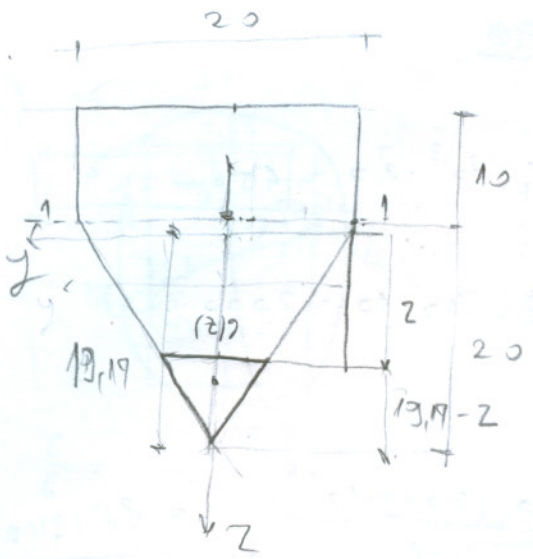
$0,104\ MPa$

$\tau_{xz} = \frac{T_z \cdot S_y^*}{I_y\ b(z)} = \frac{T_z' S_1}{I_1\ b(z)}$

$S_1 = 15 \cdot 15 \frac{1}{3} 15\sqrt{2} = \boxed{1590,99\ cm^3}$

$\tau_{xz} = \frac{3,54 \cdot 10^3 \cdot 1590,99 \cdot 10^{-6}}{33750 \cdot 10^{-8} \cdot \frac{30\sqrt{2}}{2} \cdot 10^{-2}} = 98,66\ kPa$

$0,0986\ MPa$



$$z_t = \frac{5 \cdot 200 + 16{,}67 \cdot 200}{400} = 10{,}83$$

$$I_y = \frac{1}{12} 10^3 \cdot 20 + (5 + 0{,}83)^2 \cdot 200 + \frac{1}{36} \cdot 20^3 \cdot 20 + 5{,}83^2 \cdot 200 = \boxed{19730{,}01\ cm^4}$$

$$\tau_{xz} = \frac{T_z \cdot S_y^*}{I_y \cdot b(z)}$$

$$S_y^{1-1} = 20 \cdot 10 \cdot 5{,}83 = 1166\ cm^3$$

$$S_y^T = \frac{19{,}17 \cdot b(z)}{2} \cdot 12{,}78 = 122{,}50\, b(z)\ cm^3$$

$$S_y(z) = \frac{b(z)(19{,}17 - z)}{2} \cdot \left(z + \frac{1}{3}(19{,}17 - z)\right) =$$

$$= \frac{b(z)(19{,}17 - z)}{2} \left(\frac{3z + 19{,}17 - z}{3}\right) =$$

$$= \frac{b(z)(19{,}17 - z)(2z + 19{,}17)}{6}$$

$$\tau_{xz}^{(z)} = \frac{2 \cdot 10^3 \cdot b(z)(19{,}17 - z)(2z + 19{,}17) \cdot 10^{-4}}{6\ 19730{,}01 \cdot 10^{-8}\ b(z)} = 168{,}94(19{,}17 - z)(2z + 19{,}17)$$

$(3238,5798 - 168,342)(2z + 19,17) =$

$= 6488,1596\,z + 62083,57\,z - 337,88\,z^2 - 3229,23 = 0$

$-675,76\,z = -6477,1596$

$\boxed{z = 9,58}$

$\tau_t = \dfrac{\frac{1}{2}\cdot 20h \cdot \frac{2}{3}h + 200\pi\,(h + 8,488)}{10h + 200\pi}$

h

$\tau_{xz} = \dfrac{T_z \cdot S_y^*}{I_y \cdot b(z)}$

$S_y^T = \dfrac{20^2\pi}{2} \cdot \dfrac{4}{3}\dfrac{20}{\pi} = \boxed{5333,33\ cm^3}$

