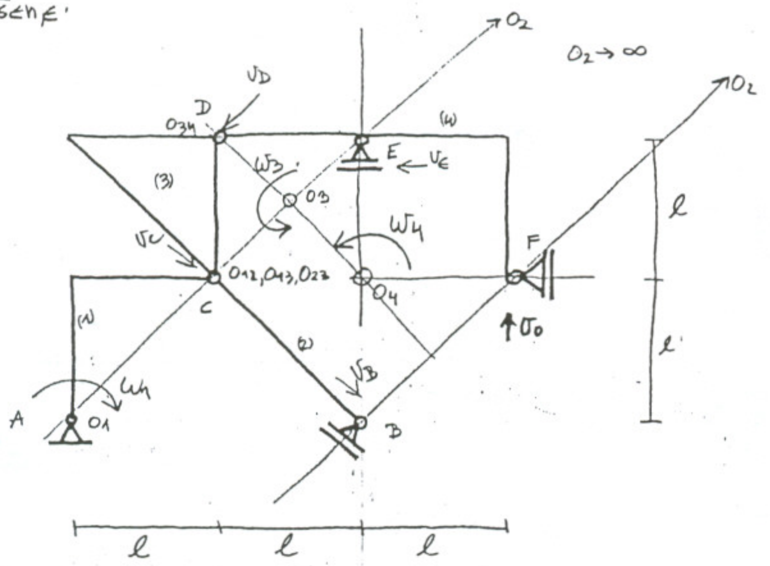
Kolokvijum 19.11.2004

Zadatak 1.

U prikazanom položaju mehanizma na slici poznati su brzina tačke F: $V_F = V_0$ i ubrzanje tačke E: $a_E = a_0$. Odrediti:

(1) Ugaone brzine svih tela sistema i brzine tačaka B, C, D, E
(2) Ugaona ubrzanja svih tela sistema i ubrzanja tačaka B, D, F

Rešenje:



(1) Brzine i ugaone brzine

$$\boxed{v_F = v_0}$$

$$v_F = \omega_4 \cdot \overline{FO_4} \Rightarrow \omega_4 = \frac{v_F}{\ell} \Rightarrow \boxed{\omega_4 = \frac{v_0}{\ell}}$$

$$v_E = \omega_4 \cdot \overline{EO_4} \Rightarrow v_E = \frac{v_0}{\ell} \cdot \ell \Rightarrow \boxed{v_E = v_0}$$

$$v_D = \omega_4 \cdot \overline{DO_4} \Rightarrow v_D = \frac{v_0}{\ell} \cdot \ell\sqrt{2} \Rightarrow \boxed{v_D = v_0\sqrt{2}}$$

$$v_D = \omega_3 \cdot \overline{DO_3} \Rightarrow \omega_3 = \frac{v_0\sqrt{2}}{\ell\sqrt{2}} \cdot 2 \Rightarrow \boxed{\omega_3 = \frac{2v_0}{\ell}}$$

$$v_C = \omega_3 \cdot \overline{CO_3} \Rightarrow v_C = \frac{2 \cdot v_0}{\ell} \cdot \frac{\ell\sqrt{2}}{2} \Rightarrow \boxed{v_C = v_0\sqrt{2}}$$

$$\omega_2 = 0 \Rightarrow v_B = v_C \Rightarrow \boxed{v_B = v_0\sqrt{2}}$$

$$v_C = \omega_1 \cdot \overline{CO_1} \Rightarrow \omega_1 = \frac{v_0\sqrt{2}}{\ell\sqrt{2}} \Rightarrow \boxed{\omega_1 = \frac{v_0}{\ell}}$$

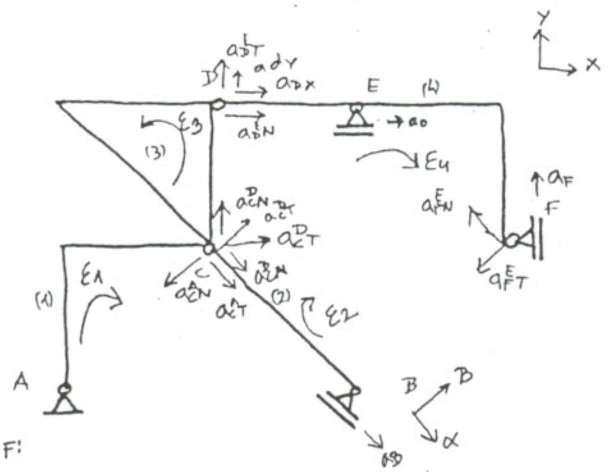
$$v_A = \omega_1 \cdot \overline{AO_1} \Rightarrow \boxed{v_A = 0}$$

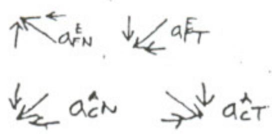
Rešenje brzina i ugaonih brzina

$$\omega_1 = \frac{v_0}{\ell} \qquad \omega_2 = 0 \qquad \omega_3 = \frac{2 \cdot v_0}{\ell} \qquad \omega_4 = \frac{v_0}{\ell}$$

$$v_A = 0 \qquad v_B = v_0\sqrt{2} \qquad v_C = v_0\sqrt{2} \qquad v_D = v_0\sqrt{2} \qquad v_E = v_0 \qquad v_F = v_0$$

2) Ubrzanja i ugaona ubrzanja





F:

$$\vec{a_F} = \vec{a_E} + \vec{a_{FN}^E} + \vec{a_{FT}^E}$$

$$a_{FN}^E = \omega_4^2 \cdot \overline{FE} \Rightarrow a_{FN}^E = \left(\frac{v_0}{l}\right)^2 \cdot l\sqrt{2} \Rightarrow a_{FN}^E = \frac{v_0^2}{l} \cdot \sqrt{2}$$

$$a_{FT}^E = \varepsilon_4 \cdot \overline{FE} \Rightarrow a_{FT}^E = \varepsilon_4 \cdot l\sqrt{2}$$

$$X: \quad 0 = a_0 - \frac{\sqrt{2}}{2} a_{FN}^E - \frac{\sqrt{2}}{2} a_{FT}^E \Rightarrow 0 = a_0 - \frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \frac{v_0^2}{l} \cancel{\sqrt{2}} - \frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \varepsilon_4 l \cancel{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \varepsilon_4 \cdot l = a_0 - \frac{v_0^2}{l} \Rightarrow \boxed{\varepsilon_4 = \frac{a_0}{l} - \left(\frac{v_0}{l}\right)^2}$$

$$Y: \quad a_F = 0 + \frac{\sqrt{2}}{2} a_{FN}^E - \frac{\sqrt{2}}{2} a_{FT}^E \Rightarrow a_F = \frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \frac{v_0^2}{l} \cancel{\sqrt{2}} - \frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot l \cancel{\sqrt{2}} \cdot \varepsilon_4 \Rightarrow$$

$$\Rightarrow a_F = \frac{v_0^2}{l} - l\left(\frac{a_0}{l} - \frac{v_0^2}{l^2}\right) \Rightarrow a_F = \frac{v_0^2}{l} - a_0 + \frac{v_0^2}{l} \Rightarrow \boxed{a_F = -a_0 + \frac{2v_0^2}{l}}$$

