Израчунати интеграл:

1. $\int \frac{x^2}{\sqrt[3]{2x+1}}dx = \frac{3}{4}\int \frac{t^{12}-2t^6+1}{\cancel{t^2}}\cancel{t^5}\overset{t^3}{}dt = \frac{3}{4}\left[\int t^{15}dt - 2\int t^9 dt + \int t^3 dt\right] =$

$2x+1=t^6$ $\quad t=\sqrt[6]{2x+1}$ $\quad = \frac{3}{48}(2x+1)^{8/3} - \frac{3}{20}(2x+1)^{5/3} + \frac{3}{16}(2x+1)^{2/3} + C$

$2dx = 6t^5dt$

$dx = 3t^5dt$

$2x = t^6-1$

$x = \frac{1}{2}(t^6-1)$

$x^2 = \frac{1}{4}(t^{12}-2t^6+1)$

2. $\int \frac{dx}{(x+1)\sqrt{1-x}} = \int \frac{-2\cancel{t}dt}{(2-t^2)\cdot\cancel{t}} = \int \frac{2dt}{t^2-2} = \int \frac{dt}{\left(\frac{t}{\sqrt{2}}\right)^2-1} = \sqrt{2}\int \frac{da}{a^2-1} =$

$1-x=t^2$ $\quad t/\sqrt{2}=a$ $\quad = \sqrt{2}\ln\left|\frac{\sqrt{(1-x)/2}-1}{\sqrt{(1-x)/2}+1}\right| + C$

$-x = t^2-1$ $\quad dt/\sqrt{2} = da$

$x = 1-t^2$ $\quad dt = \sqrt{2}da$

$x+1 = 2-t^2$ $\quad t=\sqrt{1-x}$

$-dx = 2tdt$

$dx = -2tdt$

3. $\int \frac{\sqrt{x}+3}{x+2\sqrt{x}-3}dx = \int \frac{t+3}{t^2+2t-3}\cdot 2tdt = 2\int \frac{t\cancel{(t+3)}}{\cancel{(t+3)}(t-1)}dt = 2\int \frac{t}{t-1}dt =$

$x=t^2$ $\quad = 2\int \frac{t-1+1}{t-1}dt = 2\int dt + 2\int \frac{dt}{t-1} = 2\sqrt{x} + 2\ln|\sqrt{x}-1| + C$

$dx = 2tdt$

$t=\sqrt{x}$

4. $\int \frac{\sqrt{x}}{1+\sqrt{x}}dx = \int \frac{1+\sqrt{x}-1}{1+\sqrt{x}}dx = \int dx - \int \frac{dx}{1+\sqrt{x}} = x - \int \frac{2tdt}{1+t} = x - 2\int \frac{1+t-1}{1+t}dt$

$x=t^2$ $\quad = x - 2\int dt + 2\int \frac{dt}{1+t} = x - 2\sqrt{x} + 2\ln|1+\sqrt{x}| + C =$

$dx = 2tdt$ $\quad = 2\ln|1+\sqrt{x}| - \sqrt{x} + C$

5. $\int \frac{1-2\sqrt{x}}{1+2\sqrt{x}}dx = \int \frac{1+2\sqrt{x}-4\sqrt{x}}{1+2\sqrt{x}}dx = \int dx - 2\int \frac{1+2\sqrt{x}-1}{1+2\sqrt{x}}dx =$

$= x - 2\int dx + 2\int \frac{1}{1+2\sqrt{x}}dx = x - 2x + 2\int \frac{2tdt}{1+2t} = -x + 2\int \frac{1+2t-1}{2t+1}dt =$

$x=t^2$ $\quad = -x + 2\int dt - 2\int \frac{dt}{1+2t} = -x + 2\sqrt{x} - 2\ln|1+2\sqrt{x}| + C$

$dx = 2tdt$

$t=\sqrt{x}$

6. $\int \frac{\sqrt{x+1}-2}{x+1+\sqrt{x+1}}dx = \int \frac{t-2}{t^2+t}\cdot 2tdt = 2\int \frac{\cancel{t}(t-2)}{\cancel{t}(t+1)}dt = 2\int \frac{t+1-3}{t+1}dt =$