$S_y^{(z)} = \dfrac{(40-z)\,b(z)}{2} \cdot \left(z + \dfrac{1}{3}(40-z)\right) = \dfrac{(40-z)\,b(z)}{2} \cdot \dfrac{2z+40}{3} =$

$= \dfrac{(40-z)\,b(z)\,(2z+40)}{6}$

$\tau_{xz}^{parte} = \dfrac{10\cdot 10^3 \cdot 5333,33\cdot 10^{-6}}{19732,01\cdot 10^{-8}\ 40\cdot 10^{-2}} = 88,6814\,Pa$

$\tau_{xz}^{doble} = \dfrac{-11-}{-11-} = 157,36$

$\tau_{xz} = \dfrac{10\cdot 10^3\,(40-z)\,b(z)\,(2z+40)\,10^{-4}}{6\cdot 16949,99\cdot 10^{-8}\ b(z)} = \boxed{98,33\,(40-z)(2z+40)} =$

$98,3\,(80z + 160 - 2z^2 - 40z) = 0$

$z = 10 \Rightarrow \boxed{\tau_{xz} = 176,994}$

$\boxed{7864\,z} + 15728 \boxed{-196,6\,z^2} - 3932\,z = 0$

$-393,2\,z + 3932 = 0 \Rightarrow \boxed{z = 10}$

$S_y = \frac{30 \cdot 15}{2} \cdot 23 = 4509\,cm^3$

40 : 20

2 = 30 : x

$2 = \frac{30}{x} \Rightarrow x = 15$

$\tau_{xz} = \frac{10 \cdot 10^3 \cdot 4500 \cdot 10^{-6}}{169499 \cdot 10^{-8} \cdot 15 \cdot 10^{-2}} = 176,992\,kPa$

$\tau_{max} = \frac{176,992}{\frac{1}{4}}$

$\cos\beta = \frac{40}{\sqrt{15^2 + 40^2}}$

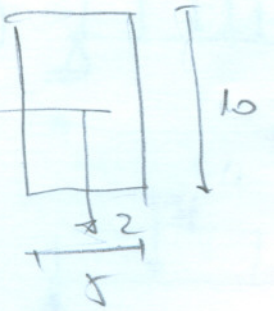
$\tau_{max} = 182,44\,kPa$

$\sigma_y^A = -\frac{M_z}{I_z} y + \frac{M_y}{I_y} z$ $\quad A(-7,5;\ 10)$

$\sigma_x^A = -\frac{40 \cdot 10^3}{69498,52 \cdot 10^{-8}} \cdot (-7,5) \cdot 10^{-2} + \frac{10 \cdot 10^3}{169499 \cdot 10^{-8}} \cdot 10 \cdot 10^{-2} = 4,9\,MPa$

$$\begin{bmatrix} 4,9 & 0 & 0,182 \\ 0 & 0 & 0 \\ 0,182 & 0 & 0 \end{bmatrix}$$

3)



$$J_y = \frac{1}{12} \cdot 10^3 \cdot 5 = 416,67\,cm^4$$

$$J_z = \frac{1}{12} \cdot 5^3 \cdot 10 = 104,17\,cm^4$$

~~$i_y = 2,88$~~

$$i_{min} = 1,44\,cm$$

$$\lambda = \frac{300}{1,44} = 208,33$$

$$\sigma_{kR,min} = \begin{cases} \sigma_{EL} = 7,28\,MPa \\ \sigma^{TET} = 14004 \\ \sigma^{T} = 32\,MPa \end{cases}$$

$$P_{kR} = 7,28 \cdot 10^6 \cdot 50 \cdot 10^{-4} = 36,4\,kN$$

5) $P_{dop} = 900$

$$900 = \frac{1}{2} P_{kR} \Rightarrow P_{kR} = 1400\,kN$$

$$P_{kR} = \sigma_{kR} \cdot A \Rightarrow \sigma_{kR} = \frac{1400}{50} \cdot 10 = 280\,MPa$$

$$\sigma_{kR}^{EL} = \frac{\pi^2 E}{\lambda^2}$$

$$\lambda^2 = \frac{\pi^2 E}{\sigma_{kR}^{EL}} = \frac{\pi^2 \cdot 32 \cdot 10^9}{280 \cdot 10^6} \Rightarrow \lambda_1 = 33,58$$