D:

$$\vec{a_D} = \vec{a_E} + \vec{a_{DN}^E} + \vec{a_{DT}^E}$$

$$a_{DN}^E = \omega_4^2 \cdot DE \Rightarrow a_{DN}^E = \left(\frac{v_0}{l}\right)^2 \cdot l \Rightarrow a_{DN}^E = \frac{v_0^2}{l}$$

$$a_{DT}^E = \varepsilon_4 \cdot DE \Rightarrow a_{DT}^E = \varepsilon_4 \cdot l \Rightarrow a_{DT}^E = a_0 - \frac{v_0^2}{l}$$

$$X: \quad \boxed{a_{Dx} = a_0 + \frac{v_0^2}{l}}$$

$$Y: \quad a_{Dy} = 0 + a_{DT}^E \Rightarrow \boxed{a_{Dy} = a_0 - \frac{v_0^2}{l}}$$

$$\boxed{a_D = \left(a_0 + \frac{v_0^2}{l}\right)\vec{i} + \left(a_0 - \frac{v_0^2}{l}\right)\vec{j}}$$

$$\left.\begin{array}{l}\vec{a_C} = \vec{a_A} + \vec{a_{CN}^A} + \vec{a_{CT}^A} \\ \vec{a_C} = \vec{a_D} + \vec{a_{CN}^D} + \vec{a_{CT}^D}\end{array}\right\} \Rightarrow \vec{a_D} + \vec{a_{CN}^D} + \vec{a_{CT}^D} = \cancel{\vec{a_A}} + \vec{a_{CN}^A} + \vec{a_{CT}^A} \qquad / \vec{i}, \vec{j}$$

$$a_{CN}^D = \omega_3^2 \overline{CD} \Rightarrow a_{CN}^D = \left(\frac{2v_0}{l}\right)^2 \cdot l \Rightarrow a_{CN}^D = \frac{4 \cdot v_0^2}{l^2} \cdot l \Rightarrow a_{CN}^D = \frac{4v_0^2}{l}$$

$$a_{CT}^D = \varepsilon_3 \cdot \overline{CD} \Rightarrow a_{CT}^D = \varepsilon_3 \cdot l$$

$$a_{CN}^A = \omega_1^2 \cdot \overline{AC} \Rightarrow a_{CN}^A = \left(\frac{v_0}{l}\right)^2 \cdot l\sqrt{2} \Rightarrow a_{CN}^A = \frac{v_0^2\sqrt{2}}{l}$$

$$a_{CT}^A = \varepsilon_1 \cdot \overline{AC} \Rightarrow a_{CT}^A = \varepsilon_1 \cdot l\sqrt{2}$$

$$X: \quad a_{Dx} + a_{CT}^D = -a_{CN}^A\frac{\sqrt{2}}{2} + a_{CT}^A\frac{\sqrt{2}}{2} \Rightarrow a_0 + \frac{v_0^2}{l} + \varepsilon_3 \cdot l = -\frac{\sqrt{2}}{2}\cdot\frac{v_0^2\sqrt{2}}{l} + \varepsilon_1 \cdot l\sqrt{2}\cdot\frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow a_0 + \frac{v_0^2}{l} + \varepsilon_3 \cdot l = -\frac{v_0^2}{l} + \varepsilon_1 \cdot l$$

$$Y: \quad a_{Dy} + a_{CN}^D = -a_{CN}^A \cdot \frac{\sqrt{2}}{2} - a_{CT}^A\frac{\sqrt{2}}{2} \Rightarrow a_0 - \frac{v_0^2}{l} + \frac{4v_0^2}{l} = -\frac{\sqrt{2}}{2}\cdot\frac{v_0^2\sqrt{2}}{l} - \frac{\sqrt{2}}{2}\cdot\varepsilon_1 l\sqrt{2} \Rightarrow$$

$$\Rightarrow \varepsilon_1 \cdot l = -a_0 - \frac{3v_0^2}{l} - \frac{v_0^2}{l} \Rightarrow \varepsilon_1 \cdot l = -a_0 - \frac{4v_0^2}{l} \Rightarrow \boxed{\varepsilon_1 = -\frac{a_0}{l} - \frac{4v_0^2}{l^2}}$$

$$\varepsilon_3 \cdot l = -a_0 - \frac{v_0^2}{l} - \frac{v_0^2}{l} - a_0 - \frac{4v_0^2}{l} \Rightarrow \varepsilon_3 \cdot l = -2a_0 - \frac{6v_0^2}{l} \Rightarrow \boxed{\varepsilon_3 = -\frac{2a_0}{l} - \frac{6v_0^2}{l^2}}$$

B:

$$\left.\begin{array}{l}\vec{a_C} = \vec{a_B} + \vec{a_{CN}^B} + \vec{a_{CT}^B} \\ \vec{a_C} = \cancel{\vec{a_A}}^{0} + \vec{a_{CN}^A} + \vec{a_{CT}^A}\end{array}\right\} \Rightarrow \vec{a_B} + \cancel{\vec{a_{CN}^B}} + \vec{a_{CT}^B} = \vec{a_{CN}^A} + \vec{a_{CT}^A}$$

$$a_{CN}^B = \omega_2^2 \overline{CB} \Rightarrow a_{CN}^B = 0$$

$$a_{CT}^B = \varepsilon_2 \cdot \overline{CB} \Rightarrow a_{CT}^B = \varepsilon_2 \cdot l\sqrt{2}$$

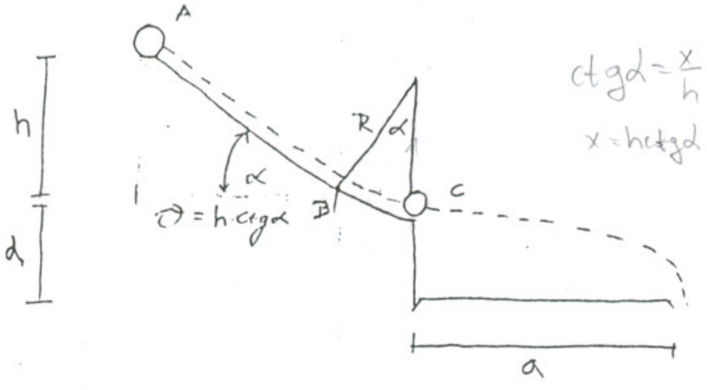
$$\alpha: \quad a_B = a_{CT}^A \Rightarrow a_B = \varepsilon_1 \cdot l\sqrt{2} \Rightarrow a_B = \left(-\frac{a_0}{l} - \frac{4v_0^2}{l^2}\right)\cdot l\sqrt{2} \Rightarrow$$

$$\Rightarrow \boxed{a_B = -a_0\sqrt{2} - 4\sqrt{2}\frac{v_0^2}{l}}$$

$$\beta: \quad 0 + a_{CT}^B = -a_{CN}^A \Rightarrow \varepsilon_2 \cdot l\sqrt{2} = -\frac{v_0^2\sqrt{2}}{l} \Rightarrow \boxed{\varepsilon_2 = -\frac{v_0^2}{l^2}}$$

Zadatak 2

Da bi se zaštitio kolovoz od mogućeg odrona kamenja, teren iznad puta je profilisan kao na slici.
Kamen mase m iz položaja A počinje da klizi bez početne brzine.
Odrediti.
(1) Pritisak na podlogu u položaju C.
(2) Maksimalnu širinu kolovoza tako da kamen padne van kolovoza.



