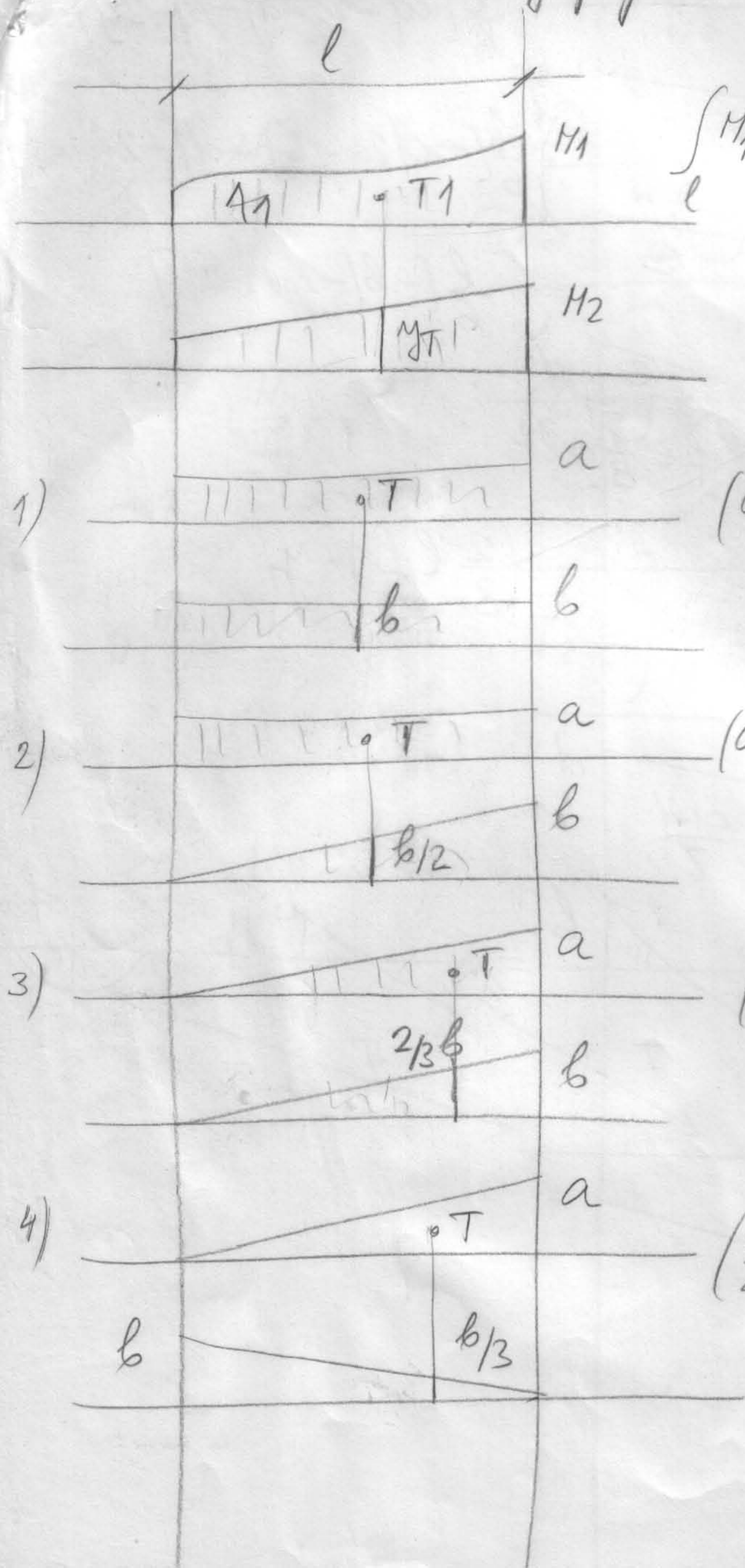
27.06.2008. 10h
04.02.2000 9h (1)

Множење дијаграма



$$\int_l M_1 M_2 \, ds = A_1 \cdot y_T$$

Н Јовановић
204/08

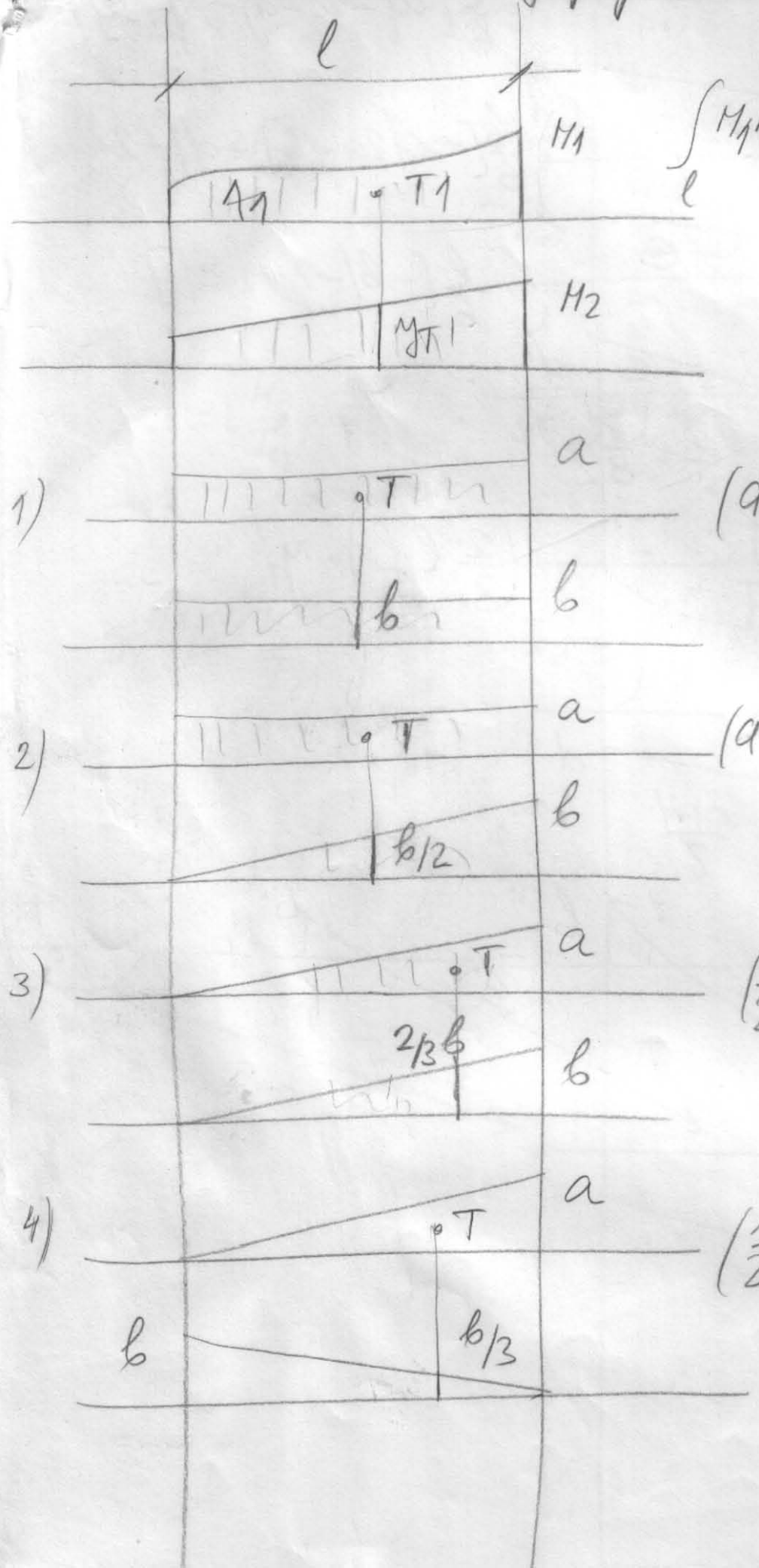
1) $(al) \cdot b$

2) $(al) \cdot \frac{b}{2}$

3) $\left(\frac{1}{2} al\right) \cdot \frac{2}{3} b$

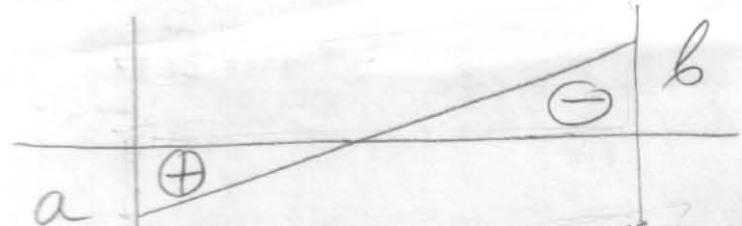
4) $\left(\frac{1}{2} al\right) \cdot \frac{b}{3}$

Mohrove gijafans

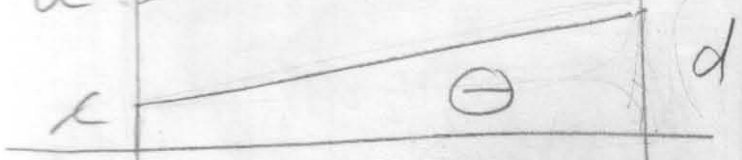


$$\int_l M_1 M_2 \, ds = A_1 \cdot y_T$$

5)

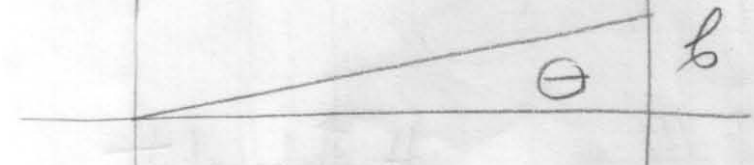


$\frac{l}{6}[a(-2c-d)-b(-2d-c$

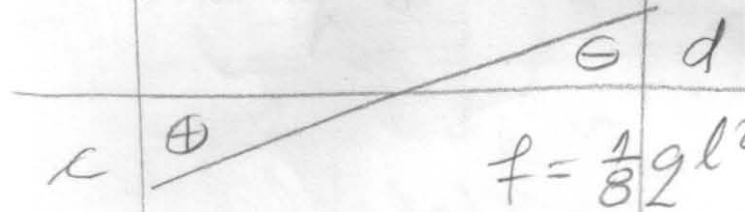


$\frac{l}{6}[-c(2a-b)-d(-2b+a)]$

6)

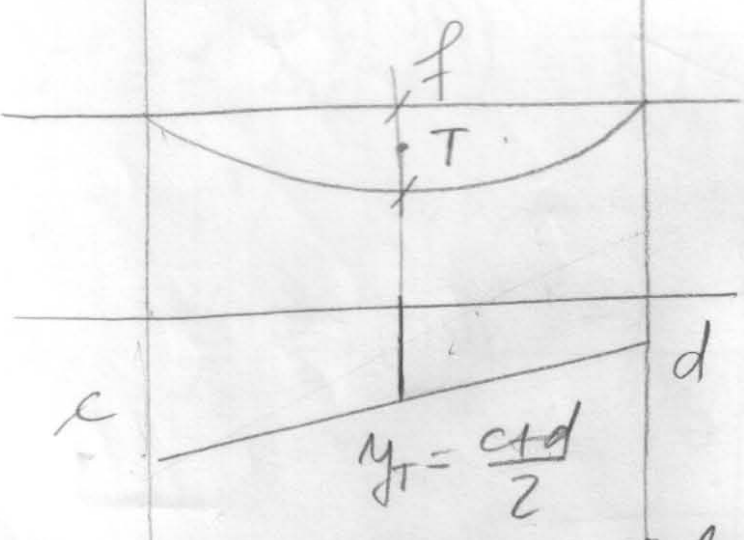


$\frac{l}{6}[-b(-2d+c)]$



$f = \frac{1}{8} q l^2$

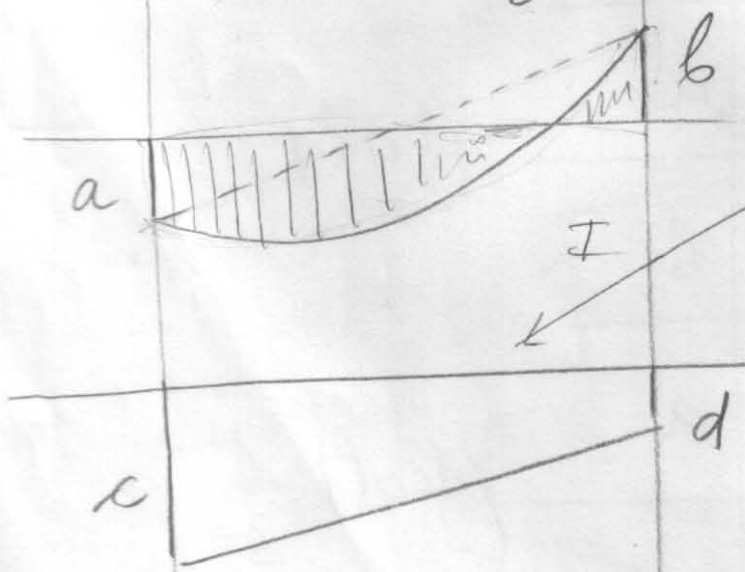
7)



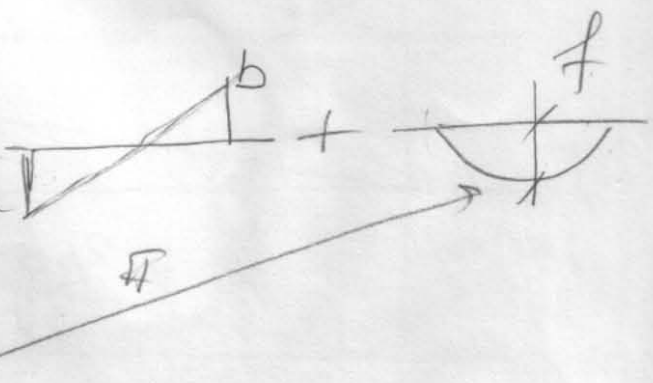
$\left(\frac{2}{3} l f\right) \cdot y_T$

$y_T = \frac{c+d}{2}$

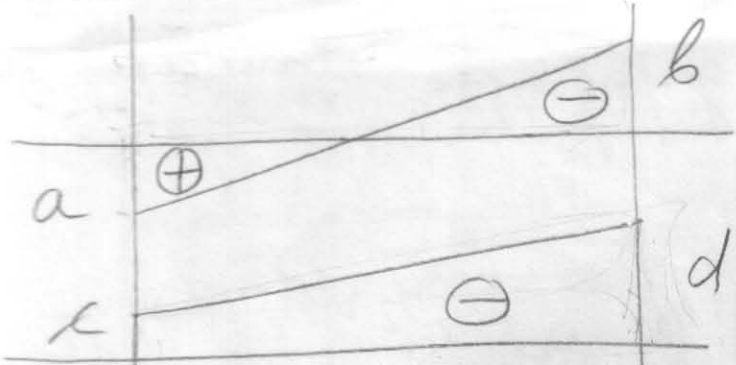
8)



$\Rightarrow$



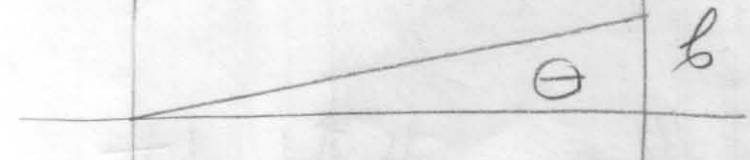
5)



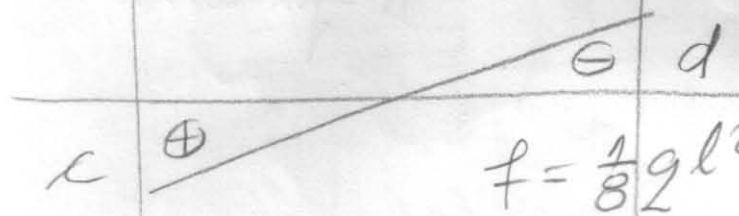
$\frac{l}{6}[a(-2c-d)-b(-2d-c)]$

$\frac{l}{6}[-c(2a-b)-d(-2b+a)]$

6)

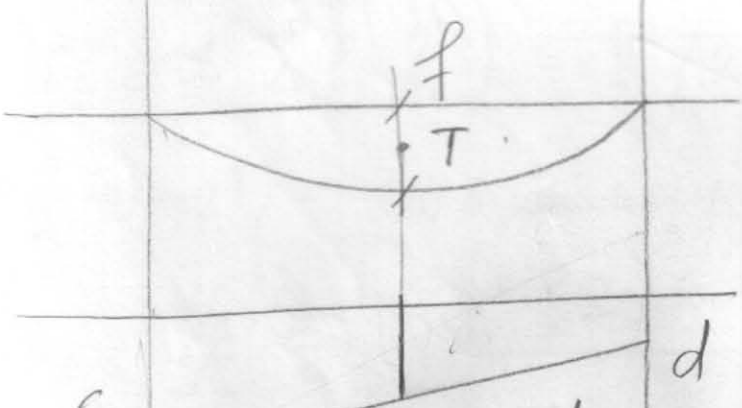


$\frac{l}{6}[-b(-2d+c)]$



$f = \frac{1}{8}ql^2$

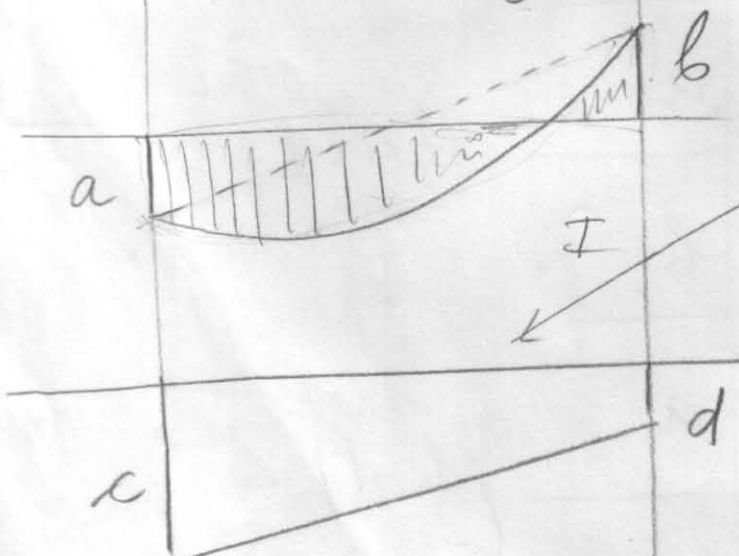
7)



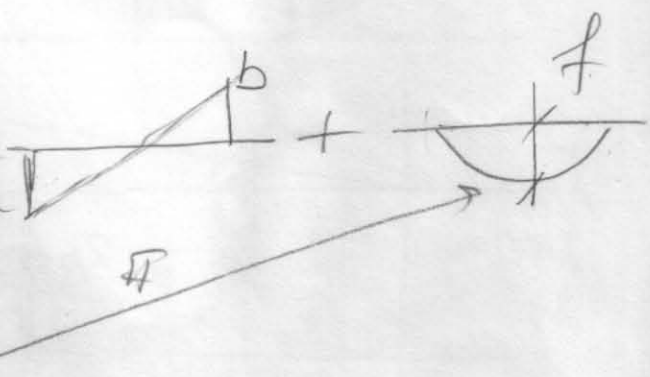
$\left(\frac{2}{3}lf\right) \cdot y_T$

$y_T = \frac{c+d}{2}$

8)



$\Rightarrow$



σ_x Нормалы компоненталы напоы

$$\sigma_x = \frac{N}{A}$$

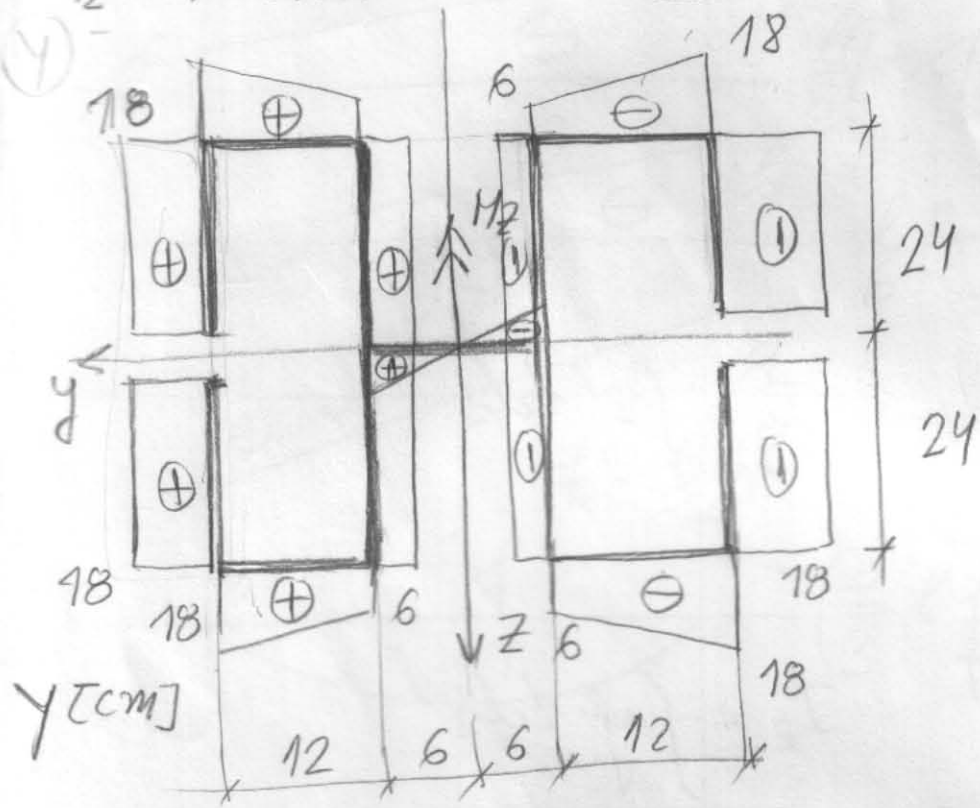
$$\sigma_x^{Hy} = \left(\frac{Hy}{Iy}\right) \cdot z^{gij} \qquad Iy = \int z^2 t\,ds$$

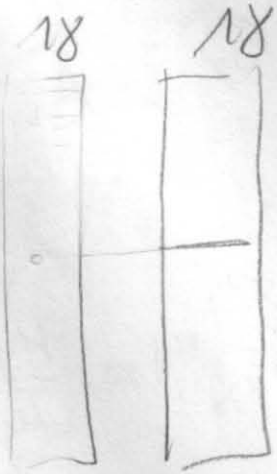
$$\sigma_x^{Hz} = -\left(\frac{Hz}{Iz}\right) \cdot y^{gij} \qquad Iz = \int y^2 t\,ds$$

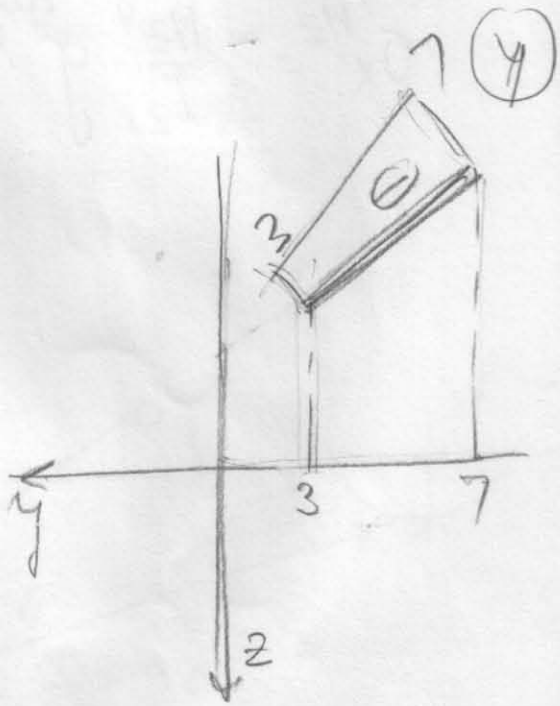
$N = 400\,kN$

$M_z = 70\,kNm$

$t = 1\,cm$







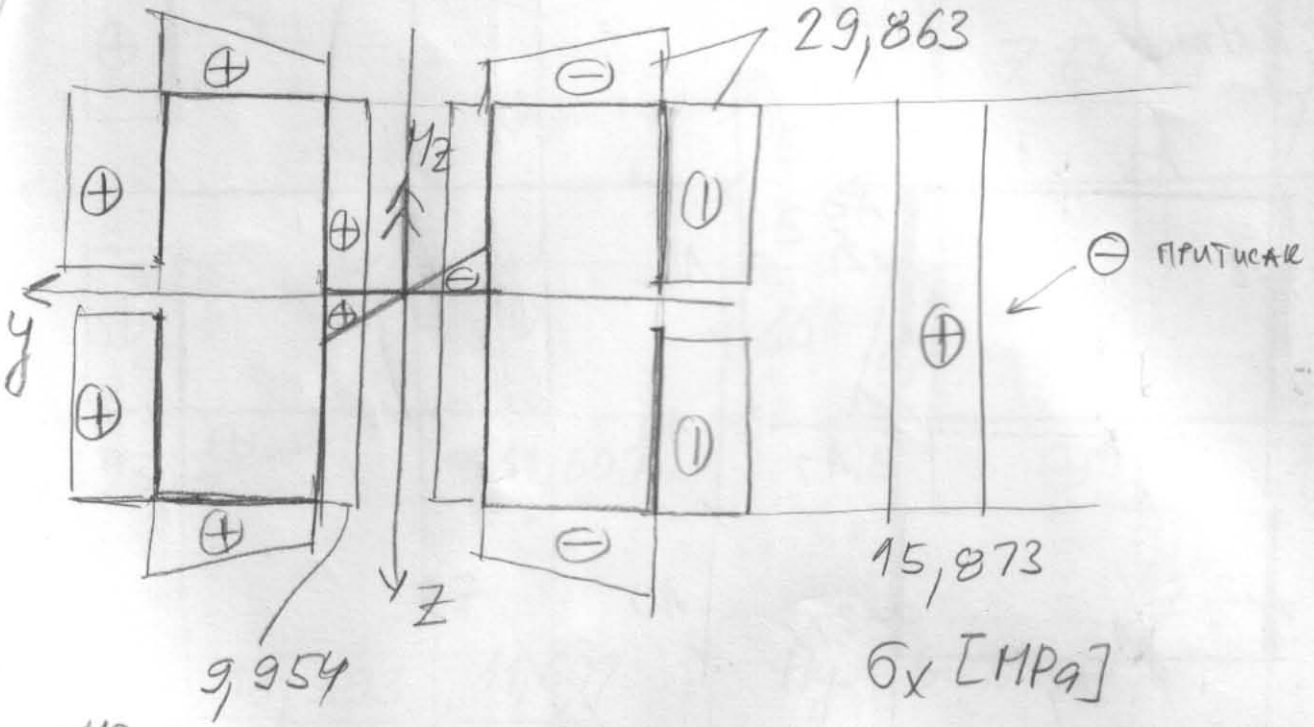
$$\sigma_x^N = \frac{N}{A}$$

$$\sigma_x^{M_z} = \left(\frac{M_z}{I_z}\right) y \qquad \text{gdje} \qquad I_z = \int y^2 t\,ds \qquad \text{gdje}$$

$$I_z = 2 \cdot 1 \cdot (18 \cdot 48) \cdot 18 + 4 \cdot 1 \cdot \frac{12}{6}\left[18\left(2 \cdot 18 + 6\right) + 6\left(2 \cdot 6 + 18\right)\right] +$$

$$+ 2 \cdot 1 \cdot (6 \cdot 48) \cdot 6 + 2 \cdot 1 \cdot \left(\frac{1}{2} \cdot 6 \cdot 6\right) \cdot \frac{2}{3} \cdot 6 = 42\,192\,cm^4$$

σ_x^{Hz} [MPa]



$$\sigma_x^{Hz} = \frac{7000\ kNcm}{42\,192\ cm^4} \cdot (6\ cm) = 9,954\ MPa$$

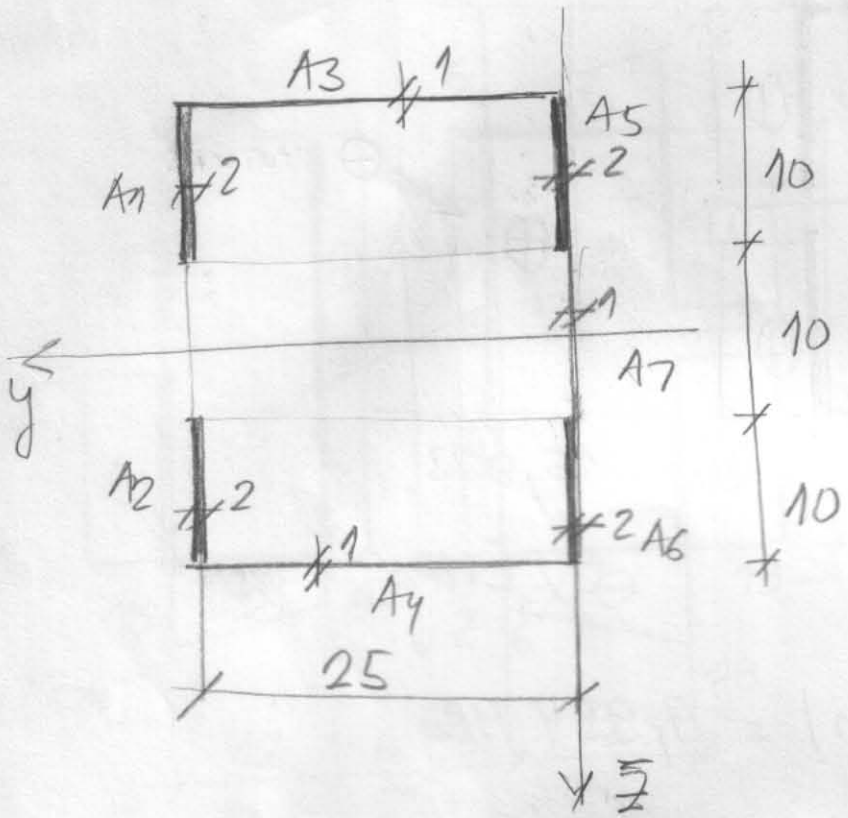
$$|18cm| = 29,863\ MPa$$

$$A = 5 \cdot 12 \cdot 1 + 4 \cdot 48 \cdot 1 = 252 cm^2$$

$$\sigma_x^{N} = \frac{400\ kN}{252 cm^2} = 15,873\ MPa$$

$M_y = 5\ kNm$

$M_z = 3\ kNm$



$A_1 = 20 \quad \bar{T}_1(25; -10)$

$A_2 = 20 \quad \bar{T}_2(25; 10)$

$A_3 = 25 \quad \bar{T}_3(12,5; -15)$

$A_4 = 25 \quad \bar{T}_4(12,5; 15)$

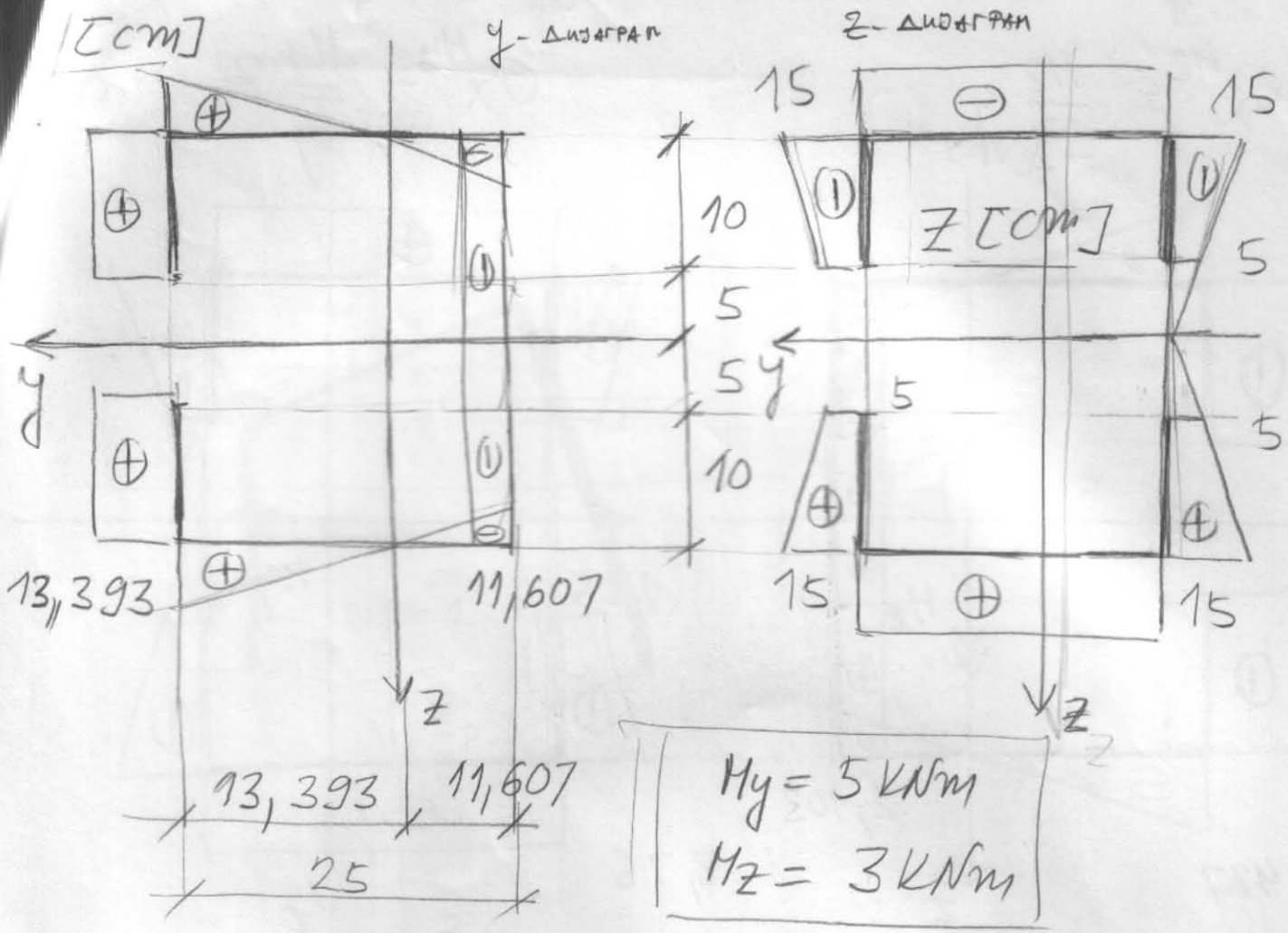
$A_5 = 20 \quad \bar{T}_5(0; -10)$

$A_6 = 20 \quad \bar{T}_6(0; 10)$

$A_7 = 10 \quad \bar{T}_7(0; 0)$

$A = 140$

$$y_T = \frac{20 \cdot 25 \cdot 2 + 25 \cdot 12,5 \cdot 2}{140} = 11,607$$



$M_y = 5\ kNm$

$M_z = 3\ kNm$

$$I_z = \int y^2 t\,ds = 2\cdot 2\left(13{,}393\cdot 10\right)\cdot 13{,}393 +$$

$$+ 2\cdot 1\cdot \frac{1}{2}\left(13{,}393 - 13{,}393\right)\cdot \frac{2}{3}\cdot 13{,}393 +$$

$$+ 2\cdot 1\cdot \frac{1}{2}\cdot\left(11{,}607\cdot 11{,}607\right)\cdot \frac{2}{3}\cdot 11{,}607 +$$

$$+ 2\cdot 2\cdot\left(11{,}607\cdot 10\right)\cdot 11{,}607 + 1\cdot\left(11{,}607\cdot 10\right)\cdot 11{,}607$$

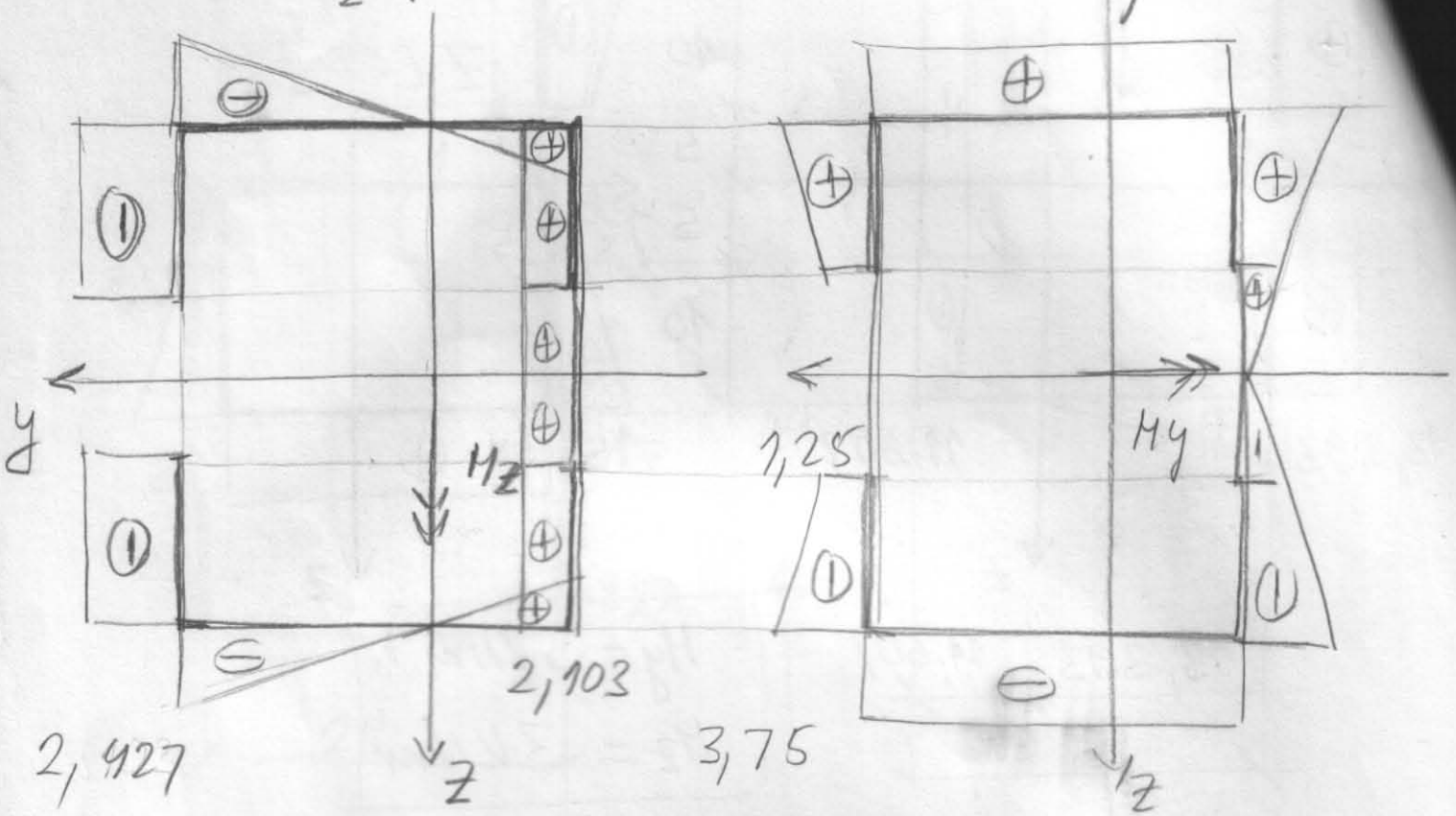
$$I_z = 16\,555{,}06\ cm^4$$

$$I_y = 4\cdot 2\cdot \frac{10}{6}\left[5\left(2\cdot 5 + 15\right) + 15\left(2\cdot 15 + 5\right)\right] +$$

$$+ 2\cdot 1\cdot\left(15\cdot 25\right)\cdot 15 + 2\cdot 1\cdot\left(\frac{1}{2}\cdot 5\cdot 5\right)\cdot\frac{2}{3}5 = \underline{20\,000\ cm^4}$$

$\sigma_x^{Mz} = \frac{Mz}{I_z} y$

$\sigma_x^{My} = \frac{My}{I_y} z$



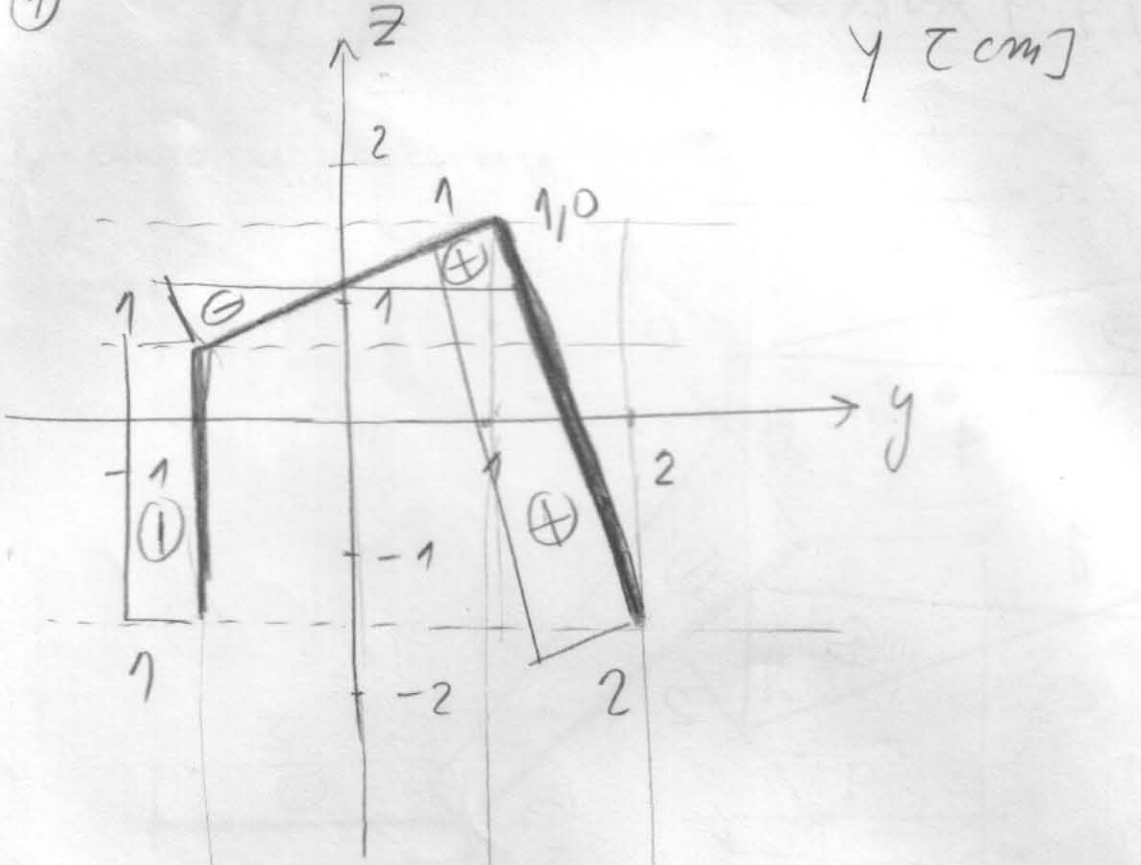
$\sigma_x^{Mz} = \frac{300\,kNcm}{16\,555,060\,cm^4} \cdot 11,607\,cm = 2,103\,MPa$