$$\lambda = \frac{l_i}{i_{min}}$$

$$0,12\lambda = 37 - 32$$

~~$\lambda = \frac{5}{0,12} = 41,67$~~

~~$l_i = \frac{i_{min}}{A}$~~

$$l_i = 33,58 \cdot 1,44$$

$$l_i = 48,35$$

$$l_i = 0,5\,l \Rightarrow l = \frac{48,35}{0,5} = 96,7\,cm$$

$i_{min} = 11,18$ $\quad A = 342,48 m^2$

$P_y = \frac{1}{2} P_{kR} \Rightarrow \boxed{P_{kR} = 2 \cdot 700 = 1400 kN}$

$\sigma_{kR} = \frac{1400}{342,48} \cdot 10 = \boxed{14,85 MPa}$

$\sigma_{kR}^{EL} = \pi^2 \frac{E}{\lambda^2}$

$\lambda^2 = \frac{\pi^2 \cdot 32 \cdot 10^9}{14,85 \cdot 10^6} \Rightarrow \boxed{\lambda_1 = 145,83}$

$14,85 = 37 - 0,12 \cdot \lambda$

$0,12\lambda = 37 - 14,85 \Rightarrow \boxed{\lambda_2 = 184,58}$

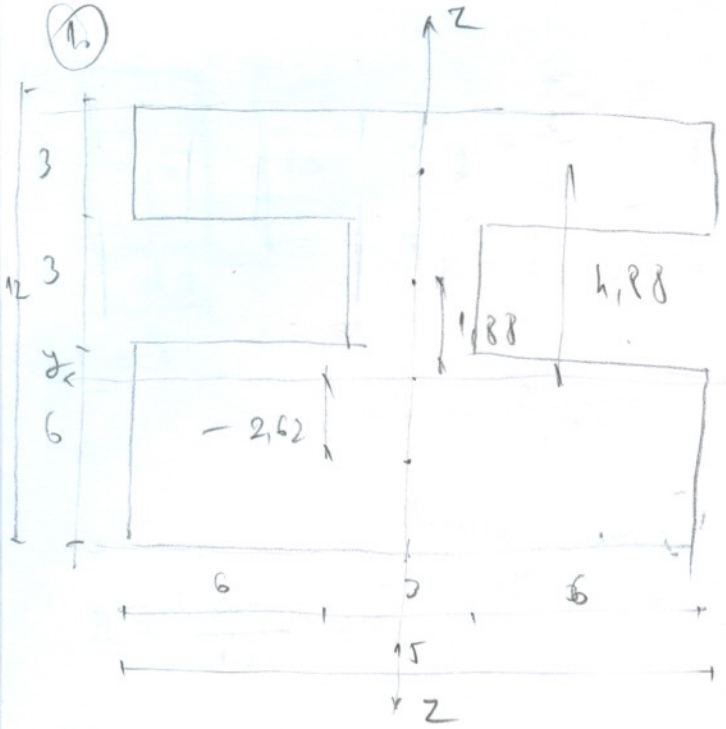
$\lambda_1 = \frac{l_i}{i_{min}} \Rightarrow l_{i_1} = 145,83 \cdot 11,18 = 1630,38 cm$

$l_{i_2} = 184,58 \cdot 11,18 = 2178,044 cm$

$l_i = 0,802 l \Rightarrow l_1 = \frac{1630,38}{0,707} = 2306,05 cm$

$l_2 = \frac{2178,044}{0,707} = 3080,08 cm$

1.