$ctg\alpha = \frac{x}{h}$

$x = h ctg\alpha$

AB – Hrapavo
BC – Idealno glatko

$m = 2\,kg$ $\quad g = 10\,m/s^2$
$h = 4\,m$
$R = 3\,m$ $\quad d = 4\,m$
$\alpha = 30°$ $\quad \mu = 0{,}1$

Rešenje

Deo A–B

$$T_B - T_A = A_{A-B} \Rightarrow \frac{1}{2}\cancel{m}\cdot v_B^2 - \cancel{\frac{1}{2}m v_A^2} = \cancel{m}\cdot g\cdot h - \cancel{m}\cdot g\cdot \mu x$$ $\Rightarrow$

$$\Rightarrow \frac{v_B^2}{2} = g\cdot h - g\mu\cdot h\cdot ctg\alpha \Rightarrow v_B^2 = 2\cdot(10\cdot 4 - 10\, 0{,}1\cdot 4\cdot ctg\,30°) \Rightarrow$$

$$\Rightarrow v_B^2 = 80 - 8\cdot\frac{\cos 30}{\sin 30} \Rightarrow v_B^2 = 80 - 13{,}8564 \Rightarrow \boxed{v_B^2 = 66{,}1436}$$

Deo B–C

$$T_C - T_B = A_{B-C} \Rightarrow \frac{1}{2}\cancel{m}\cdot v_C^2 - \frac{1}{2}\cancel{m}v_B^2 = \cancel{m}\cdot g\cdot R(1-\cos\alpha) \Rightarrow$$

$$\Rightarrow \frac{v_C^2}{2} = \frac{1}{2}\cdot 66{,}1436 + 10\cdot 3\cdot(1-\cos 30) \Rightarrow v_C^2 = 2\cdot(33{,}0718 + 4{,}0192) \Rightarrow$$

$$\Rightarrow v_C^2 = 74{,}1821 \Rightarrow \boxed{v_C = 8{,}613\ m/s}$$

$m a_n = N(\varphi) - mg$

$N(\varphi) = m\frac{v^2}{R} + mg$

(1) Pritisak na podlogu u tački C.

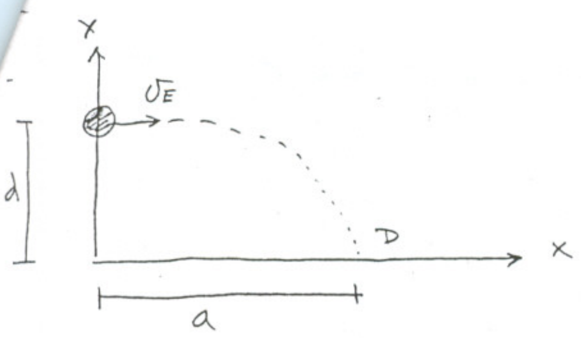
$\downarrow mg$
$\downarrow a_n$
$\uparrow N_C$

$$N_C = a_n + mg$$

$$N_C = \frac{m\cdot v_C^2}{R} + mg \Rightarrow N_C = \frac{2\cdot 74{,}1821}{3} + 2\cdot 10 \Rightarrow$$

$$\Rightarrow \boxed{N_C = 69{,}455\ N}$$

2) Da padne van kolovoza



Njutnov zakon

$$m\cdot\vec{a}=\vec{F_R}\ /\vec{i},\vec{j}\ \Rightarrow\ \begin{array}{l} x:\ \ m\cdot\ddot{x}=0\ \Rightarrow\ \dot{x}=C_1\ \Rightarrow\ x=C_1t+C_2 \\ y:\ \ m\cdot\ddot{y}=-mg\ \Rightarrow\ \dot{y}=-gt+C_3\ \Rightarrow\ y=-\frac{1}{2}gt^2+C_3t+C_4 \end{array}$$

Početni uslovi

$t=0\ \Rightarrow\ x(0)=0 \qquad \dot{x}(0)=v_E$

$y(0)=d\ \Rightarrow\ y(0)=4 \qquad \dot{y}(0)=0$

Iz početnih uslova i njutnovog zakona dobijamo vrednosti konstanti $\Rightarrow$

$\dot{x}(0)=v_E \qquad \dot{x}=C_1\Rightarrow C_1=v_E\ \Rightarrow\ \boxed{C_1=8{,}613}$

$\dot{y}(0)=0 \qquad \dot{y}=-\cancel{gt}^{0}+C_3\ \Rightarrow\ \boxed{C_3=0}$

$x(0)=0 \qquad x=\cancel{C_1t}^{0}+C_2\ \Rightarrow\ \boxed{C_2=0}$

$y(0)=4 \qquad y(0)=-\cancel{\frac{1}{2}gt^2}^{0}+\cancel{C_3t}^{0}+C_4\ \Rightarrow\ \boxed{C_4=4}$

Konačne jednačine

$x(t)=8{,}613\cdot t \qquad \dot{x}(t)=8{,}613$

$y(t)=-\frac{1}{2}\cdot g\cdot t^2+4 \qquad \dot{y}(t)=-gt\ \Rightarrow\ \dot{y}(t)=-10\cdot t$

Da padne van kolovoza $\quad y(t_D)=0$

$-\frac{1}{2}\cdot 10\,t^2+4=0\ \Rightarrow\ t_D^2=\frac{4}{5}\ \Rightarrow\ \boxed{t_D=0{,}894}$

$x(t_D)=8{,}613\cdot 0{,}894\ \Rightarrow\ x(t_D)=7{,}70\ \Rightarrow\ \boxed{a_{max}=7{,}70}$

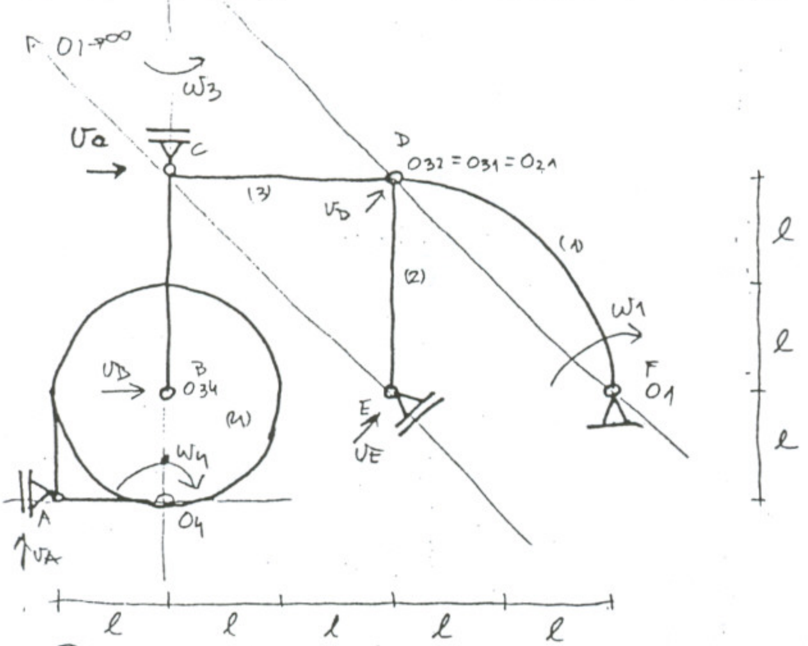
KOLOKVIJUM 2.12.2005

Zadatak 1.

U prikazanom položaju mehanizma na slici poznati su brzina i ubrzanje tačke C. $a_C = a_0$ i $v_C = v_0$. Odrediti:

(1) Ugaone brzine svih tela sistema i brzine tačaka A, B, D, E

(2) Ugaona ubrzanja svih tela sistema i ubrzanja tačaka B, E



(1) Brzine i ugaone brzine

$\boxed{v_C = v_0}$

$v_C = \omega_3 \cdot \overline{CO_3} \Rightarrow \boxed{\omega_3 = \dfrac{v_0}{2l}}$

$v_D = \omega_3 \cdot \overline{DO_3} \Rightarrow v_D = \dfrac{v_0}{\cancel{2l}} \cdot 2\sqrt{2}\,l \Rightarrow \boxed{v_D = v_0\sqrt{2}}$

$v_B = \omega_3 \cdot \overline{BO_3} \Rightarrow v_B = \dfrac{v_0}{2l} \cdot 4l \Rightarrow \boxed{v_B = 2v_0}$

$v_B = \omega_4 \cdot \overline{BO_4} \Rightarrow \boxed{\omega_4 = \dfrac{2v_0}{l}}$

$v_A = \omega_4 \cdot \overline{AO_4} \Rightarrow v_A = \dfrac{2 \cdot v_0}{l} \cdot l \Rightarrow \boxed{v_A = 2v_0}$