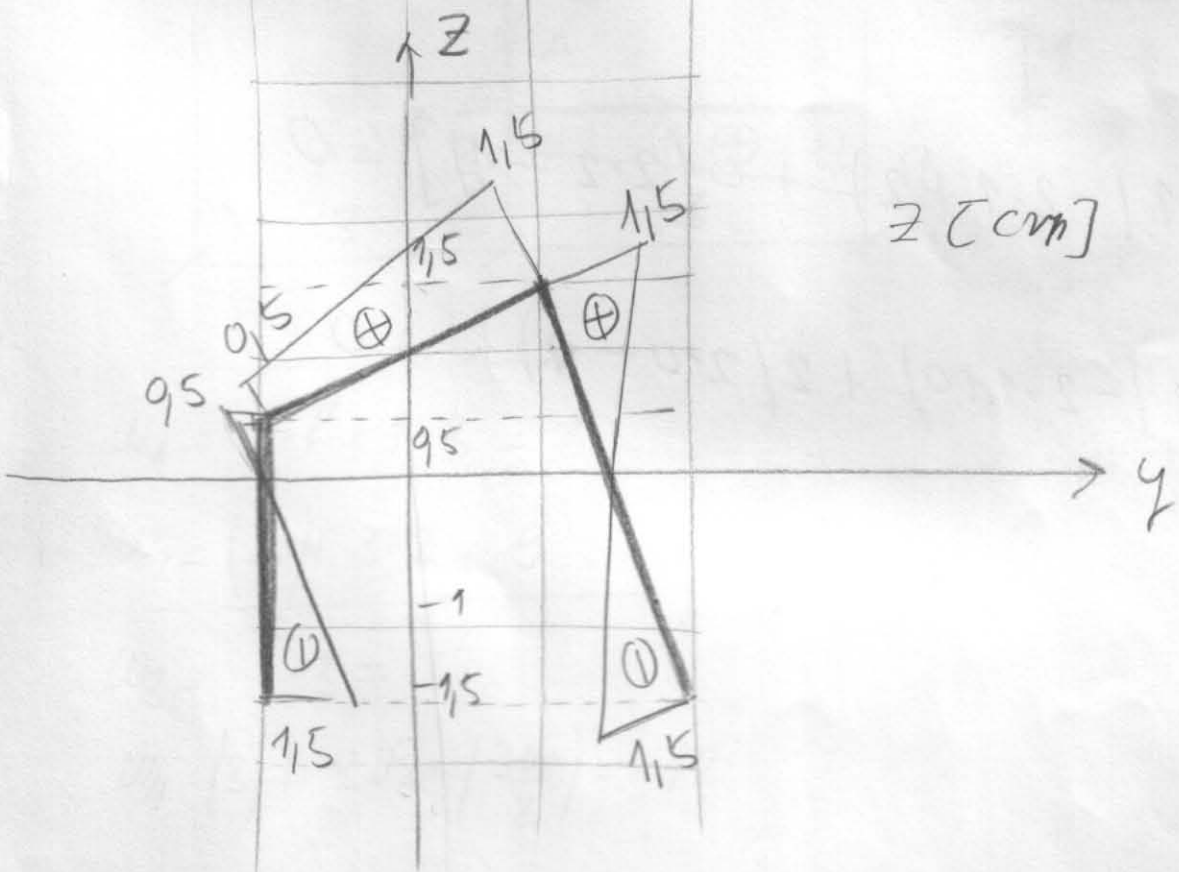
$13,393\,cm = 2,427\,MPa$

$\sigma_x^{My} = \frac{500\,kNcm}{20\,000\,cm^4} \cdot 5\,cm = 1,25$

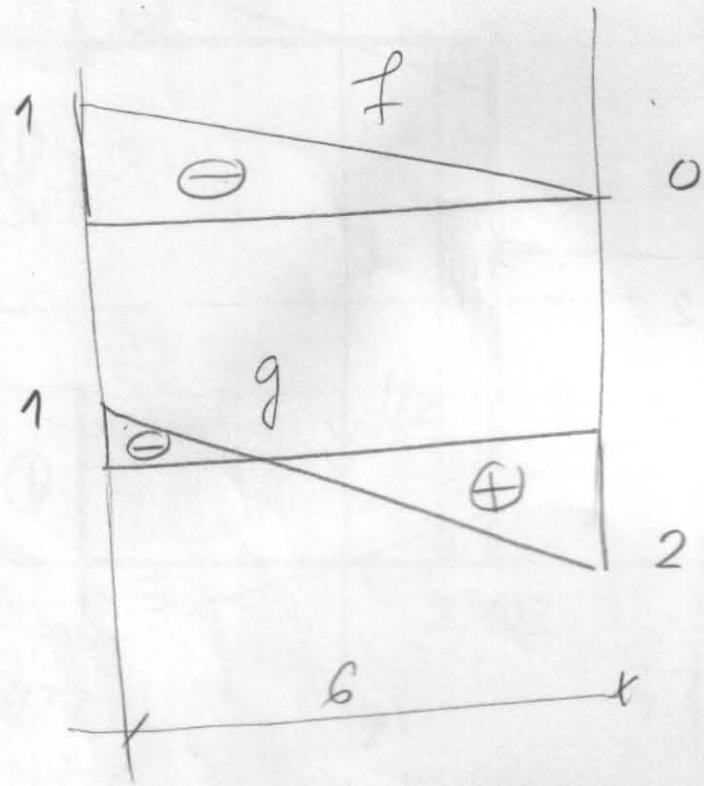
$\cdot 15\,cm = 3,75$

①





(2) $\int f \cdot g \, ds$



$$\frac{6}{6}[-1(-2\cdot 1+2) + 0\cdot(2\cdot 2-1)] = 0$$

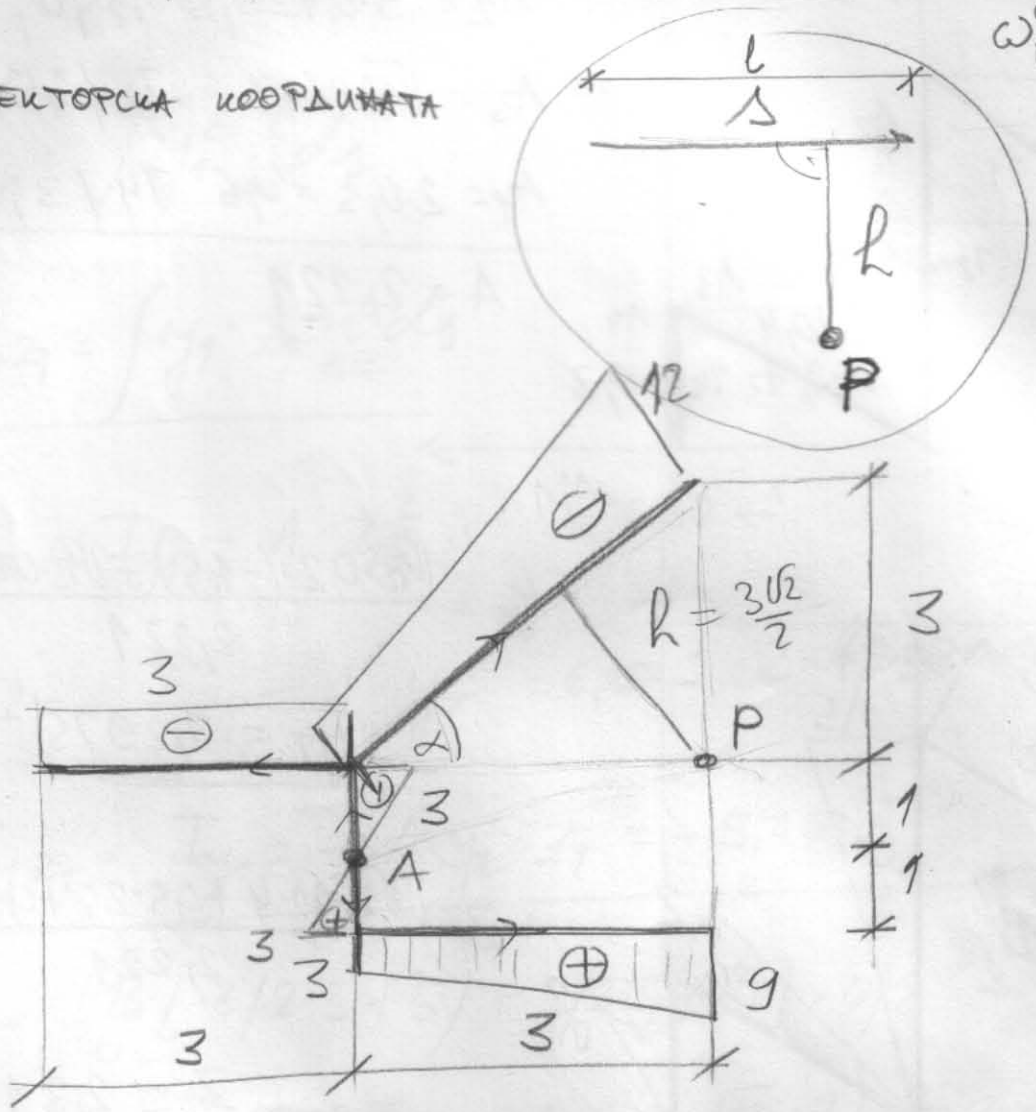
$$\frac{6}{6}[-1(-2\cdot 1+0) + 2(2\cdot 0-1)] = 0$$

⊕ ← → ⊖

3. *gijarom* $\omega_p^{(A)} = s \cdot h$

$\omega_p^{(A)} = s \cdot h = l \cdot h$

ω_p - СЕКТОРСКА КООРДИНАТА



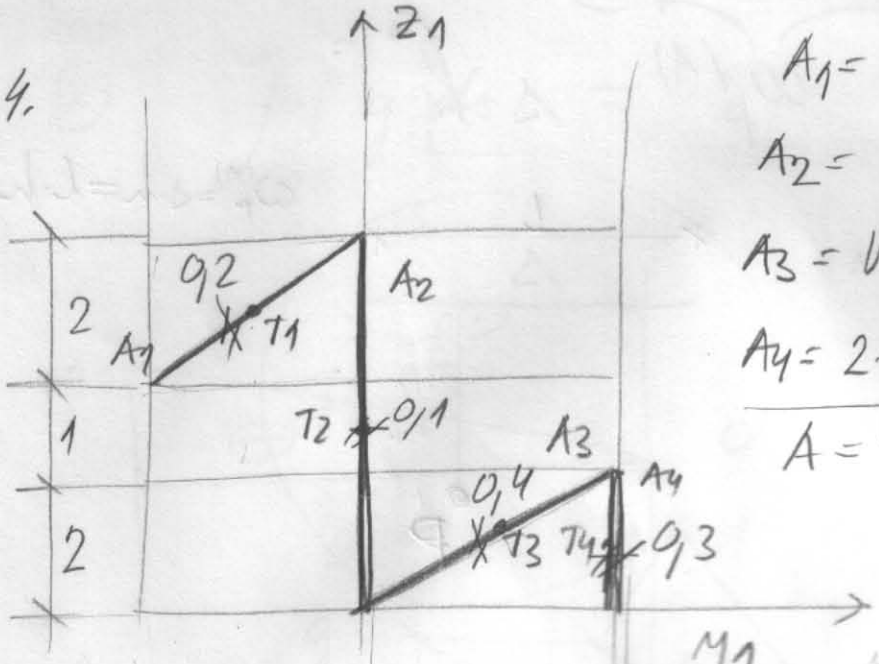
$\omega_1 = s \cdot h = 1 \cdot 3 = 3$

$\omega_2 = \left(\frac{3}{2}\right) + 3 \cdot 2 = 9$

$\omega_3 = 1 \cdot 3 = 3$

$\omega_4 = \left(\frac{3}{2}\right) + 3\sqrt{2} \cdot \left(\frac{3\sqrt{2}}{2}\right) = 12$

4.



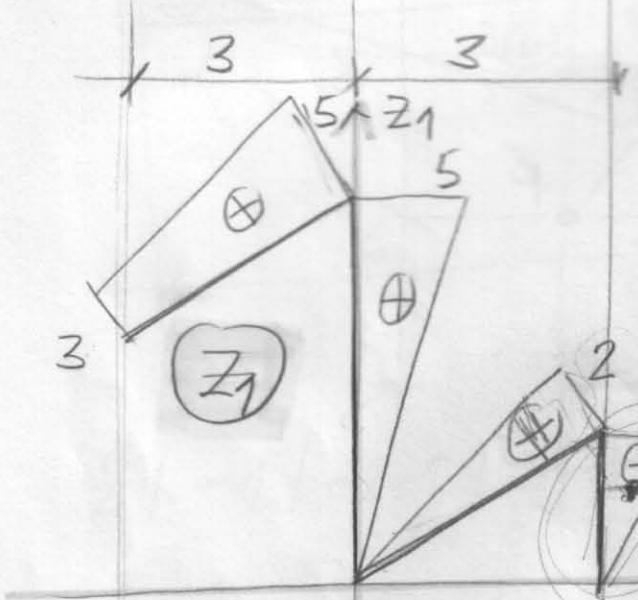
$A_1 = \sqrt{13} \cdot 0,2 \quad T_1 [-1,5; 4$

$A_2 = 5 \cdot 0,1 = 0,5 \quad T_2 [0; 2,5)$

$A_3 = \sqrt{13} \cdot 0,4 \quad T_3 [1,5; 1)$

$A_4 = 2 \cdot 0,3 = 0,6 \quad T_4 (3; 1)$

$A = 2,221$

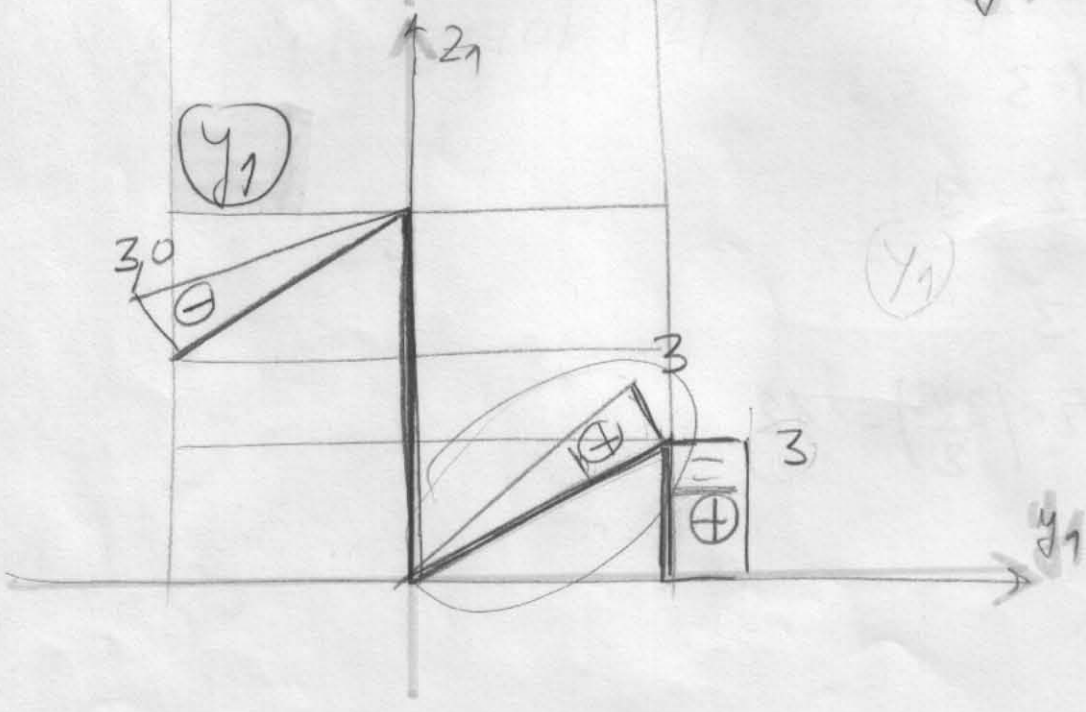


$$y_T = \frac{\sqrt{13} \cdot 0,2 \cdot (-1,5) + \sqrt{13} \cdot 0,4 \cdot 1,5 + 0,6}{2,221}$$

$$y_T = 1,2975$$

$$z_T = \frac{\sqrt{13} \cdot 0,2 \cdot 4 + 0,5 \cdot 2,5 + 0,4 \cdot 1 + 0,6}{2,221}$$

$$z_T = 2,312$$



$$I_{y_1} = \int z_1^2 \underline{\underline{t}}\, ds$$

$$I_{z_1} = \int y_1^2 \underline{\underline{t}}\, ds$$

$$I_{y_1 z_1} = \int y_1 \cdot z_1 \underline{\underline{t}}\, ds$$

$$I_{y_T} = I_{y_1} - A \cdot z_T^2 \qquad = 6{,}80\ \text{cm}^4$$

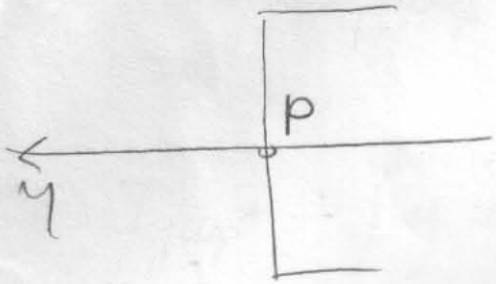
$$I_{z_T} = I_{z_1} - A \cdot y_T^2 = 6{,}35\ \text{cm}^4 \quad \text{zaokrąglam!}$$

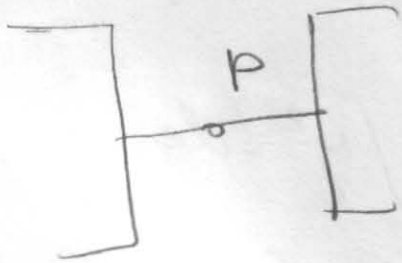
$$I_{y z_T} = I_{y_1 z_1} - A\, y_T \cdot z_T = -5{,}15\ \text{cm}^4$$

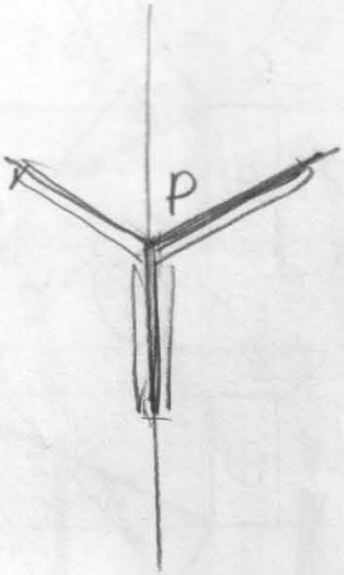
$$I_{y_1} = \frac{\sqrt{13}}{6}\left[3(2\cdot 3+5) + 5(2\cdot 5+3)\right]\cdot 0{,}2 +$$
$$+\left(\frac{1}{2}\cdot 5\cdot 5\right)\cdot\frac{2}{3}\cdot 5\cdot 0{,}1 + \left(\frac{1}{2}\sqrt{13}\cdot 2\right)\cdot\frac{2}{3}\cdot 2\cdot 0{,}4 +$$
$$+\left(\frac{1}{2}\cdot 2\cdot 2\right)\cdot\frac{2}{3}\cdot 2\cdot 0{,}3 = 18{,}67\ \text{cm}^4 \checkmark$$

$$I_{z_1} = \left(\frac{1}{2}\sqrt{13}\cdot 3\right)\cdot\frac{2}{3}\cdot 3\cdot 0{,}2 + \left(\frac{1}{2}\sqrt{13}\cdot 3\right)\cdot\frac{2}{3}\cdot 3\cdot 0{,}4 +$$
$$+ (2\cdot 3)\cdot 3\cdot 0{,}3 = 10{,}09\ \text{cm}^4 \checkmark$$

$$I_{y_1 z_1} = -\frac{\sqrt{13}}{6}\left[3(2\cdot 3+5)\right]\cdot 0{,}2 + \left(\frac{1}{2}\sqrt{13}\cdot 3\right)\cdot\frac{2}{3}\cdot 2\cdot 0{,}4 +$$
$$+\left(\frac{1}{2}\cdot 2\cdot 2\right)\cdot 3\cdot 0{,}3 = 0{,}718\ \text{cm}^4 \checkmark$$


P
y


P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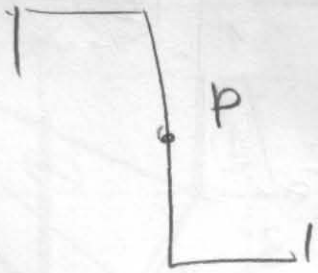
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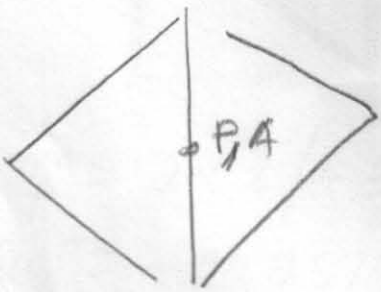
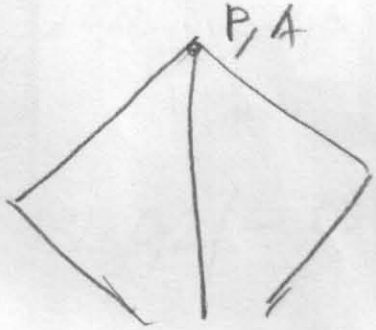
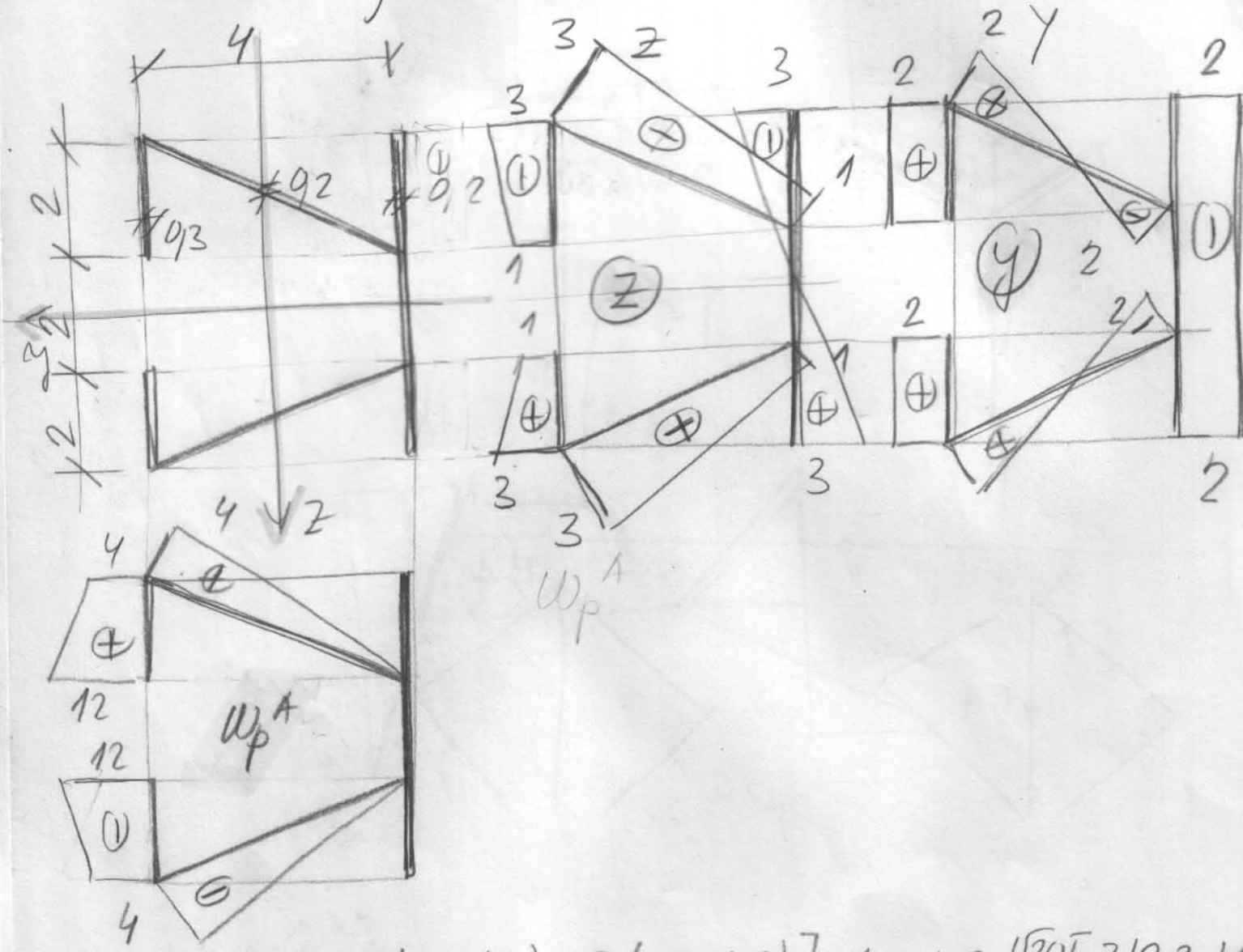

P, 4


P, 4

6. $I_y = \int z^2 t\,ds$

$I_z = \int y^2 t\,ds$

$I_{zw} = \int z \cdot w \cdot t\,ds$



$I_y = 2 \cdot \frac{2}{6}[1(2 \cdot 1 + 3) + 3(2 \cdot 3 + 1)] \cdot 0{,}3 + 2 \cdot \frac{\sqrt{20}}{6}[3(2 \cdot 3 + 1$

$+ 1(2 \cdot 1 + 3)] \cdot 0{,}2 + 2 \cdot \left(\frac{1}{2} \cdot 3 \cdot 3\right) \cdot \frac{2}{3} \cdot 3 \cdot 0{,}2 = 16{,}55\ cm^4$

$I_z = 2 \cdot (2 \cdot 2) \cdot 2 \cdot 0{,}3 + 2 \cdot \frac{\sqrt{20}}{6}[2(2 \cdot 2 - 2) - 2(-2 \cdot 2 + 2)] \cdot 0{,}2$

$+ (2 \cdot 6) \cdot 2 \cdot 0{,}2 = 11{,}985$

$I_{zw} = -\frac{2}{6}[12 \cdot (2 \cdot 1 + 3) + 4(2 \cdot 3 + 1)] \cdot 0{,}3 \cdot 2 - \frac{\sqrt{20}}{6}[4(2 \cdot 3 + 1$

$\cdot 2 \cdot 0{,}2 = -25{,}35\ cm^4$

8. y, z, ω_p^*

$$I_{y\omega_p^*} = \int y \cdot \omega_p^* \cdot t \, ds$$

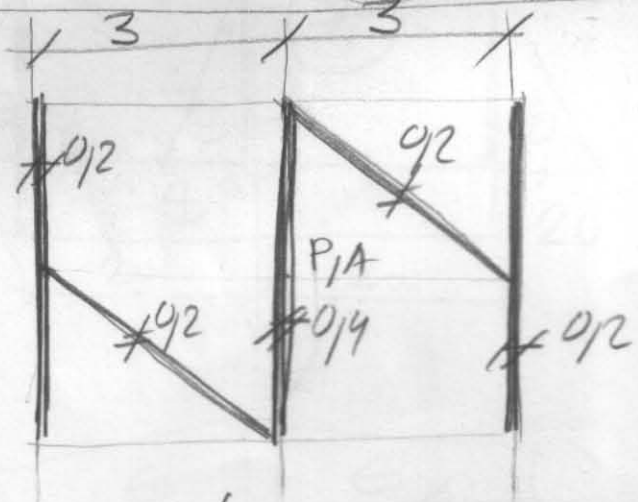
$$I_{z\omega_p^*} = \int z \omega_p^* \cdot t \, ds$$

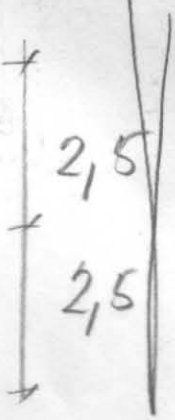
СЕКТОРСКИ ЦЕНТРИФУГАЛНИ МОМЕНТ ИНЕРЦИЈЕ

центрирамо симетрични пресек

$$\omega = \omega_p^* - \frac{S\omega_p^*}{A}$$ дај цифра

$$S\omega_p^* = \int \omega_p^* t \, ds$$



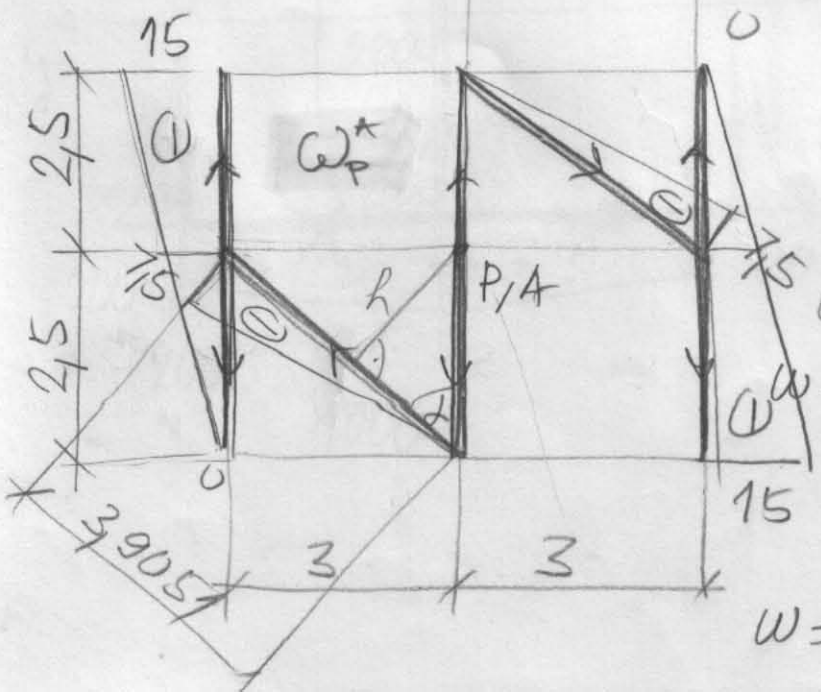


$A = (5 \cdot 0,2) \cdot 2 + 5 \cdot 0,4 + 2 \cdot 3,9051 \cdot 0,2 = 5,1562 \text{ cm}^2$

$S\omega_p^* = 2 \cdot 0,2 \left(-\frac{1}{2} 15 \cdot 5\right) + 2 \cdot 0,2 \left(-\frac{1}{2} \cdot 3,9051 \cdot 7,5\right)$

$S\omega_p^* = -20,85765 \text{ cm}^4$

$\omega_p^* = s \cdot h$



$\sin\alpha = \frac{h}{2,5} = \frac{3}{3,9051}$

$h = 1,9206$

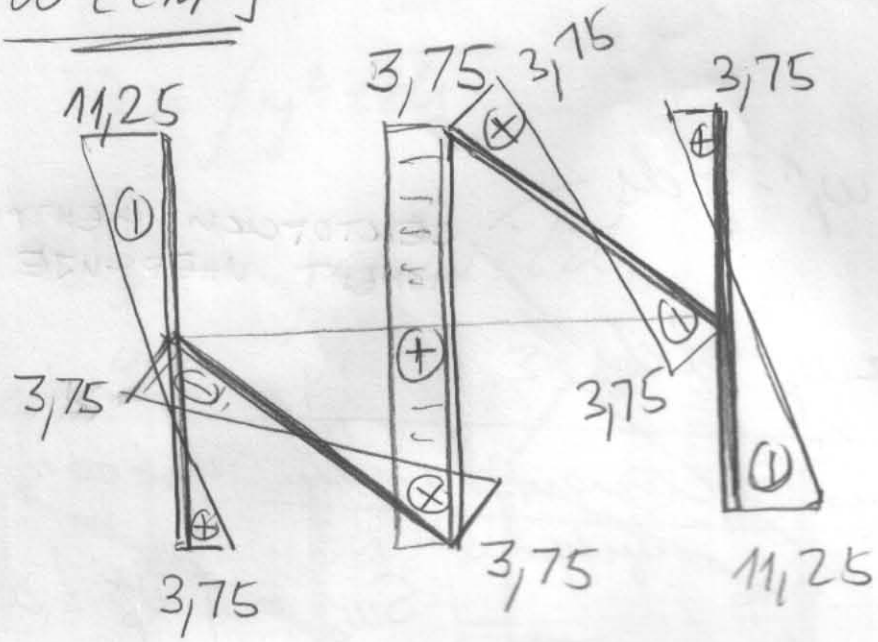
$\omega = sh = 3,9051 \cdot 1,9206 = 7,5$

$\omega = 7,5 + 2,5 \cdot 3 =$

$$\omega = \omega_p^* - \frac{-20,85765 \text{ cm}^4}{5,1562 \text{ cm}^2}$$

$\omega = \omega_p^* + 3,75$

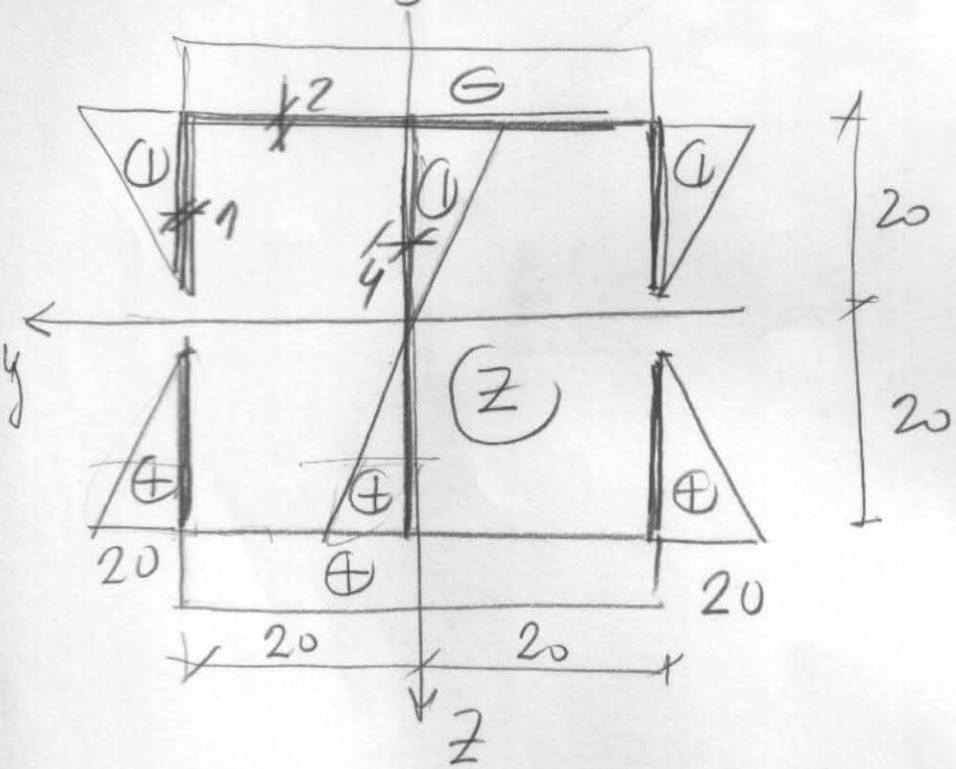
$W\ [cm^2]$

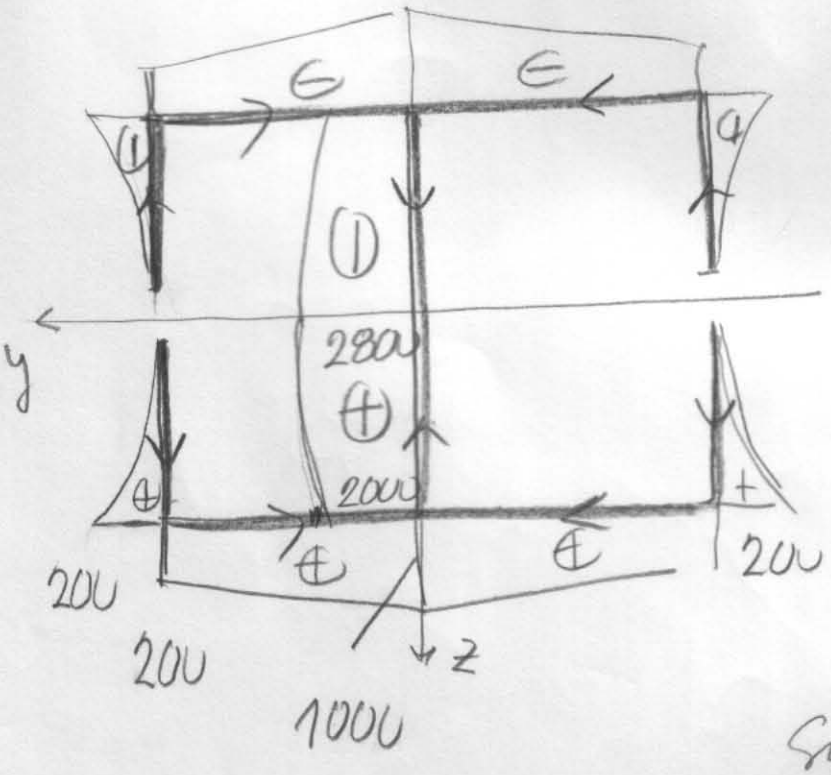


$S_y = \int z\,t\,ds$ → од слободних крајева

$S_z = \int y\,t\,ds$ — СТАТИЧКИ МОМЕНТИ ИНЕРЦИЈЕ

$S_\omega = \int \omega\,t\,ds$ — НОРМИРАНА СЕКТОРСКА КООРДИНАТА (СЕКТОРСКИ СТАТИЧКИ МОМЕНАТ)