$A_1 = 90\,cm^2$

$A_2 = 9\,cm^2$

$A_3 = 45\,cm^2$

$A = 144\,cm^2$

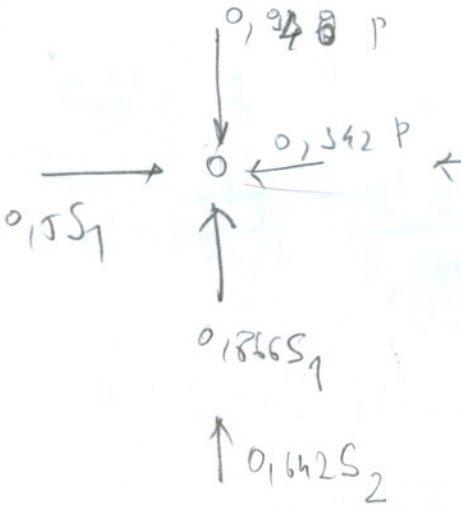
$$z_T = \frac{3 \cdot 90 + 7{,}5 \cdot 9 + 10{,}5 \cdot 45}{144} = 5{,}62$$

$$I_y = \frac{1}{12} \cdot 6^3 \cdot 15 + 2{,}62^2 \cdot 90 + \frac{1}{12} \cdot 3^4 + 1{,}88^2 \cdot 9 + \frac{1}{12} \cdot 3^3 \cdot 15 + 4{,}88^2 \cdot 45$$

$$I_y = 2031{,}75\,cm^4$$

$$I_z = \frac{1}{12} \cdot 15^3 \cdot 6 + \frac{1}{12} \cdot 3^4 + \frac{1}{12} \cdot 15^3 \cdot 3 = 2538\,cm^4$$

$I_{min} = I_y = 2031{,}75$ | $i_{min} = 3{,}76$



$\Sigma x = 0: \quad 0{,}5 S_1 - 0{,}342 P - 0{,}766 S_2 = 0$

$$0{,}5 S_1 + 0{,}766 S_2 = 0{,}342 P$$

$\Sigma y = 0$

$$0{,}866 S_1 + 0{,}642 S_2 = 0{,}940 P$$

$S_1 = 1{,}462 P$

$S_2 = -0{,}$

$S_1 = 0{,}953\,P$

$S_2 = 0{,}196\,P$

$$\lambda_1 = \frac{300}{3{,}86} = 93{,}09$$

$$\sigma^{FL} = \pi^2 \frac{E}{\lambda^2} = \pi^2 \frac{210 \cdot 10^9}{93{,}09^2} = 239{,}17\,MPa$$

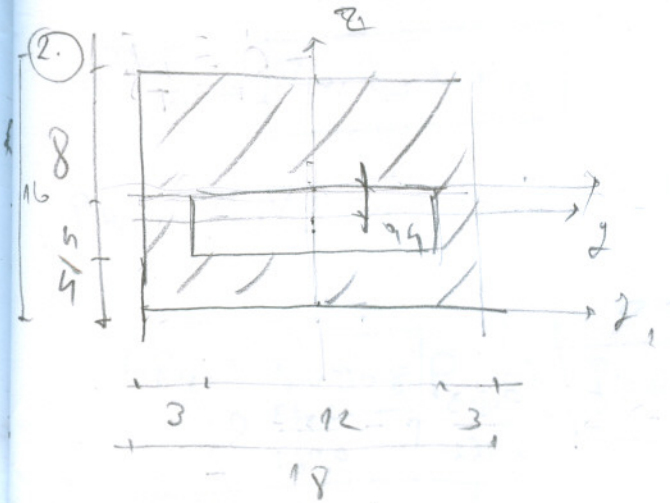
$$\sigma^{TET} = 310 - 1{,}32 \cdot 93{,}09 = 191{,}26\,MPa$$

$$P_{KR} = 191{,}26 \cdot 10^6 \cdot 144 \cdot 10^{-4} = 1890{,}144\,kN$$

$$1890{,}144 = 0{,}196\,P \Rightarrow P_{KR} = 10\,739{,}45\,kN$$

$$P_{dop} = 3068{,}41\,kN$$

2.