$\boxed{\omega_2 = 0}$ $\quad v_E = v_D \Rightarrow \boxed{v_E = v_0\sqrt{2}}$

$v_D = \omega_1 \cdot \overline{DO_1} \Rightarrow \omega_1 = \dfrac{v_0\sqrt{2}}{2l\sqrt{2}} \Rightarrow \boxed{\omega_1 = \dfrac{v_0}{2l}}$

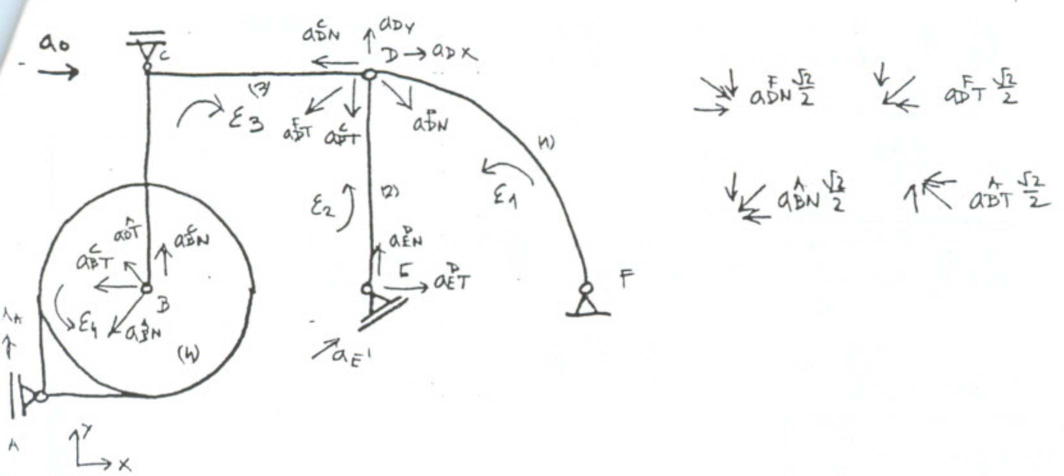
$v_F = \omega_1 \overline{FO_1} \Rightarrow \boxed{v_F = 0}$

Rešenja:

$\omega_1 = \dfrac{v_0}{2l} \qquad \omega_2 = 0 \qquad \omega_3 = \dfrac{v_0}{2l} \qquad \omega_4 = \dfrac{2v_0}{l}$

$v_A = 2v_0 \qquad v_B = 2v_0 \qquad v_C = v_0 \qquad v_D = v_0\sqrt{2} \qquad v_E = v_0\sqrt{2} \qquad v_F = 0$

Ubrzanja i ugaona ubrzanja.



D:

$$\left.\begin{aligned} \vec{a}_D &= \vec{a}_C + \vec{a}_{DN}^C + \vec{a}_{DT}^C \\ \vec{a}_D &= \cancel{\vec{a}_F}^{\,0} + \vec{a}_{DN}^F + \vec{a}_{DT}^F \end{aligned}\right\} \Rightarrow \vec{a}_C + \vec{a}_{DN}^C + \vec{a}_{DT}^C = \vec{a}_{DN}^F + \vec{a}_{DT}^F \qquad / \vec{i}, \vec{j}$$

$$a_{DN}^C = \omega_3^2 \cdot CD \Rightarrow a_{DN}^C = \left(\frac{v_0}{2l}\right)^2 \cdot 2l \Rightarrow a_{DN}^C = \frac{v_0^2}{2l}$$

$$a_{DT}^C = \varepsilon_3 \cdot CD \Rightarrow a_{DT}^C = \varepsilon_3 \cdot 2l$$

$$a_{DN}^F = \omega_1^2 \cdot DF \Rightarrow a_{DN}^F = \left(\frac{v_0}{2l}\right)^2 \cdot 2l\sqrt{2} \Rightarrow a_{DN}^F = \frac{v_0^2}{2l} \cdot \sqrt{2}$$

$$a_{DT}^F = \varepsilon_1 \cdot DF \Rightarrow a_{DT}^F = \varepsilon_1 \cdot 2l\sqrt{2}$$

$$X: \ a_0 - a_{DN}^C = + a_{DN}^F \frac{\sqrt{2}}{2} - a_{DT}^F \frac{\sqrt{2}}{2} \Rightarrow a_0 - \frac{v_0^2}{2l} = \frac{v_0^2}{2l} \cdot \cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} - \varepsilon_1 \cdot 2l \cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} \Rightarrow$$

$$\Rightarrow \varepsilon_1 \cdot 2l = -a_0 + \frac{v_0^2}{2l} + \frac{v_0^2}{2l} \Rightarrow \varepsilon_1 = -\frac{a_0}{2l} + \frac{2v_0^2}{4l^2} \Rightarrow \boxed{\varepsilon_1 = -\frac{a_0}{2l} + \frac{v_0^2}{2l^2}}$$

$$Y: \ 0 - a_{DT}^C = - a_{DN}^F \frac{\sqrt{2}}{2} - a_{DT}^F \frac{\sqrt{2}}{2} \Rightarrow -\varepsilon_3 \cdot 2l = -\frac{v_0^2}{2l} \cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} - \varepsilon_1 \cdot 2l \cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} \Rightarrow$$

$$\Rightarrow -\varepsilon_3 \cdot 2l = -\frac{v_0^2}{2l} - 2l \cdot \left(-\frac{a_0}{2l} + \frac{v_0^2}{2l^2}\right) \Rightarrow -\varepsilon_3 \cdot 2l = -\frac{v_0^2}{2l} + a_0 - \frac{v_0^2}{l} \Rightarrow$$

$$\Rightarrow -\varepsilon_3 \cdot 2l = a_0 - \frac{3}{2}\frac{v_0^2}{l} \Rightarrow \boxed{\varepsilon_3 = -\frac{a_0}{2l} + \frac{3}{4} \cdot \frac{v_0^2}{l^2}}$$

B:

$$\left.\begin{aligned} \vec{a}_B &= \vec{a}_C + \vec{a}_{BN}^C + \vec{a}_{BT}^C \\ \vec{a}_B &= \vec{a}_A + \vec{a}_{BN}^A + \vec{a}_{BT}^A \end{aligned}\right\} \Rightarrow \vec{a}_C + \vec{a}_{BN}^C + \vec{a}_{BT}^C = \vec{a}_A + \vec{a}_{BN}^A + \vec{a}_{BT}^A \qquad / \vec{i}, \vec{j}$$

$$a_{BN}^C = \omega_3^2 \cdot BC \Rightarrow a_{BN}^C = \left(\frac{v_0}{2l}\right)^2 \cdot 2l \Rightarrow a_{BN}^C = \frac{v_0^2}{2l}$$

$$a_{BT}^C = \varepsilon_3 \cdot BC \Rightarrow a_{BT}^C = \left(-\frac{a_0}{2l} + \frac{3}{4} \cdot \frac{v_0^2}{l^2}\right) \cdot 2l \Rightarrow a_{BT}^C = -a_0 + \frac{3v_0^2}{2l}$$

$$a_{BN}^A = \omega_4 \cdot AB \Rightarrow a_{BN}^A = \left(\frac{2v_0}{l}\right)^2 \cdot l\sqrt{2} \Rightarrow a_{BN}^A = \frac{4v_0^2}{l}\sqrt{2}$$

$$a_{BT}^A = \varepsilon_4 \cdot AB \Rightarrow a_{BT}^A = \varepsilon_4 \cdot l\sqrt{2}$$