$x+1=t^2$ $\quad = 2\int dt - 6\int \frac{dt}{1+t} = 2\sqrt{x+1} - 6\ln|1+\sqrt{x+1}| + C$

$dx = 2tdt$

$t = \sqrt{x+1}$

7. $\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx = \int \frac{\sqrt{x+1}-1+2}{\sqrt{x+1}-1}dx = \int dx + 2\int \frac{dx}{\sqrt{x+1}-1} =$

$x+1=t^2$
$dx=2t\,dt$
$\sqrt{x+1}=t$

$= x+2\int\frac{2t\,dt}{t-1} = x+4\int\frac{t-1+1}{t-1}dt = x+4\int dt + 4\int\frac{dt}{t-1} =$

$= x+4\sqrt{x+1}+4\ln|\sqrt{x+1}-1|+C$

8. $\int\frac{2\sqrt{2-x}+1}{8-x-5\sqrt{2-x}}dx = \int\frac{2\sqrt{2-x}+1}{6+(2-x)-5\sqrt{2-x}}dx = \int\frac{(2t+1)(-2t\,dt)}{6+t^2-5t} =$

$2-x=t^2$
$-dx=2t\,dt$
$dx=-2t\,dt$
$\sqrt{2-x}=t$

$=-2\int\frac{2t^2+t}{(t-2)\cdot(t-3)}dt = 4\ln|\sqrt{2-x}-2| - 6\ln|\sqrt{2-x}-3|+C$

$2t^2+t = At-3A+Bt-2B$
$A+B=1$ $\quad B=1-A$
$-3A-2B=0$
$-3A-2+2A=0$
$-A=+2$
$A=-2$ $\quad B=3$

9. $\int\frac{\sqrt[3]{x}}{\sqrt[3]{x}+1}dx = \int\frac{\sqrt[3]{x}+1-1}{\sqrt[3]{x}+1}dx = \int dx - \int\frac{dx}{\sqrt[3]{x}+1} = x-\int\frac{3t^2dt}{t+1} =$

$x=t^3$
$dx=3t^2dt$
$t=\sqrt[3]{x}$

$=x-3\int\frac{t^2+2t+1-2t-1}{t+1}dt = x-3\int\frac{(t+1)^2-(2t+1)}{t+1}dt$

$=x-3\int(t+1)dt+3\int\frac{2t+1}{t+1}dt = x-3\int a\,da+3\int\frac{t+1+t}{t+1}dt=$

$t+1=a$
$dt=da$

$=x-\frac{3}{2}(\sqrt[3]{x}+1)^2+3\int dt+3\int\frac{t+1-1}{t+1}dt=$

$=x-\frac{3}{2}(\sqrt[3]{x}+1)^2+3\sqrt[3]{x}+3\int dt-3\int\frac{dt}{t+1}=$

$=x-\frac{3}{2}(\sqrt[3]{x}+1)^2+6\sqrt[3]{x}-3\ln|\sqrt[3]{x}+1|+C$

10. $\int\frac{\sqrt[3]{3x+4}}{1+\sqrt[3]{3x+4}}dx = \int\frac{1+\sqrt[3]{3x+4}-1}{1+\sqrt[3]{3x+4}}dx = \int dx - \int\frac{dx}{1+\sqrt[3]{3x+4}} =$

$=x-\int\frac{t^2\,dt}{1+t} = x-\int\frac{t^2+2t+1-2t-1}{t+1}dt = x-\int\frac{(t+1)^2-(2t+1)}{t+1}dt$

$3x+4=t^3$
$3dx=3t^2dt$
$dx=t^2dt$
$t+1=a$
$dt=da$
$t=\sqrt[3]{3x+4}$

$=x-\int(t+1)dt+\int\frac{2t+1}{t+1}dt=$

$=x-\int a\,da+\int\frac{t+1+t}{t+1}dt=$

$=x-\frac{1}{2}(\sqrt[3]{3x+4}+1)+\sqrt[3]{3x+4}\int\frac{t+1-1}{t+1}dt=$

$=x-\frac{1}{2}(\sqrt[3]{3x+4}+1)+2\sqrt[3]{3x+4}-\ln|\sqrt[3]{3x+4}+1|+C$

$$\int \frac{\dots}{\sqrt[3]{x+2}}\,dx = 3\int \frac{t^3\,dt}{(t^3-2)+t} =$$

$$\int \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}-2}\,dx = 3\int \frac{t^2+1}{t-2}\cdot t^2\,dt = 3\int \frac{t^4+t^2}{t-2}\,dt =$$