$A_1 = 288$

$A_2 = 48$

$A = 240$

$$z_t = \frac{0 + 2 \cdot 48}{240} = -0{,}4$$

$$I_{y_1} = \frac{1}{12} 16^3 \cdot 18 + 0{,}4^2 \cdot 288 - \left(\frac{1}{12} 4^3 \cdot 12 + 1{,}6^2 \cdot 48 \right)$$

$$I_z = \frac{1}{12} \cdot 18^3 \cdot 16 - \frac{1}{12} 12^3 \cdot 4 = 7200\,cm^4$$

$A_1 = 16 \cdot 18 = 288$ $\quad T_1(0;8)$ $\quad T_1'(0;-2{,}8)$

$A_2 = 48$ $\quad T_2(0;6)$ $\quad T_2'(0;-4{,}8)$

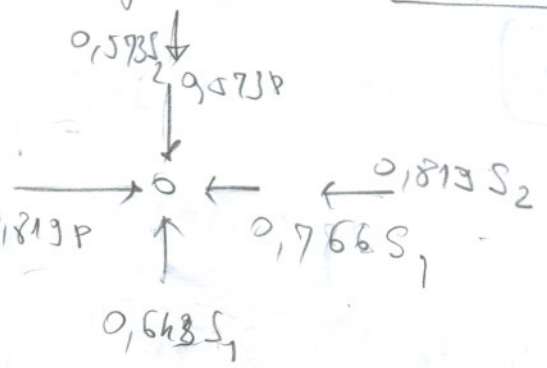
$A = 240$

$$z_T = \frac{8 \cdot 288 + 6 \cdot 48}{240} = 10{,}8$$

$$I_y = \frac{1}{12} 16^3 \cdot 18 + 2{,}8^2 \cdot 288 - \left(\frac{1}{12} \cdot 4^3 \cdot 12 + 4{,}8^2 \cdot 48\right) = 7232\ cm^4$$

$$\boxed{I_y = 5849{,}6\ cm^4}$$

$$i_{min} = \sqrt{\frac{5849{,}6}{240}} = \boxed{4{,}94\ cm}$$



$$\Sigma x = 0: \quad 0{,}819P - 0{,}819S_2 - 0{,}766S_1 = 0$$

(1) $\boxed{0{,}766S_1 + 0{,}819S_2 = 0{,}819P} \Rightarrow S_1 = \frac{0{,}819}{0{,}766}P - \frac{0{,}819}{0{,}766}S_2$

$$\Sigma y = 0: \quad \text{(2)} \ \boxed{0{,}643S_1 - 0{,}573S_2 = 0{,}573P}$$

$$\Delta = \begin{vmatrix} 0{,}766 & 0{,}819 \\ 0{,}643 & -0{,}573 \end{vmatrix} = -0{,}9655$$

$$\Delta_1 = \boxed{\begin{array}{l} S_1 = 0{,}972P \\ S_2 = 0{,}908P \end{array}}$$

$$\Delta = \begin{vmatrix} 0,766 & 0,819 \\ 0,643 & -0,573 \end{vmatrix} = A \qquad S_1 = \frac{\Delta_1}{\Delta} = \frac{B}{A} = \boxed{0,972\ P}$$

$$\Delta_1 = \begin{vmatrix} 0,819 & 0,819 \\ 0,573 & -0,573 \end{vmatrix} = B \qquad S_2 = \frac{\Delta_2}{\Delta} = \frac{C}{A} = \boxed{0,0908\ P}$$

$$\Delta_2 = \begin{vmatrix} 0,766 & 0,819 \\ 0,643 & 0,573 \end{vmatrix} = C$$

$\boxed{S_2 = 0,0908\ P}$

$$\lambda = \frac{600}{4,94} = \boxed{121,46}$$

$$\sigma_{kr}^{EL} = \pi^2 \frac{210 \cdot 10^9}{121,46^2} = 140,49\ MPa$$

$$\sigma^{TET} = \boxed{76,79\ MPa}$$

$$P_{kr} = 76,79 \cdot 10^6 \cdot 240 \cdot 10^{-4} = \boxed{1,84\ MN}$$

$$1,84 = \underline{0,0908}\ P \Rightarrow \boxed{P_{kr} = 20,26\ MN}$$

$\boxed{P = 5,98\ MN}$