$$X: \; a_0 - a_{BT}^{C} = 0 - a_{BN}^{A}\frac{\sqrt{2}}{2} - a_{BT}^{A}\frac{\sqrt{2}}{2} \Rightarrow a_0 - \left(-a_0 + \frac{3}{2}\frac{v_0^2}{l}\right) = -\frac{4v_0^2\sqrt{2}}{l}\cdot\frac{\sqrt{2}}{2} - \varepsilon_4 \cdot l\sqrt{2}\cdot\frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow a_0 + a_0 - \frac{3}{2}\frac{v_0^2}{l} = -\frac{4v_0^2}{l} - \varepsilon_4 \cdot l \Rightarrow \varepsilon_4 \cdot l = -\frac{4v_0^2}{l} + \frac{3}{2}\frac{v_0^2}{l} - 2a_0 \Rightarrow$$

$$\Rightarrow \varepsilon_4 \cdot l = -2a_0 - \frac{5}{2}\cdot\frac{v_0^2}{l} \Rightarrow \boxed{\varepsilon_4 = -\frac{2a_0}{l} - \frac{5}{2}\cdot\frac{v_0^2}{l^2}}$$

$$Y: \; 0 + a_{BN}^{C} = a_A - a_{BN}^{A}\frac{\sqrt{2}}{2} + a_{BT}^{A}\frac{\sqrt{2}}{2} \Rightarrow \frac{v_0^2}{2l} = a_A - \frac{4v_0^2\sqrt{2}}{l}\cdot\frac{\sqrt{2}}{2} + \varepsilon_4 \cdot l\sqrt{2}\cdot\frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow a_A = \frac{v_0^2}{2l} + \frac{4v_0^2}{l} - \varepsilon_4 \cdot l \Rightarrow a_A = \frac{9v_0^2}{2l} + 2a_0 + \frac{5}{2}\cdot\frac{v_0^2}{l} \Rightarrow$$

$$\Rightarrow a_A = 2a_0 + \frac{14}{2}\cdot\frac{v_0^2}{l} \Rightarrow \boxed{a_A = 2a_0 + 7\frac{v_0^2}{l}}$$

$$B: \; \vec{a_B} = \vec{a_C} + \vec{a_{BN}^{C}} + \vec{a_{BT}^{C}} \quad \Big/ \begin{smallmatrix}\vec{i}\\ \vec{j}\end{smallmatrix}$$

$$X: \; a_{Bx} = a_0 - a_{BT}^{C} \Rightarrow a_{Bx} = a_0 - \left(-a_0 + \frac{3}{2}\frac{v_0^2}{l}\right) \Rightarrow \boxed{a_{Bx} = 2a_0 - \frac{3}{2}\frac{v_0^2}{l}}$$

$$Y: \; a_{By} = 0 + a_{BN}^{C} \Rightarrow \boxed{a_{By} = \frac{v_0^2}{2l}}$$

$$\boxed{a_B = \left(2a_0 - \frac{3}{2}\frac{v_0^2}{l}\right)\vec{i} + \left(\frac{v_0^2}{2l}\right)\cdot\vec{j}}$$

$$D: \; \vec{a_D} = \vec{a_C} + \vec{a_{DN}^{C}} + \vec{a_{DT}^{C}} \quad \Big/ \begin{smallmatrix}\vec{i}\\ \vec{j}\end{smallmatrix}$$

$$\boxed{a_D = \left(a_0 - \frac{v_0^2}{2l}\right)\vec{i} + \left(a_0 - \frac{3}{2}\frac{v_0^2}{l}\right)\vec{j}}$$

$$X: \; a_{Dx} = a_0 - a_{DN}^{C} \Rightarrow \boxed{a_{Dx} = a_0 - \frac{v_0^2}{2l}}$$

$$Y: \; a_{Dy} = 0 - a_{DT}^{C} \Rightarrow a_{Dy} = -2l\cdot\left(-\frac{a_0}{2l} + \frac{3}{4}\cdot\frac{v_0^2}{l^2}\right) \Rightarrow \boxed{a_{Dy} = a_0 - \frac{3}{2}\frac{v_0^2}{l}}$$

$$E: \; \vec{a_E} = \vec{a_D} + \vec{a_{EN}^{D}} + \vec{a_{ET}^{D}}$$

$$a_{EN}^{D} = \omega_2^2 \cdot ED \Rightarrow a_{EN}^{D} = 0$$

$$a_{ET}^{D} = \varepsilon_2 \cdot ED \Rightarrow a_{ET}^{D} = \varepsilon_2 \cdot 2l$$

$$X: \; a_E\frac{\sqrt{2}}{2} = a_{Dx} + a_{ET}^{D} \Rightarrow a_E\frac{\sqrt{2}}{2} = a_0 - \frac{v_0^2}{2l} + \varepsilon_2 \cdot 2l$$

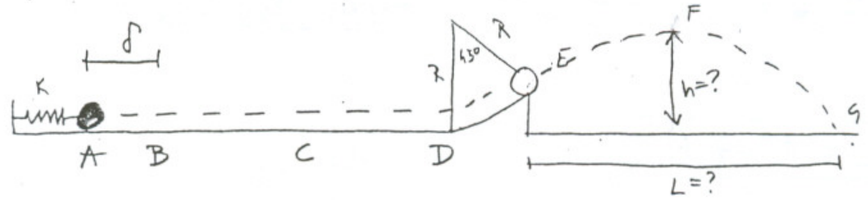
$$Y: \; a_E\frac{\sqrt{2}}{2\sqrt{2}} = a_{Dy} \Rightarrow a_E\frac{\sqrt{2}}{2} = a_0 - \frac{3}{2}\frac{v_0^2}{l} \Rightarrow \boxed{a_E = a_0\sqrt{2} - \frac{3}{2}\cdot\frac{v_0^2}{l}\sqrt{2}}$$

$$\varepsilon_2 \cdot 2l = a_0 - \frac{3}{2}\frac{v_0^2}{l} - a_0 + \frac{v_0^2}{2l} \Rightarrow \varepsilon_2 \cdot 2l = -\frac{2}{2}\frac{v_0^2}{l} \Rightarrow \boxed{\varepsilon_2 = -\frac{v_0^2}{2l^2}}$$

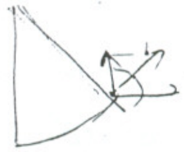
Zadatak 2.

Materijalna tačka mase m nalazi se u položaju A, kao što je na skici. Opruga krutosti k je sabijena za δ i po njenom puštanju tačka počinje da se kreće po podlozi A-B-C-D-E. U položaju E tačka napušta podlogu i započinje kretanje. Odrediti:

1) Mesto pada materijalne tačke na podlogu $L=?$
2) Maksimalnu visinu koju tačka dostigne tokom kretanja $h=?$



ABC - Idealno glatko	$\delta = 0,2\ell$	$k = 50 \cdot \frac{m \cdot g}{\ell}$	$\mu = 0,1$
CD - hrapavo	$BC = CD = 0,5\ell$		
DE - Idealno glatko	$R = 0,6\ell$	$g = 10$	



Rešenje:

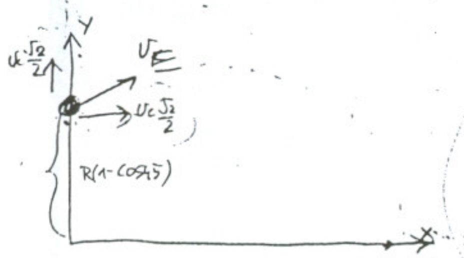
Deo A-E

$$T_E - T_A = A_{A-E} \Rightarrow \frac{1}{2} m v_E^2 - \cancel{\frac{1}{2} m v_A^2} = \frac{1}{2} k\delta^2 - m g \mu \overline{CD} - m \cdot g \cdot R(1-\cos 45°) \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cancel{m} v_E^2 = \frac{1}{2} \cdot 50 \frac{\cancel{m} \cdot 10}{\ell} \cdot (0,2\ell)^2 - \cancel{m} \cdot 10 \cdot 0,1 \cdot 0,5\ell - \cancel{m} \cdot 10 \cdot 0,6\ell\,(1-\cos 45°) \Rightarrow$$

$$\Rightarrow \frac{v_E^2}{2} = 10\ell - 0,5\ell - 1,7576\ell \Rightarrow v_E^2 = 2 \cdot 7,7424\ell \Rightarrow v_E^2 = 15,4848\ell \Rightarrow \boxed{v_E = 3,935\sqrt{\ell}}$$