$S_y\,[cm^3]$

$S_{y1} = \frac{1}{2} \cdot 20 \cdot 20 \cdot 1 =$

$S_{y1} = 200$

$S_{y2} = 200 + 20 \cdot 20 \cdot 2 =$

$S_{y2} = 1000$

$S_{y3} = 2000 + \frac{1}{2} \cdot 20 \cdot 20 \cdot 4$

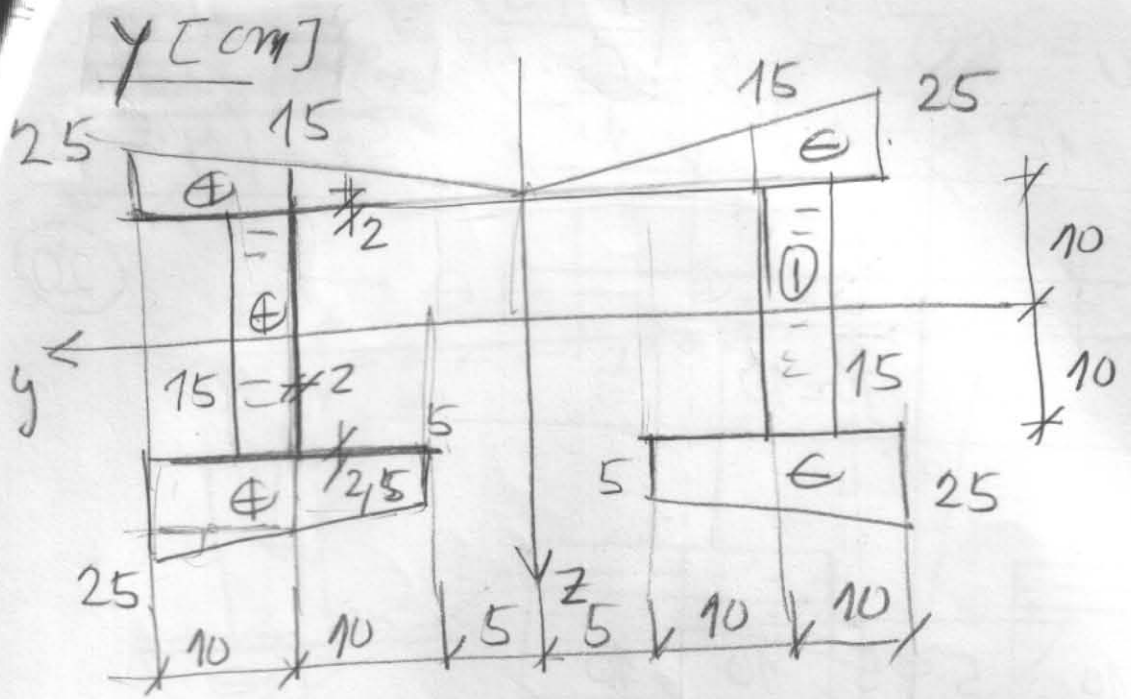
τ_x компоненти на напони

$$\tau_x^{T_y \, \text{дој}} = -\frac{T_y \cdot S_z^{*\,\text{дој}}}{I_z \cdot t}$$

$$S_z^* = \int y\, t\, ds \qquad I_z = \int y^2\, t\, ds$$

$$\tau_x^{T_z \, \text{дој}} = -\frac{T_z \cdot S_y^{*\,\text{дој}}}{I_y \cdot t}$$

$$S_y^* = \int z\, t\, ds \qquad I_y = \int z^2\, t\, ds$$

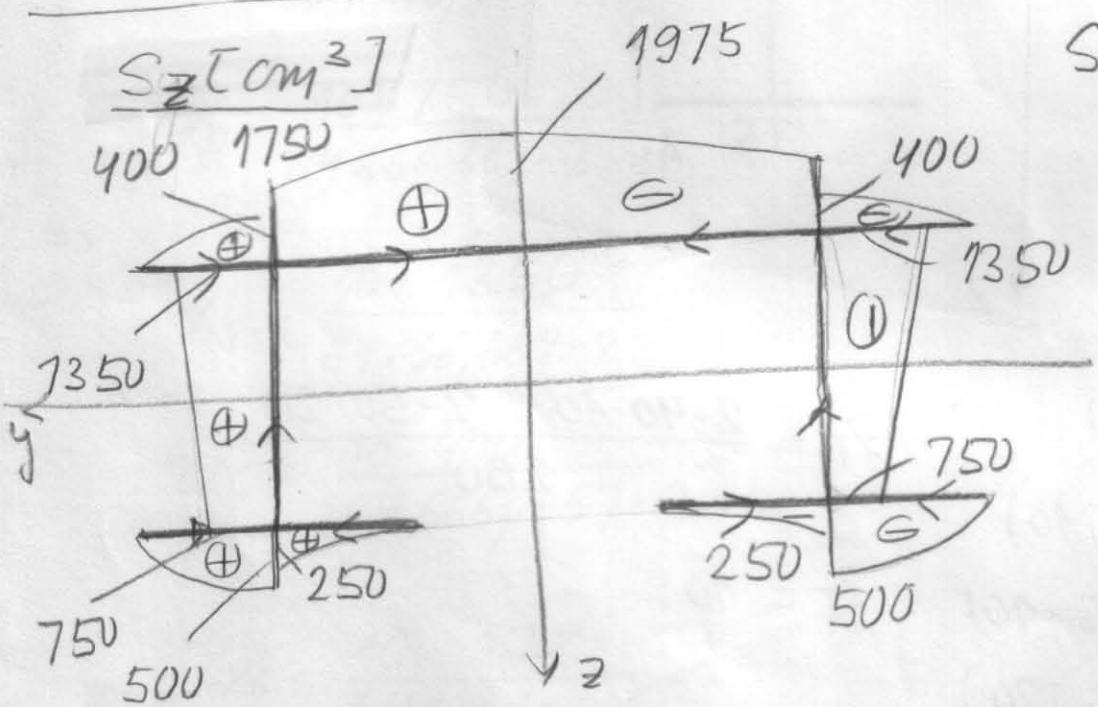


$$I_z = \int y^2 t\,ds = 2\left(\frac{1}{2}\cdot 25\cdot 25\right)\cdot\frac{2}{3}\cdot 25\times 2 +$$

$$+ 2\cdot(20\cdot 15)\cdot 15\times 2 + 2{,}5\cdot\frac{20}{6}\left[25\cdot(2\cdot 25+5)+\right.$$

$$\left.5(2\cdot 5+25)\right]\times 2 = 64\,666{,}667\ cm^4$$

$S_z = \int y\,t\,ds$



$$S_{z_1} = \frac{15+25}{2}\cdot 10\cdot 2{,}5 = 500 \qquad S_{z_2} = \frac{15+15}{2}\cdot 10\cdot 2{,}5 = 250$$

$$S_{z_3} = 750 + 15\cdot 20\cdot 2 = 1350 \qquad S_{z_4} = \frac{15+25}{2}\cdot 10\cdot 2 = 400$$

$$S_{z_5} = 1750 + \frac{1}{2}\,15\cdot 15\cdot 2 = 1975$$

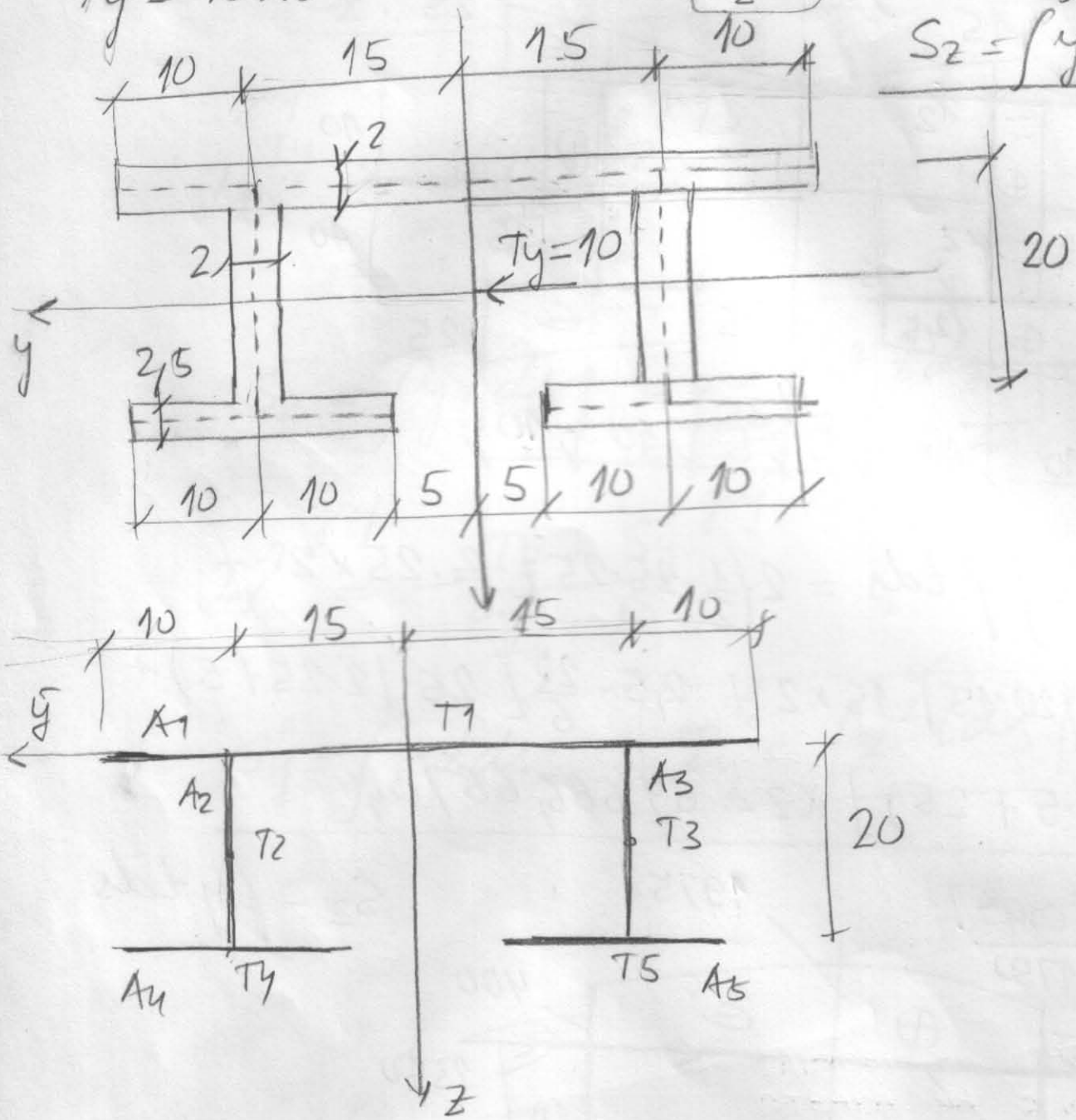
$T_y = 10\,kN$

$\tau_{xy} = \frac{T_y \cdot S_z}{I_z \cdot t}$ gdzie T_y

$I_z = \int y^2 t$

$S_z = \int y \cdot t \, ds$

(20)



$A_1 = 100 \quad T_1(0;0)$

$A_2 = 40 \quad T_2(15;10)$

$A_3 = 40 \quad T_3(-15;10)$

$A_4 = 50 \quad T_4(15;20)$

$A_5 = 50 \quad T_5(-15;20)$

$A = 280$

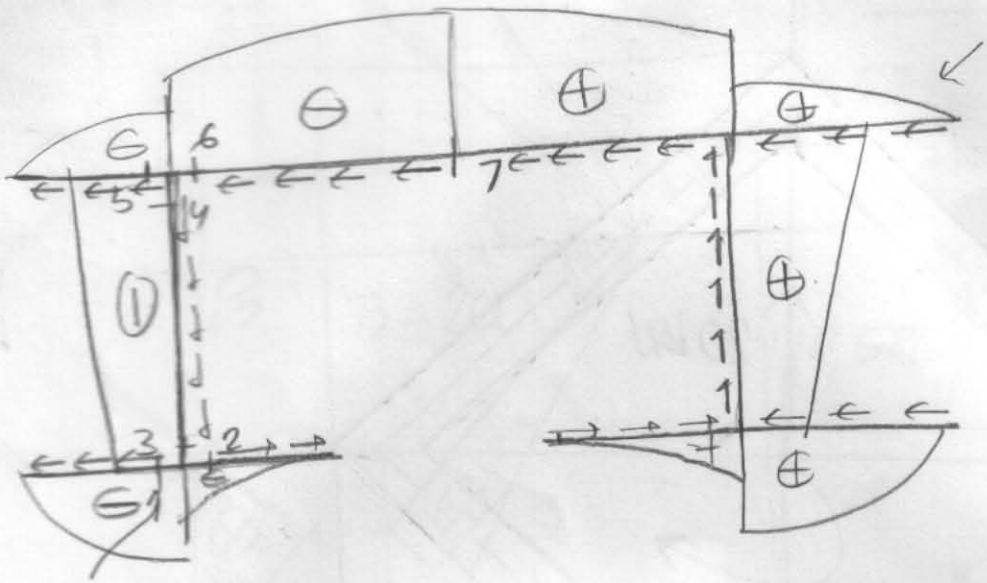
$z_T = \frac{2 \cdot 40 \cdot 10 + 2 \cdot 50 \cdot 20}{280}$

$z_T = 10$

τ_x^{Ty} [MPa]

$$\tau_x = -\frac{T_y \cdot S_z}{I_z \cdot t}$$

(21)



0,309

$$\tau_{x1} = \frac{10\,kN \cdot 500\,cm^3}{64\,666,667\,cm^4 \cdot 2,5\,cm} = 0,309\,MPa$$

$$\tau_{x2} = \frac{10 \cdot 250}{64\,666,667 \cdot 2,5} =$$

$$\tau_{x3} = \frac{10 \cdot 750}{64\,666,667 \cdot 2} =$$

$$\tau_{x4} = \frac{10 \cdot 1350}{64\,666,667 \cdot 2} =$$

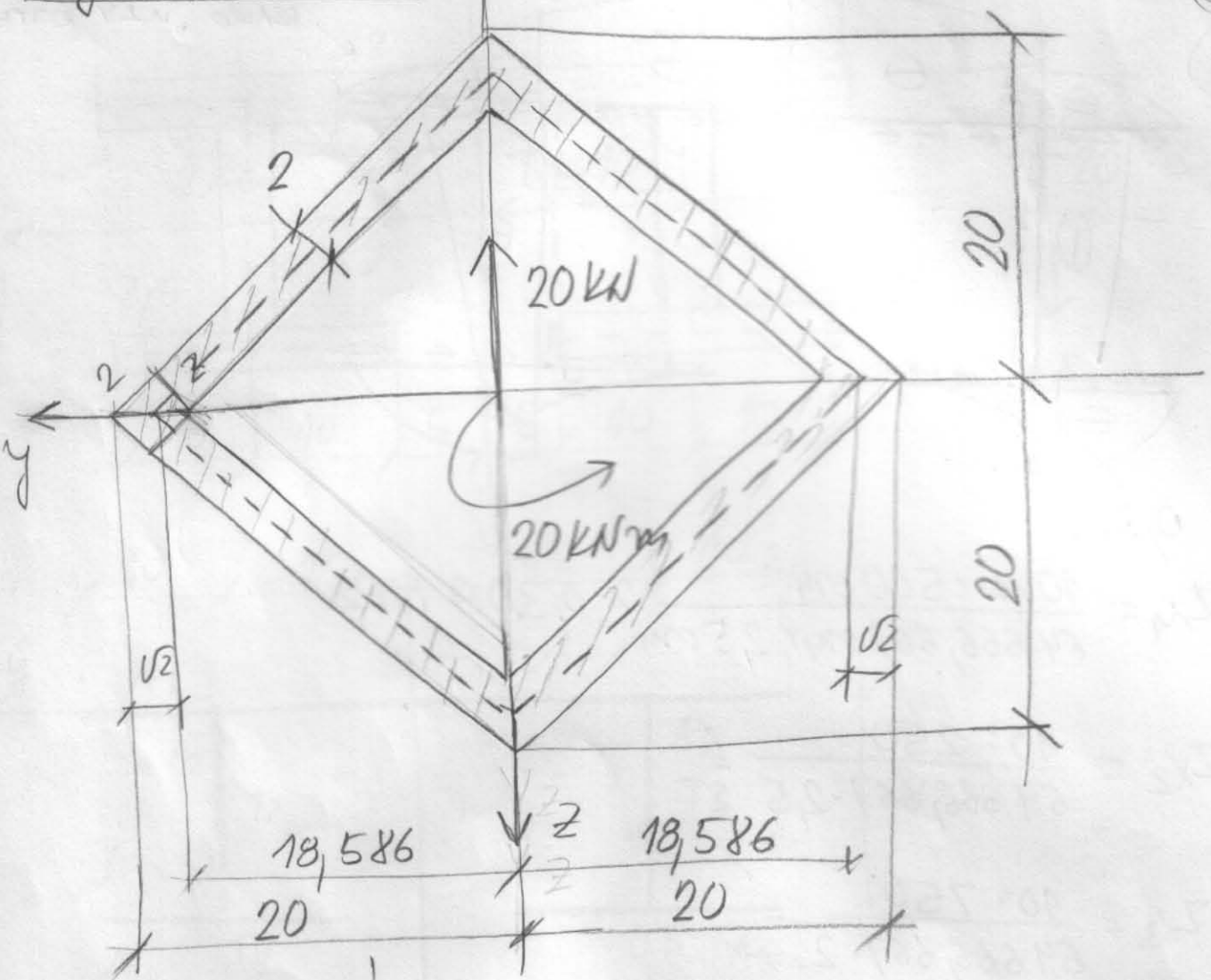
$$\tau_{x5} = \frac{10 \cdot 400}{64\,666,667 \cdot 2} =$$

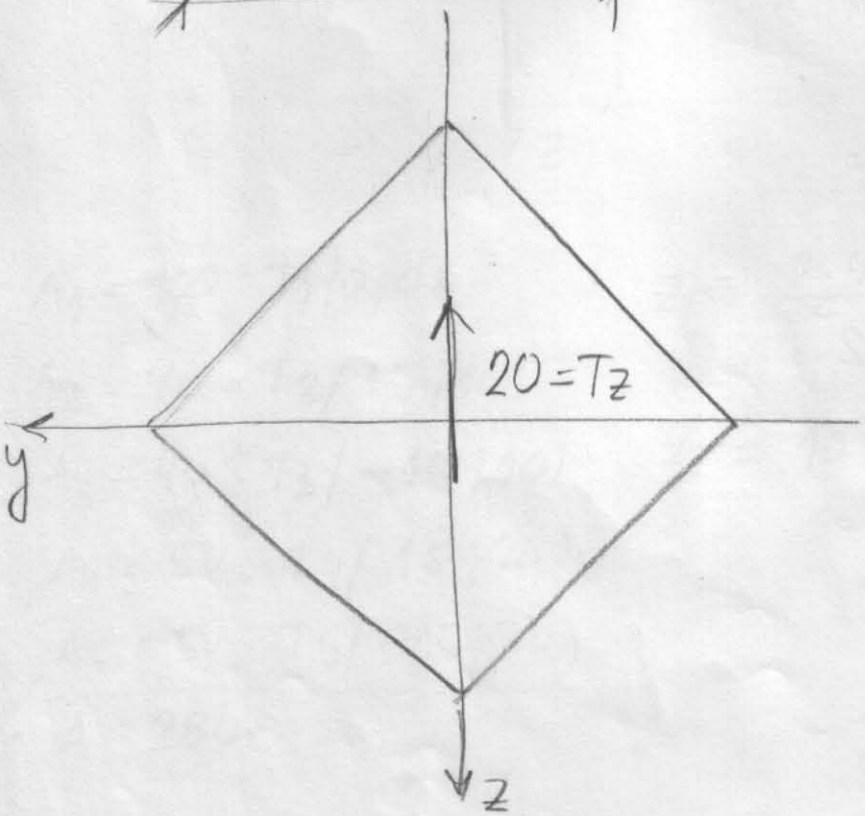
$$\tau_{x6} = \frac{10 \cdot 1750}{64\,666,667 \cdot 2} =$$

$$\tau_{x7} = \frac{10 \cdot 1975}{64\,666,667 \cdot 2} =$$

Затворени танкостидни пресеци

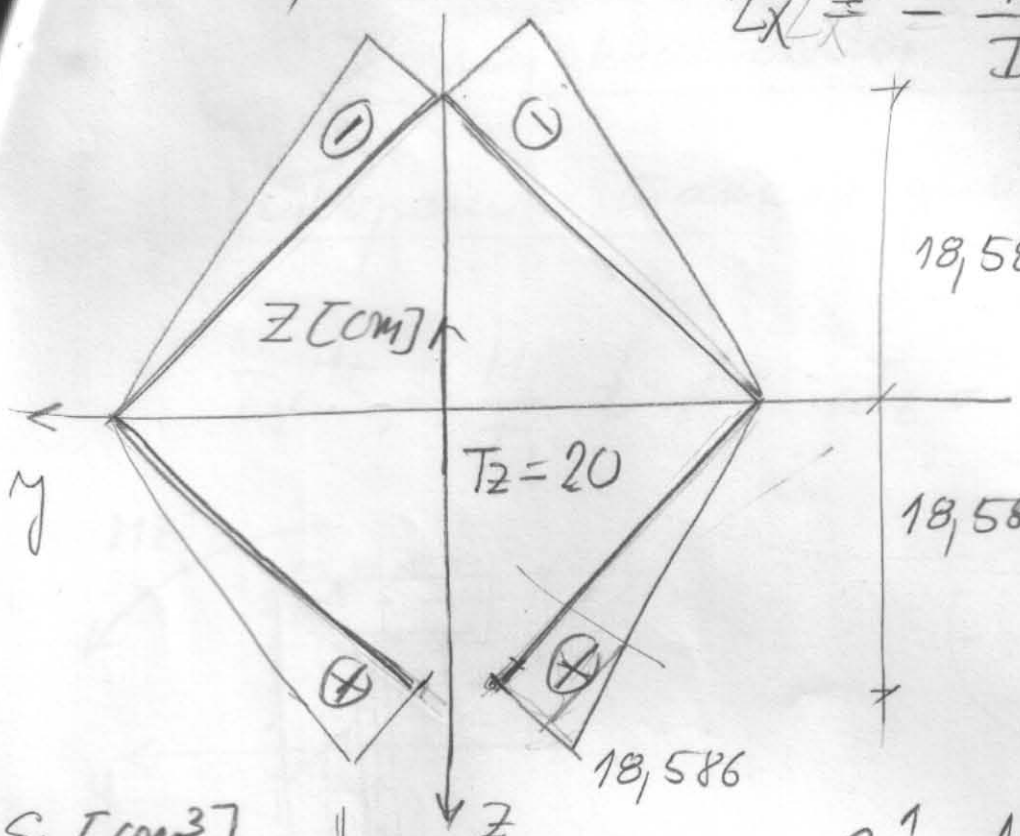
$I_y = 24395{,}5\ cm^4$



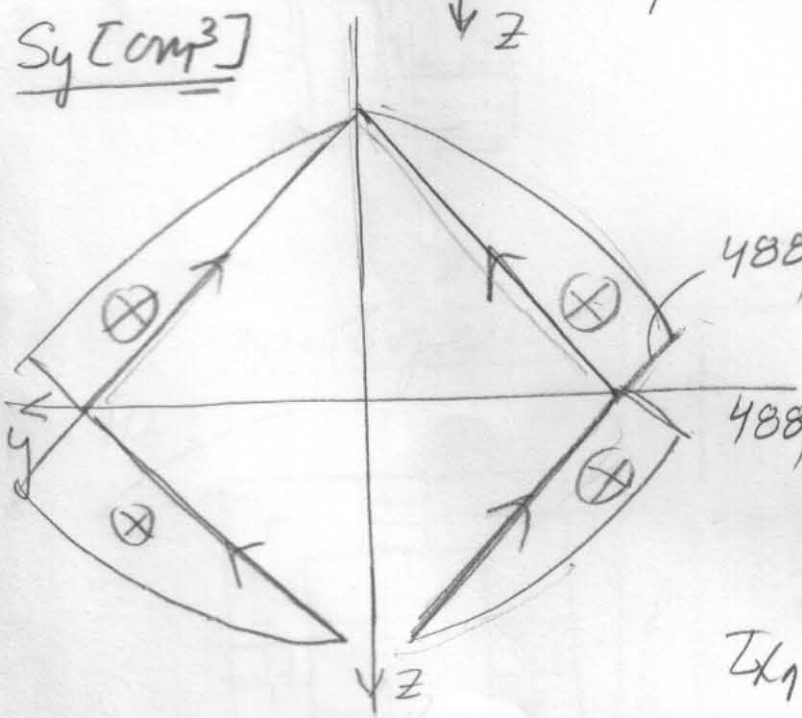


ТАНКОСИДНИ ПРЕСЕК СЕ ОТВАРА ~~НА~~ ОСИ ~~НА~~ КОЈОЈ СЕ НАЛАЗИ ТРАНСВЕРЗАЛНА СИЛА

$$\tau_{zx} = -\frac{T_z \cdot S_y}{I_y \cdot t} \qquad S_y = \int z\,t\,ds$$



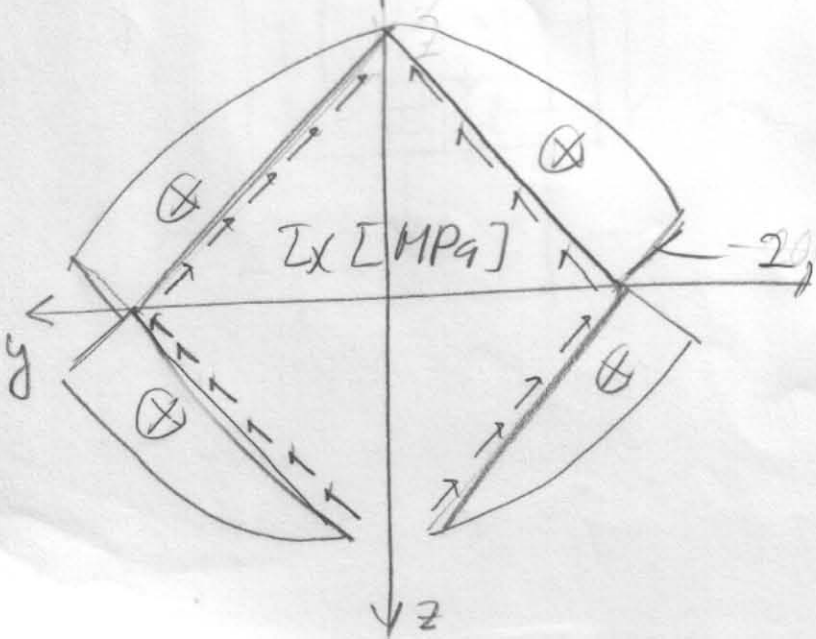
S_y [cm³]



$$S_y^1 = \frac{1}{2}\left(18{,}586\sqrt{2}\right) \cdot 18{,}586^2$$

$$\tau_{x1} = \frac{20\,kN \cdot 488{,}525\,cm^3}{24395{,}5\,cm^4 \cdot 2\,cm}$$

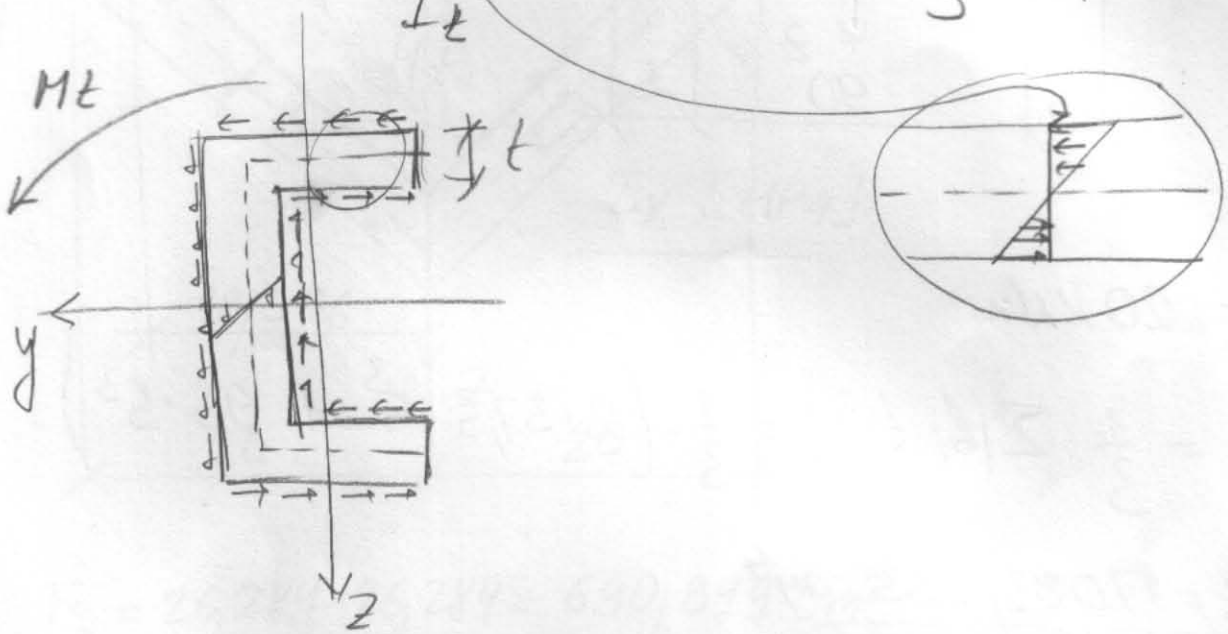
$$\tau_{x1} = 2{,}003\,MPa$$



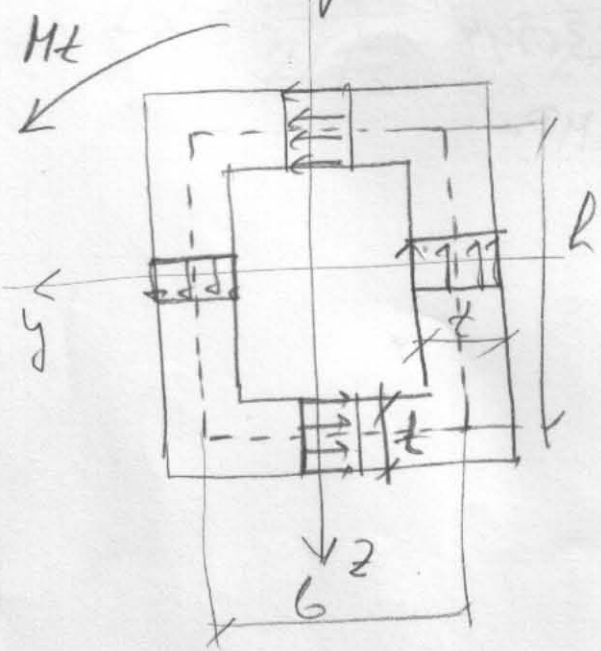
Контролни напони τ_x од M_t

отворени танкоѕидни пресеци

$$\tau_x^{M_t} = \frac{M_t}{I_t} \cdot t \qquad I_t = \frac{1}{3} \sum (b_i \cdot t_i^3)$$

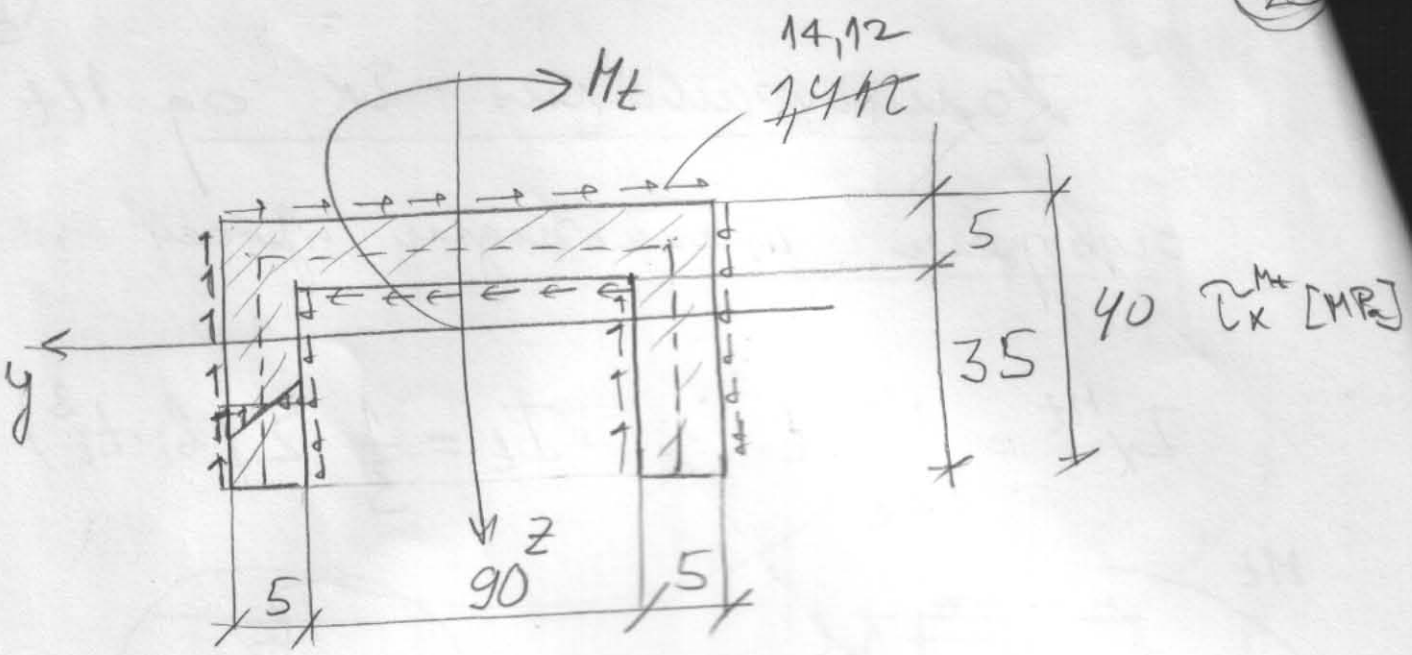


затворени



$$\tau_x = \frac{M_t}{2t \cdot F_S}$$

$$F_S = b \cdot h$$



$M_t = 20\,kNm$

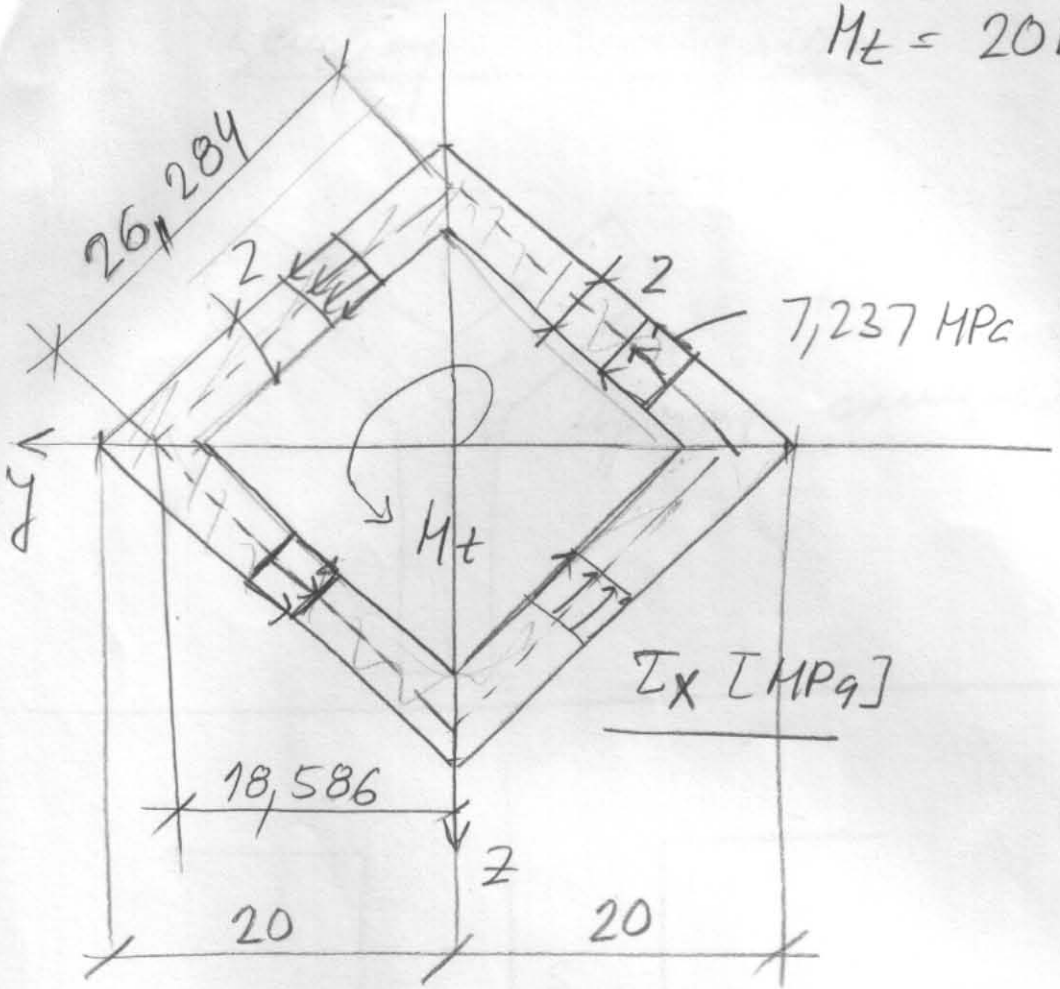
$$I_t = \frac{1}{3}\sum(b_i \cdot t_i^3) = \frac{1}{3}\cdot\left(2\cdot 37{,}5\cdot 5^3 + 95\cdot 5^3\right)$$

$$\underline{I_t = 7083{,}333\,cm^4}$$

$$\tau_{x1} = \frac{M_t}{I_t}\cdot t = \frac{2000\,kNcm}{7083{,}333\,cm^4}\cdot 5\,cm$$

$$\tau_{x1} = 1{,}411\,\frac{kN}{cm^2} = 14{,}12\,MPa$$

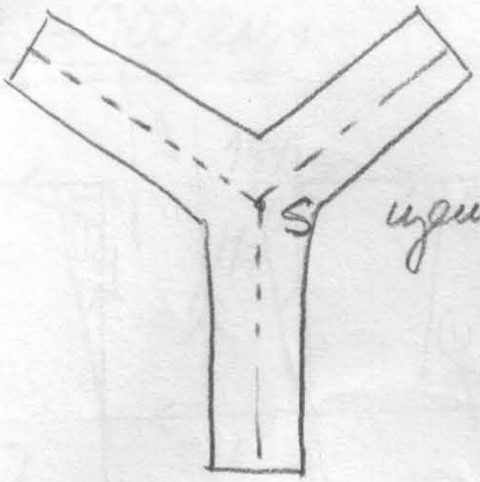
$M_t = 20\,kNm$



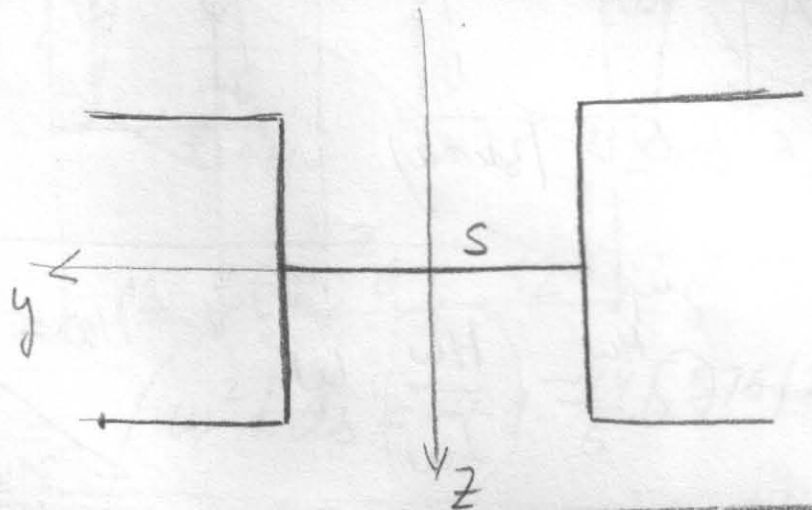
$$F_S = 26{,}284 \cdot 26{,}284 = 690{,}849\ cm^2$$

$$\tau_x = \frac{M_t}{2 t F_S} = \frac{2000\ kNcm}{2 \cdot 2 \cdot 690{,}849\ cm^2} = 7{,}237\ MPa$$

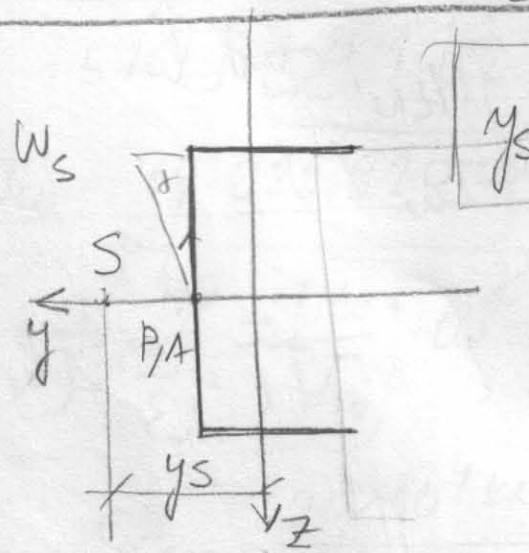
Центр сдвига



центр сдвига



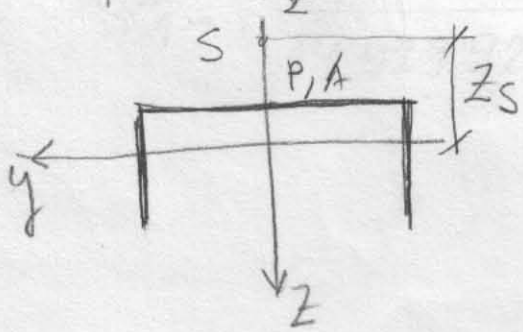
$$w = s \cdot h$$



$$y_s = y_p + \frac{I_{zw_p^\Delta}}{I_y}$$

$$I_y = \int z^2 t\,ds$$

$$I_{zw_p^\Delta} = \int z\, w_p^\Delta t\,ds$$



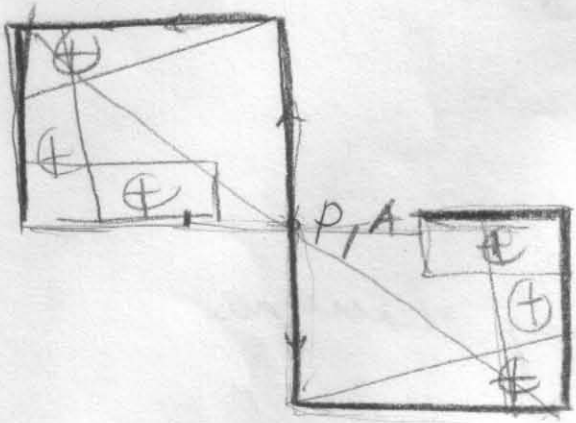
$$z_s = z_p - \frac{I_{yw_p^\Delta}}{I_z}$$

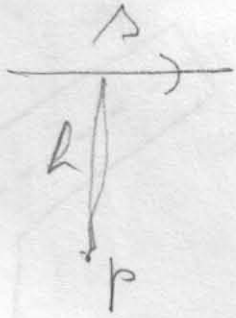
$$I_{yw_p^\Delta} = \int y \cdot w_p^\Delta \cdot t\,ds$$

$$I_z = \int y^2 t\,ds$$

$\oplus_t \rightarrow \ominus$

$\underline{w_p^A\ [cm^2]} = s \cdot l$





цифр.

$$w = w_p^A - \frac{Sw_p^A}{A}$$

gij gij

$$Sw_p^A = \int w_p^A \cdot t\,ds \quad \text{(znak)}$$

$\oplus$ $\ominus$ $M_w\ [kNm^2]$

$$\sigma_x^{M_w} = \frac{M_w}{I_w} \cdot w$$

gij gij

$$M_w = \sum (P_i \cdot w_{znak})$$

znak

$M_{tw}\ [kNm]$

$$\tau_x^{M_{tw}} = -\frac{M_{tw} \cdot Sw}{I_w \cdot t}$$

gij gij

$M_{ts}\ [kNm]$

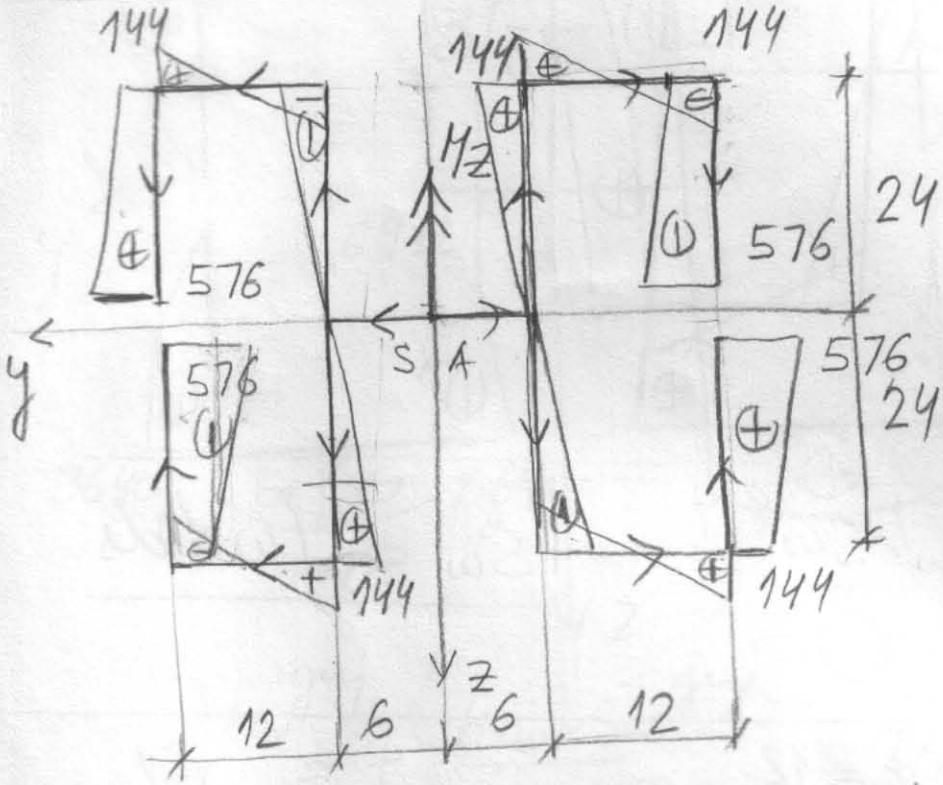
$$\tau_x = \frac{M_{ts}}{I_t} \cdot t \qquad I_t = \frac{1}{3} \sum (b_i \cdot t_i^3$$