$t=\sqrt[3]{x}$

$$= 3/4\,\sqrt[3]{x^4} + 2x + \frac{15}{2}\sqrt[3]{x^2} + 30\sqrt[3]{x} + 60\ln|\sqrt[3]{x}-2| \dots$$

$dx = 3t^2\,dt$

$(t^4+t^2):(t-2) = t^3+2t^2+5t+10$

$-t^4 \mp 2t^3$

$2t^3+t^2$

$-2t^3 \mp 4t^2$

$5t^2$

$-5t^2 \mp 10t$

$10t$

$-10t \mp 20$

$+20$

13. $$\int \frac{\sqrt{x-1}-\sqrt{x+1}}{\sqrt{x-1}+\sqrt{x+1}}\,dx = \int \frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{x-1}+\sqrt{x+1}}\,dx - 2\int \frac{\sqrt{x+1}}{\sqrt{x-1}+\sqrt{x+1}}\,dx = \int dx - 2\int \frac{\sqrt{x+1}\,dx}{\sqrt{x-1}+\sqrt{x+1}}$$

$$= x - 2\int \frac{t\cdot 2t\,dt}{\sqrt{t^2-2}+t}$$

$x+1=t^2$ $\quad x=t^2-1$

$dx = 2t\,dt$ $\quad x-1=t^2-2$

$x-1=t^2$

14. $$\int \frac{\sqrt[3]{x+1}+2}{\sqrt[3]{(x+1)^2}+2}\,dx = \int \frac{t+2}{t^2+2}\cdot 3t^2\,dt = 3\int \frac{t^3+2t^2}{t^2+2}\,dt =$$

$x+1=t^3$

$dx=3t^2\,dt$

$$= \frac{3}{2}\sqrt[3]{(x+1)^2} + 6\sqrt[3]{x+1} + 6\int \frac{t-2}{t^2+2}\,dt =$$

$(t^3+2t^2):(t^2+2) = t+2$

$-t^3 \mp 2t$

$2t^2-2t$

$-2t^2 \mp 4$

$-2t-4$

$t=\sqrt[3]{x+1}$

$$= \frac{3}{2}\sqrt[3]{(x+1)^2} + 6\sqrt[3]{x+1} + 3\int \frac{2t\,dt}{t^2+2} - 8\int \frac{dt}{t^2+2} =$$

$$+3\int \frac{da}{a} - 4\int \frac{dt}{\left(\frac{t}{\sqrt{2}}\right)^2+1}$$

$$3\ln\left(\sqrt[3]{(x+1)^2}+1\right) - 4\sqrt{2}\int \frac{db}{b^2+1}$$

$$-4\sqrt{2}\,\mathrm{arctg}\left(\sqrt[3]{x+1}/\dots\right) + C$$

$t^2+2=a$ $\quad t/\sqrt{2}=b$

$2t\,dt=da$ $\quad dt/\sqrt{2}=db$

$dt=\sqrt{2}\,db$

15. $\int \frac{dx}{\sqrt[3]{2-x}+1} = -3\int \frac{t^2 dt}{t+1} = -\frac{3}{2}\sqrt[3]{(2-x)^2} + 3\sqrt[3]{2-x} - 3\ln|\sqrt[3]{2-x}+1| + C$

$2-x = t^3 \qquad t = \sqrt[3]{(2-x)}$

$-dx = 3t^2 dt$

$dx = -3t^2 dt$

$\cancel{t^2} : (t+1) = t-1$

$\cancel{t^2} + t$

$-t$

$-t-1$

1

16. $\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx = 6\int \frac{t^6+t^4+t}{\cancel{t^6}(1+t^2)}\cancel{t^5}dt = 6\int \frac{t^5+t^3+1}{1+t^2}dt =$

$x = t^6$

$dx = 6t^5 dt \qquad = \frac{3}{2}\sqrt{x} + 6\operatorname{arctg}\sqrt[6]{x} + C$