Deo, E-G



Njutnov zakon

$$m \cdot \vec{a} = \vec{F}_R \; / \vec{i}, \vec{j} \Rightarrow$$

$$x: \; m \cdot \ddot{x} = 0 \Rightarrow \dot{x} = C_1 \Rightarrow x = C_1 t + C_2$$

$$y: \; m \cdot \ddot{y} = -mg \Rightarrow \dot{y} = -gt + C_3 \Rightarrow y = -\frac{1}{2} g t^2 + C_3 t + C_4$$

Početni uslovi

$$t = 0 \Rightarrow x(0) = 0 \qquad \dot{x}(0) = v_c \frac{\sqrt{2}}{2}$$

$$y(0) = R(1-\cos 45°) \qquad \dot{y}(0) = v_c \frac{\sqrt{2}}{2}$$

Iz početnih uslova i njutnovog zakona za $t=0 \Rightarrow$

N(4)

$$\dot{x} = C_1 \Rightarrow C_1 = v_c \frac{\sqrt{2}}{2} \Rightarrow \boxed{C_1 = 2,782\sqrt{\ell}}$$

$$x = \cancel{C_1 t} + C_2 \Rightarrow \boxed{C_2 = 0}$$

$$\dot{y} = -\cancel{gt} + C_3 \Rightarrow \boxed{C_3 = 2,782\sqrt{\ell}}$$

$$y = -\cancel{\frac{1}{2} g t^2} + \cancel{C_3 t} + C_4 \Rightarrow C_4 = R(1-\cos 45) \Rightarrow \boxed{C_4 = 0,1758\,\ell}$$

Konačne jednačine

$$X = 2{,}782\sqrt{l}\cdot t$$

$$Y = -5t^2 + 2{,}782\sqrt{l}\,t + 0{,}1758\,l$$

(1) Tačka pada u G

$$Y(t_G)=0 \Rightarrow -5t_G^2 + 2{,}782\,t_G + 0{,}1758 = 0 \Rightarrow$$

$$\Rightarrow t_{G1,2} = \frac{-2{,}782 \pm \sqrt{(2{,}782)^2 + 5\cdot 4\cdot 0{,}1758}}{-10} \Rightarrow t_{G1,2} = \frac{-2{,}782 \pm 3{,}355}{-10} \Rightarrow \boxed{t_G = 0{,}6137}$$

$$X(t_G) = 2{,}782\sqrt{l}\cdot 0{,}6137 \Rightarrow \boxed{X = L = 1{,}707\,l}$$

(2) Maksimalna visina

$$\dot{Y}(t_F) = 0 \qquad -10t + 2{,}782\,l = 0 \Rightarrow \boxed{t_F = 0{,}2782}$$

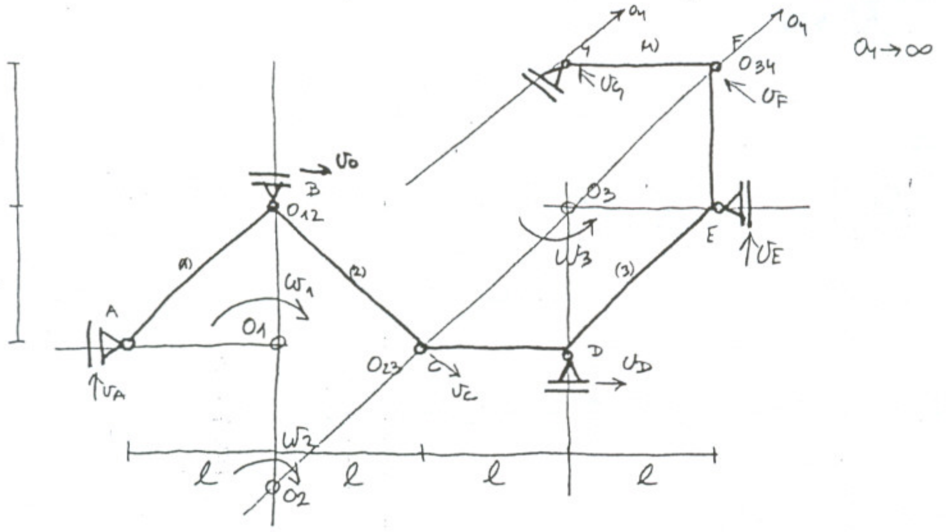
$$Y(t_F) = -5(0{,}2782)^2 + 2{,}782\cdot 0{,}2782 + 0{,}1758 \Rightarrow h = Y(t_F) \Rightarrow \boxed{h = 0{,}563\,l}$$

Kolokvijum

Zadatak 1.

U prikazanom položaju mehanizma na slici poznati su brzina tačke B: $v_B = v_0$, ubrzanje tačke D $a_D = a_0$.

(1) Ugaone brzine svih tela sistema, brzine tačaka A, D, E, G
(2) Ugaona ubrzanja svih tela sistema i ubrzanja tačaka A, B, E, G



Rešenje:

(1) Brzine i ugaone brzine

$\boxed{v_B = v_0}$

$v_B = \omega_1 \cdot \overline{O_1B} \Rightarrow \boxed{\omega_1 = \frac{v_0}{l}}$

$v_A = \omega_1 \cdot \overline{O_1A} \Rightarrow v_A = \frac{v_0}{l} \cdot l \Rightarrow \boxed{v_A = v_0}$

$v_B = \omega_2 \cdot \overline{O_2B} \Rightarrow \boxed{\omega_2 = \frac{v_0}{2l}}$

$v_C = \omega_2 \cdot \overline{CO_2} \Rightarrow v_C = \frac{v_0}{2l} \cdot l\sqrt{2} \Rightarrow \boxed{v_C = \frac{v_0 \cdot \sqrt{2}}{2}}$

$v_C = \omega_3 \cdot \overline{CO_3} \Rightarrow \omega_3 = \frac{v_0\sqrt{2}}{2l\sqrt{2}} \Rightarrow \boxed{\omega_3 = \frac{v_0}{2l}}$

$v_D = \omega_3 \cdot \overline{DO_3} \Rightarrow v_D = \frac{v_0}{2l} \cdot l \Rightarrow \boxed{v_D = v_0/2}$

$v_E = \omega_3 \cdot \overline{EO_3} \Rightarrow v_E = \frac{v_0}{2l} \cdot l \Rightarrow \boxed{v_E = v_0/2}$

$v_F = \omega_3 \cdot \overline{FO_3} \Rightarrow v_F = \frac{v_0}{2l} \cdot l\sqrt{2} \Rightarrow \boxed{v_F = v_0/\sqrt{2}}$

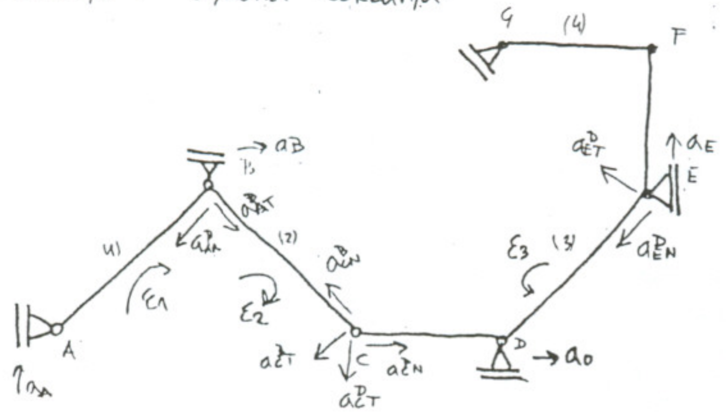
$\boxed{\omega_4 = 0}$ $\quad v_G = v_F \Rightarrow \boxed{v_G = v_0/\sqrt{2}}$

Rešenje:

$\omega_1 = \frac{v_0}{l}$ $\quad \omega_2 = \frac{v_0}{2l}$ $\quad \omega_3 = \frac{v_0}{2l}$ $\quad \omega_4 = 0$

$v_A = v_0$ $\quad v_B = v_0$ $\quad v_C = \frac{v_0}{\sqrt{2}}$ $\quad v_D = \frac{v_0}{2}$ $\quad v_E = \frac{v_0}{2}$ $\quad v_F = \frac{v_0}{\sqrt{2}}$ $\quad v_G = \frac{v_0}{\sqrt{2}}$

4) Ubrzanja i ugaona ubrzanja.