$\oplus \leftarrow \omega \rightarrow \ominus \quad \omega = s \cdot h$

$M_\omega = 20\ kNm^2$

$M_{t\omega} = -100\ kNm$

$\omega\ [cm^2]$



$\omega_1 = 24 \cdot 6 = 144$

$\omega_2 = 144 - 12 \cdot 24 = -144$

$\omega_3 = -144 - 24 \cdot 18$

$t = 1\ cm$

$$I_\omega = \int \omega^2 t\, ds = 4 \cdot 1 \cdot \frac{24}{6}\left[576\left(2 \cdot 576 + 144\right) + 144\left(2 \cdot 144 + 576\right)\right] + 8 \cdot 1\left(\frac{1}{2} \cdot 6 \cdot 144\right) \cdot \frac{2}{3} \cdot 144 + 4 \cdot 1 \cdot \left(\frac{1}{2} \cdot 24 \cdot 144\right) \frac{2}{3} 144$$

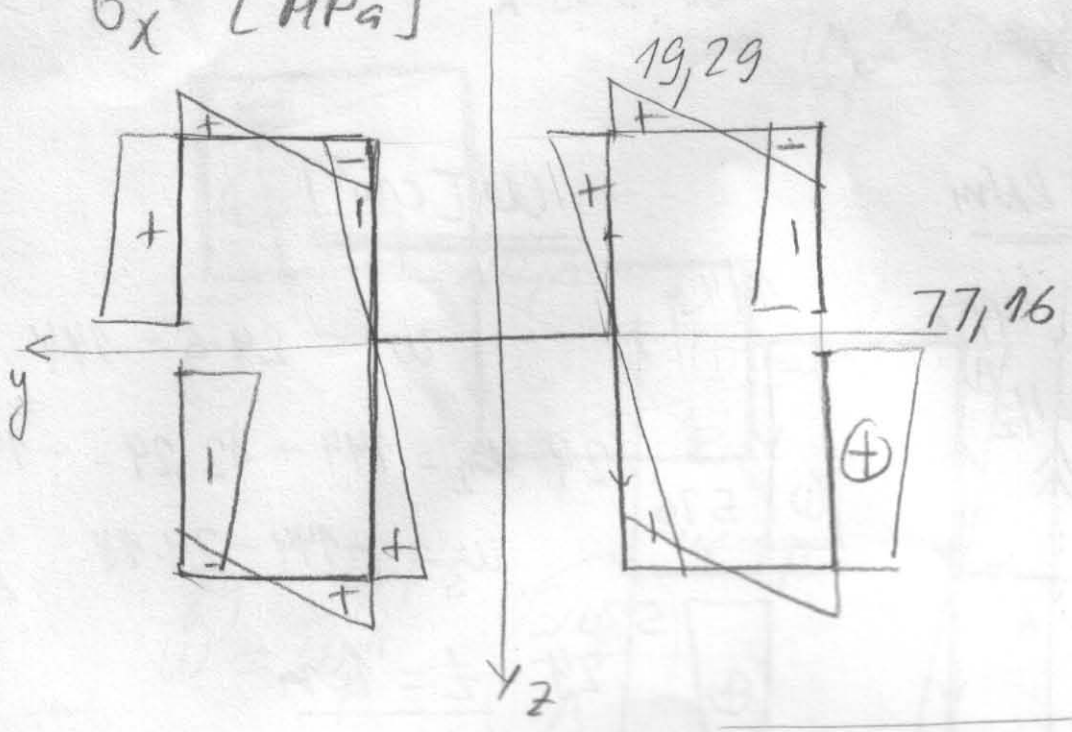
$I_\omega = 14\,929\,920\ cm^6$

$$\sigma_x^{gij}{}_{M_\omega} = \left(\frac{M_\omega}{I_\omega}\right) \cdot \omega^{gij}$$

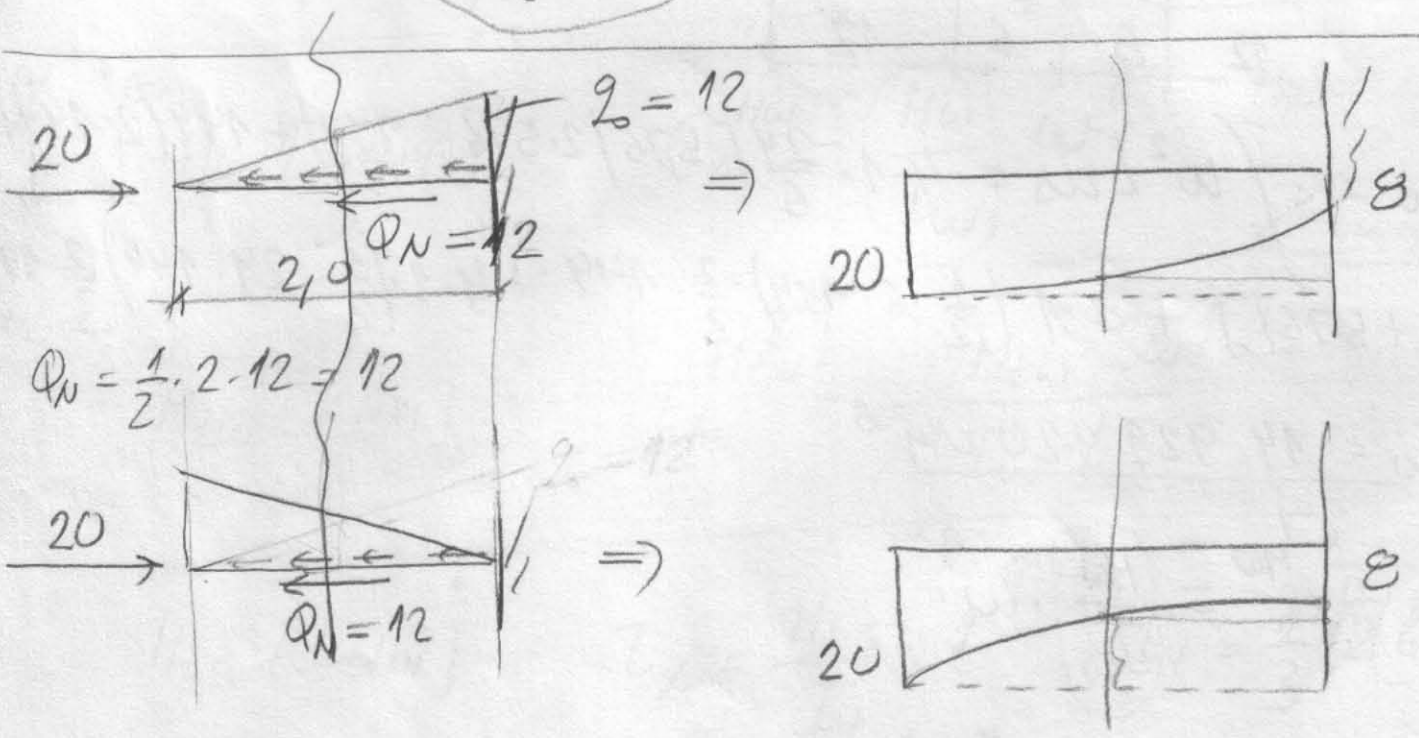
$$\sigma_{x_1} = \frac{20 \cdot 10^4\ kNcm^2}{14\,929\,920} \cdot (144) = 19{,}29\ MPa$$

$$(576) = 77{,}16\ MPa$$

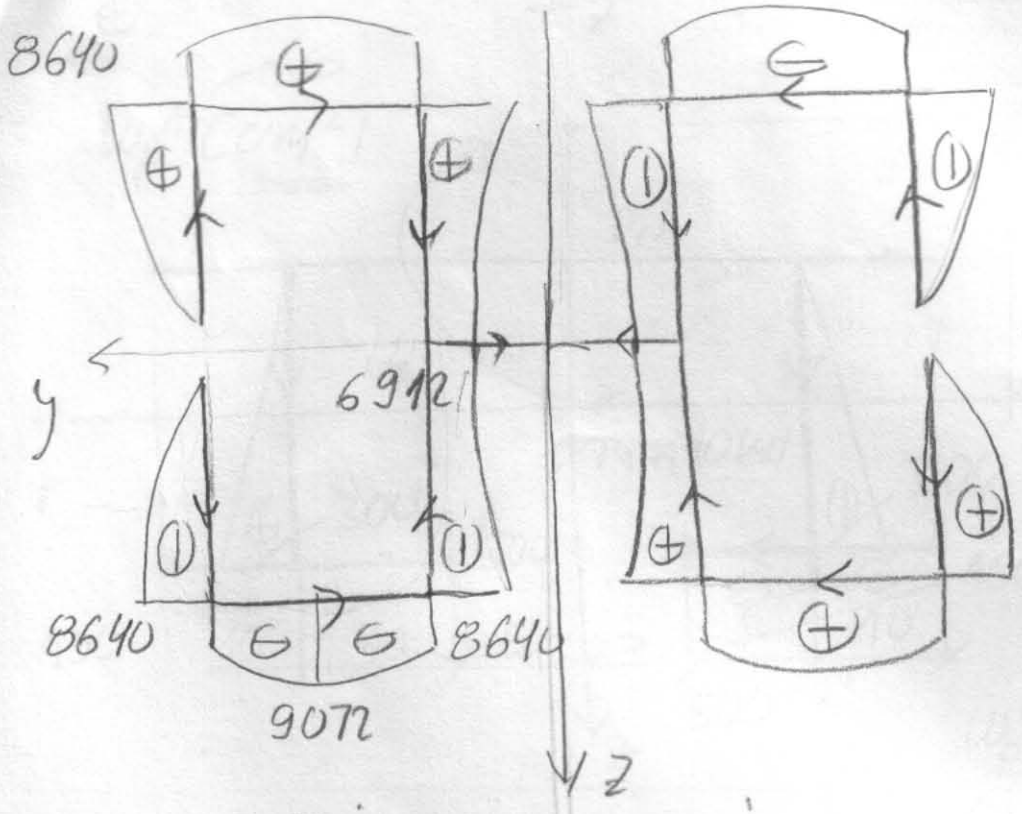
σ_x^{Mw} [MPa]



$$\tau_{Mw}^{gij} = -\frac{M_{tw} \cdot S_w^{gij}}{I_w \cdot t} \qquad S_w = \int \omega \cdot t \, ds$$



$S_w = \int w\,t\,ds$ $\quad S_w[cm^4]$



$$S_{w_1} = \frac{576+144}{2}\cdot 24$$

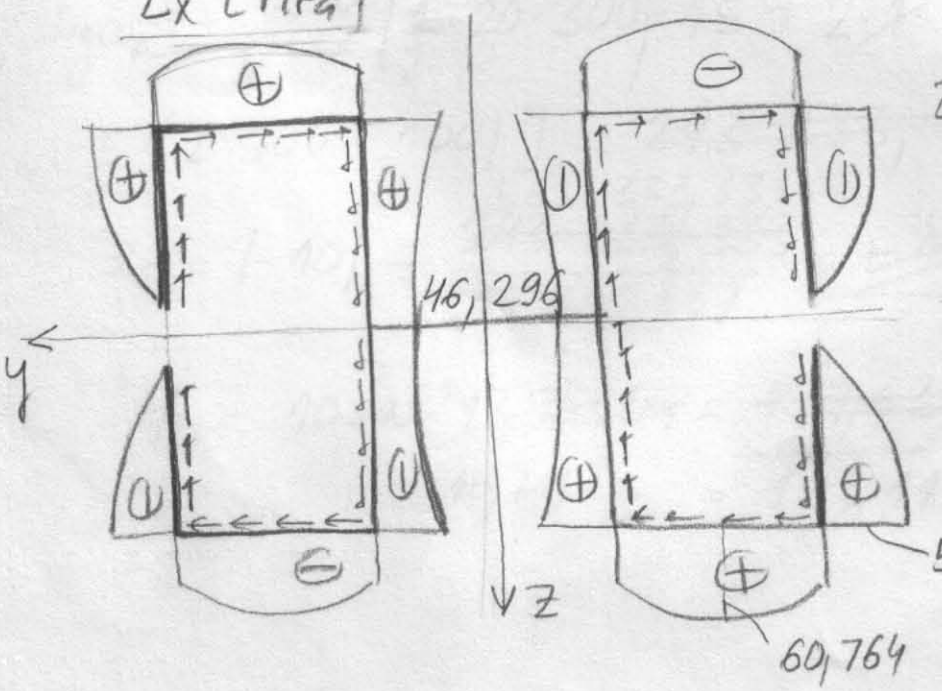
$$S_{w_2} = 8640 + \frac{1}{2}\cdot 6\cdot 144$$

$$S_{w_3} = -8640 + \frac{1}{2}\cdot 24\cdot 144$$

$$I_x^{M_{zw}} = -\frac{M_{zw}^{grij}\cdot S_w^{grij}}{I_w \cdot t}$$

$$I_{x_1} = \frac{10000\,kNcm \cdot 8640}{14\,929\,920\,cm^6 \cdot 1\,cm}$$

M_{zw}
$I_x\,[MPa]$



$I_{x_1} = 57{,}870\ MPa$

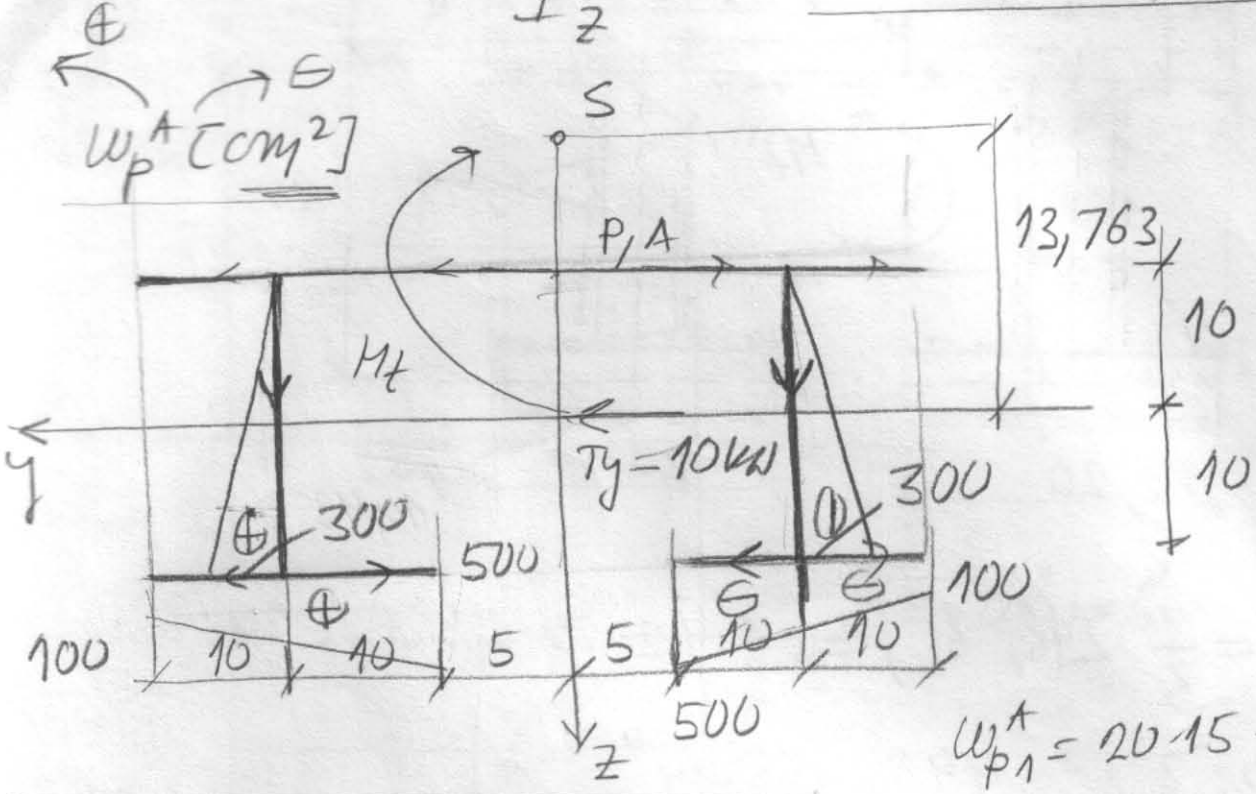
$I_{x_2}^{(9072)} = 60{,}764\ MPa$

$I_{x_3}^{(6912)} = 46{,}296\ MPa$

$$z_s = z_p - \frac{I_y \omega_p^A}{I_z}$$

$I_z = 64\,666,667\ cm^4$

$\omega_p^A\ [cm^2]$



$\omega_{p1}^A = 20 \cdot 15 = 300$

$\omega_{p2}^A = 300 + 10 \cdot 20 = 500$

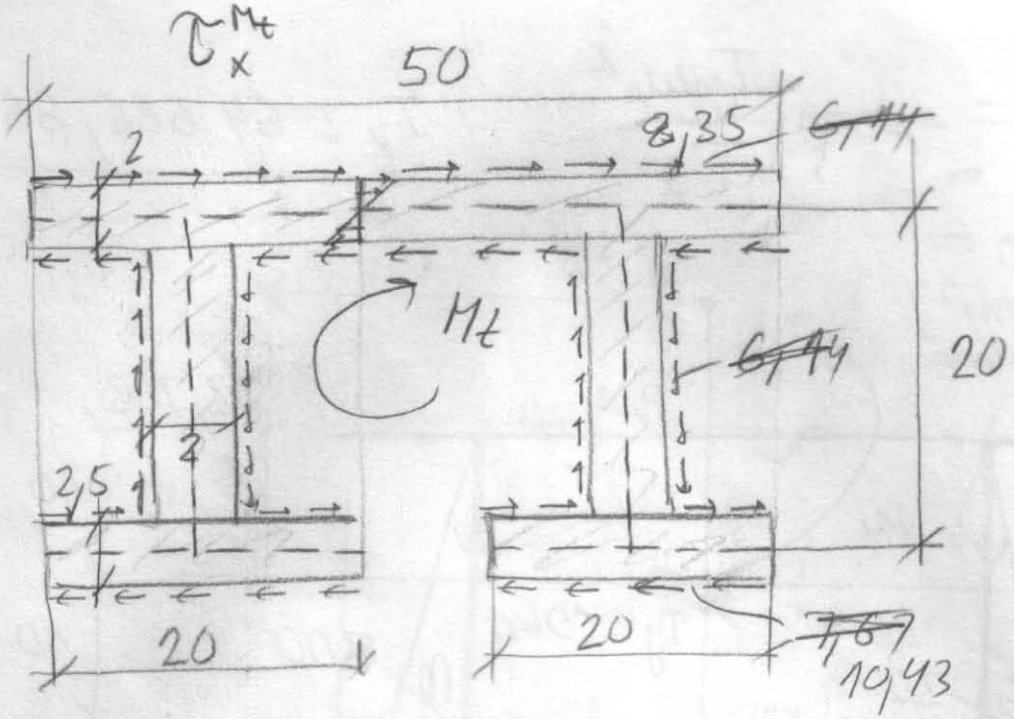
$\omega_{p3}^A = 300 - 10 \cdot 20 = 100$

$$I_y \omega_p^A = \int y \cdot \omega_p^A \cdot t\, ds$$

$$I_y \omega_p^A = 2\left(\frac{1}{2} \cdot 20 \cdot 300\right) \cdot 15 + 2 \cdot \frac{20}{6}\left[25\,(2 \cdot 100 + 500) + 5\,(2 \cdot 500 + 100)\right] = 563\,333,333\ cm^5$$

$$z_S = (-10) - \frac{563\,333,333}{64\,666,667} = -18,711\ cm$$

$$M_t = 10\,kN \cdot 18,711\ cm = 187,11\ kNcm$$

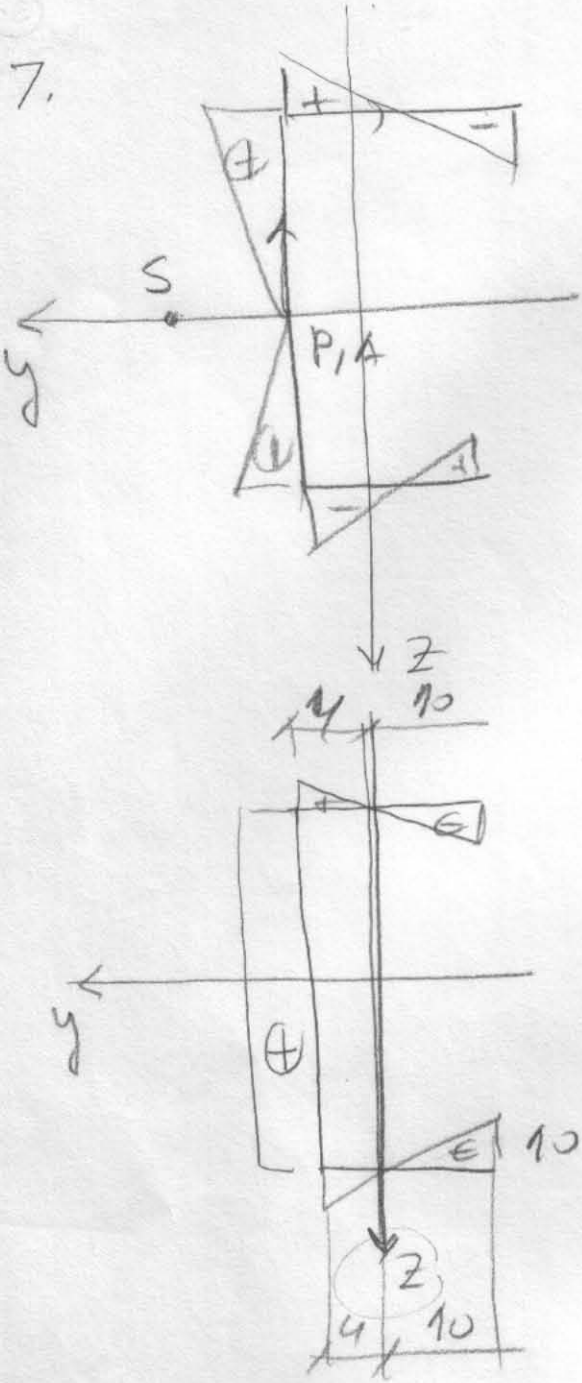


$$I_t = \frac{1}{3}\sum(b_i \cdot t_i^3) = \frac{1}{3}\left(2\cdot 20\cdot 2{,}5^3 + 2\cdot 20\cdot 2^3 + 50\cdot 2^3\right) = 448{,}333\ cm^4$$

$$\tau_x = \frac{Mt}{I_t}\cdot t \qquad \tau_{x1} = \frac{187{,}113\ kNcm}{448{,}333\ cm^4}\cdot 2\,cm = 8{,}35\ MPa$$

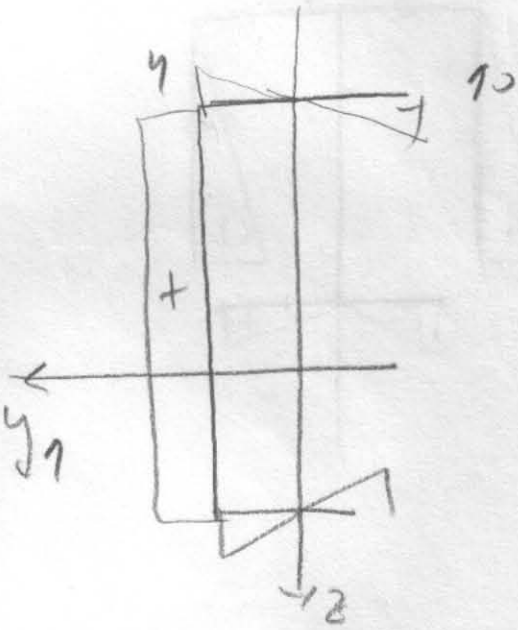
$$(2{,}5\,cm) = 10{,}43\ MPa$$

7.



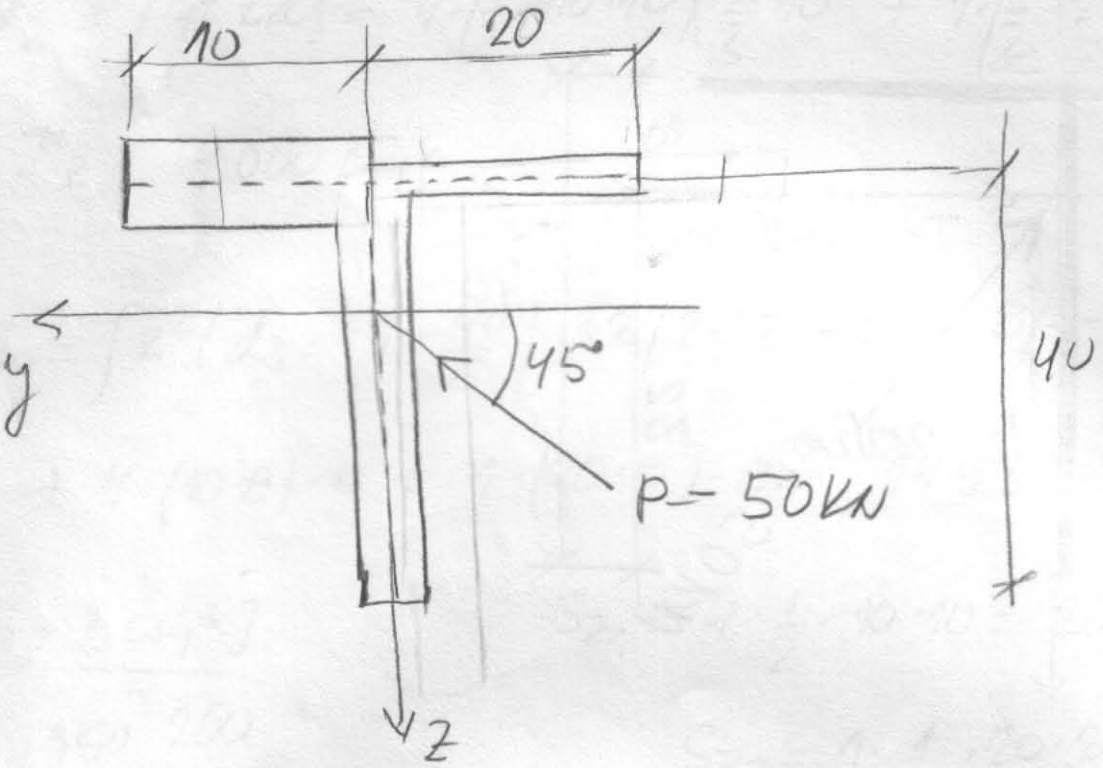
$\omega\,[cm^2]$

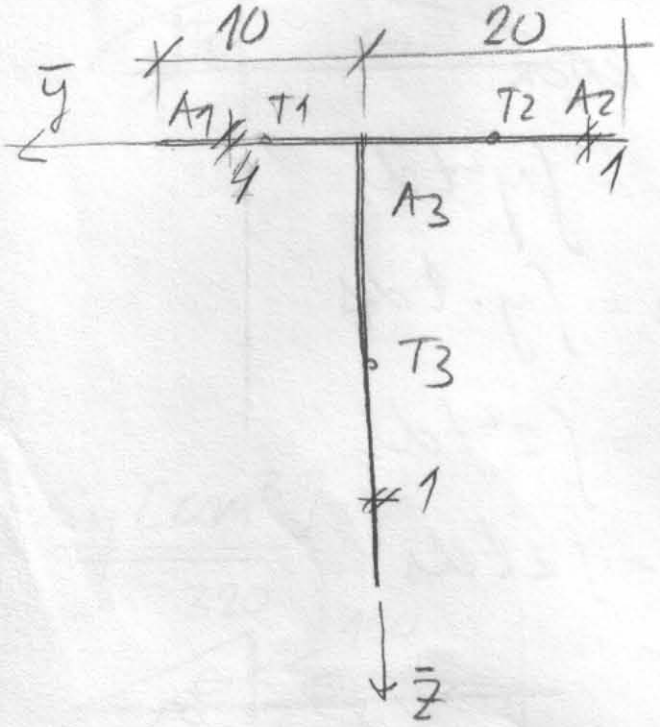
$Y\,[cm]$

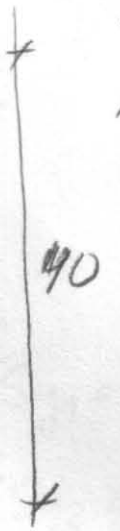


Y grej

$$I_{y\omega_p} = \int y \cdot \omega \cdot b \, dA = const$$







$A_1 = 40 \quad T1(5;0)$

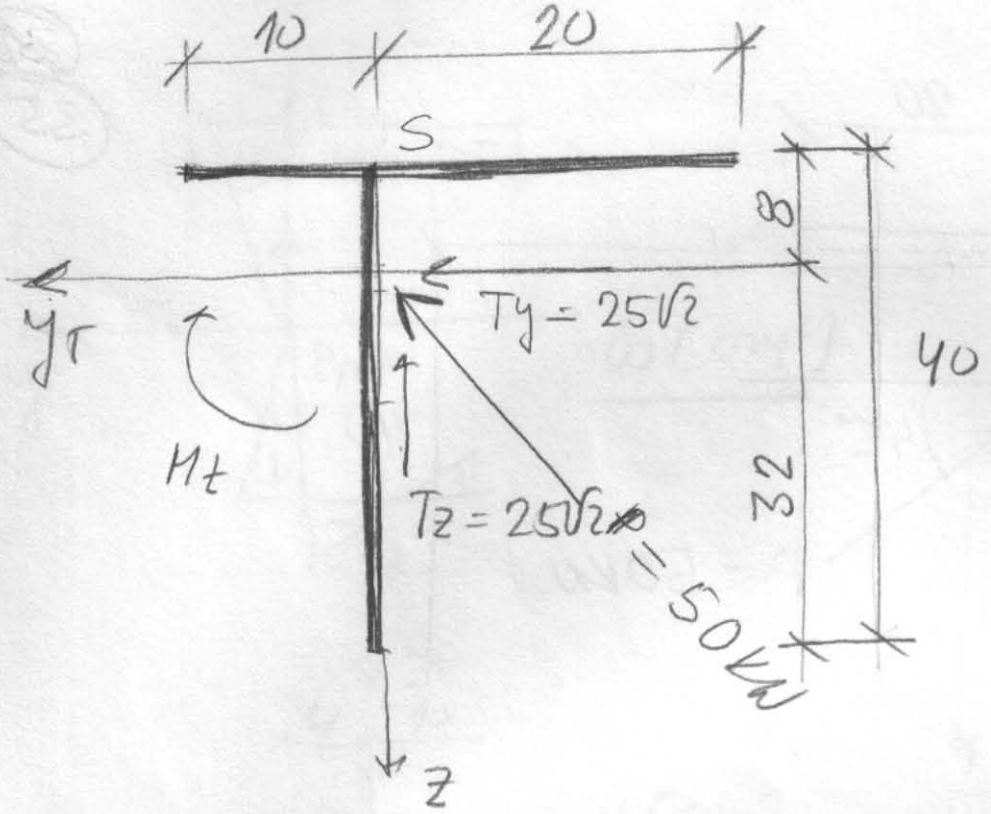
$A_2 = 20 \quad T2(-10;0)$

$A_3 = 40 \quad T3(0;20)$

$A = 100$

$$z_T = \frac{40 \cdot 20}{100} = 8\,cm$$

$$y_T = \frac{40 \cdot 5 + 20(-10)}{100} = 0$$



$M_t = 25\sqrt{2} \cdot 8\ kNcm = 200\sqrt{2}\ kNcm$

$$\tau_x^{T_y} = -\frac{T_y \cdot S_z}{I_z \cdot t}$$

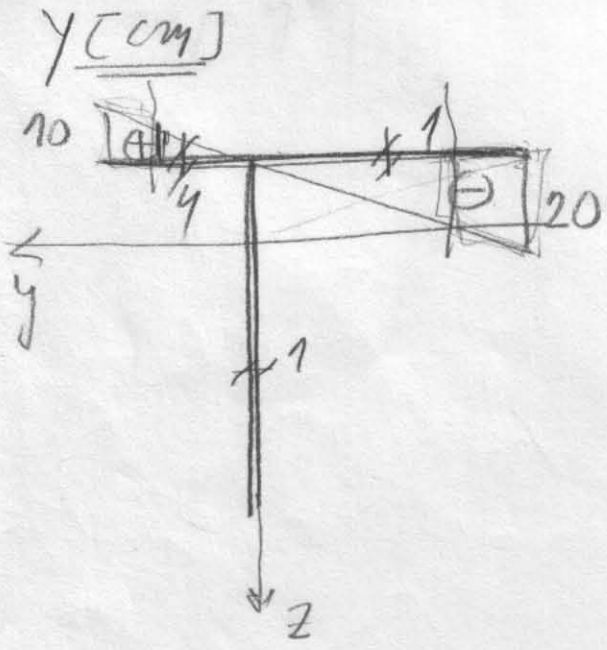
$$\tau_x^{T_z} = -\frac{T_z \cdot S_y^{guj}}{I_y \cdot t}$$

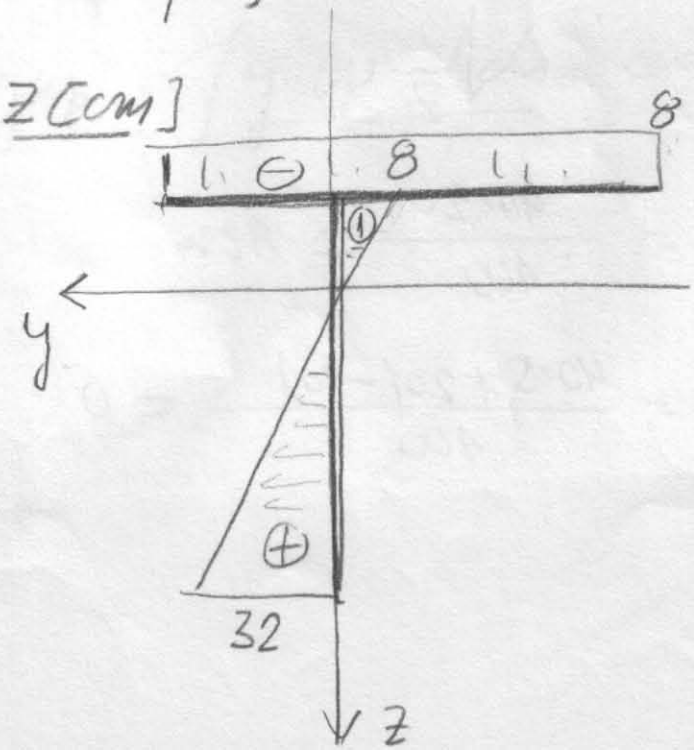
$$I_z = \int y^2 t\,ds$$

$$S_z = \int y \cdot t\,ds$$

$$I_y = \int z^2 t\,ds$$

$$S_y = \int z\,t\,ds$$





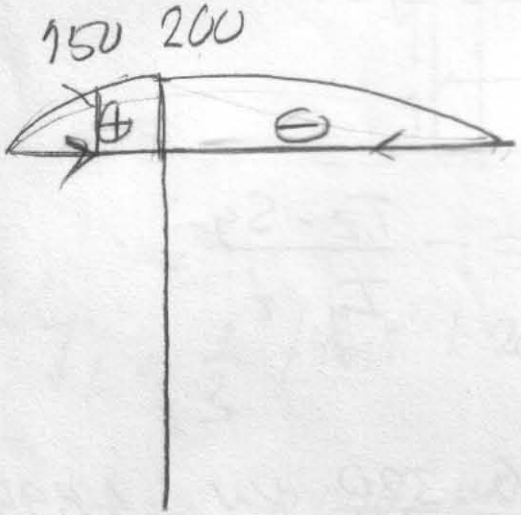
$$I_z = \int y^2 t\,ds = 4\cdot\left(\frac{1}{2}\cdot 10\cdot 10\right)\cdot\frac{2}{3}10 + 1\cdot\left(\frac{1}{2}\cdot 20\cdot 20\right)\cdot\frac{2}{3}20$$

$$I_z = 4000\ cm^4$$

$$I_y = \int z^2 t\,ds = 1\cdot\frac{40}{6}\left[32\left(2\cdot 32 - 8\right) - 8\left(-2\cdot 8 + 32\right)\right]$$

$$+ 4\cdot(10\cdot 8)\cdot 8 + 1\cdot(20\cdot 8)\cdot 8 = 14933{,}333\ cm^4$$

$S_z\ [cm^3]$

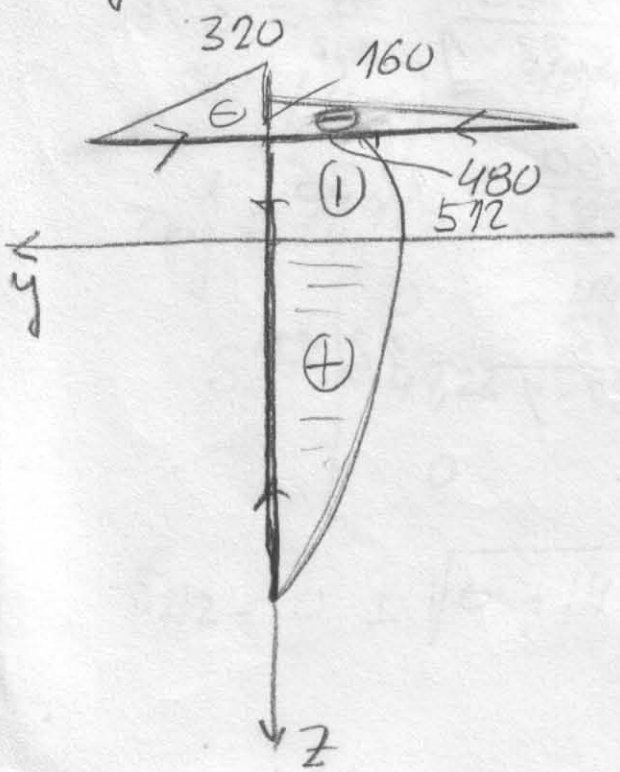


$$S_{z_1} = 4\cdot\frac{1}{2}\cdot 10\cdot 10 = 200$$

$$S_{z_2} = 1\cdot\frac{1}{2}\cdot 20\cdot 20 = 200$$

$$S_A = 4\cdot\frac{1}{2}(10+5)\cdot 5$$

$S_y\ [cm^3]$



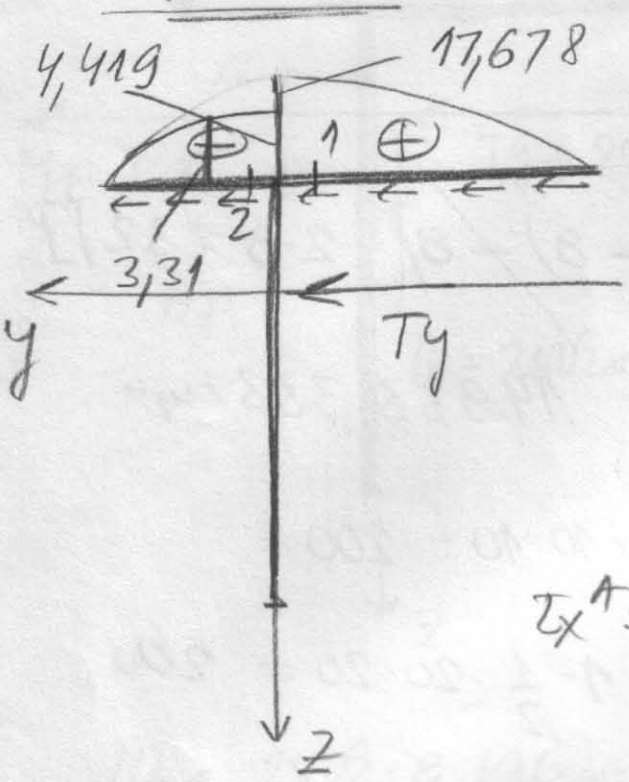
$$S_{y_1} = 4\cdot(8\cdot 10) = 320$$

$$S_{y_2} = 1\cdot(8\cdot 20) = 160$$

$$S_{y_3} = 480 + 1\cdot\frac{1}{2}\cdot 8\cdot 8 = 512$$

$$S_{y_4} = 1\cdot\frac{1}{2}\cdot 32\cdot 32 = 512$$

$\tau_x^{T_y}$ [MPa]