$(\cancel{t^5} + \cancel{t^3} + 1) : (t^2+1) = t^3$

$-\cancel{t^5} + \cancel{t^3}$

1

$t = \sqrt[6]{x}$

17. $\int \frac{1+\sqrt[4]{x}}{x+\sqrt{x}}dx = \int \frac{1+t}{t^4+t^2}\cdot 4t^3 dt = 4\int \frac{t^3(t+1)}{\cancel{t^2}(t^2+1)}dt =$

$x = t^4$

$dx = 4t^3 dt \qquad = 4\int \frac{t^2+t}{t^2+1}dt = 4\sqrt[4]{x} + 2\ln(\sqrt{x}+1) - 4\operatorname{arctg}\sqrt[4]{x} + C$

$(t^2+t) : (t^2+1) = 1$

$-t^2+1$

$t-1 \qquad t = \sqrt[4]{x}$

18. $\int \frac{dx}{\sqrt[4]{2x-1}-1} = 2\int \frac{t^3 dt}{t-1} = \frac{2}{3}\sqrt[4]{(2x-1)^3} + \sqrt{2x-1} + 2\sqrt[4]{2x-1} +$

$2x-1 = t^4 \qquad t = \sqrt[4]{(2x-1)} \qquad + 2\ln|\sqrt[4]{2x-1} - 1| + C$

$2dx = 4t^3 dt$

$dx = 2t^3 dt$

$t^3 : (t-1) = t^2 + t + 1$

$-t^3 - t^2$

t^2

$-t^2 - t$

t

$-t - 1$

1

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}}\,dx = 6\int \frac{t^6}{1+t^2}\,dt = \frac{6}{5}\sqrt[6]{x^5} - 3\sqrt{x} + 6\sqrt[6]{x} - 6\,\mathrm{arctg}\sqrt[6]{x} + C$$

$t = \sqrt[6]{x}$

$\ldots = t^4 - t^2 + 1$

t^2

$+1$

-1

20. $$\int \frac{dx}{x(\sqrt[5]{x} + \sqrt[5]{x^2})} = 5\int \frac{t^4\,dt}{t^5(t+t^2)} = 5\int \frac{dt}{t^2(t+1)} = -5\ln|\sqrt[5]{x}| - \frac{1}{\sqrt[5]{x}} + 5\ln|\sqrt[5]{x}+1| + C$$

$x = t^5 \qquad t = \sqrt[5]{x}$

$dx = 5t^4\,dt$

$1 = At^2 + At + Bt + B + Ct^2$

$A + C = 0 \qquad C = -A$

$A + B = 0 \qquad B = -A$

$B = 1 \qquad \underline{A = -1} \qquad \underline{C = 1}$

21. $$\int \frac{dx}{x + 2\sqrt[3]{x^2} + \sqrt[3]{x^4}} = 3\int \frac{t^2\,dt}{t^3 + 2t^2 + t^4} = 3\int \frac{dt}{t^2+t+2} = 3\int \frac{dt}{(t+\frac{1}{2})^2 + \frac{7}{4}} =$$

$x = t^3 \qquad t = \sqrt[3]{x}$

$dx = 3t^2\,dt$

$$= 3\cdot\frac{4}{7}\int \frac{dt}{\left(\frac{t+1/2}{\sqrt{7}/2}\right)^2 + 1} = 3\cdot\frac{4}{7}\cdot\frac{\sqrt{7}}{2}\int \frac{da}{a^2+1} =$$

$\frac{t+1/2}{\sqrt{7}/2} = a$

$$= \frac{6}{\sqrt{7}}\,\mathrm{arctg}\left(\frac{\sqrt[3]{x} + 1/2}{\sqrt{7}/2}\right) + C$$

$\frac{dt}{\sqrt{7}/2} = da$

$dt = \frac{\sqrt{7}}{2}\,da$

22. $$\int \sqrt[3]{\frac{2-x}{2+x}}\,\frac{dx}{(2-x)^2} = \int \left(\frac{2+x}{2-x}\right)^{-1/3}\frac{dx}{(2-x)^2} = \frac{1}{4}\int \frac{dt}{t} = \frac{1}{4}\ln\left|\sqrt[3]{\frac{2+x}{2-x}}\right| + C$$