$a_{EN}^D \frac{\sqrt{2}}{2}$ $\quad a_{ET}^D \frac{\sqrt{2}}{2}$

$a_{CN}^B \frac{\sqrt{2}}{2}$ $\quad a_{CT}^B \frac{\sqrt{2}}{2}$

$a_{BN}^A \frac{\sqrt{2}}{2}$ $\quad a_{BT}^A \frac{\sqrt{2}}{2}$

E: $\vec{a}_E = \vec{a}_D + \vec{a}_{EN}^D + \vec{a}_{ET}^D \quad / \vec{i}, \vec{j}$

$$a_{EN}^D = \omega_3^2 \cdot \overline{ED} \Rightarrow a_{EN}^D = \left(\frac{v_0}{2\ell}\right)^2 \cdot \ell\sqrt{2} \Rightarrow a_{EN}^D = \frac{v_0^2\sqrt{2}}{4\ell}$$

$$a_{ET}^D = \varepsilon_3 \cdot ED \Rightarrow a_{ET}^D = \varepsilon_3 \cdot \ell\sqrt{2}$$

X: $0 = a_0 - a_{EN}^D \frac{\sqrt{2}}{2} - a_{ET}^D \frac{\sqrt{2}}{2} \Rightarrow 0 = a_0 - \frac{v_0^2 \cancel{\sqrt{2}}}{4\ell} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} - \varepsilon_3 \cdot \ell \cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} \Rightarrow$

$$\Rightarrow \varepsilon_3 \cdot \ell = a_0 - \frac{v_0^2}{4\ell} \Rightarrow \boxed{\varepsilon_3 = \frac{a_0}{\ell} - \frac{v_0^2}{4\ell^2}}$$

Y: $a_E = 0 - a_{EN}^D \frac{\sqrt{2}}{2} + a_{ET}^D \frac{\sqrt{2}}{2} \Rightarrow a_E = -\frac{v_0^2 \cancel{\sqrt{2}}}{4\ell} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} + \varepsilon_3 \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} \ell \cancel{\sqrt{2}} \Rightarrow$

$$\Rightarrow a_E = -\frac{v_0^2}{4\ell} + \left(\frac{a_0}{\ell} - \frac{v_0^2}{4\ell^2}\right) \cdot \ell \Rightarrow \boxed{a_E = a_0 - \frac{v_0^2}{2\ell}}$$

C: $\left.\begin{array}{l} \vec{a}_C = \vec{a}_D + \vec{a}_{CN}^D + \vec{a}_{CT}^D \\ \vec{a}_C = \vec{a}_B + \vec{a}_{CN}^B + \vec{a}_{CT}^B \end{array}\right\} \Rightarrow \vec{a}_D + \vec{a}_{CN}^D + \vec{a}_{CT}^D = \vec{a}_B + \vec{a}_{CN}^B + \vec{a}_{CT}^B \quad / \vec{i}, \vec{j}$

$$a_{CN}^D = \omega_3^2 \cdot CD \Rightarrow a_{CN}^D = \left(\frac{v_0}{2\ell}\right)^2 \cdot \ell \Rightarrow a_{CN}^D = \frac{v_0^2}{4\ell}$$

$$a_{CT}^D = \varepsilon_3 \cdot CD \Rightarrow a_{CT}^D = \varepsilon_3 \cdot \ell \Rightarrow a_{CT}^D = a_0 - \frac{v_0^2}{4\ell}$$

$$a_{CN}^B = \omega_2^2 \cdot BC \Rightarrow a_{CN}^B = \left(\frac{v_0}{2\ell}\right)^2 \cdot \ell\sqrt{2} \Rightarrow a_{CN}^B = \frac{v_0^2 \cdot \sqrt{2}}{4\ell}$$

$$a_{CT}^B = \varepsilon_2 \cdot BC \Rightarrow a_{CT}^B = \varepsilon_2 \cdot \ell\sqrt{2}$$

X: $a_0 + a_{CN}^D = a_B - a_{CN}^B \frac{\sqrt{2}}{2} - a_{CT}^B \frac{\sqrt{2}}{2} \Rightarrow a_0 + \frac{v_0^2}{4\ell} = a_B - \frac{v_0^2 \cancel{\sqrt{2}}}{4\ell} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} - \varepsilon_2 \cdot \ell \cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} \Rightarrow$

$$\Rightarrow a_0 + \frac{v_0^2}{4\ell} = a_B - \frac{v_0^2}{4\ell} - \varepsilon_2 \ell$$

Y: $0 - a_{CT}^D = 0 + a_{CN}^B \frac{\sqrt{2}}{2} - a_{CT}^B \frac{\sqrt{2}}{2} \Rightarrow -a_0 + \frac{v_0^2}{4\ell} = + \frac{v_0^2 \cancel{\sqrt{2}}}{4\ell} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} - \varepsilon_2 \cdot \ell \cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} \Rightarrow$

$$\Rightarrow \varepsilon_2 \cdot \ell = a_0 - \cancel{\frac{v_0^2}{4\ell}} + \cancel{\frac{v_0^2}{4\ell}} \Rightarrow \boxed{\varepsilon_2 = \frac{a_0}{\ell}} \qquad \boxed{a_B = 2a_0 + \frac{v_0^2}{2\ell}}$$

A: $\vec{a_A} = \vec{a_B} + \overrightarrow{a^B_{AN}} + \overrightarrow{a^B_{AT}} \quad / \vec{x}, y$

$$a^B_{AN} = \omega_1^2 \cdot AB \Rightarrow a^B_{AN} = \left(\frac{v_0}{l}\right)^2 \cdot l\sqrt{2} \Rightarrow a^B_{AN} = \frac{v_0^2}{l} \cdot \sqrt{2}$$

$$a^B_{AT} = \varepsilon_1 \cdot AB \Rightarrow a^B_{AT} = \varepsilon_1 \, l\sqrt{2}$$