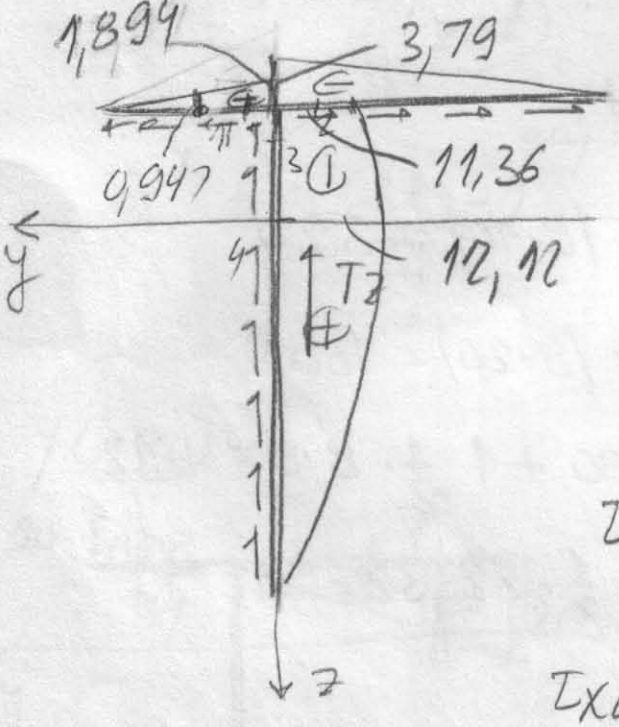
$$\tau_x^{T_y} = -\frac{T_y \cdot S_z}{I_z \cdot t}$$ gdy gdy

$$\tau_{x1} = \frac{25\sqrt{2}\,kN \cdot 200\,cm^3}{4000\,cm^4 \cdot 1\,cm} = 17{,}678\,MPa$$

$$\tau_{x2} = \frac{25\sqrt{2}\,kN \cdot 200\,cm^3}{4000\,cm^4 \cdot 4\,cm} = 4{,}419\,MPa$$

$$\tau_x^{\uparrow} = \frac{25\sqrt{2} \cdot 150}{4000 \cdot 4} = 3{,}31$$

$\tau_x^{T_z}$ [MPa]



$$\tau_x^{T_z} = -\frac{T_z \cdot S_y}{I_y \cdot t}$$

$$\tau_{x1} = \frac{25\sqrt{2} \cdot 320}{14933{,}33 \cdot 4}\,\frac{kN}{cm^2} = 1{,}894\,MPa$$

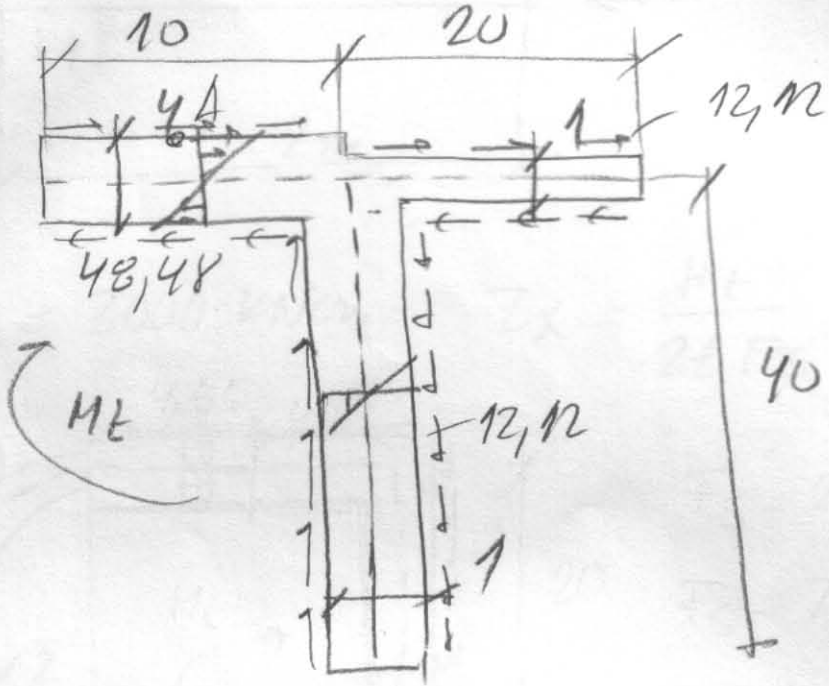
$$\tau_{x2} = \frac{25\sqrt{2} \cdot 160}{14933{,}33 \cdot 1}\,\frac{kN}{cm^2} = 3{,}79$$

$$\tau_{x3} = \frac{25\sqrt{2} \cdot 480}{14933{,}33 \cdot 1} = 11{,}36$$

$$\tau_{x4} = \frac{25\sqrt{2} \cdot 512}{14933{,}33 \cdot 1} = 12{,}12$$

τ_x^{Mt} [MPa]

Н Довантовити 204/08



$$I_t = \frac{1}{3}\left(40\cdot 1^3 + 20\cdot 1^3 + 10\cdot 4^3\right) = 233{,}33\ cm^4$$

$$\tau_{x1} = \frac{200\sqrt{2}\ kNcm}{233{,}33\ cm^4} \cdot 1\ cm = 12{,}12\ MPa$$

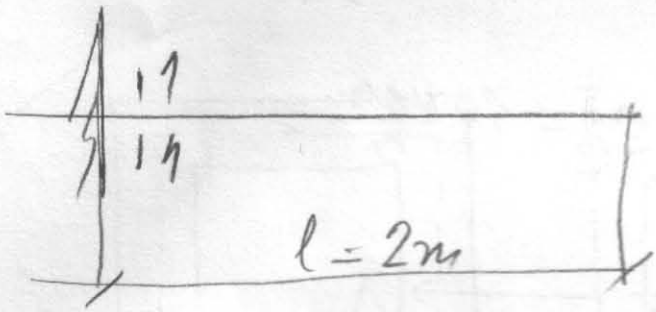
$$\tau_{x2}(4\ cm) = 48{,}48\ MPa$$

$$\tau_{xy}^{A} = 4{,}419 + 0{,}947 - 48{,}48 = -44{,}22$$

$$S^A = \begin{bmatrix} 0 & -44{,}22 & 0 \\ -44{,}22 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa \quad \begin{matrix} \rightarrow x \\ \rightarrow y \end{matrix}$$

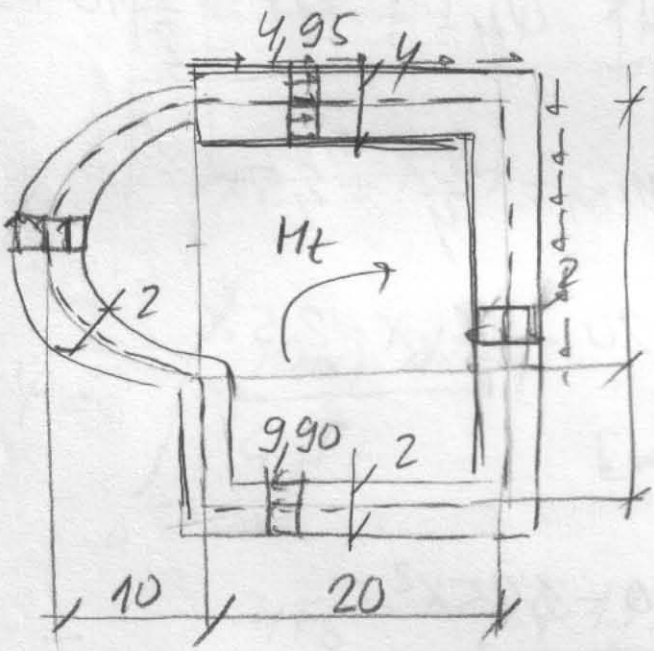
$$\sigma_{1,2} = 0 \pm \sqrt{0 + (44{,}22)^2} = \begin{matrix} \boxed{\sigma_1 = 44{,}22\ MPa} \\ \sigma_2 = -44{,}22 \end{matrix}$$

$G = 80\,GPa = 80\,000\,MPa = 8000\,kN/cm^2$



$M_t = 30\,kNm$

$M_t = 3000\,kNcm$ $\quad I_x = \dfrac{M_t}{2t F_S}$



$F_S = 20 \cdot 30 + \frac{1}{2} 10^2 \pi$

$F_S = 757{,}08\,cm^2$

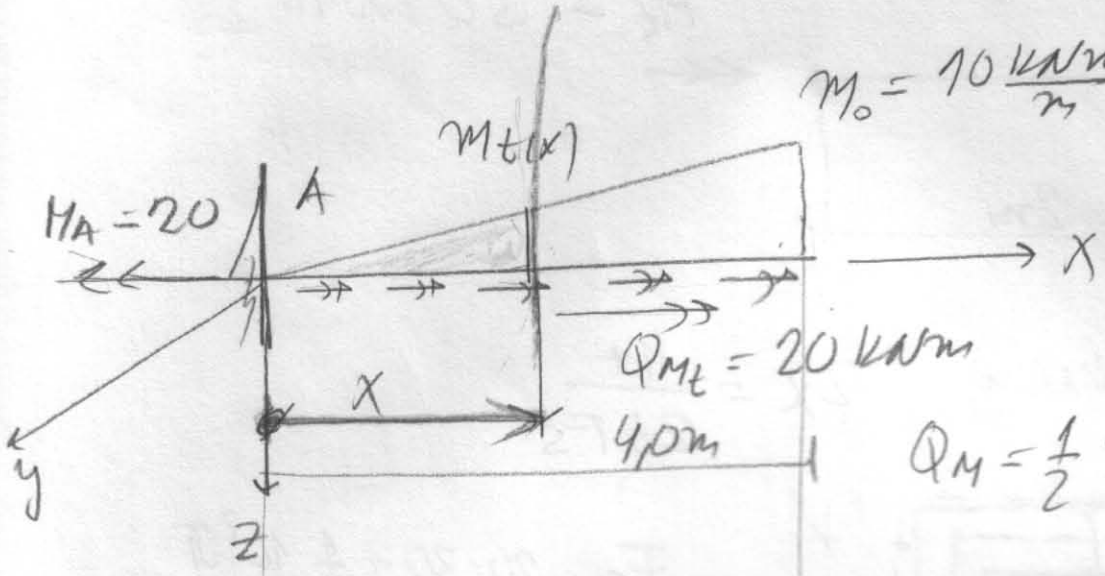
$I_{x_1} = \dfrac{3000}{2 \cdot 2 \cdot 757{,}08} = 9{,}90\,MPa$

$I_{x_2} = \dfrac{3000}{2 \cdot 4 \cdot 757{,}08} = 4{,}95\,MPa$

$$I_t = \frac{4F_S^2}{\sum \frac{Bi}{ti}} = \frac{4 \cdot 757{,}08^2}{\left(\frac{20}{2} + \frac{30}{2} + \frac{20}{4} + \frac{10}{2} + \frac{10\pi}{2}\right)} = 45\,213{,}42\,cm^4$$

$$\varphi_{max} = \frac{M_t l}{G I_t} = \frac{3000\,kNcm \cdot 200cm}{8000 \frac{kN}{cm^2} \cdot 45\,213{,}42\,cm^4} = 1{,}65 \cdot 10^{-3}\,rad$$

a) Направете дијаграм M_t

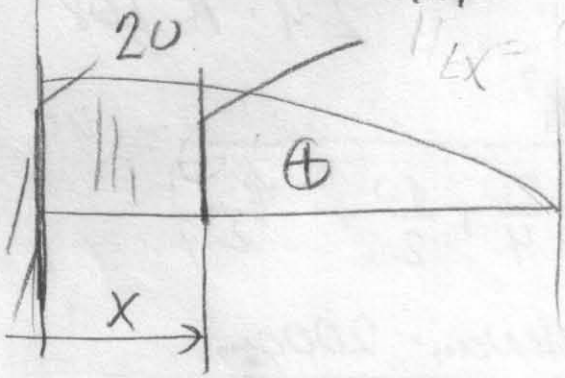


$$Q_M = \frac{1}{2} m l = \frac{1}{2} 10 \cdot 4 = 20$$

$$M_{t(x)} : x = 10 : 4 \qquad M_{t(x)} = \frac{10x}{4} = 2{,}5x$$

$$M_{t(x)} = 20 - \frac{1}{2} \cdot x \cdot m_{t(x)} = 20 - \frac{1}{2} \cdot x \cdot 2{,}5x$$

$$M_{t(x)} = 20 - 1{,}25x^2\ [kNm]$$



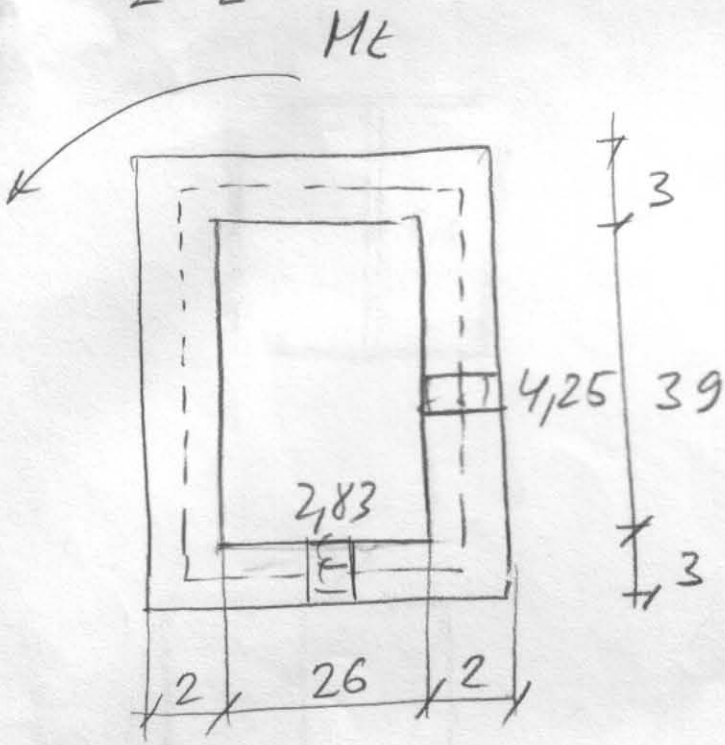
$$M_{t(x=0)} = 20$$

$$M_{t(x=2)} = 15$$

$$E = 20\ GPa = 20 \cdot 10^6\ kPa \left[\frac{kN}{m^2}\right] \qquad \nu = 0{,}15$$

$$G = \frac{E}{2(1+\nu)} = \frac{20 \cdot 10^6}{2(1+0{,}15)} = 8{,}69 \cdot 10^6 \frac{kN}{m^2}$$

2-2



$F_S = 28 \cdot 42 = 1176 cm^2$

$I_{x1} = \frac{2000\ kN/cm}{2 \cdot 2cm \cdot 1176}$

$I_{x2} = \frac{2000\ kN/cm}{2 \cdot 3 \cdot 1176}$

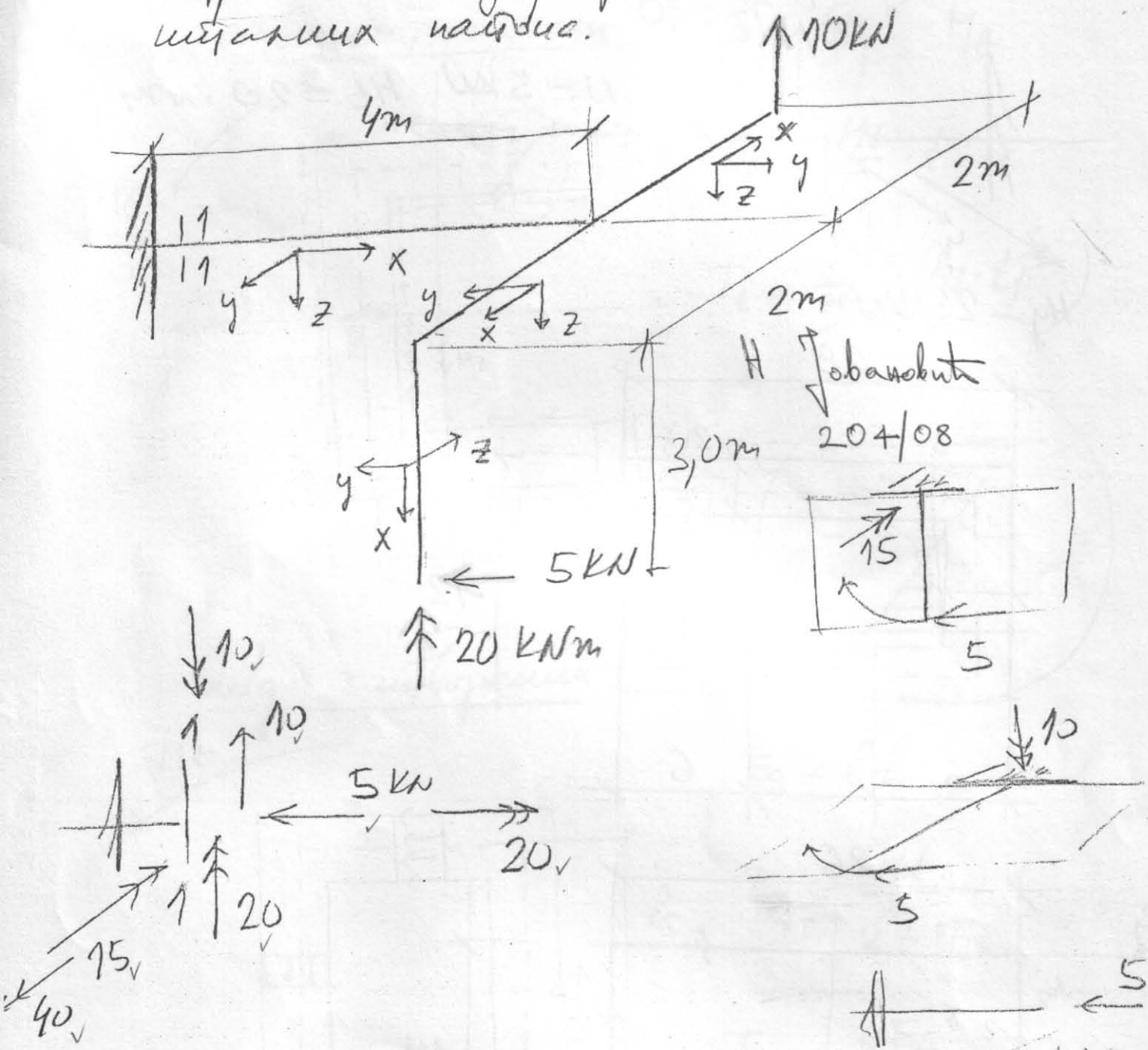
$$\varphi = \int_0^4 \frac{Mt(x)\, dx}{G\, I_t}$$

$$I_t = \frac{4 F_S^2}{\sum \frac{b_i}{G_i}} = \frac{4 \cdot 1176^2}{\left(\frac{42}{2} \cdot 2 + \frac{28}{3} \cdot 2\right)} = 91\,185{,}23\ cm^4$$

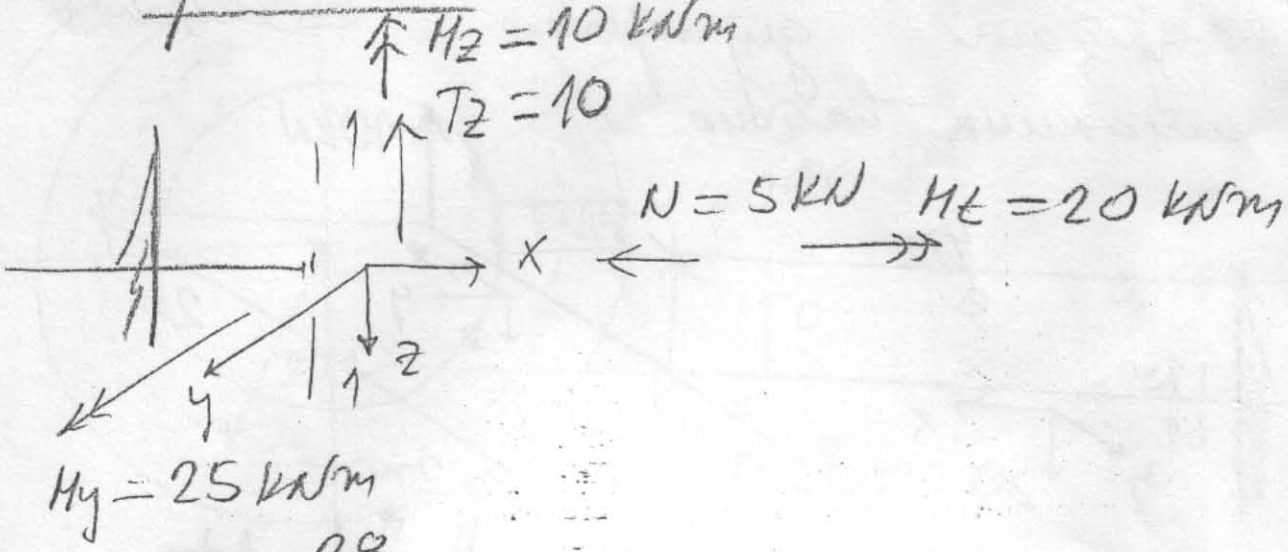
$$\varphi = \int_0^4 \frac{(20 - 1{,}25 x^2)\, dx}{8{,}69 \cdot 10^6 \frac{kN}{m^2} \cdot 91\,185{,}23 \cdot 10^{-8}}$$

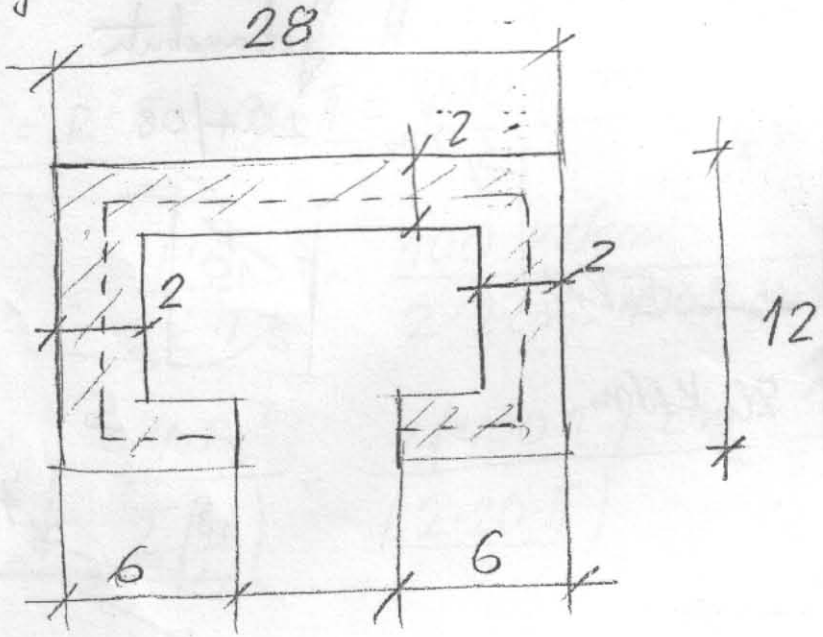
$$\varphi = \frac{1}{7929{,}15} \cdot \left| 20x - 1{,}25 \frac{x^3}{3} \right|_0^4 = 6{,}73 \cdot 10^{-3}\ rad$$

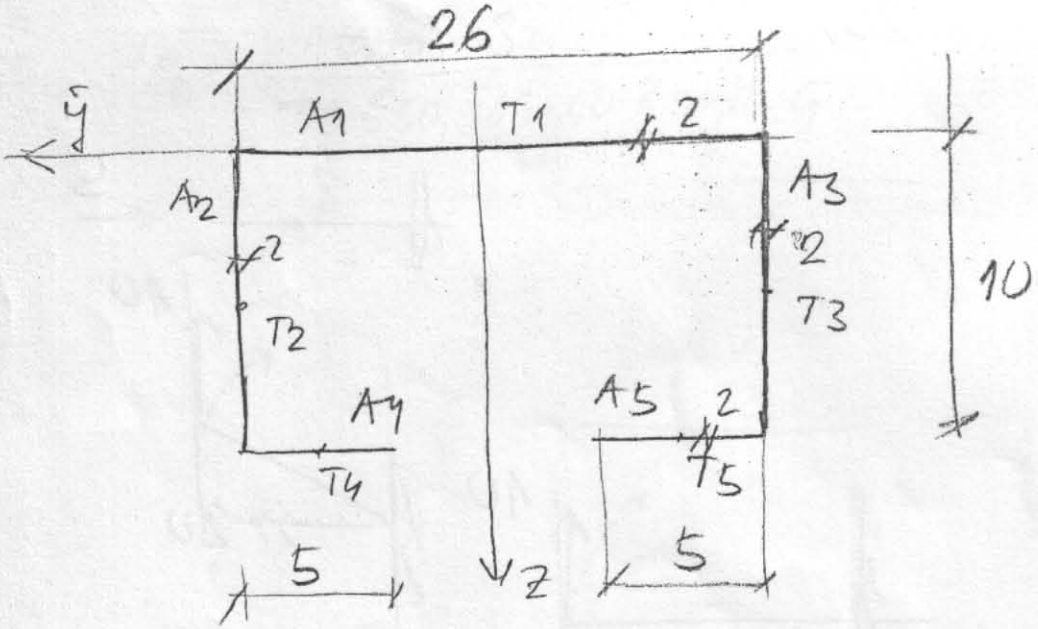
1. За пресек 1-1 у унутрашњу нацртати дијаграме комплементалних напона.



Пресеч 1-1


Mz = 10 kNm
Tz = 10
1
1
N = 5kN
Mt = 20 kNm
x
z
y
My = 25 kNm


28
2
2
2
12
6
6

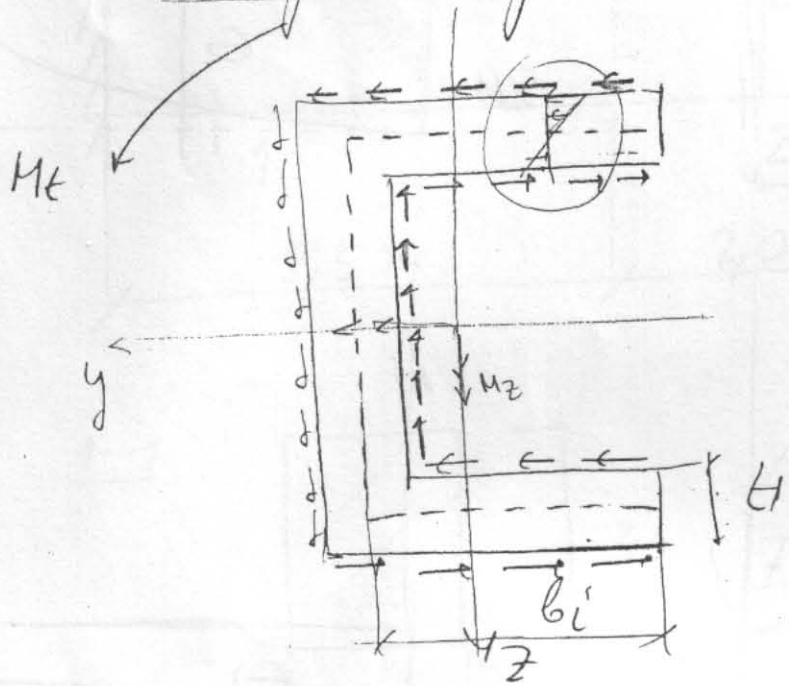

26
y
A1
T1
2
A2
2
T2
A3
2
T3
A4
T4
A5
2
T5
10
5
z
5

τ_x од M_t

* ⊖ M_t

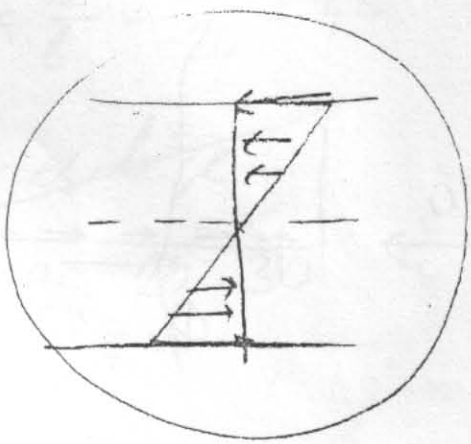
⊕ M_t

отворени пресеци

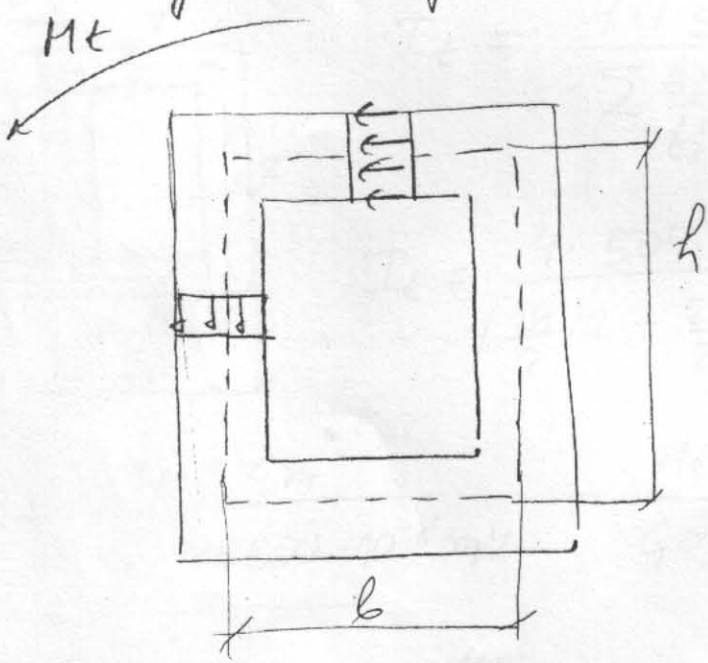


$$\tau_x = \frac{M_t}{I_t} \cdot t$$

$$I_t = \frac{1}{3} \sum \left(b_i \cdot t_i^3 \right)$$



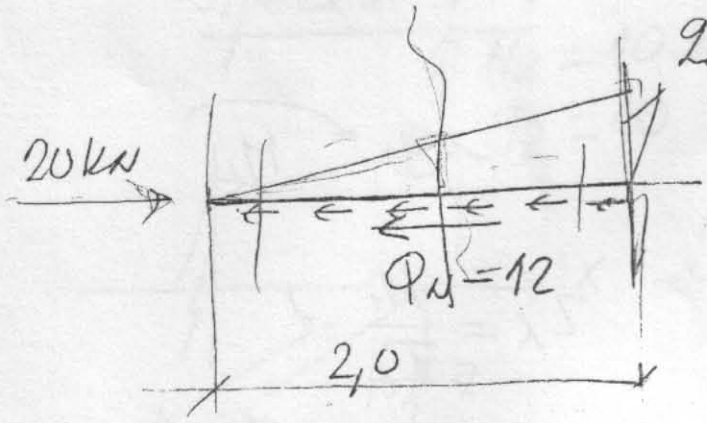
под затворени



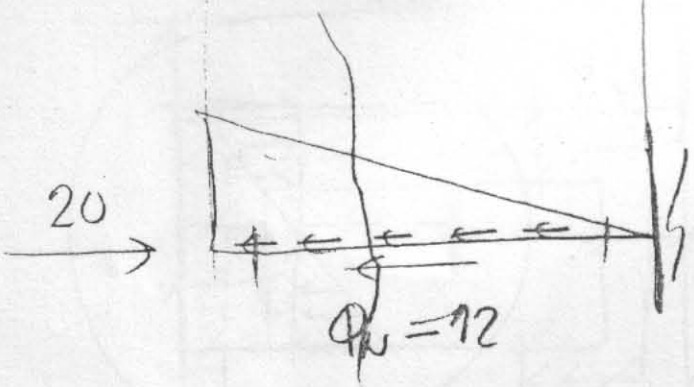
$$F_S = b\,h$$

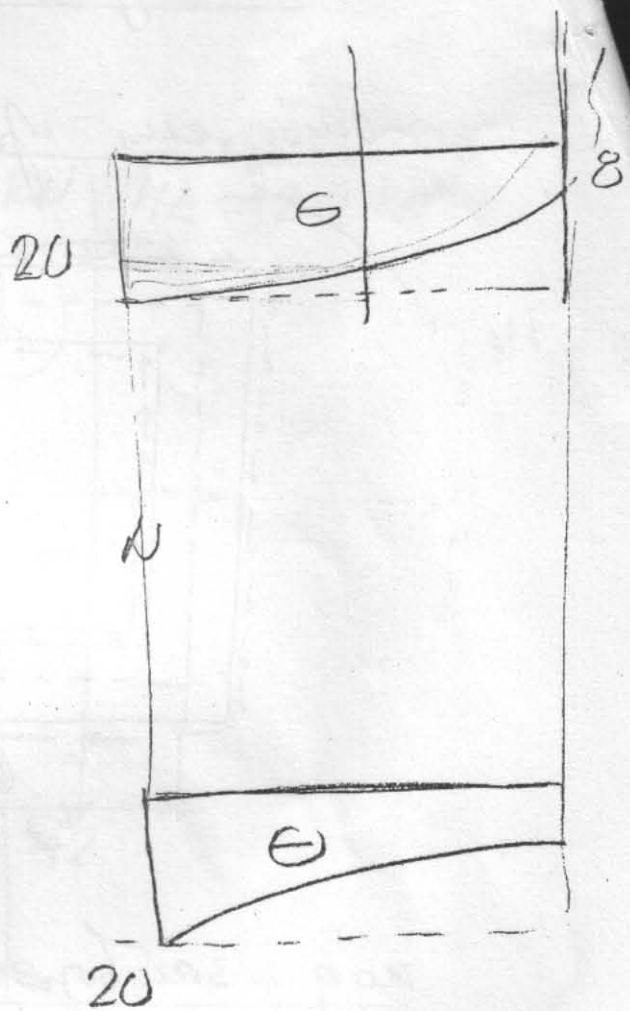
$$\tau_x = \frac{M_t}{2t \cdot F_S}$$

$$I_t = \frac{4F_S^2}{\sum \left(\frac{b_i}{t_i} \right)}$$



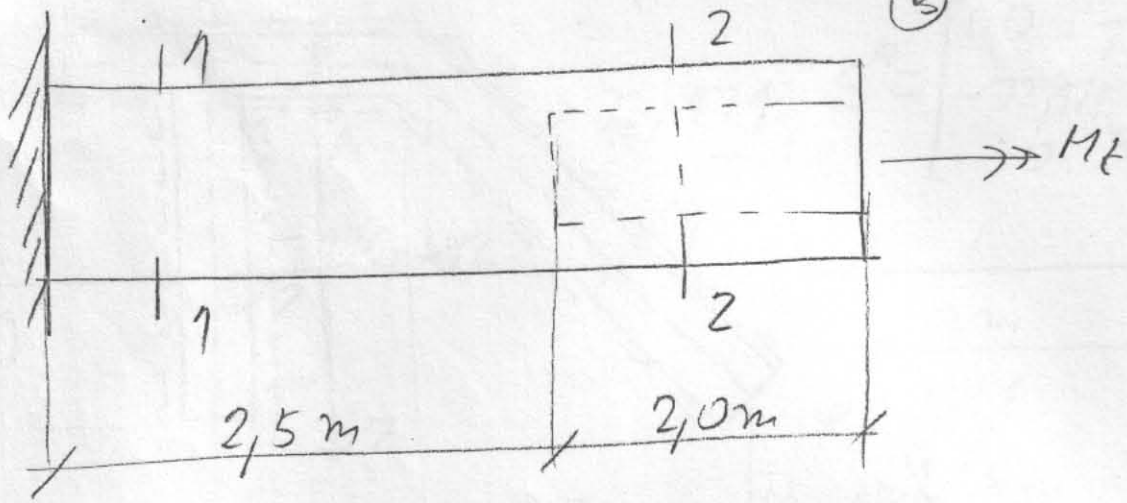
$$Q_N = \frac{1}{2} \cdot 12 \cdot 2 = 12$$



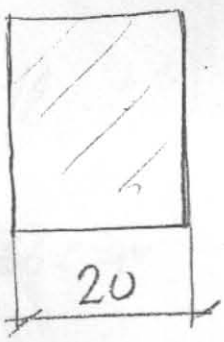


$$\varphi = \frac{M_t \cdot l}{G \cdot I_t}$$

(B)



1–1



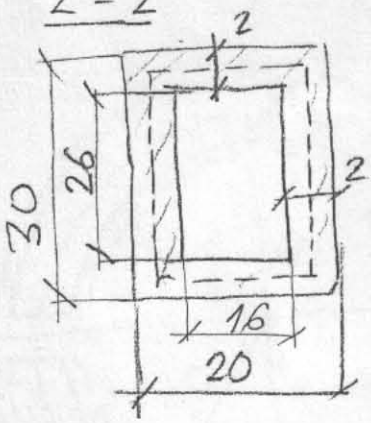
$$\mu = \frac{c}{b} = \frac{30}{20} = 1{,}5$$

$$I_t = \alpha \cdot b^3 \cdot c$$

$$I_t = 0{,}195 \cdot 20^3 \cdot 30$$

$$I_t = 46\,800\ cm^4$$

2–2

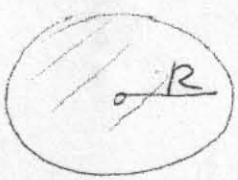


$$I_t = \frac{4 \cdot F_S^2}{\sum \frac{b_i}{t_i}} \qquad F_S = 18 \cdot 28$$

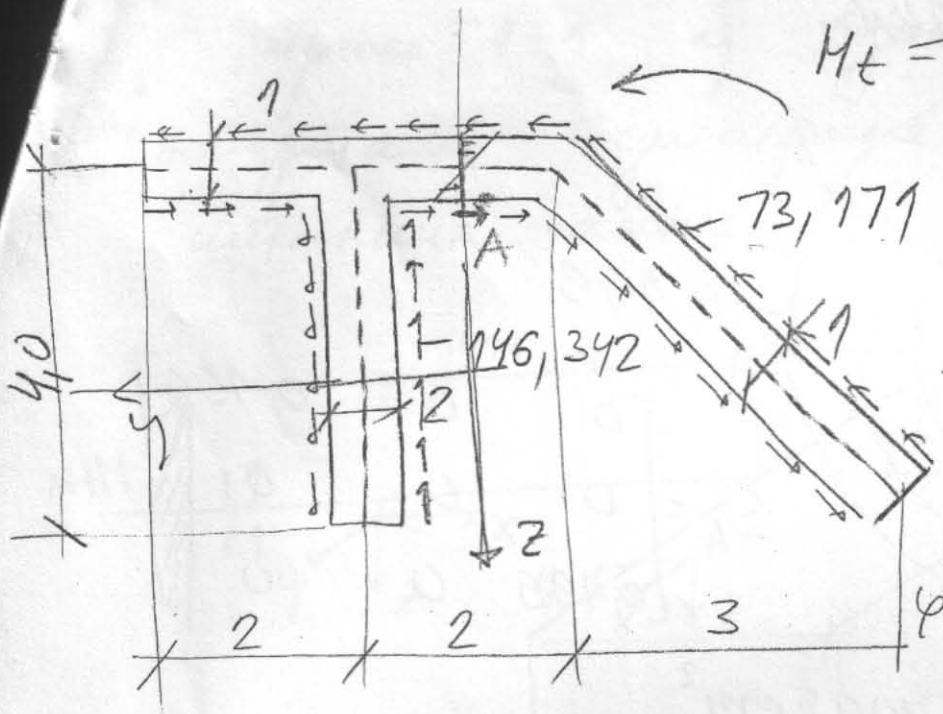
$$F_S = 504\ cm^2$$

$$I_t = \frac{4 \cdot 504^2}{\left(\frac{18}{2} \cdot 2 + \frac{28}{2} \cdot 2\right)} = 22\,088{,}35\ cm^4$$

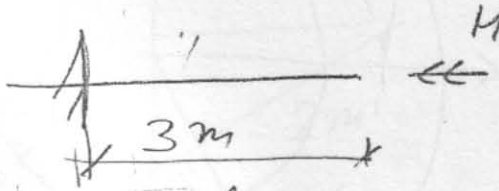
$$\varphi_B = \frac{M_t \cdot 2{,}5\,m}{G \cdot 46800 \cdot 10^{-8}\ m^4} + \frac{M_t \cdot 2{,}0\,m}{G \cdot 22088{,}35 \cdot 10^{-8}\ m^4}$$



$$I_t = \frac{R^4 \pi}{2}$$



$$S^A = \begin{bmatrix} 0,0 & -73,171 & 0 \\ -73,171 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$$



$$\varphi = \frac{M_t \cdot l}{G\, I_t}$$

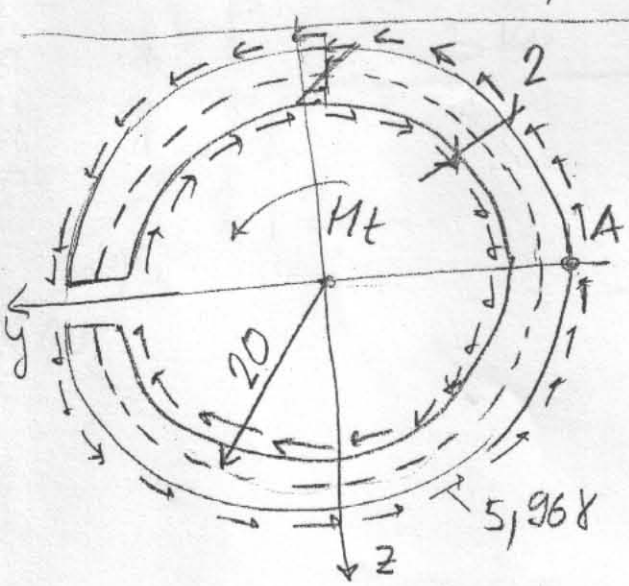
$$\varphi = \frac{1\,kNm \cdot 3\,m}{G \cdot 13,666 \cdot 10^{-8}\,m^4}$$

$$I_t = \frac{1}{3} \Sigma (b_i \cdot t_i^3) = \frac{1}{3} \cdot (4 \cdot 2^3 + 4 \cdot 1^3 + 5 \cdot 1^3)$$

$$I_t = 13,666\,cm^4$$

$$\tau_x = \frac{M_t}{I_t} \cdot t = \frac{100\,kNcm}{13,666\,cm^4} \cdot (1\,cm) = 73,171\,MPa$$

$$(2\,cm) = 146,342\,MPa$$



$$I_t = \frac{1}{3} (b_i \cdot t_i^3) \qquad M_t = 1\,kNm$$

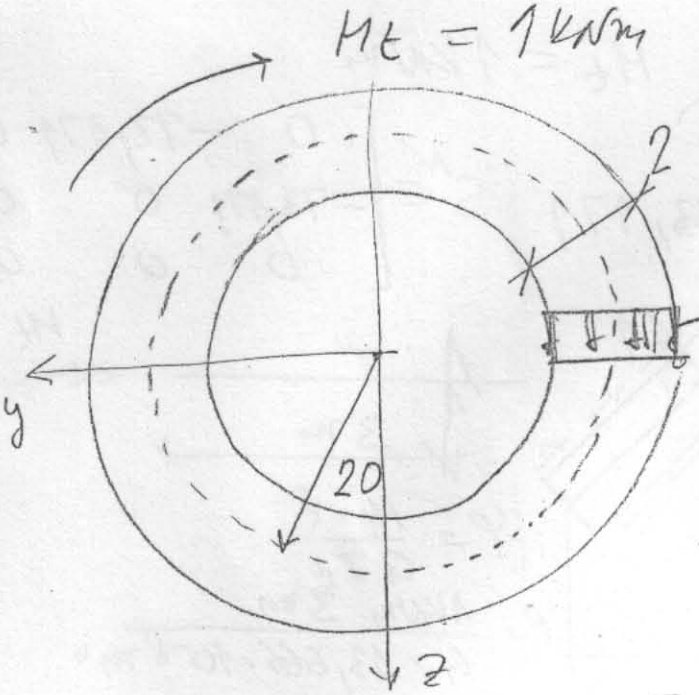
$$I_t = \frac{1}{3} \cdot (2 \cdot 20\pi \cdot 2^3)$$

$$I_t = 335,103\,cm^4$$

$$S^A = \begin{bmatrix} 0 & 0 & -5,968 \\ 0 & 0 & 0 \\ -5,968 & 0 & 0 \end{bmatrix} MPa$$

$$\tau_x^{Mt} = \frac{100\,kNcm}{335,103\,cm^4} \cdot 2\,cm = 5,968\,MPa$$

$$\varphi = \frac{1\,kNm \cdot 3\,m}{G \cdot 335,103 \cdot 10^{-8}\,m^4}$$



$$S_A = \begin{bmatrix} 0 & 0 & 0{,}199 \\ 0 & 0 & 0 \\ 0{,}199 & 0 & 0 \end{bmatrix} \text{ MPa}$$

$$F_S = R^2\pi = 20^2\pi = 400\pi\,cm^2$$

$$\tau_X = \frac{Mt}{2t \cdot F_S} = \frac{100\ kNcm}{2 \cdot 2cm \cdot 400\pi\,cm^2} = 0{,}199\,MPa$$

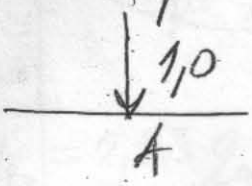
$$I_t = \frac{4 \cdot F_S^2}{\sum\left(\frac{b_i}{t_i}\right)} = \frac{4\,(400\pi)^2\,cm^4}{\left(\frac{2 \cdot 20 \cdot \pi}{2}\right)} = 100530{,}965\,cm^4$$

$$\varphi_B = \frac{1\,kNm \cdot 3m}{100530{,}965 \cdot 10^{-8}\,m^4 \cdot G}$$

ОМ2

Помагала

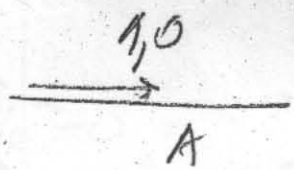
1. вертикално помагало – уред



Н.Г.