$\frac{2+x}{2-x} = t^3$

$\frac{2-x+2+x}{(2-x)^2}\,dx = dt$

$\frac{dx}{(2-x)^2} = \frac{dt}{4}$

1.23. $\int \sqrt[3]{\frac{1-x}{1+x}} \frac{dx}{(1+x)^2} = -\frac{1}{2}\int t\,dt = -\frac{1}{4}\sqrt[3]{\left(\frac{1-x}{1+x}\right)^2} + c$

$\frac{1-x}{1+x} = t^3$

$\frac{-1-x-1+x}{(1+x)^2}dx = dt$

$\frac{dx}{(1+x)^2} = \frac{dt}{-2}$

24. $\int \sqrt[5]{\left(\frac{x+1}{x-1}\right)^2} \frac{dx}{(x-1)^2} = -\frac{1}{2}\int t^2 dt = -\frac{1}{6}\sqrt[5]{\left(\frac{x+1}{x-1}\right)^3} + c$

$\frac{x+1}{x-1} = t^5$

$\frac{x-1-x-1}{(x-1)^2}dx = dt$

$\frac{dx}{(x-1)^2} = -\frac{dt}{2}$

25. $\int \frac{\sqrt[3]{x+1}}{x-1}dx = \int \frac{\sqrt[3]{t^3+2}}{\cancel{t}} \cdot 3t^2 dt$

$x-1 = t^3 \quad x = t^3+1$
$dx = 3t^2 dt$

26. $\int \sqrt{\frac{1-x}{1+x}} \frac{dx}{x} = -\int \sqrt{\frac{1-\frac{1}{a}}{1+\frac{1}{a}}} \cdot \frac{da}{a} =$

$\frac{1}{x} = a \qquad = -\int \sqrt{\frac{a-1}{a+1}} \cdot \frac{da}{a} = -\int \frac{\sqrt{t^2-2}}{\cancel{t}} \cdot \frac{2\cancel{t}\,dt}{t^2-1}$

$\frac{1}{a} = x \qquad = -2\int \frac{\sqrt{t^2-2}}{t^2-1}dt$

$-\frac{dx}{x^2} = da$

$\frac{dx}{x} = -da \cdot x \qquad 2 + 2\int \frac{\sqrt{t^2-2}}{t^2-1}dt$

$\frac{dx}{x} = -da \cdot \frac{1}{a}$

$a+1 = t^2$
$a = t^2-1$
$a-1 = t^2-2$
$da = 2t\,dt$

$t^2-2 = b^2$
$2t\,dt = db \cdot 2b$

$$\int \frac{x-5}{3x+1}\, dx = \int \sqrt{}$$

30.)

$$\int x\sqrt{1-x^2}\,dx = \int \frac{-t\,dt}{\sqrt{1-t^2}} = \frac{1}{2}\int a^{-1/2}da = \sqrt{1-(1-x^2)^2}+C = \sqrt{2-x^2}\cdot|x|+C$$

$1-1+2x^2-x^4$

$-x^2=t^2-1$

$x^2=1-t^2$

$x=\sqrt{1-t^2}$

$2x\,dx = 2t\,dt$

$x\,dx = -t\,dt$

$dx = \frac{-t\,dt}{\sqrt{1-t^2}}$

$t^2=a$

$2t\,dt = da$

$t\,dt = da/2$

32. $$\int x^2\sqrt{4-x^2}\,dx = -\int (4-t^2)\cdot t\cdot \frac{t\,dt}{\sqrt{4-t^2}} =$$

$4-x^2=t^2 \quad -x^2=t^2-4$

$-2x\,dx = 2t\,dt \quad x^2=4-t^2$

$x\,dx = -t\,dt \quad x=\sqrt{4-t^2}$

$dx = -\frac{t\,dt}{\sqrt{4-t^2}}$

33. $$\int \sqrt{3-2x-x^2}\,dx = \int\sqrt{4-(1+x)^2}\,dx = \int\sqrt{4-t^2}\,dt = \int\frac{-a^2\,da}{\sqrt{4-a^2}} = \int \frac{4-a^2-4}{\sqrt{4-a^2}}\,da = \int\sqrt{4-a^2}\,da -$$