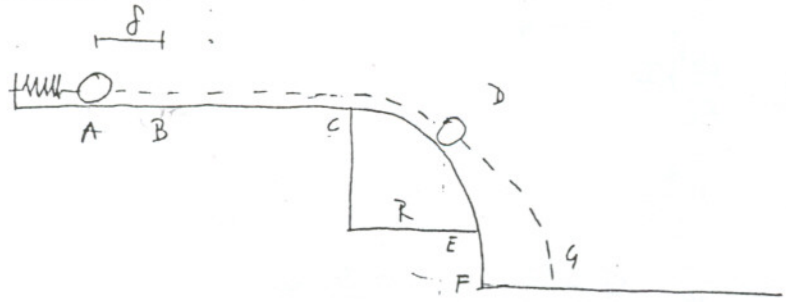
$$X: \quad 0 = a_B - \frac{\sqrt{2}}{2} a^B_{AN} + \frac{\sqrt{2}}{2} a^B_{AT} \Rightarrow 0 = 2a_0 + \frac{v_0^2}{2l} - \frac{\sqrt{2}}{\cancel{2}} \cdot \frac{v_0^2\sqrt{2}}{l} + \varepsilon_1 \, l\sqrt{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \Rightarrow$$

$$\Rightarrow \varepsilon_1 l = -2a_0 - \frac{v_0^2}{2l} + \frac{v_0^2}{l} \Rightarrow \boxed{\varepsilon_1 = -\frac{2a_0}{l} + \frac{v_0^2}{2l^2}}$$

$$Y: \quad a_A = 0 - a^B_{AN} \frac{\sqrt{2}}{2} - a^B_{AT} \frac{\sqrt{2}}{2} \Rightarrow a_A = -\frac{v_0^2}{l} \frac{\sqrt{2} \cdot \sqrt{2}}{\cancel{2}} - \varepsilon_1 \cdot l\sqrt{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \Rightarrow$$

$$\Rightarrow a_A = -\frac{v_0^2}{l} + 2a_0 - \frac{v_0^2}{2l} \Rightarrow \boxed{a_A = 2a_0 - \frac{3}{2}\frac{v_0^2}{l}}$$

Zadatak 2



AB - Idealno glatko
BC - Hrapavo
CDEF - Idealno glatko

$\delta = 0{,}2R$ $\qquad k = 15 \cdot \frac{m \cdot g}{R}$

$BC = 2R$

$EF = 0{,}25R$ $\qquad \mu = 0{,}1$

Rešenje:

Deo A-B

$$T_B - T_A^{\circ} = A_{A-B} \Rightarrow \frac{1}{2} m \cdot v_B^2 - \cancel{\frac{1}{2} m v_A^2} = \frac{1}{2} \cdot k \cdot \delta^2 \Rightarrow m v_B^2 = k \cdot \delta^2 \Rightarrow v_B^2 = \frac{k \cdot \delta^2}{m} \Rightarrow$$

$$\Rightarrow v_B^2 = \frac{15 \cdot \cancel{m} \cdot g}{R} \cdot \frac{(0{,}2R)^2}{\cancel{m}} \Rightarrow \boxed{v_B^2 = 0{,}6gR}$$

Deo B-C

$$T_C - T_B = A_{B-C} \Rightarrow \frac{1}{2} \cancel{m} v_C^2 - \frac{1}{2} \cancel{m} v_B^2 = -\cancel{m} \cdot g \cdot \mu \cdot \overline{BC} \Rightarrow \frac{v_C^2}{2} = \frac{0{,}6gR}{2} - g \cdot 0{,}1 \cdot 2R \Rightarrow$$

$$\Rightarrow v_C^2 = 0{,}6gR - 0{,}4gR \Rightarrow \boxed{v_C^2 = 0{,}2gR}$$

Deo C-D

$$T_D - T_C = A_{C-D} \Rightarrow \frac{1}{2} \cancel{m} v_D^2 - \frac{1}{2} \cancel{m} v_C^2 = \cancel{m} g \cdot R(1-\cos\alpha) \Rightarrow$$

$$\Rightarrow \frac{v_D^2}{2} = \frac{0{,}2gR}{2} + gR(1-\cos\alpha) \Rightarrow v_D^2 = 0{,}2gR + 2gR - 2gR\cos\alpha \Rightarrow$$

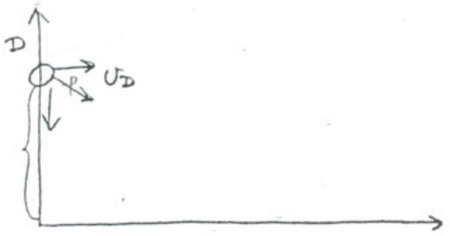
$$\Rightarrow \boxed{v_D^2 = 2{,}2gR - 2gR\cos\alpha}$$

Mesto gde napušta podlogu

$$N(\alpha) = \cancel{m} g \cdot \cos\alpha - \frac{\cancel{m} \cdot v_D^2}{R} = 0 \Rightarrow g \cdot \cos\alpha - \frac{(2{,}2g\cancel{R} - 2g\cancel{R}\cos\alpha)}{\cancel{R}} = 0 \Rightarrow$$

$$\Rightarrow \cos\alpha - 2{,}2 + 2\cos\alpha = 0 \Rightarrow 3\cos\alpha = 2{,}2 \Rightarrow \boxed{\alpha = 42{,}83}$$

$$v_D^2 = 2{,}2gR - 1{,}467gR \Rightarrow \boxed{v_D^2 = 0{,}733gR}$$



$$m\cdot\vec{a}=\vec{F_R} \Rightarrow \Big/ \begin{matrix}\vec{i}\\ \vec{j}\end{matrix} \qquad \begin{matrix} m\ddot{x}=0 & \Rightarrow \dot{x}=C_1 & \Rightarrow x=C_1t+C_2 \\ m\ddot{y}=-mg & \Rightarrow \dot{y}=-gt+C_3 & \Rightarrow y=-\frac{1}{2}gt^2+C_3t+C_4 \end{matrix}$$

Početni uslovi

$t=0 \Rightarrow \quad x(0)=0$

$$y(0)=\overline{EF}+R\cdot\sin\alpha \Rightarrow y(0)=0{,}25R+0{,}6798R \Rightarrow y(0)=0{,}9298\,R$$

$$\dot{x}(0)=v_D\cdot\cos\alpha = 0{,}856\sqrt{gR}\cos 42{,}83 \Rightarrow \dot{x}(0)=0{,}6277\sqrt{gR}$$

$$\dot{y}(0)=-v_D\sin\alpha = -0{,}856\sqrt{gR}\cdot\sin 42{,}83 \Rightarrow \dot{y}(0)=-0{,}5819\sqrt{gR}$$

$\dot{x}=C_1 \Rightarrow \boxed{C_1=0{,}6277\sqrt{gR}}$

$\dot{y}=\cancel{-gt}+C_3 \Rightarrow \boxed{C_3=-0{,}5819\sqrt{gR}}$

$x=C_1t+C_2 \Rightarrow \boxed{C_2=0}$

$y=-\frac{1}{2}gt+C_3\cdot t+C_4 \Rightarrow \boxed{C_4=0{,}9298\,R}$

Konačne jednačine

$$x(t)=0{,}6277\sqrt{gR}\cdot t$$

$$y(t)=-\frac{1}{2}gt^2-0{,}5819\sqrt{gR}\,t+0{,}9298\,R$$

Da padne $\quad y(t_G)=0$

$$-\frac{1}{2}gt_G^2-0{,}5819\sqrt{gR}\cdot t_G+0{,}9298R=0 \Rightarrow$$

$$\Rightarrow t_G=\frac{0{,}5819\sqrt{gR}\pm\sqrt{0{,}3386\cdot g\cdot R+\frac{4}{2}\cdot 0{,}9298\,R\cdot(\frac{1}{2}g)}}{-g} \Rightarrow$$

$$\Rightarrow t_{G_{1,2}}=\frac{0{,}5819\sqrt{gR}\pm\sqrt{0{,}3386\cdot g\cdot R+1{,}8596\,Rg}}{-g} \Rightarrow \frac{0{,}5819\sqrt{gR}\pm 1{,}4826\sqrt{gR}}{-g} \Rightarrow$$

$$\Rightarrow \boxed{t_G=0{,}9007\frac{\sqrt{R}}{\sqrt{g}}}$$

$$X(t_q) = 0{,}6277\sqrt{g}\sqrt{R}\cdot t_q \Rightarrow X(t_q) = 0{,}6277\sqrt{gR}\cdot 0{,}9007\cdot\frac{\sqrt{R}}{\sqrt{g}} \Rightarrow$$

$$\Rightarrow \boxed{X(t_q) = 0{,}565\,R}$$