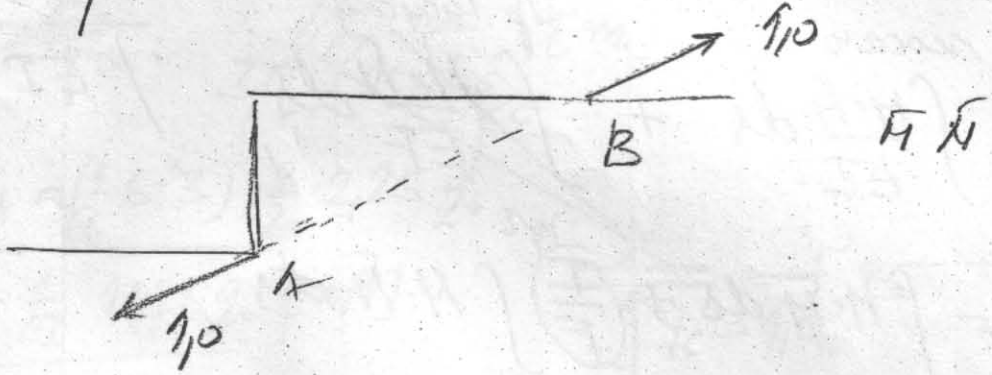
Н. Јованова
204/08

2. хор помагало



Н.Г.

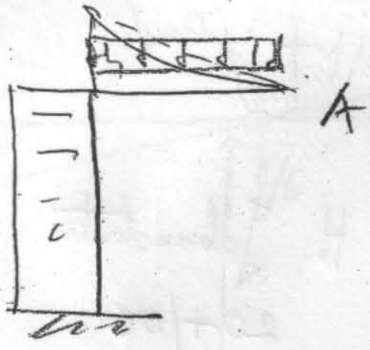
3 разлика страна А и В



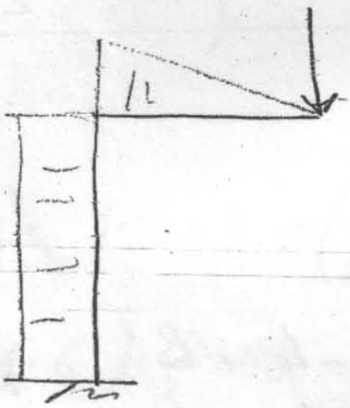
Н.Г.

4. обратно – ротација – нагиб

угол станке A



$M \quad N$



$\bar{M} \quad \bar{N}$

носан $\qquad\qquad$ пр щтан

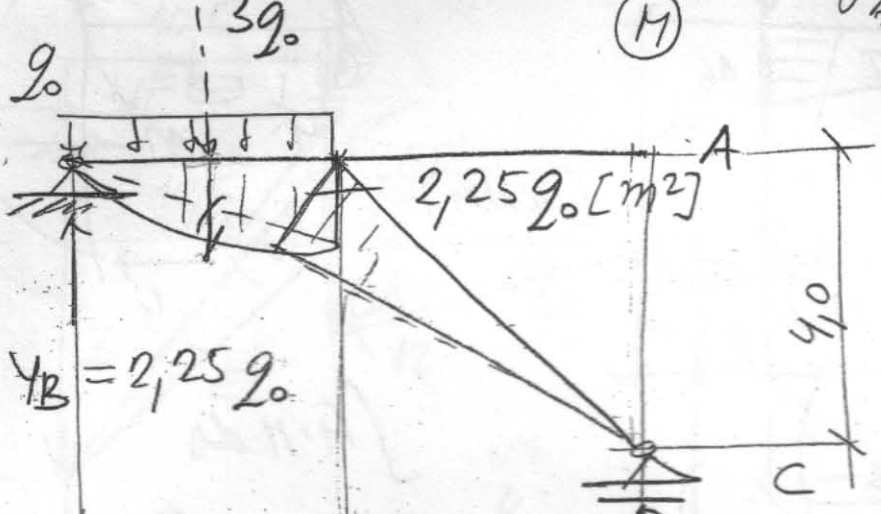
$$v_A = \int \frac{M\cdot\bar{M}}{EI}\,ds + \int \frac{N\cdot\bar{N}}{EF}\,ds \cdot \Big/ EI$$

$$EI v_A = \int M\cdot\bar{M}\,ds + \overset{m^2}{\frac{I}{F}} \int N\cdot\bar{N}\,ds$$

$$[m]\, v_A = \frac{1}{EI}\cdot\Big\{ \qquad\qquad \Big\}$$

$U_A = \int M \cdot \bar{M} \, ds$

$EI = const$



$f = \frac{1}{8} q l^2$

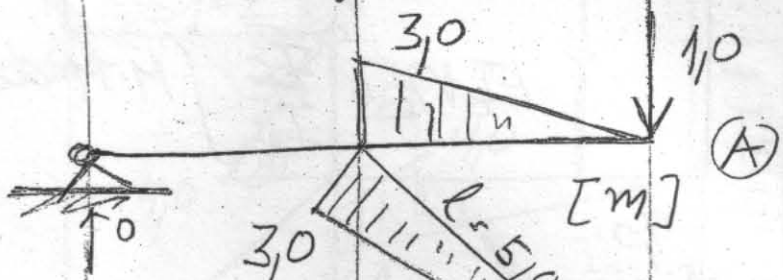
$f = \frac{1}{8} q_0 \cdot 3^2 = \frac{9}{8} q_0$

$Y_B = 2{,}25\, q_0$

$Y_C = 0{,}75\, q_0$

$\curvearrowright \Sigma M_B = 0 \qquad 3q_0 \cdot 1{,}5 - Y_C \cdot 6 = 0$

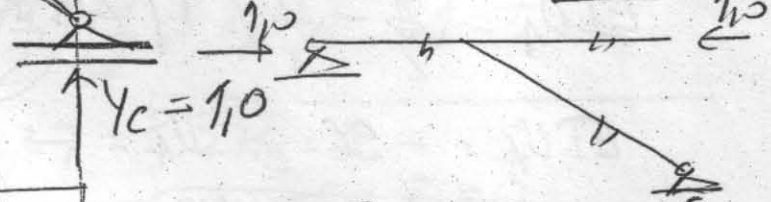
(M) za yoxl



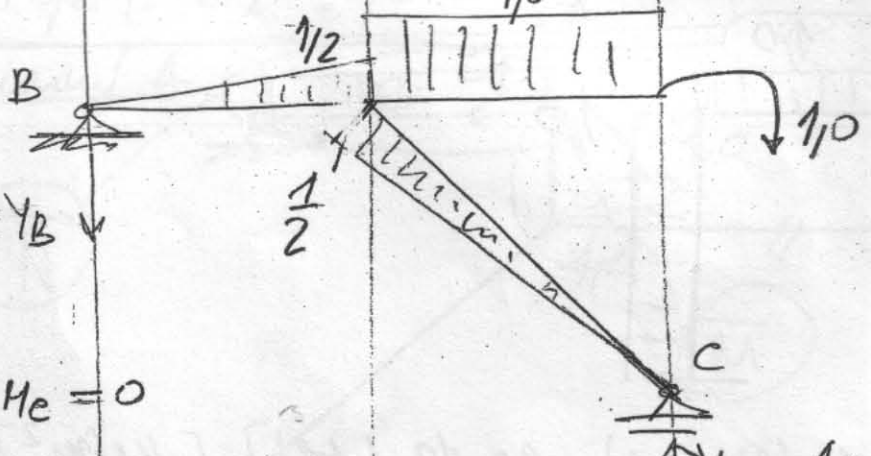
$EI\, U_A = \left(\frac{1}{2} 5 \cdot 3\right) \cdot \frac{2}{3} \cdot 2{,}25\, q_0$

$U_A = \frac{1}{EI} 11{,}25\, q_0 \; [m^4]$

$\frac{kN}{m}$ $U_A = 0$



(M) za nadir



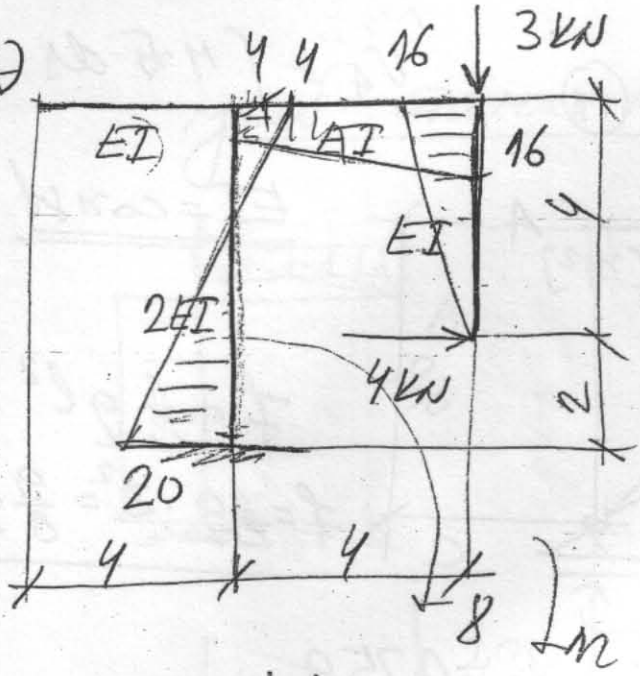
$\Sigma M_C = 0$

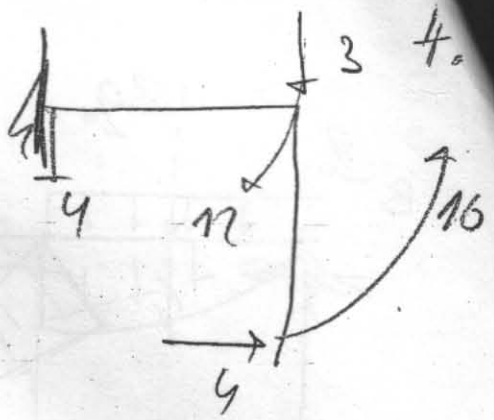
$-Y_B \cdot 6 + 1{,}0 = 0$

$EI\, \varphi_A = \left(\frac{1}{2} \cdot 5 \cdot \frac{1}{2}\right) \cdot \frac{2}{3} \cdot 2{,}25\, q_0 - \left(\frac{1}{2} \cdot 3 \cdot \frac{1}{2}\right) \cdot \frac{2}{3} \cdot 2{,}25\, q_0 -$

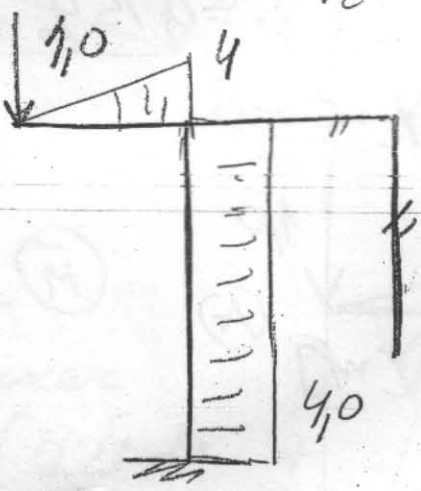
$- \frac{2}{3} 3 \cdot \frac{9}{8} q_0 \cdot \frac{1}{4} \; [m^4]$

(b) (A)





$\int \bar{M} \cdot M \, ds$



szukam U_A (A)

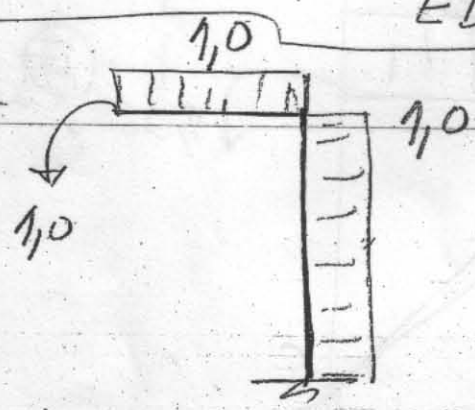
$$U_A = \int \frac{M \cdot \bar{M}}{EI} ds \quad / \cdot EI_c$$

$$EI_c U_A = \frac{I_c}{I_i} \int M \cdot \bar{M} \, ds$$

$$EI U_A = \frac{1}{2} \cdot \frac{6}{6} [4(2 \cdot 4 + 4) - 20(2 \cdot 4 + 4)]$$

$$EI U_A = -96 \qquad U_A = -\frac{1}{EI} \cdot 96 \; [kNm^3]$$

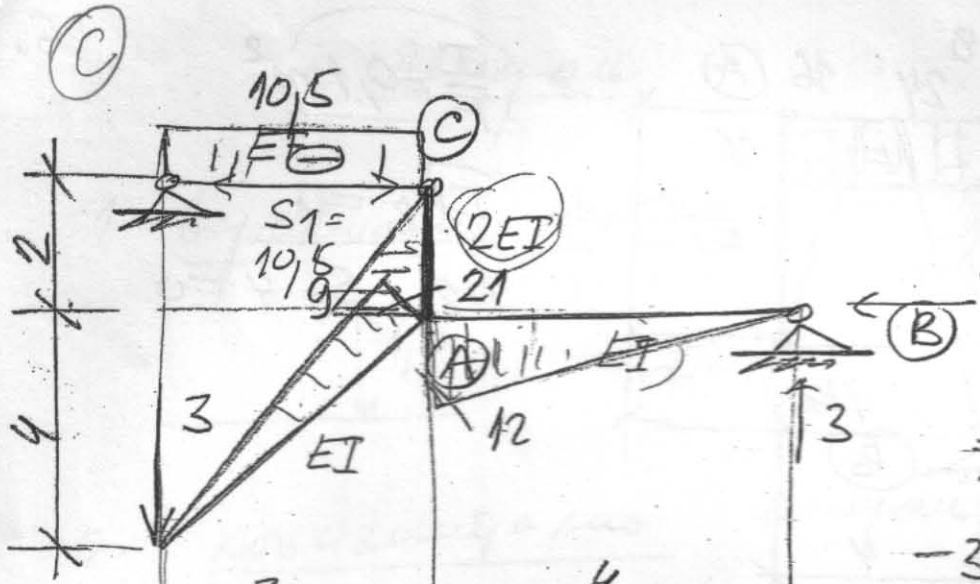
szukam φ_A



$$E \cdot \varphi_A = \frac{1}{2} \cdot \frac{6}{6} [4 \cdot (2 \cdot 1 + 1) - 20 \cdot (2 \cdot 1 + 1)] \; [kNm^2]$$

$$\varphi_A = \frac{1}{EI} \; / -24 \; kNm^2]$$

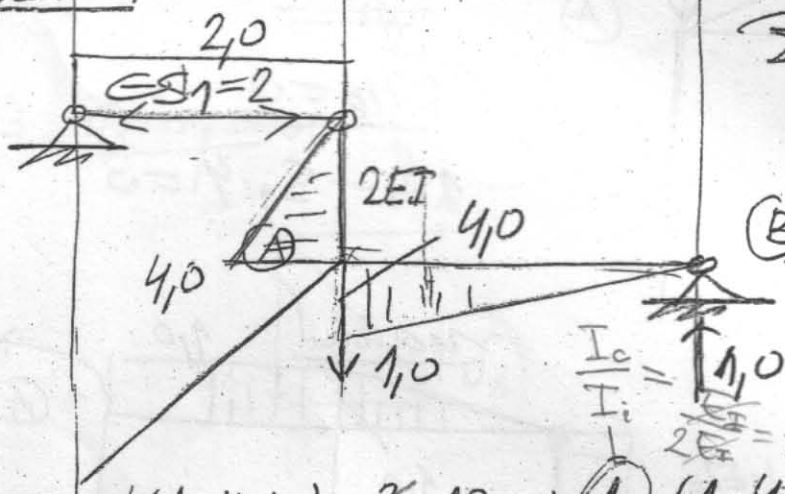
$\frac{I}{F} = 0{,}2\,m^2$



$\Sigma M_B = 0$

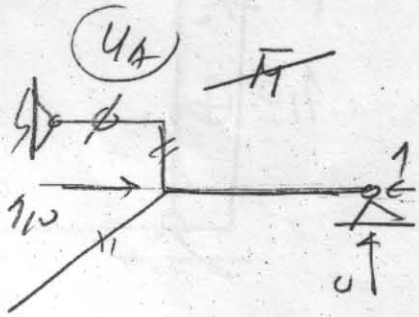
$-3 \cdot 7 + S_1 \cdot 2 = 0$

układ A



$\Sigma M_B = 0$

$S_1 \cdot 2 - 1 \cdot 4 = 0$



$\frac{I_c}{I_i} = \frac{EI}{2EI} = \frac{1}{2}$

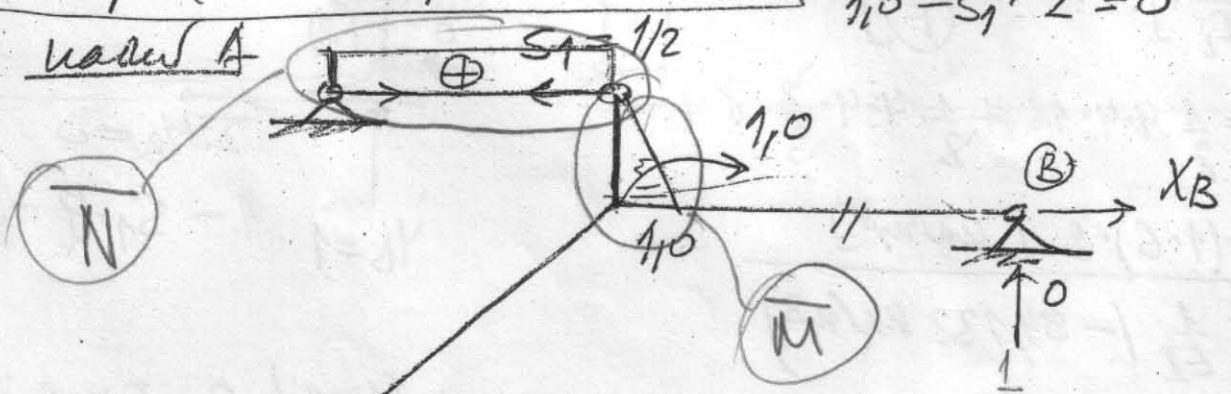
$EI\,u_A = \left(\frac{1}{2} \cdot 4 \cdot 4\right) \cdot \frac{2}{3} \cdot 12 + \frac{1}{2} \cdot \left(\frac{1}{2} \cdot 4 \cdot 2\right) \cdot \frac{2}{3} \cdot 21 +$

$+ 0{,}2 \cdot (2 \cdot 3) \cdot 10{,}5 \ [kNm^3]$

$\Sigma M_B = 0$

$1{,}0 - S_1 \cdot 2 = 0$

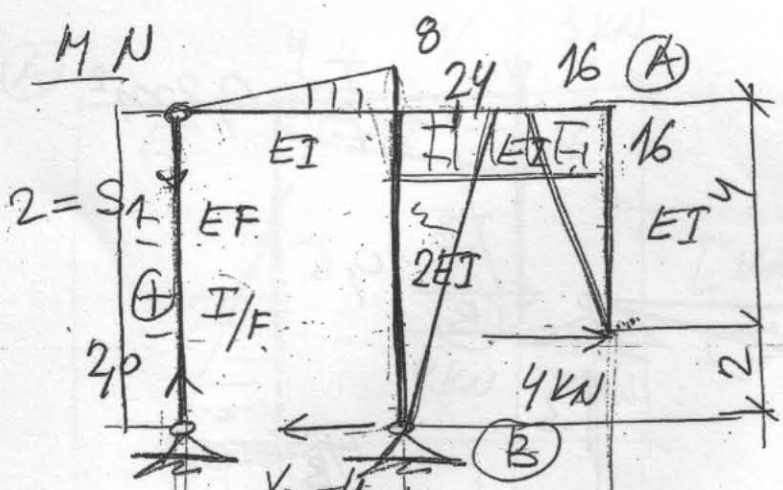
układ A



$EI\,u_A = -\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2 \cdot 1\right) \cdot \frac{2}{3} \cdot 21 - 0{,}2 \cdot (3 \cdot 1) \cdot 10{,}5 \ [kNm^3]$

$\frac{I}{F}$

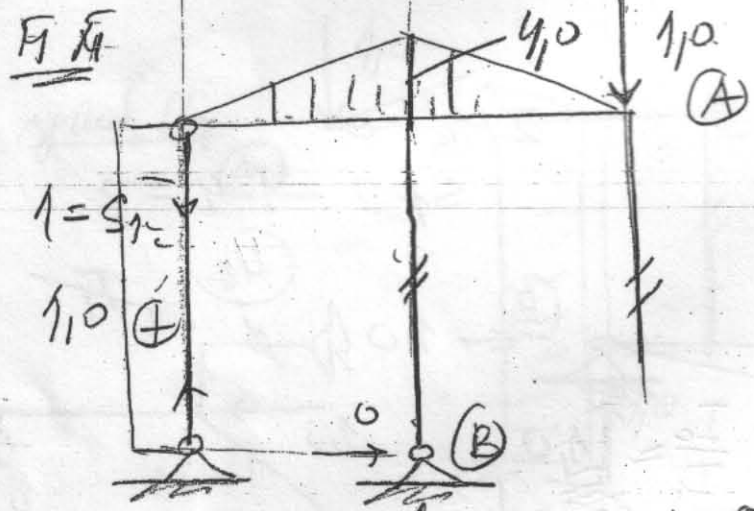
d



$\frac{I}{F} = 0{,}1\ m^2$

$\overrightarrow{\Sigma M_B = 0}$

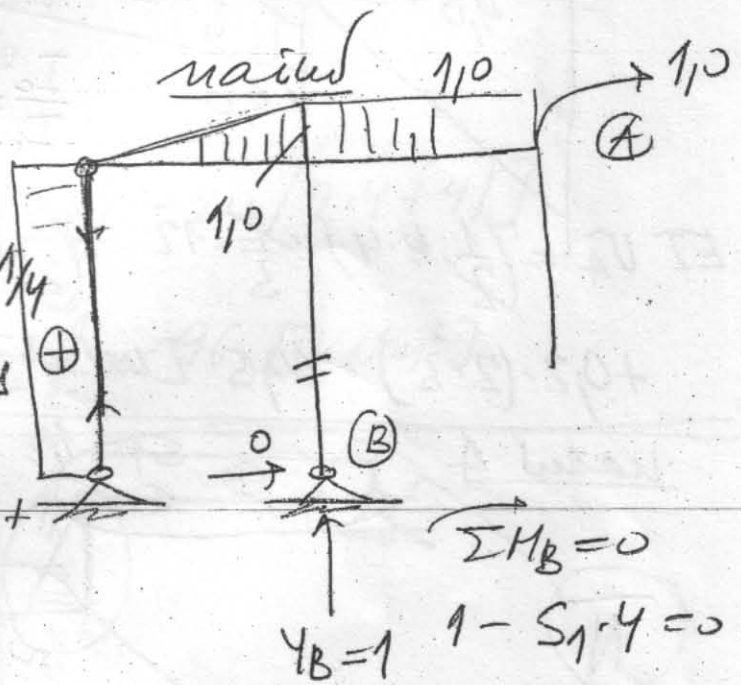
$4 \cdot 2 - S_1 \cdot 4 = 0$



усил

$\overrightarrow{\Sigma M_B = 0}$

$1 \cdot 4 - S_1 \cdot 4 = 0$



$$U_A = \int \frac{M \cdot \bar{M}}{EI_i} ds + \int \frac{N \cdot \bar{N}}{EF} ds \quad / \cdot EI_c \qquad S_1 = 1/4$$

$$EI_c U_A = \frac{I_c}{I_i} \cdot \int M \cdot \bar{M}\, ds + \frac{I_c}{F} \int N \cdot \bar{N}\, ds$$

$$EI U_A = -\frac{1}{2} 4 \cdot 4 \cdot 16 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 8 +$$

$$+ 0{,}1 \cdot (1 \cdot 6) \cdot 2 \ [kN/m^3]$$

$$U_A = \frac{1}{EI} (-84{,}133\ kN/m^3)$$

$$EI \varphi_A = -1 \cdot 4 \cdot 16 + \frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{2}{3} \cdot 8 + 0{,}1 \cdot \left(\frac{1}{4} \cdot 6\right) \cdot 2 \ [kN/m^2]$$

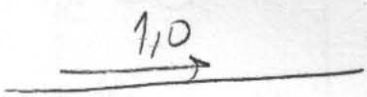
$$\varphi_A = -\frac{1}{EI}\ 53{,}03 s$$

Померы

1. вертикальные померы – угол (V_A)

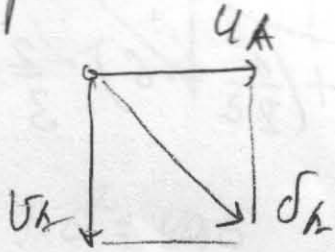
Н Н

2. горизонтальные померы (U_A)

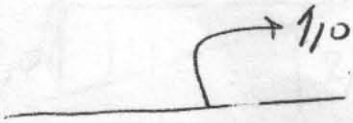


Н Н

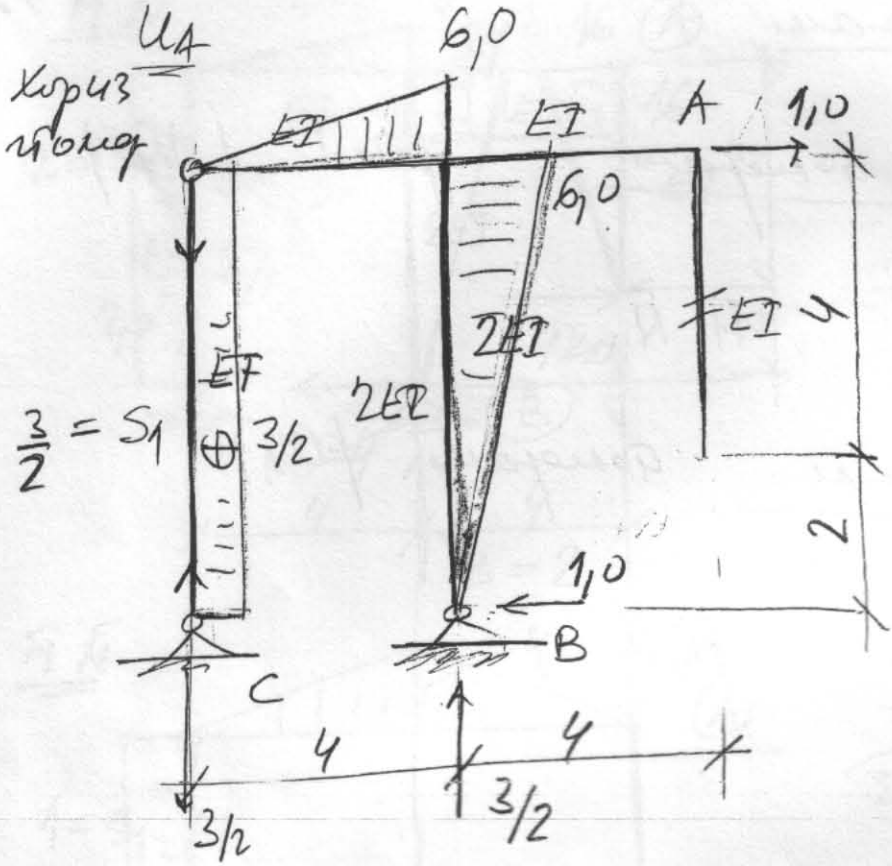
3. померы



4. обратные – ротационные – наклон



u_A — horiz. elmoz.



M M

$\overrightarrow{\Sigma} M_B = 0$

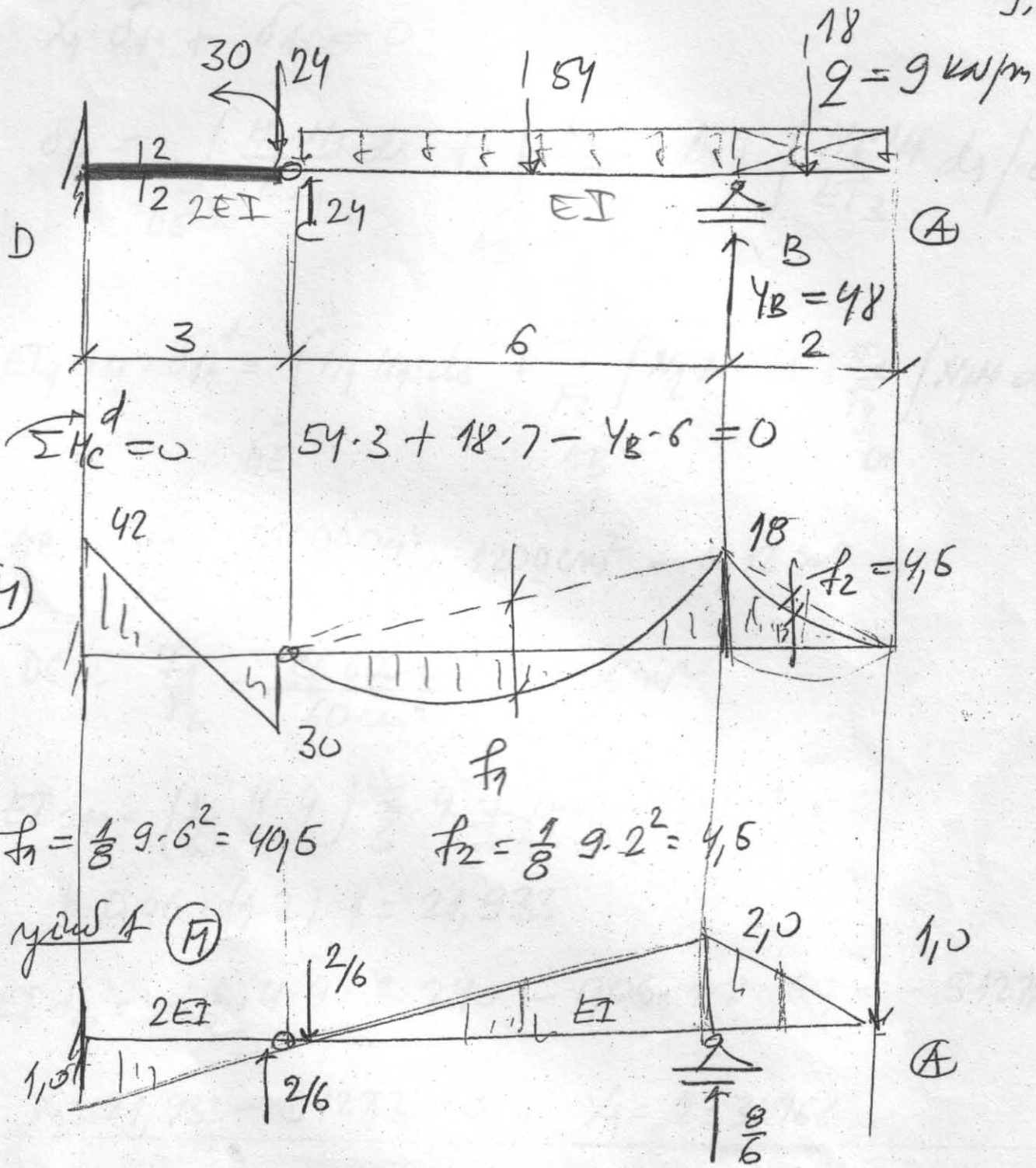
$1 \cdot 6 - S_1 \cdot 4 = 0$

$S_1 = \frac{6}{4} = \frac{3}{2}$

$$u_A = \frac{1}{EI}\left\{ \frac{1}{2} \cdot \left(\frac{1}{2} \cdot 6 \cdot 6\right) \cdot \frac{2}{3} \cdot 24 + \left(\frac{1}{2} \cdot 4 \cdot 6\right) \cdot \frac{2}{3} \cdot 8 + \right.$$

$$\left. + 0{,}1 \cdot \left(\frac{3}{2} \cdot 6\right) \cdot 2 \right\}$$

$$\delta_4 = \sqrt{u_A^2 + v_A^2}$$



$$EI\,v_A = \left(\frac{1}{2} \cdot 2 \cdot 2\right) \cdot \frac{2}{3} \cdot 18 - \frac{2}{3} \cdot 2 \cdot 4{,}5 \cdot 1{,}0 +$$

$$+\left(\frac{1}{2} \cdot 6 \cdot 2\right) \cdot \frac{2}{3} \cdot 18 - \frac{2}{3} \cdot 6 \cdot 40{,}5 \cdot 1{,}0 +$$

$$+\frac{1}{2} \cdot \frac{3}{6} \left[1 \,|\, -242 + 30\right] \; kNm^3$$

$$X_1 \cdot \delta_{11} + \delta_{10} = 0$$

$$\delta_{11} = \int_{DE} \frac{M_1 \cdot M_1}{EI_1} ds + \int_{AB} \frac{N_1 \cdot N_1}{EF_2} ds + \int_{DC} \frac{N_1 \cdot N_1}{EF_3} ds \quad / \cdot E$$

$$EI_1 \cdot \delta_{11} = \delta_{11}^* = \int_{DE} M_1 \cdot M_1 \, ds + \frac{I_1}{F_2} \int_{AB} N_1 \cdot N_1 \, ds + \frac{I_1}{F_3} \int_{DC} N_1 N_1 \, ds$$

$$AB: \quad \frac{I_1}{F_2} = \frac{36000 \, cm^4}{30 \, cm^2} = 1200 \, cm^2 = 0{,}12 \, m^2$$

$$DC: \quad \frac{I_1}{F_3} = \frac{36000 \, cm^4}{60 \, cm^2} = 0{,}06 \, m^2$$

$$EI\delta_{11} = \left(\frac{1}{2} \cdot 4 \cdot 4\right) \cdot \frac{2}{3} \cdot 4 + 0{,}12 \cdot (1 \cdot 4) \cdot 1 +$$

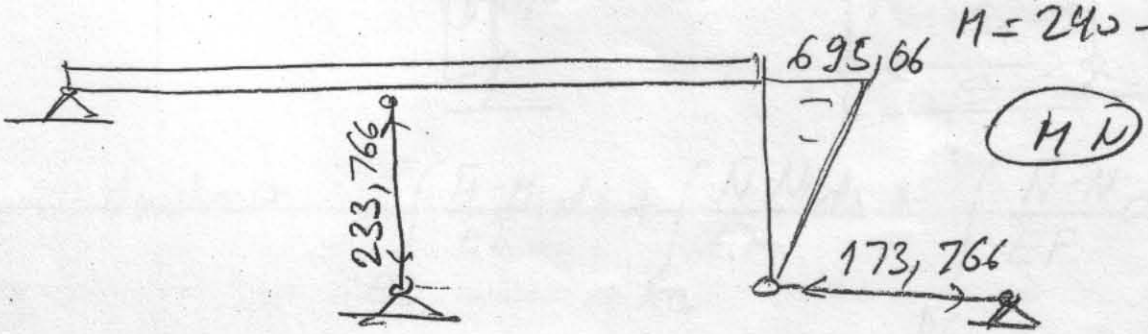
$$+ 0{,}06 \cdot (1 \cdot 2) \cdot 1 = 21{,}933$$

$$EI\delta_{10} = -\left(\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} 240 - 0{,}06 \cdot 1 \cdot 2 \cdot 60 = -5127{,}2\right.$$

$$X_1 \cdot 21{,}933 - 5127{,}2 = 0 \qquad \underline{X_1 = 233{,}766}$$

$$\underline{M = M_0 + X_1 \cdot M_1} \qquad \underline{N = N_0 + X_1 N_1}$$

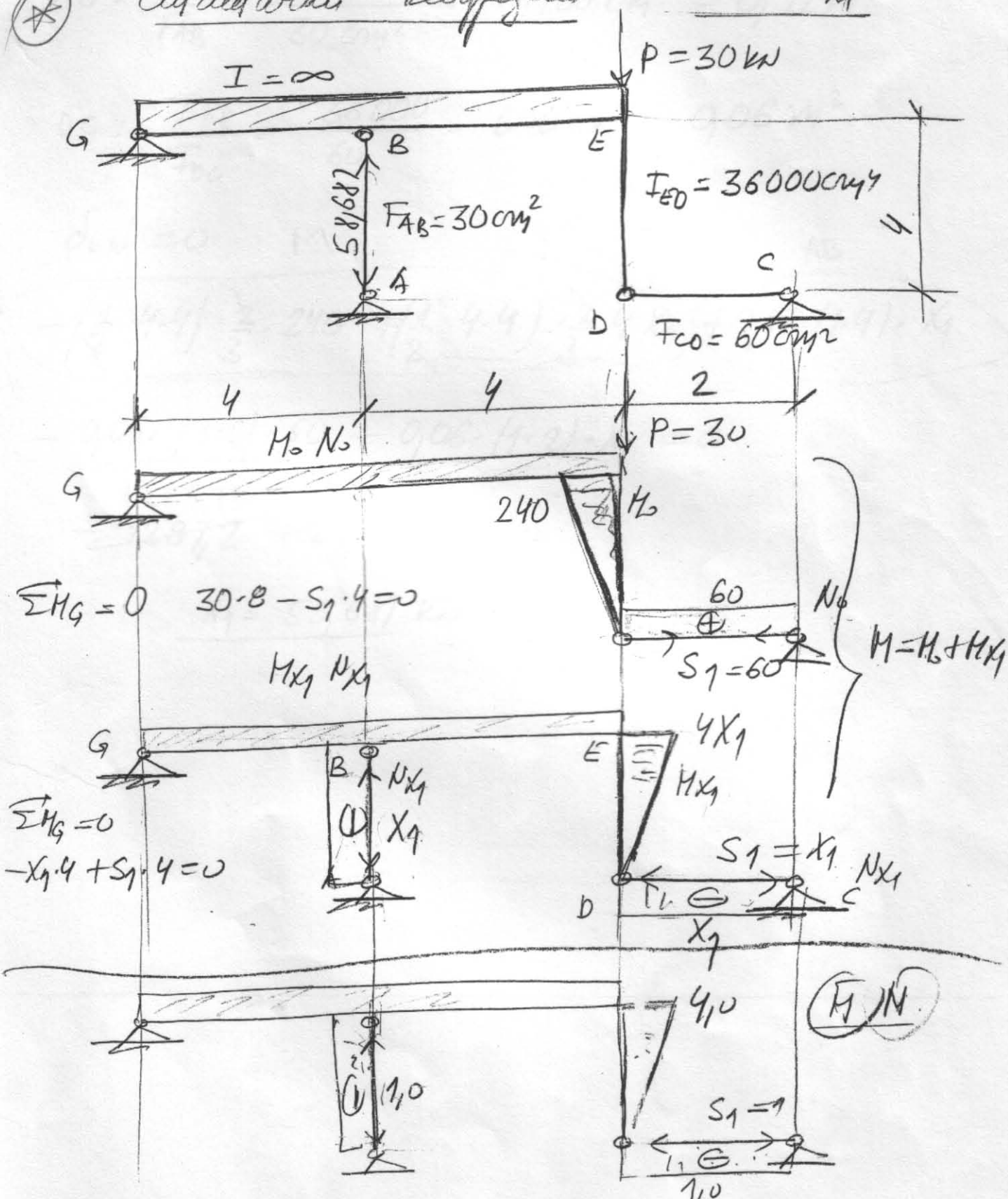
$$M = 240 - 4 \cdot 233{,}76$$



$$S = 60 - 1 \cdot 233{,}766 = -173{,}766$$

Статически неопределимая система

$M = M_0 + M_{X_1}$



$$\delta_{kk} = 0 \qquad \int_{EB} \frac{\bar{M} \cdot M}{EI} ds + \int_{AB} \frac{\bar{N} \cdot N}{EF} ds + \int_{DC} \frac{\bar{N} \cdot N}{EF} ds = 0$$

$M = M_0 + M_{X_1}$ $\qquad N = N_0 + N_{X_1}$

$$AB: \frac{I_{DE}}{F_{AB}} = \frac{36\,000\ cm^4}{30\ cm^2} = 1200\ cm^2 = 0{,}12\ m^2$$

$$DC: \frac{I_{DE}}{F_{DC}} = \frac{36000}{60} = 600\ cm^2 = 0{,}06\ m^2$$

$\delta_{kd} = 0$ M

AB

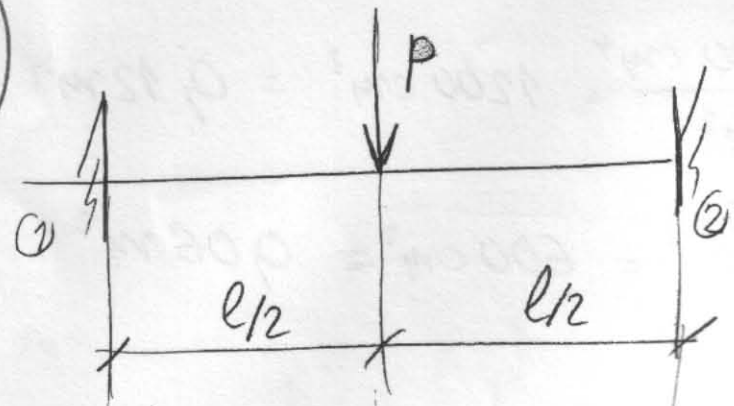
$$-\left(\frac{1}{2}\cdot 4\cdot 4\right)\cdot\frac{2}{3}\cdot 240 + \left(\frac{1}{2}\cdot 4\cdot 4\right)\cdot\frac{2}{3}\cdot 4X_1 + 0{,}12\cdot(1\cdot 4)\cdot X_1$$

$$-0{,}06\cdot(1\cdot 2)\cdot 60 + 0{,}06\cdot(1\cdot 2)\cdot X_1 = 0$$

$$-1286{,}72 + 21{,}933\,X_1 = 0$$

$$X_1 = 58{,}687\ kN$$

1



Общий условие равновесия

$r = 2k = 2 \cdot 2 = 4$

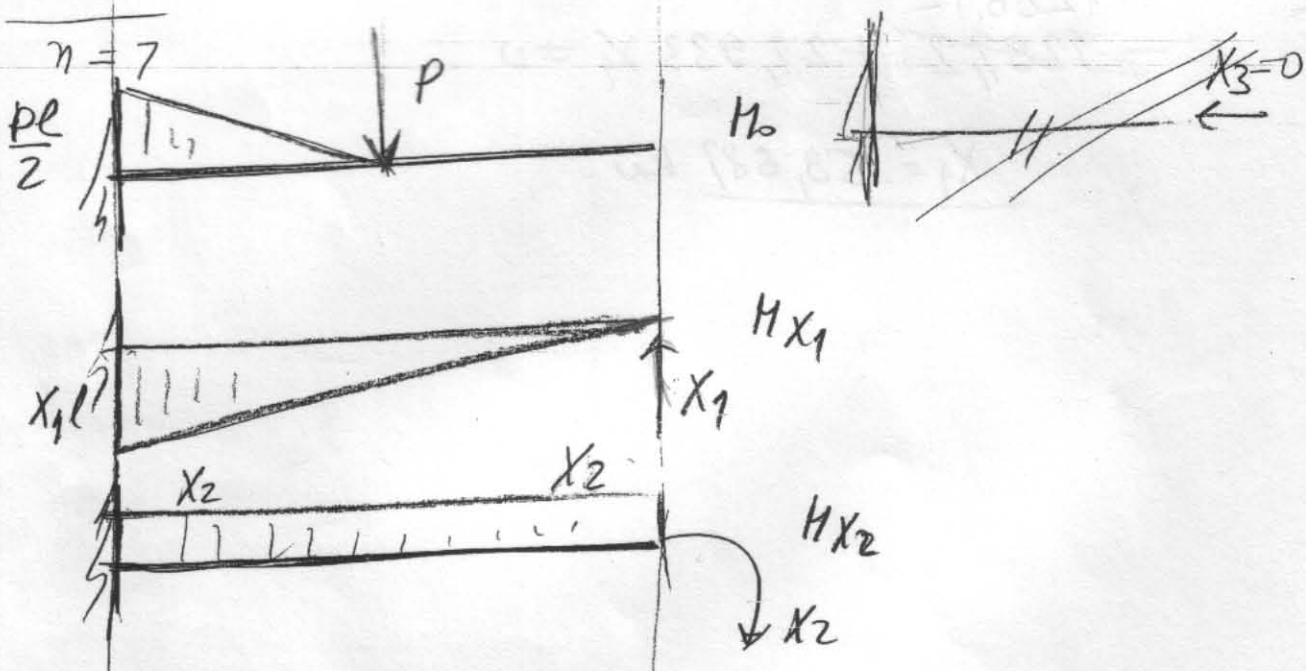
$Z_5 = 1$

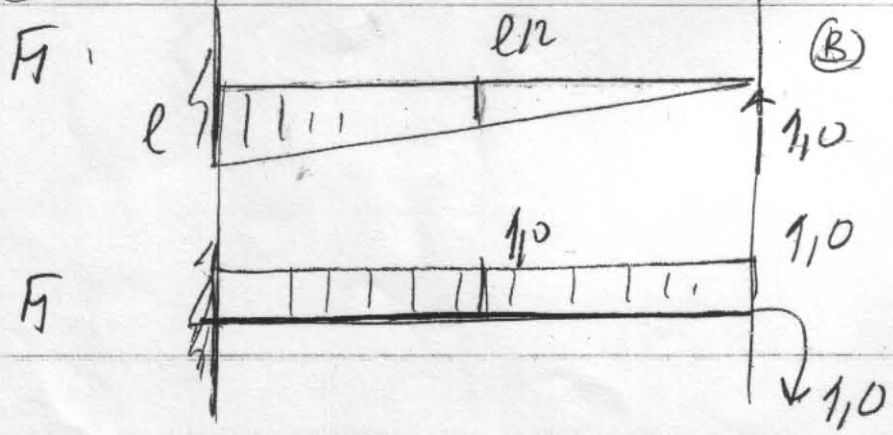
$Z_6 = 4$

$Z_4 = 2$

$Z_k = 0$

$n = 7$





Ⓑ $W_B = 0$

$\varphi_B = 0$

1) $W_B = 0$ $\qquad M = M_0 + M_{X_1} + M_{X_2}$

$$\int \frac{\bar{M} \cdot M}{EI} ds = 0$$

$$-\frac{1}{6}\frac{l}{2}\left[\frac{Pl}{2}\left(2l + \frac{l}{2}\right)\right] + \frac{1}{2} \cdot l l \cdot \frac{2}{3} X_1 l - \frac{1}{2} l l \cdot X_2 = 0$$

2) $\varphi_B = 0$

$$\frac{1}{2}\frac{l}{2} \cdot \frac{Pl}{2} \cdot 1 - \frac{1}{2} l \cdot X_1 l \cdot 1 + 1l \cdot X_2 = 0$$

$$-\frac{5}{48} Pl^3 + \frac{1}{3} X_1 l^3 - \frac{1}{2} X_2 l^2 = 0 \quad /: l^2 \quad /. 48$$

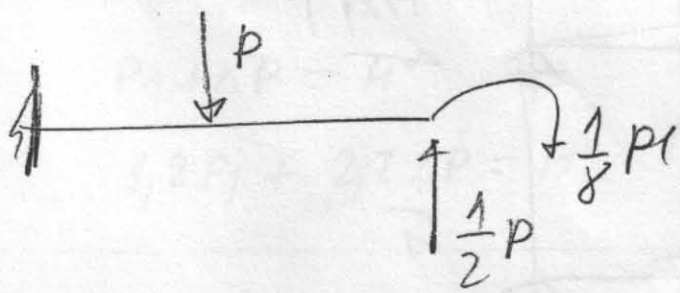
$$\frac{1}{8} Pl^2 - \frac{1}{2} X_1 l^2 + X_2 l = 0 \quad /: l \quad /. 8$$

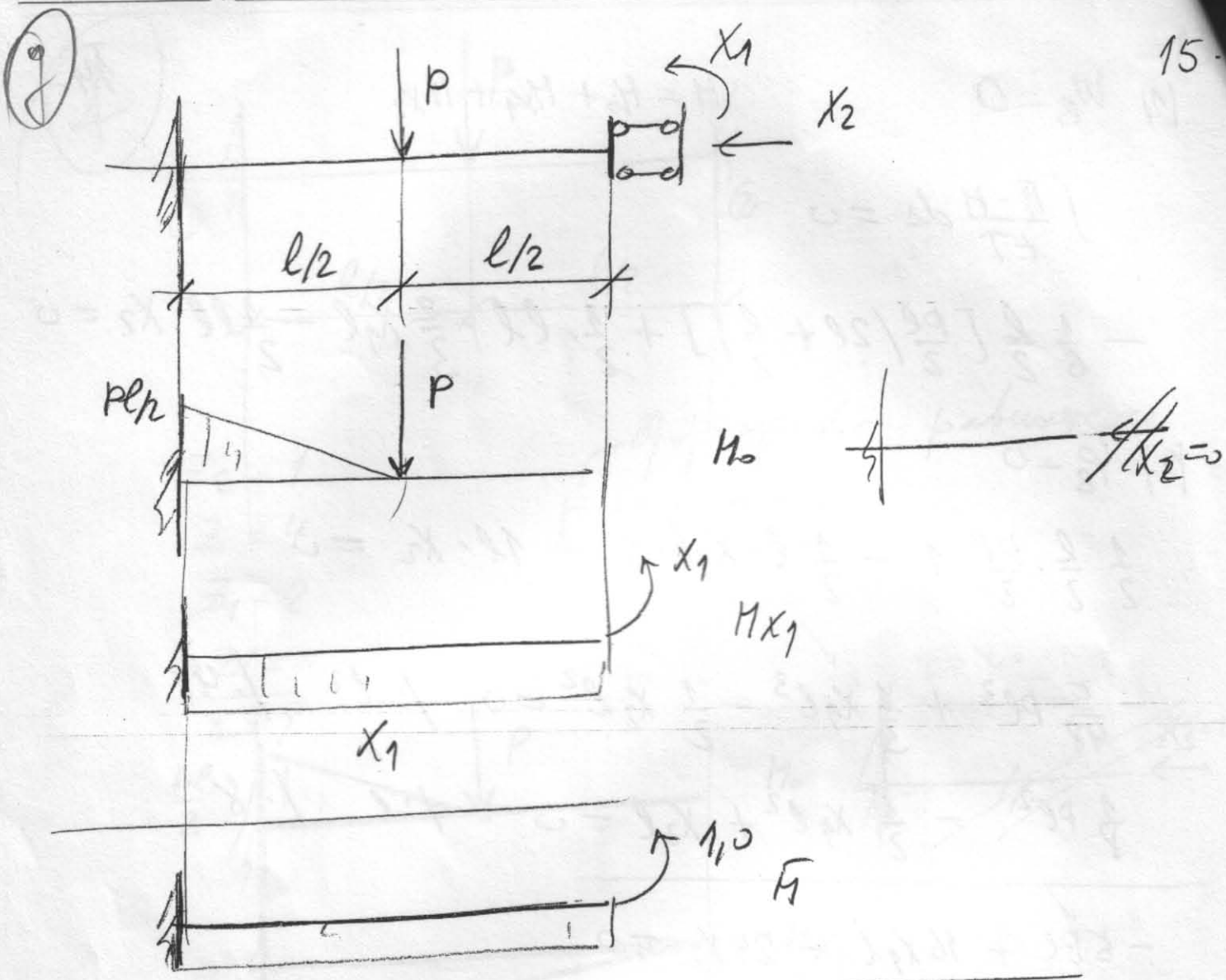
$$-5Pl + 16 X_1 l - 24 X_2 = 0$$

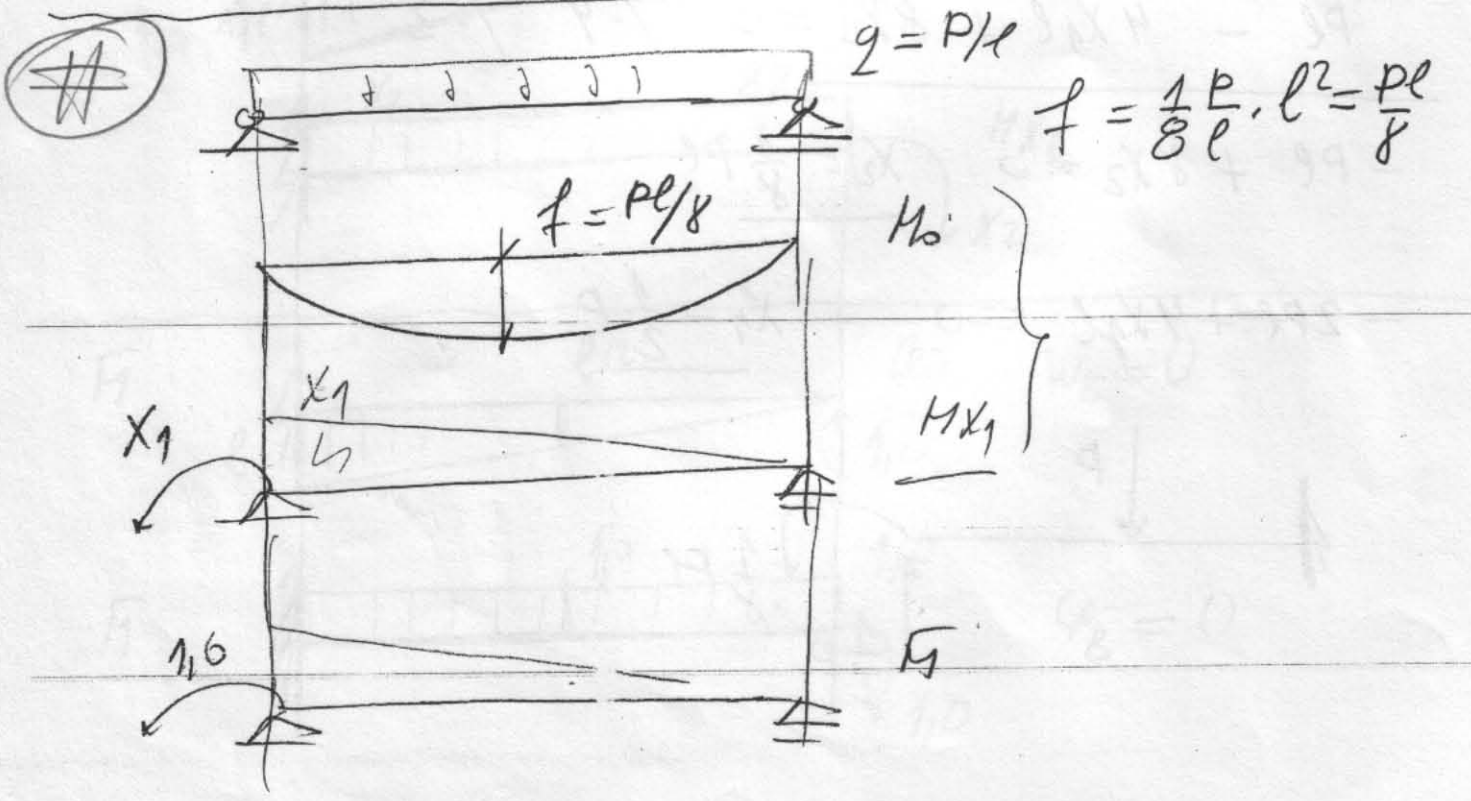
$$Pl - 4 X_1 l + 8 X_2 = 0 \quad /. 4 \quad /. 3$$

$$-Pl + 8 X_2 = 0 \qquad \underline{X_2 = \frac{1}{8} Pl}$$

$$-2Pl + 4 X_1 l = 0 \qquad \underline{X_1 = \frac{1}{2} P}$$




P
X1
X2
l/2
l/2
Pl/2
P
M0
X2=0
X1
MX1
X1
1,0
M

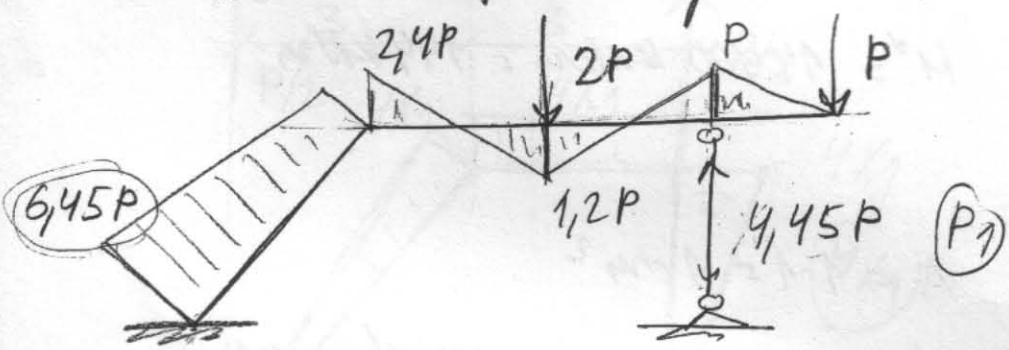

q = P/l
f = 1/8 P/l · l² = Pl/8
f = Pl/8
M0
MX1
X1
X1
1,0
M

Селс лонс P^* методс кораи кораи

по кораи кораи

$W^* = \Sigma A \cdot d_{n-n}$ $\quad M^* = W^* \cdot \sigma_T$ градиш косан

$N^* = A \cdot \sigma_T$ зс просеч шупалов

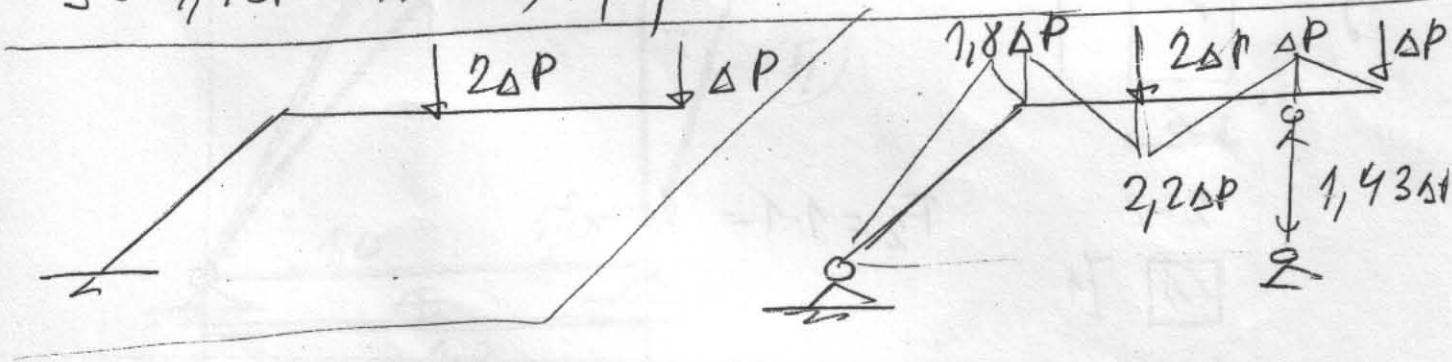


Први кораи

$\max M\ 6{,}45P = M^* \Rightarrow P_1$

$S = 4{,}45P = N^* \Rightarrow P_1$

$\min P_1 = \underline{P_1}$



$4{,}45P_1 + 1{,}43\Delta P = N^*$

$P_1 + \Delta P = M^*$

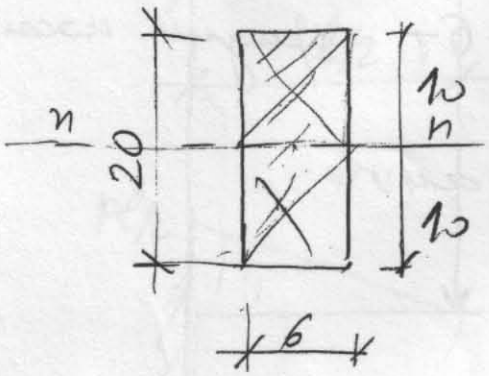
$1{,}2P_1 + 2{,}2\Delta P = M^*$

$\min \Delta P = \Delta P$

$\underline{P^* = P_1 + \Delta P}$

$\sigma_T = 240\,MPa = 24\,\frac{kN}{cm^2}$

1–1



$W^* = (6 \cdot 10 \cdot 5) \cdot 2 = 600\,cm^3$

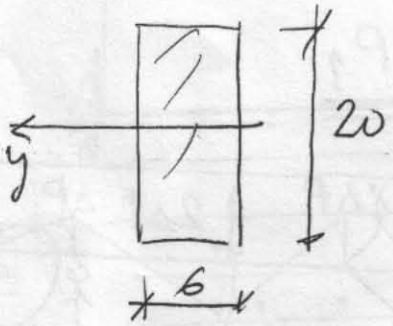
$M^* = W^* \cdot \sigma_T = 600\,cm^3 \cdot 24\,\frac{kN}{cm^2}$

$M^* = 14400\,kNcm = 144\,kNm$

$A = 1 \cdot 1 = 1\,cm^2$

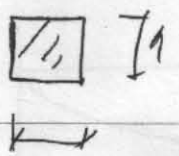
$N^* = A\,\sigma_T = 1\,cm^2 \cdot 24\,\frac{kN}{cm^2} = 24\,kN$

1–1



$I_y = \frac{6\,20^3}{12} = 4000\,cm^4$

2–2

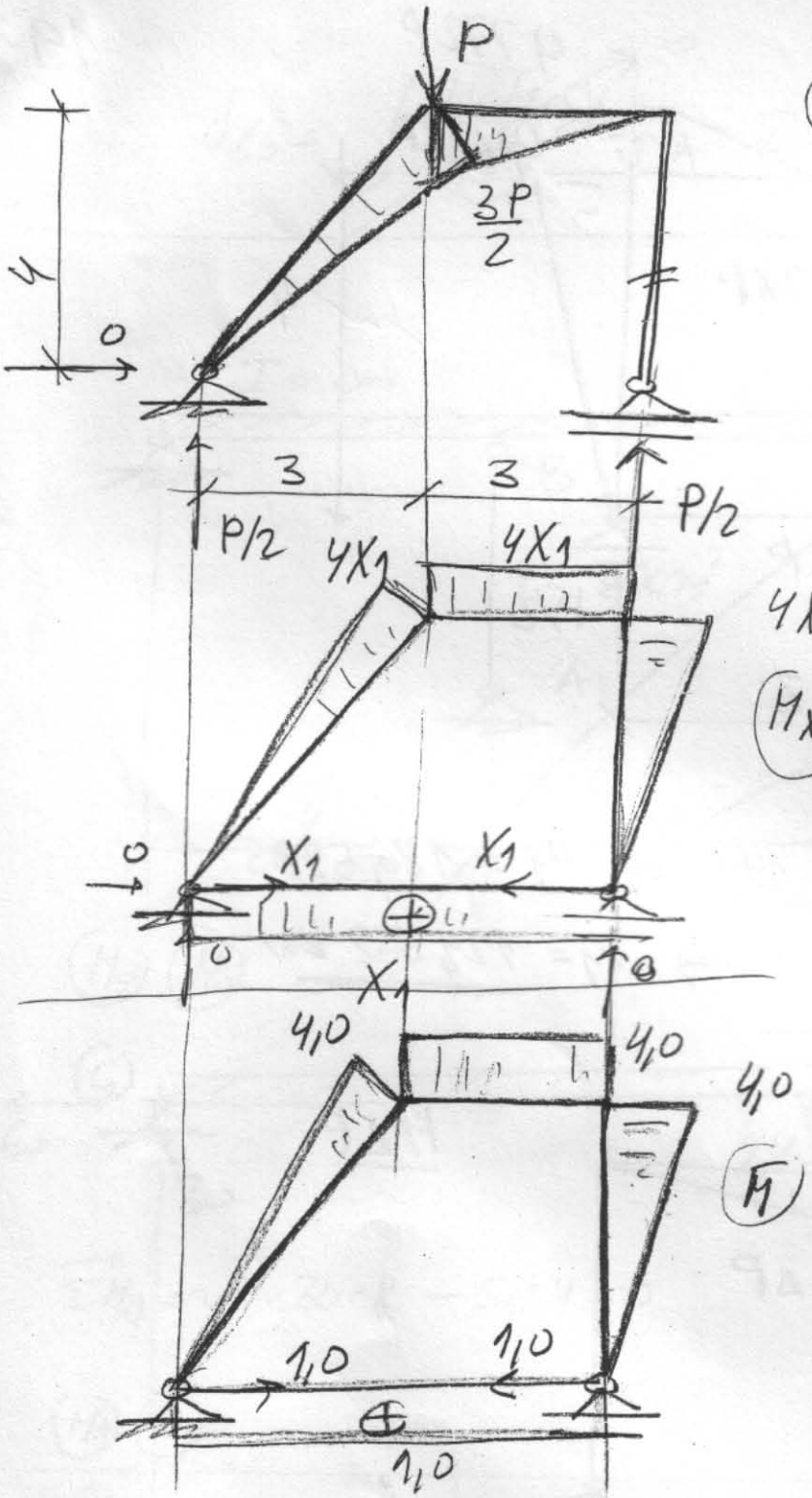


$F_2 = 1 \cdot 1 = 1\,cm^2$

$\frac{I}{F} = \frac{4000\,cm^4}{1\,cm^2} = 4000\,cm^2 = 0{,}4\,m^2$

$$\varepsilon_i = \int_s \frac{N\bar{N}}{EA}\,ds + \int_s \frac{M M_y}{EI_y}\,ds \quad / \cdot EI_y$$

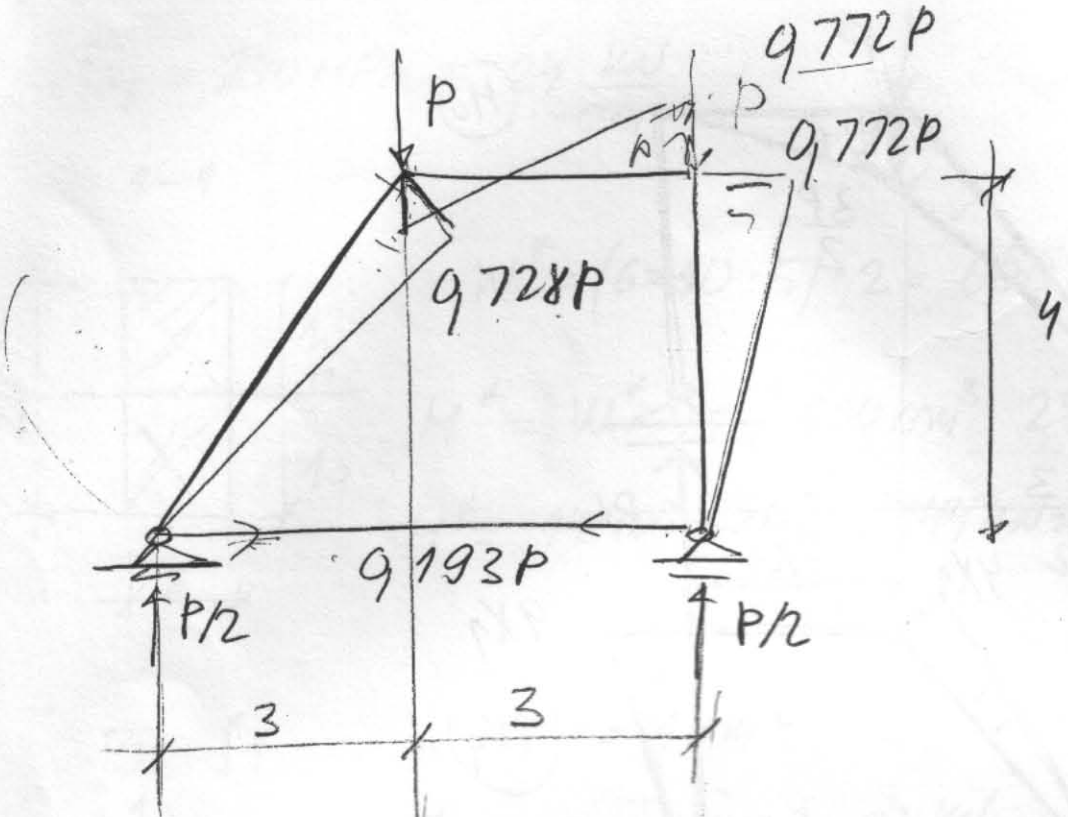
$$= \int_s M M_y\,ds + \frac{EI_y}{A} \int N\bar{N}$$



$$-\left(\tfrac{1}{2}\cdot 3\right)\cdot\tfrac{3}{4}P-\left(\tfrac{1}{2}\cdot 5\cdot 4\right)\tfrac{2}{3}\cdot\tfrac{3}{2}P+\tfrac{1}{2}\cdot 4\cdot 4\cdot\tfrac{2}{3}\cdot 4X_1+$$

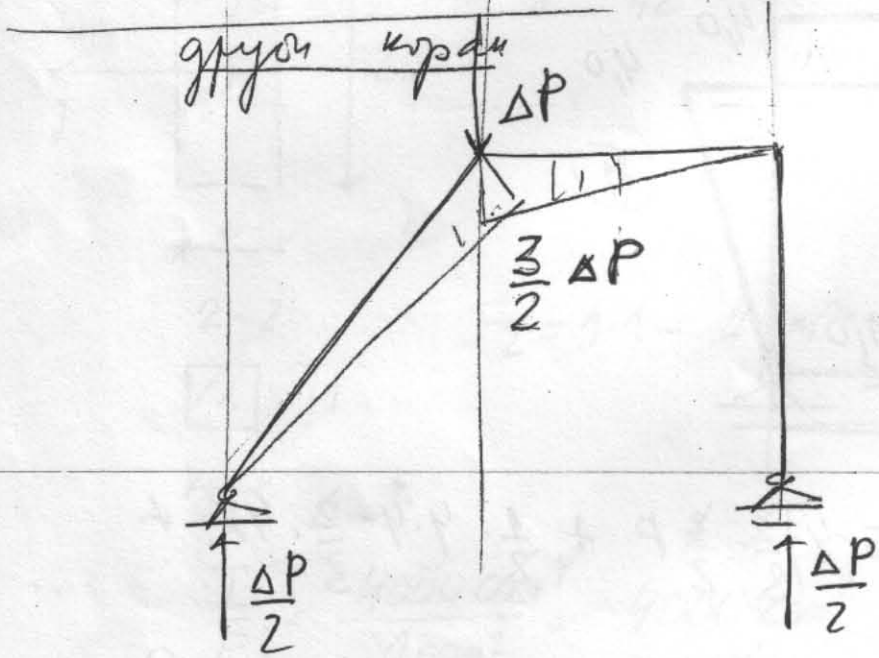
$$+(4\cdot 3)\cdot 4X_1+\tfrac{1}{2}\cdot 5\cdot 4\cdot\tfrac{2}{3}\cdot 4X_1-9{,}4\cdot(1{,}6)\cdot X_1=0$$

$$-19P+98{,}4X_1=0 \qquad \underline{X_1=0{,}193P}$$



$\max H = 0{,}772P = 144 \Rightarrow P_1 = 186{,}5285$

$S = 0{,}193P = 24 \Rightarrow P_1 = \underline{124{,}352\ kN}$



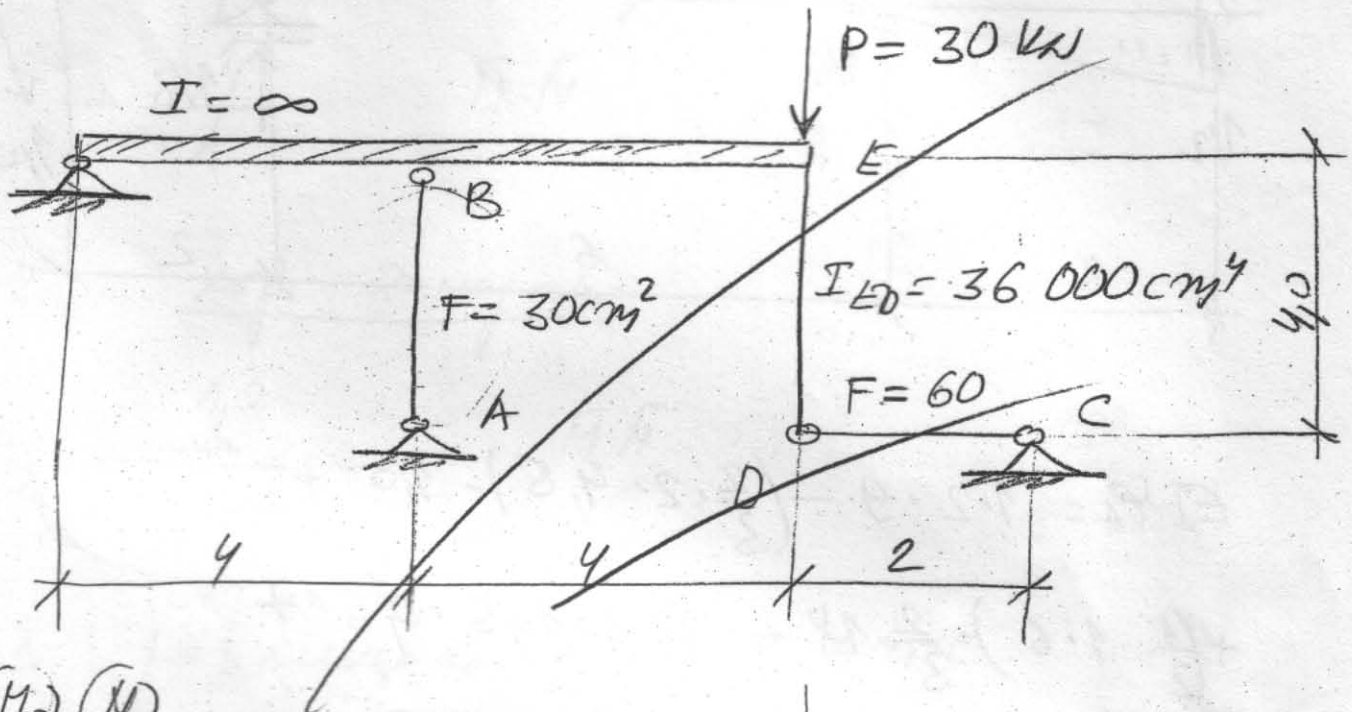
$M_{\Delta P}$

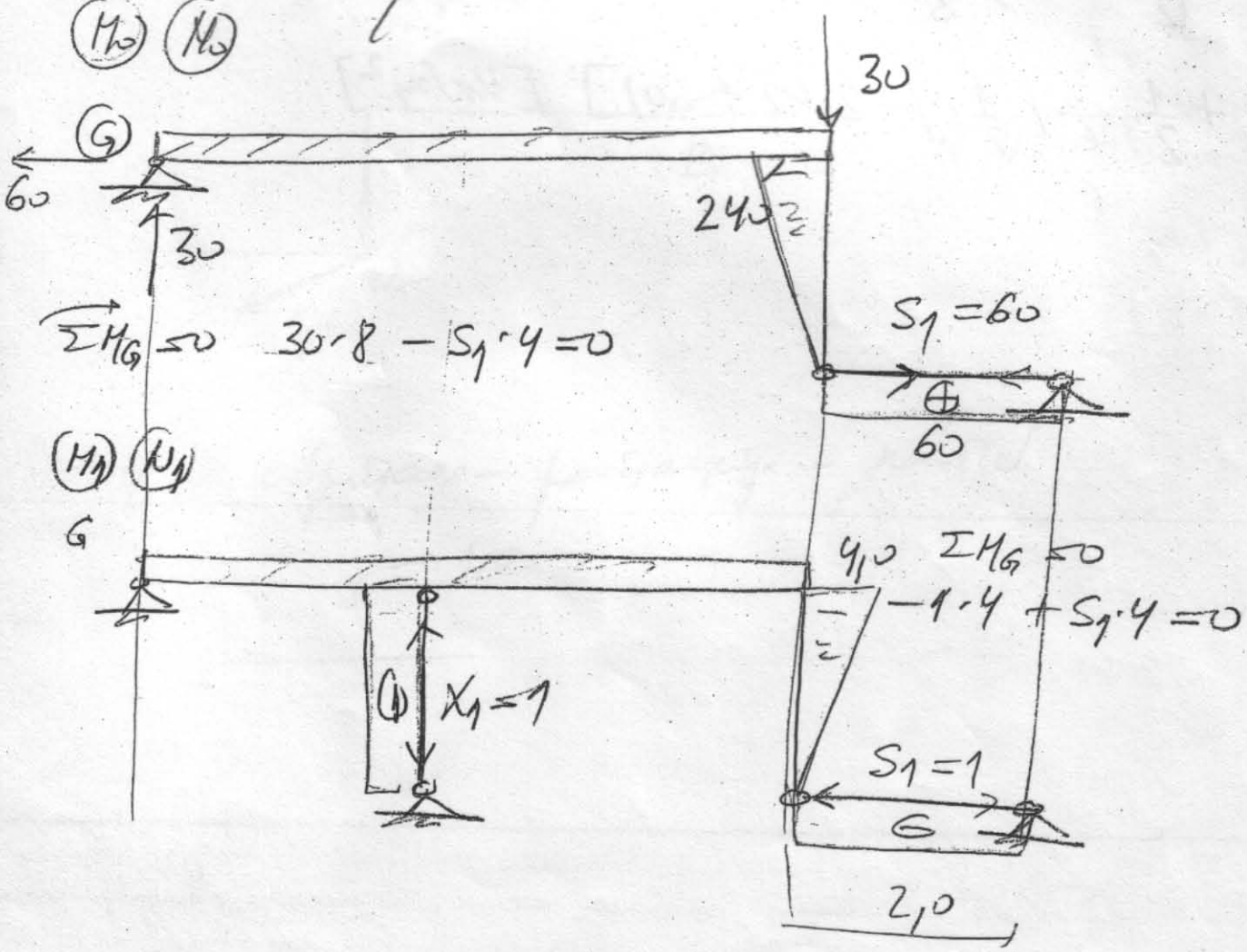
$0{,}728P_1 + \frac{3}{2}\Delta P = 144$

$0{,}728 \cdot (124{,}352) + \frac{3}{2}\Delta P = 144 \qquad \underline{\Delta P = 35{,}6478}$

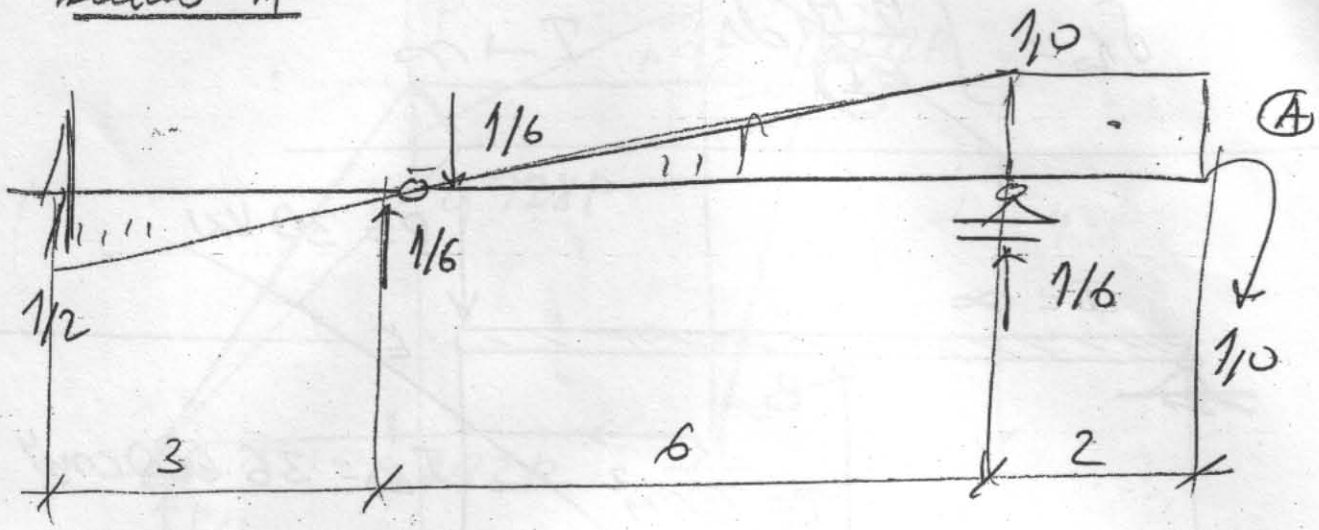
$P^* = P_1 + \Delta P = 160\ kN$

$$\delta_{10} = \int \frac{M \cdot M_1}{EI} ds \qquad I \to \infty$$





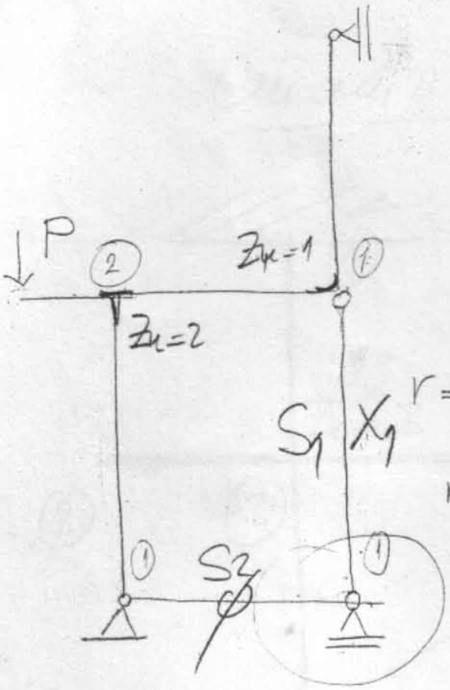
Nosník A



$$EI\varphi_A = 1 \cdot 2 \cdot 9 - \left(\frac{2}{3} \cdot 2 \cdot 4{,}5\right) \cdot 1{,}0 +$$

$$+\left(\frac{1}{2} \cdot 1 \cdot 6\right) \cdot \frac{2}{3} \cdot 18 - \frac{2}{3} \cdot 6 \cdot 4{,}05 - 9{,}5 +$$

$$+\frac{1}{2} \cdot \frac{3}{6}\left[\frac{1}{2} \cdot \left(-2 \cdot 42 + 30\right)\right] \quad [kNm^2]$$



$Z_s = 6$ ✓

$Z_o = 4$ ✓

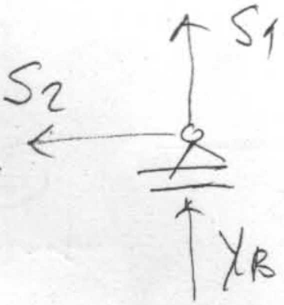
$Z_u = 0$ ✓

$Z_k = 3$ ✓

$n = 13$

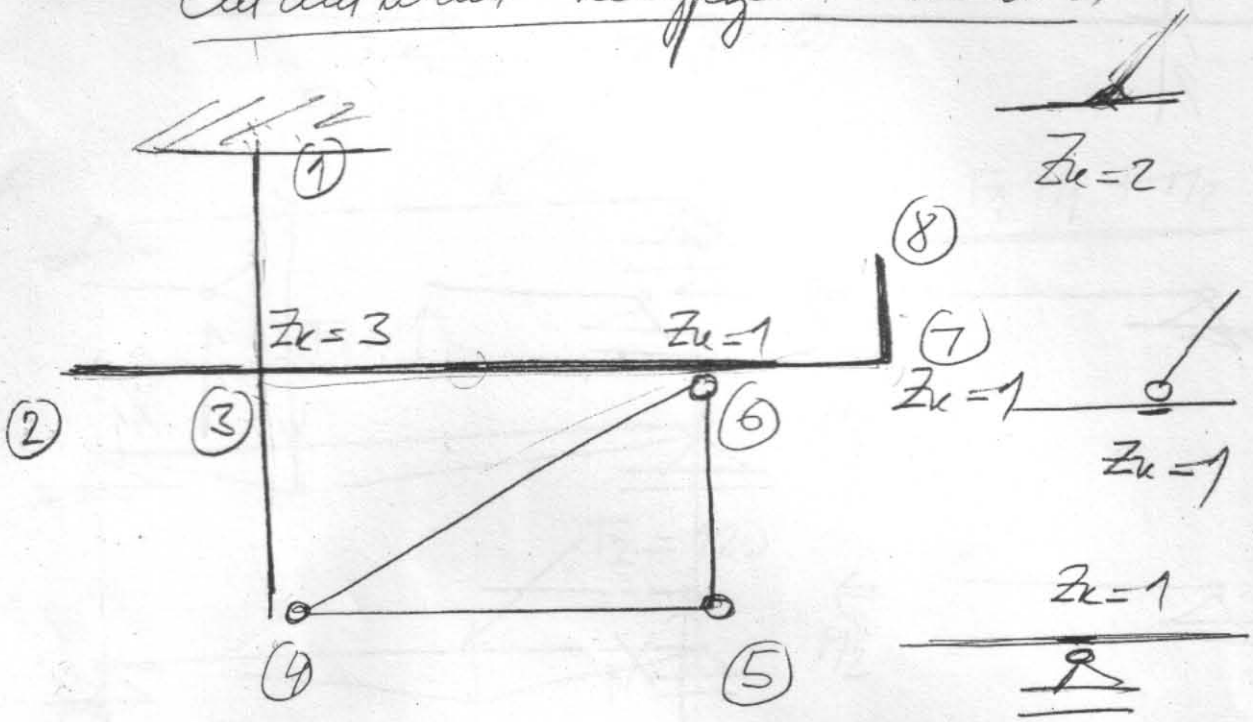
$r = 2k = 2 \cdot 6 = 12$

$n - r = 1 \times$ стат неод.



Брзо питање

Статички неодређени носач



непознате су:

Z_s = број свих штапова = 9

Z_o = број основачких веза = 2

Z_u = број могућих унутрашњих = 1

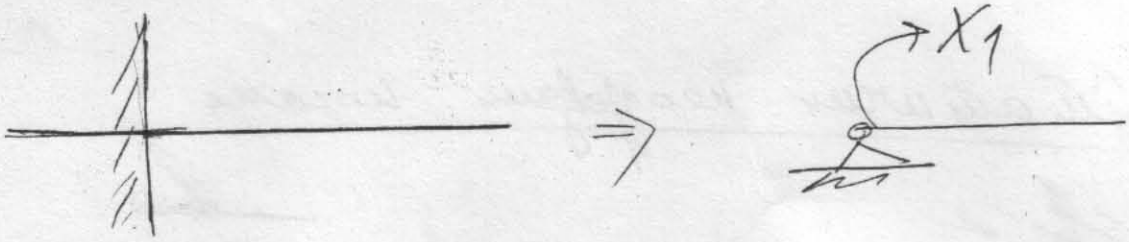
Z_k = број крутих чворова = 5

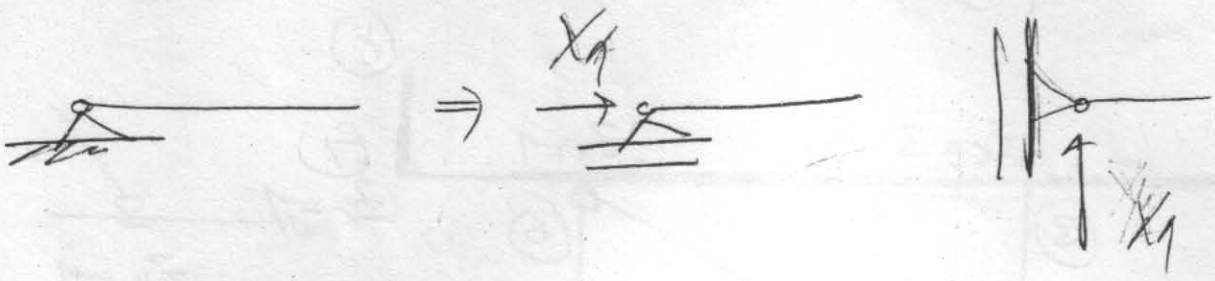
$n = 17$

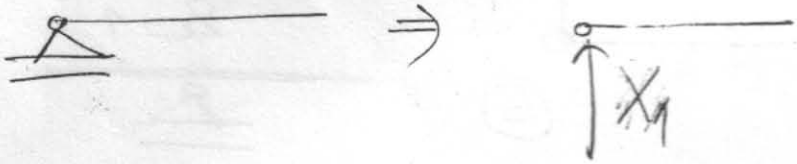
број чворова равнотеже

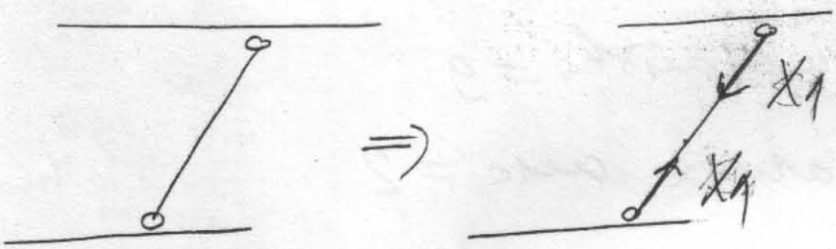
$r = 2k = 2 \cdot 8 = 16$

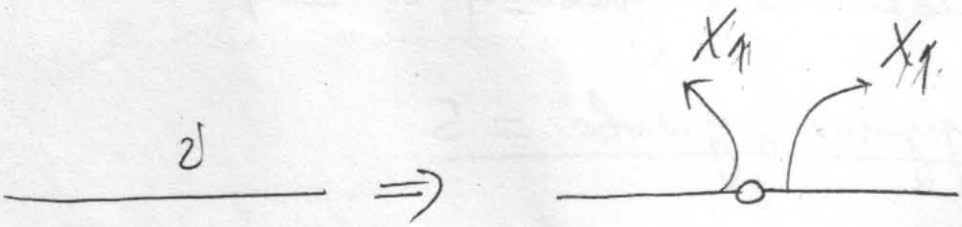
$n - r = 17 - 16 = 1 \times$ стан неодр


X1

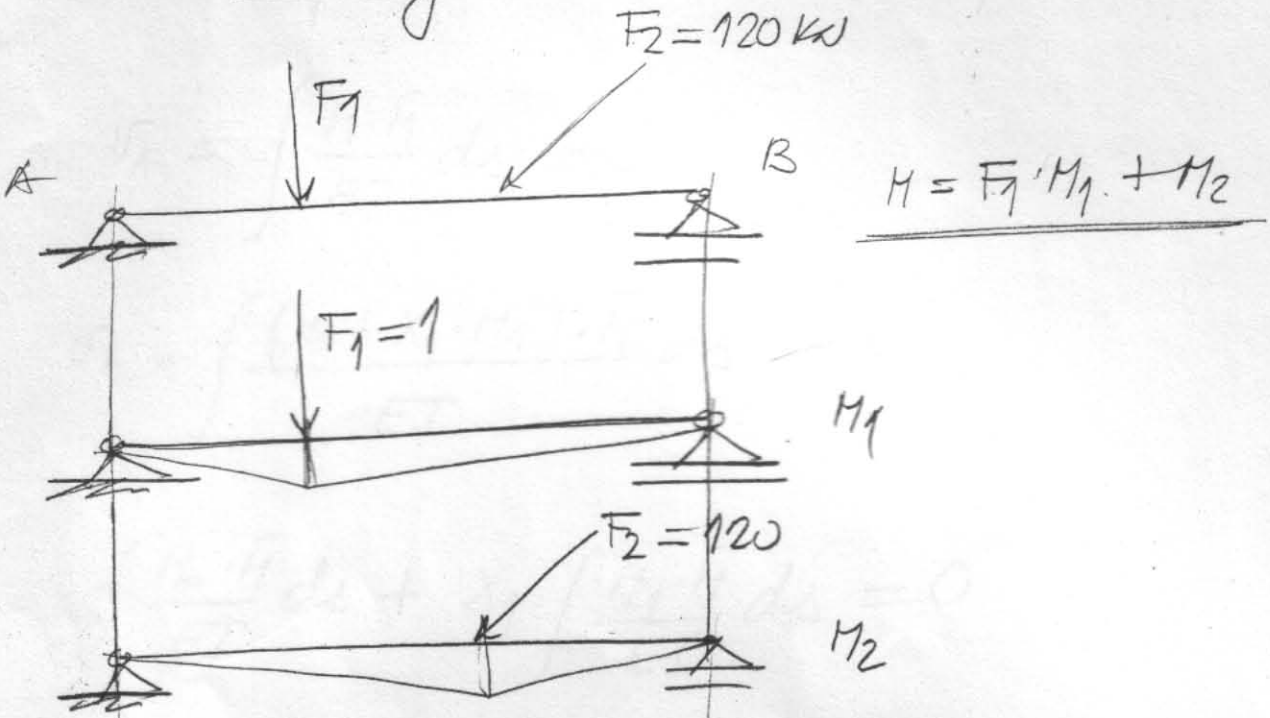

X1
X1


X1

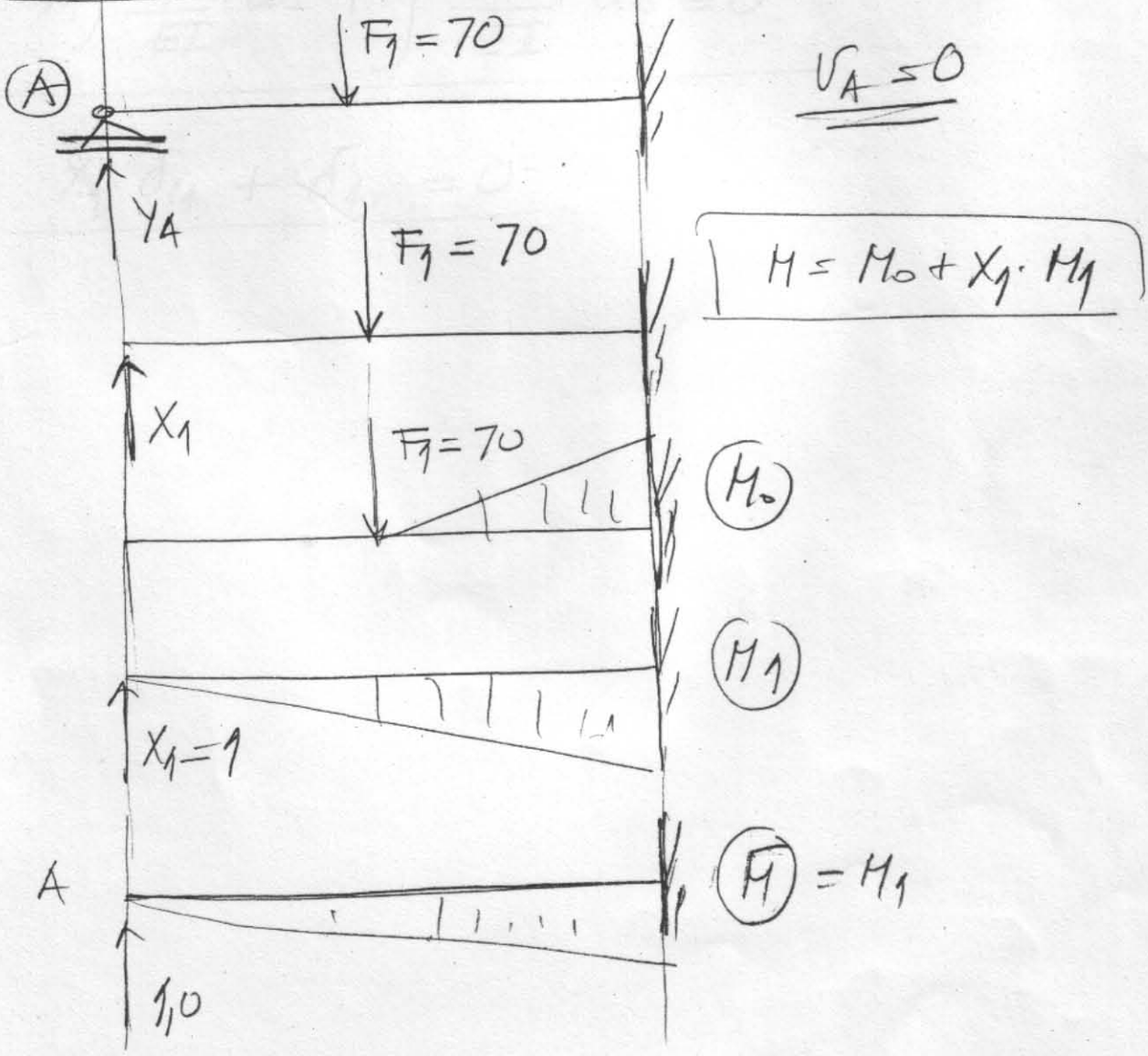

X1
X1


d
X1
X1

Метода сила



$$M = F_1 \cdot M_1 + M_2$$



$$V_A = 0$$

$$M = M_0 + X_1 \cdot M_1$$

Померање по Кастиљану

$$v_A = \int \frac{M \cdot \bar{M}}{EI} ds = 0$$

$$v_A = \int \frac{(M_0 + X_1 \cdot M_1) \cdot \bar{M}}{EI} ds = 0$$

$$\int \frac{M_0 \cdot \bar{M}}{EI} ds + X_1 \int \frac{M_1 \cdot \bar{M}}{EI} ds = 0$$

$$X_1 \int \frac{M_1 \cdot M_1}{EI} ds + \int \frac{M_0 \cdot M_1}{EI} ds = 0$$

$$X_1 \cdot \delta_{11} + \delta_{10} = 0$$

$$X_1 \cdot \delta_{11} + \delta_{10} = 0 \quad \Rightarrow X_1 \qquad / \cdot EI$$

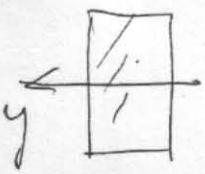
$$\left.\begin{aligned} X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} + \delta_{10} &= 0 \\ X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} + \delta_{20} &= 0 \end{aligned}\right\} \begin{matrix} X_1 \\ X_2 \end{matrix}$$

$$\left.\begin{aligned} X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} + X_3 \cdot \delta_{13} + \delta_{10} &= 0 \\ X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} + X_3 \cdot \delta_{23} + \delta_{20} &= 0 \\ X_1 \cdot \delta_{31} + X_2 \cdot \delta_{32} + X_3 \cdot \delta_{33} + \delta_{30} &= 0 \end{aligned}\right\} \begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix}$$

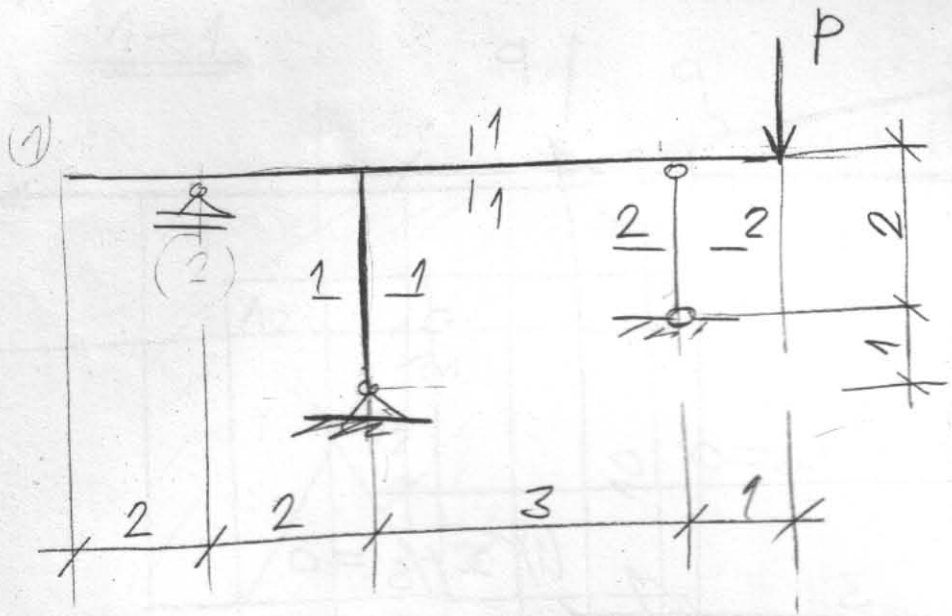
$$\delta_{11} = \int \frac{\overset{\text{носач}}{M_1 \cdot M_1}}{EI} ds + \int \frac{\overset{\text{профиль шайба}}{N_1 \cdot N_1}}{EF} ds \qquad / \cdot EI_c$$

$$\delta_{11}' = EI\,\delta_{11} = \int \frac{I_c}{I_i} M_1 \cdot M_1 \, ds + \left(\frac{I_c}{F_i}\right)^{m^2} \int N_1 \cdot N_1 \, ds$$

$$\delta_{12}' = \delta_{21}' =$$

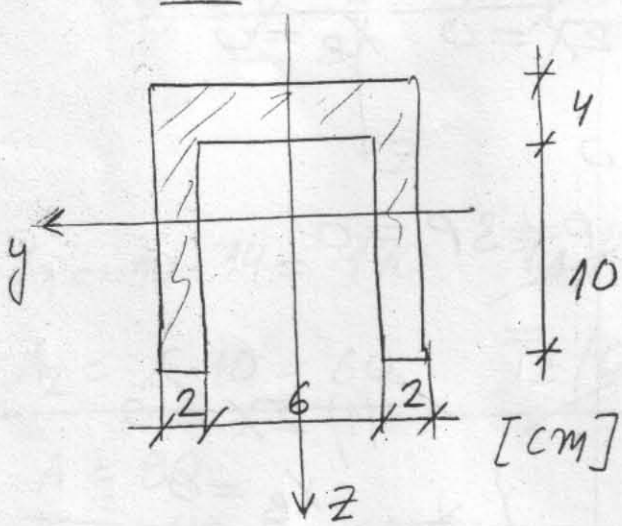
$I = I_y$ 

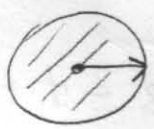
$F = b \cdot h$ 



1 – 1

2 – 2





$\sigma_T = 24 MPa$

$\sigma_T = 240\ MPa$

$Z_5 = 6$

$Z_6 = 5$

$Z_4 = 0$

$Z_k = 4$

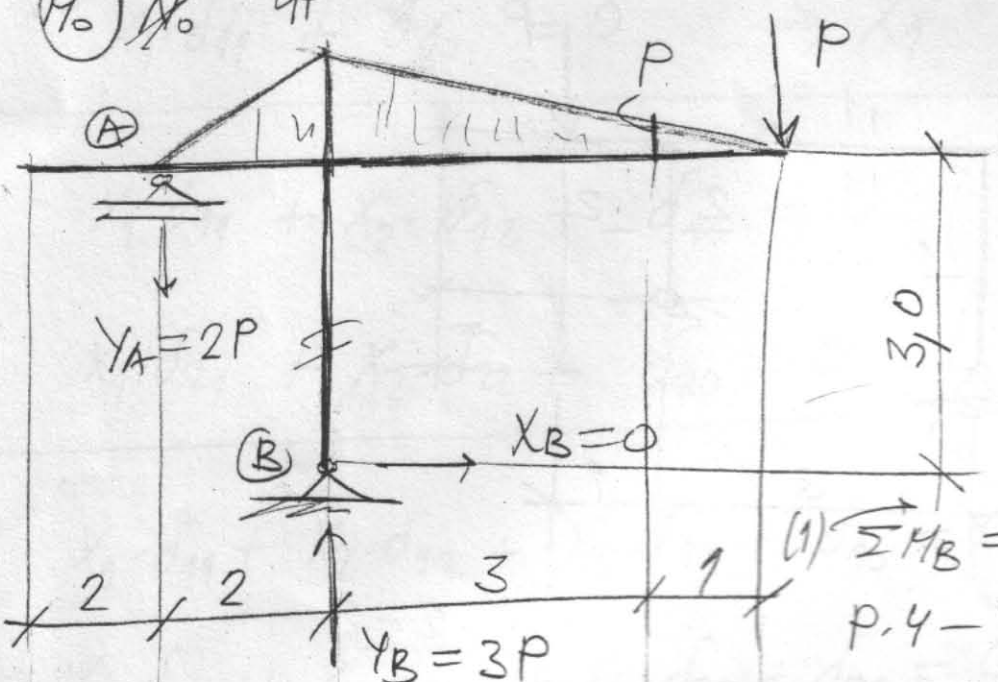
$n = 15$

(број чворова)

$r = 2k = 2 \cdot 7 = 14$

1 × статички неодр

M_0 ~~N_0~~ 4P



(1) $\overrightarrow{\Sigma M_B} = 0$

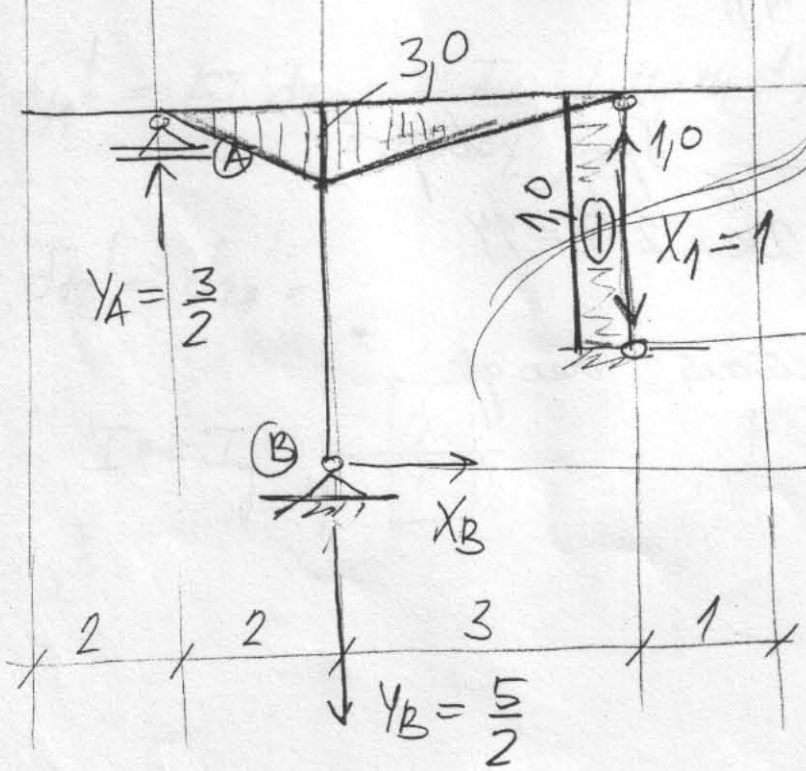
$P \cdot 4 - Y_A \cdot 2 = 0$

(2) $\Sigma X = 0$ $X_B = 0$

(3) $\overrightarrow{\Sigma M_A} = 0$ $-Y_B \cdot 2 + P \cdot 6 = 0$

Провер. $\Sigma Y = 0$ $-2P - P + 3P = 0$

M_1 N_1



(1) $\Sigma X = 0$

$X_B = 0$

(2) $\overrightarrow{\Sigma M_B^+} = 0$

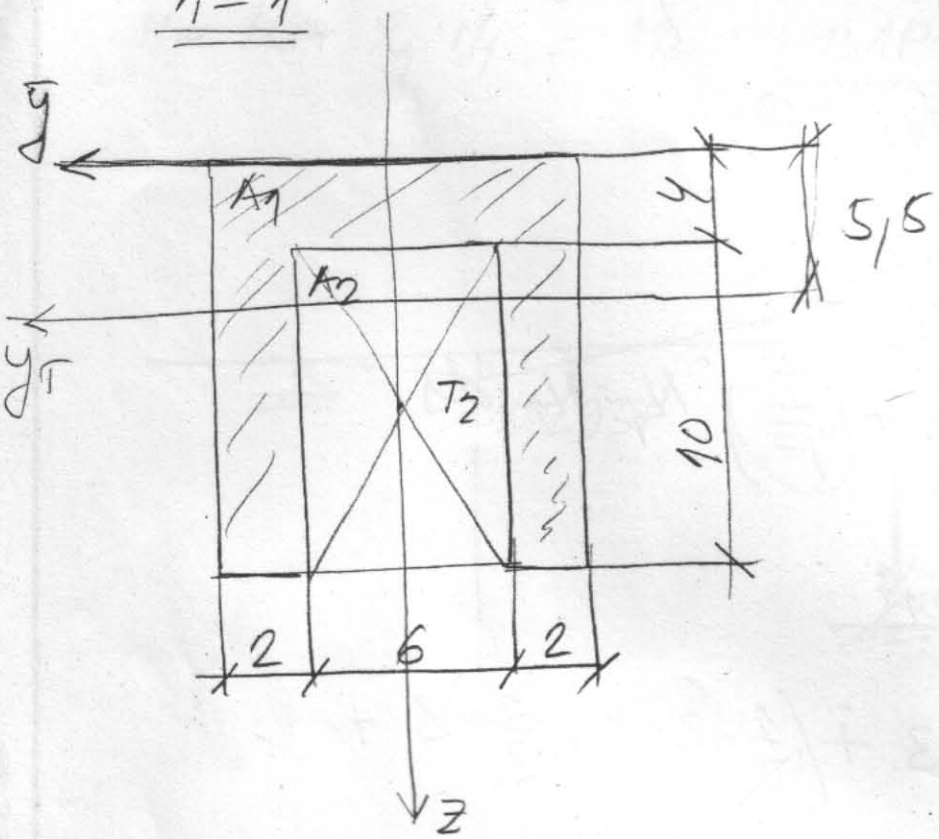
$Y_A \cdot 2 - 1 \cdot 3 = 0$

$Y_A = \frac{3}{2}$

(3) $\overrightarrow{\Sigma M_A} = 0$

$-Y_B \cdot 2 - 1 \cdot 5 = 0$

1 – 1



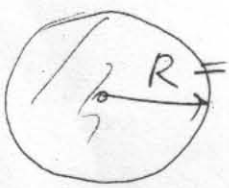
$A_1 = 10 \cdot 14 = 140$ ~~$\overline{T_1}(0;7)$~~ $T_1[0;\ 1,5]$

$A_2 = 6 \cdot 10 = 60$ ~~$\overline{T_2}(0;9)$~~ $T_2[0;\ 3,5]$

$A = 80$

$$z_T = \frac{140 \cdot 7 - 60 \cdot 9}{80} = 5,5$$

$$I_y = \frac{10 \cdot 14^3}{12} + 140 \cdot 1,5^2 - \left[\frac{6 \cdot 10^3}{12} + 60 \cdot 3,5^2\right] = 1366,667 cm^4$$



$R = 2cm \qquad A = R^2\pi = 2^2\pi = \underline{4\pi cm^2}$

$$\frac{I_y}{F} = \frac{1366,667 cm^4}{4\pi cm^2} = 108,755878 cm^2 = 0,01088 m^2$$

$$X_1 \cdot \delta_{11} + \delta_{10} = 0 \quad / \cdot EI$$

$$X_1 \cdot EI\delta_{11} + EI\delta_{10} = 0$$

(1) $X_1 \cdot \delta_{11}' + \delta_{10}' = 0$

$$\delta_{11}' = \int M_1 \cdot M_1 \, ds + \frac{I}{F} \int N_1 \cdot N_1 \, ds$$

$\frac{I}{F} = 0{,}01088\ m^2$

$$\delta_{11}' = \left(\frac{1}{2} \cdot 2 \cdot 3\right) \cdot \frac{2}{3} \cdot 3 + \left(\frac{1}{2} \cdot 3 \cdot 3\right) \cdot \frac{2}{3} \cdot 3 +$$

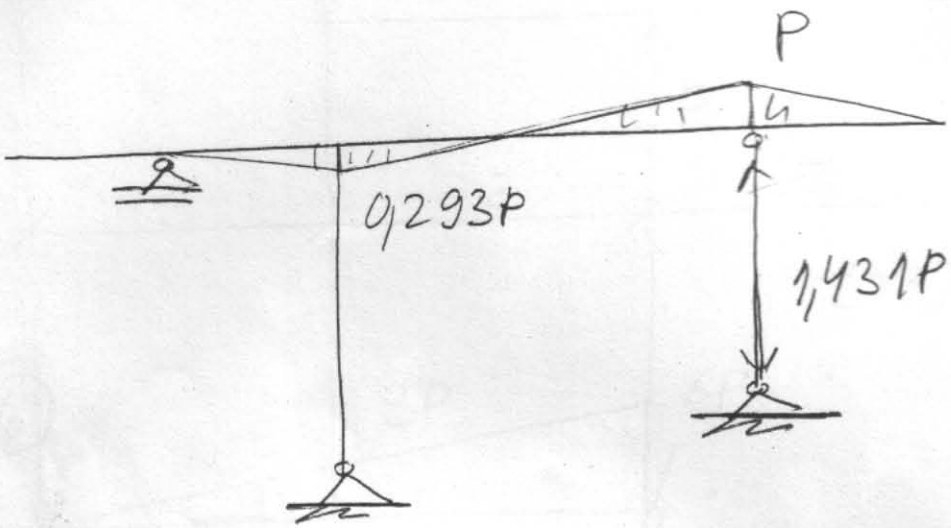
$$+ 0{,}01088 \cdot (2 \cdot 1) \cdot 1 = 15{,}022$$

$$\delta_{10}' = \int M_1 \cdot M_0 \, ds = -\left(\frac{1}{2} \cdot 2 \cdot 3\right) \frac{2}{3} \cdot 4P -$$

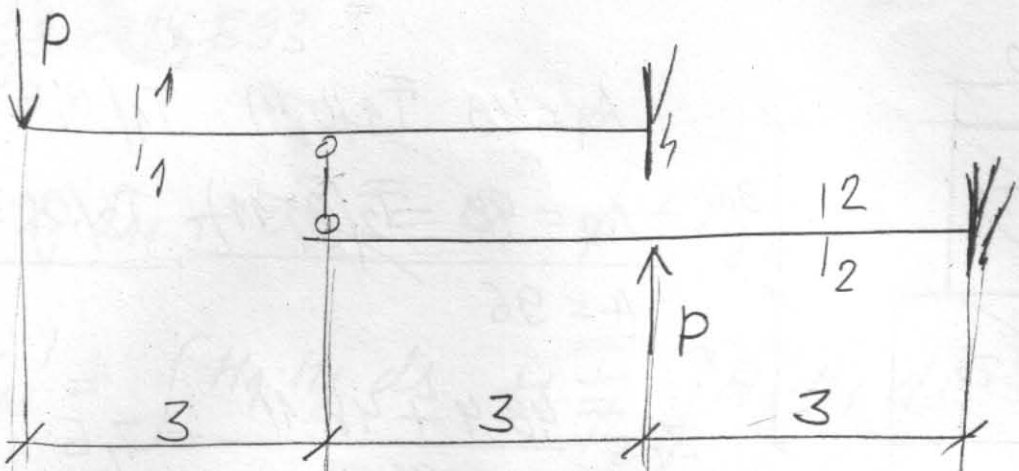
$$- \frac{3}{6}\left[3\,(2 \cdot 4P + P)\right] = -21{,}5P$$

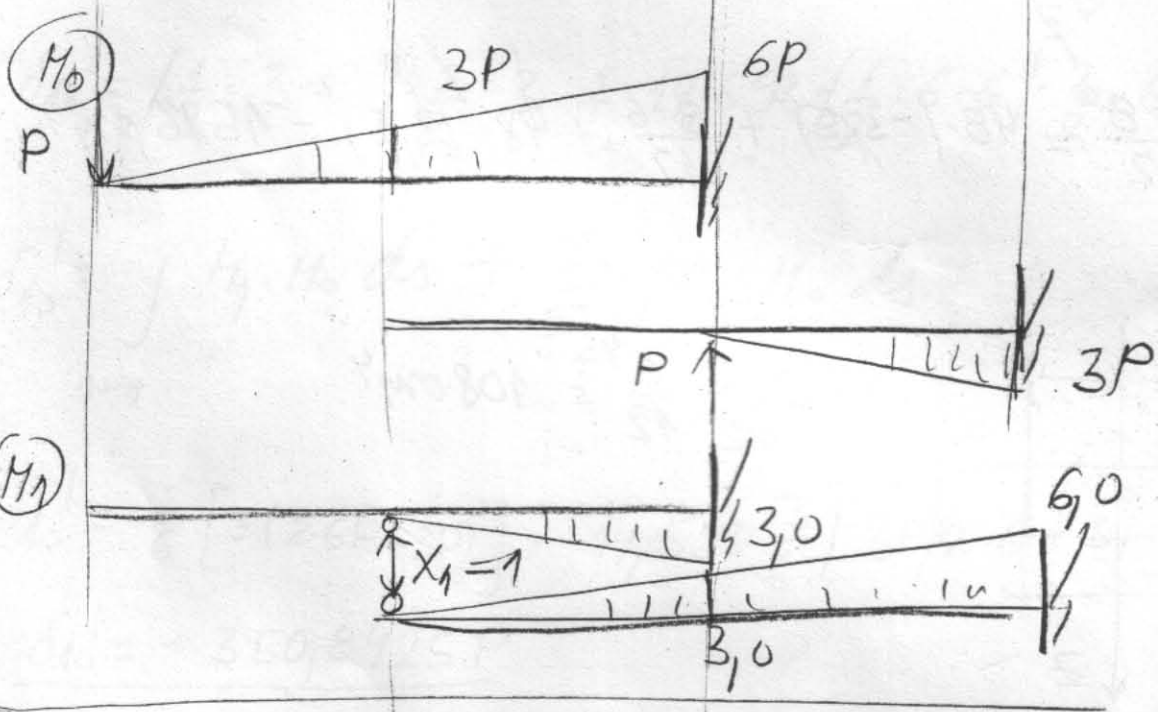
(1) $X_1 \cdot 15{,}022 - 21{,}5P = 0 \qquad \underline{X_1 = 1{,}431P}$

$$M = M_0 + X_1 \cdot M_1 = \underline{M_0 + 1{,}431P \cdot M_1}$$

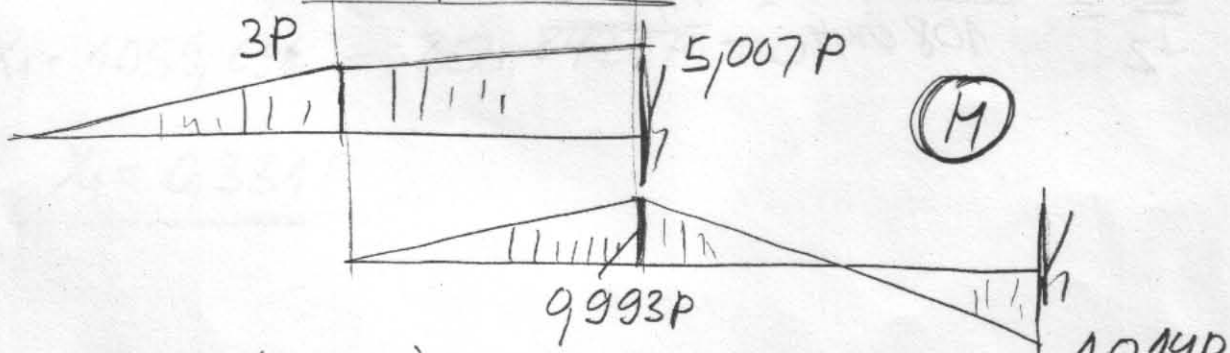


$$M = -4P + 1{,}431P \cdot (3) = 0{,}293P$$





$$M = M_0 + X_1 \cdot M_1 = M_0 + 0{,}331P \cdot M_1$$

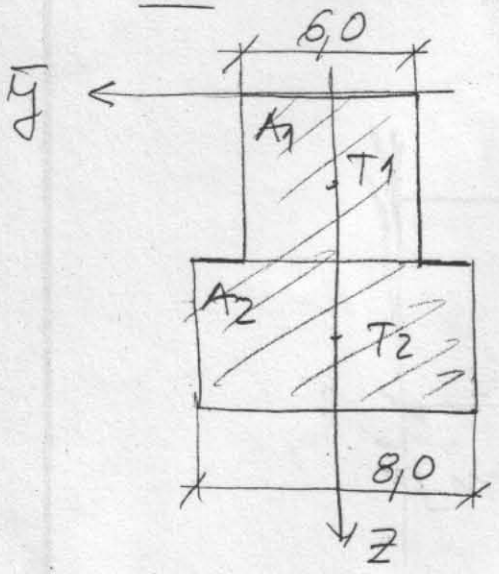


$M = -6P + 3 \cdot (0{,}331P) = -5{,}007$

$M = -3 \cdot 0{,}331P = -0{,}993P$

$M = 3P - 6 \cdot 0{,}331P = 1{,}014P$

1–1



$A_1 = 48 \quad T_1(0; 4) \quad T_1(0; -3,5)$

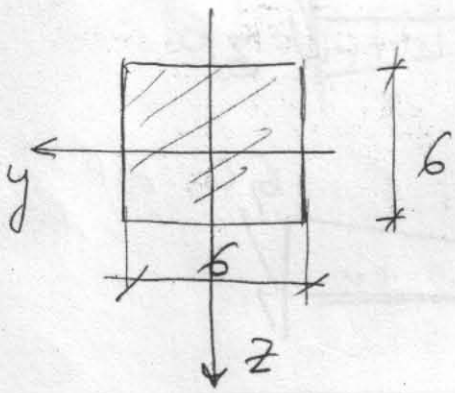
$A_2 = 48 \quad T_2(0; 11) \quad T_2(0; 3,5)$

$A = 96$

$$z_T = \frac{48 \cdot 4 + 48 \cdot 11}{96} = 7,5$$

$$I_{y_1} = \frac{6 \cdot 8^3}{12} + 48 \cdot (-3,5)^2 + \frac{8 \cdot 6^3}{12} + 48 \cdot (3,5)^2 = 1576 \text{ cm}^4$$

2–2



$$I_{y_2} = \frac{6^4}{12} = 108 \text{ cm}^4$$

$$\frac{I_1}{I_2} = \frac{1576 \text{ cm}^4}{108 \text{ cm}^4} = 14,593$$

$\frac{I_1}{I_2} = 14{,}593$

$$X_1 \cdot \delta_{11} + \delta_{10} = 0 \quad / \cdot EI_{y1}$$

$$\delta_{11}^1 = \int_{1-1} M_1 \cdot M_1 \, ds + \frac{I_1}{I_2} \int_{2-2} M_1 \cdot M_1 \, ds$$

$$\delta_{11}^1 = \left(\frac{1}{2} \cdot 3 \cdot 3\right) \cdot \frac{2}{3} \cdot 3 + 14{,}593 \left(\frac{1}{2} \cdot 6 \cdot 6\right) \cdot \frac{2}{3} \cdot 6 = 1059{,}696$$

$$\delta_{10}^1 = \int_{1-1} M_1 \cdot M_0 \, ds + \frac{I_1}{I_2} \int M_1 \cdot M_0 \, ds$$

$$\delta_{10}^1 = -\frac{3}{6}\left[3 \cdot (2 \cdot 6P + 3P)\right] - 14{,}593 \frac{3}{6}\left[3P/2 \cdot 6 + 3)\right]$$

$$\delta_{10}^1 = -350{,}8425\,P$$

$$X_1 \cdot \delta_{11}^1 + \delta_{10}^1 = 0$$

$$X_1 \cdot 1059{,}696 - 350{,}8425P = 0$$

$$X_1 = 0{,}331P$$