$3-2x-x^2=t^2$

$(-2-2x)dx = 2t\,dt$

$(-1-x)dx = t\,dt$

$dx = \frac{t\,dt}{-1}$

$1+x=t$

$dx=dt$

$4-t^2=a^2 \quad -t^2=a^2-4$

$-2t\,dt = 2a\,da \quad t^2=4-a^2$

$dt = -\frac{a\,da}{t} \quad t=\sqrt{4-a^2}$

$dt = -\frac{a\,da}{\sqrt{4-a^2}}$

34. $$\int\frac{dx}{x\sqrt{x^2-9}} = \int\frac{t\,dt}{(t^2+9)|t|} = +\int\frac{dt}{t^2+9} = \pm\frac{1}{9}\int\frac{dt}{(\frac{t}{3})^2+1} = \pm\frac{1}{3}\int\frac{da}{a^2+1} =$$

$$-\int\frac{dt}{t^2+9} = \pm\frac{1}{3}\operatorname{arctg}\left(\frac{\sqrt{x^2-9}}{3}\right)+C$$

$x^2-9=t^2 \quad x^2=t^2+9$

$2x\,dx = 2t\,dt \quad t=\sqrt{x^2-9}$

$dx = \frac{t}{x}\,dt$

$t/3 = a$

$dt/3 = da$

$dt = 3\,da$

35. $\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{dt \cdot t}{(t^2+1)|t|} = \pm \int \frac{dt}{t^2+1} = \pm \operatorname{arctg}\sqrt{x^2-1} + C$

$x^2-1=t^2 \quad x^2=t^2+1$
$2x\,dx = 2t\,dt$
$dx = \frac{t}{x}dt$

36. $\int \frac{2x+3}{\sqrt{11-2x-x^2}}dx = \int \frac{-(-2x-2)+1}{\sqrt{11-2x-x^2}}dx = -\int \frac{-2x-2}{\sqrt{11-2x-x^2}}dx + \int \frac{dx}{\sqrt{12-(x+1)^2}} =$

$11-2x-x^2=t$
$(-2x-2)dx=dt$

$= -\int t^{-1/2}dt + \int \frac{dx}{\sqrt{1-\left(\frac{x+1}{\sqrt{12}}\right)^2}} \cdot \frac{1}{\sqrt{12}} =$

$= -2\sqrt{11-2x-x^2} + \frac{1}{\cancel{\sqrt{12}}} \cdot \cancel{\sqrt{12}} \int \frac{da}{\sqrt{1-a^2}} = -2\sqrt{11-2x-x^2} + \arcsin\left(\frac{x+1}{\sqrt{12}}\right) + C$

$\frac{x+1}{\sqrt{12}} = a$

$\frac{dx}{\sqrt{12}} = da$

$dx = \sqrt{12}\,da$

37. $\int x^2\sqrt{x^2+4}\,dx = \int \sqrt{t^2-4}\,|t|\,dt$

$x^2+4=t^2 \quad x^2=t^2-4$
$2x\,dx = 2t\,dt \quad x=\sqrt{t^2-4}$
$dx = \frac{t}{x}dt$

38. $\int \frac{dx}{\sqrt{x^2-2x+8}} = \int \frac{dx}{\sqrt{(x-1)^2+7}} = \frac{1}{\sqrt{7}}\int \frac{dx}{\sqrt{\left(\frac{x-1}{\sqrt{7}}\right)^2+1}} = \int \frac{da}{\sqrt{a^2+1}} =$

$\frac{x-1}{\sqrt{7}} = a \qquad = \ln\left|\frac{x-1}{\sqrt{7}} + \sqrt{\left(\frac{x-1}{\sqrt{7}}\right)^2+1}\right| + C$

$\frac{dx}{\sqrt{7}} = da$
$dx = \sqrt{7}\,da$

39. $\int \frac{dx}{\sqrt{1-2x-x^2}} = \int \frac{dx}{\sqrt{2-(x+1)^2}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{1-\left(\frac{x+1}{\sqrt{2}}\right)^2}} = \int \frac{da}{\sqrt{1-a^2}} =$

$\frac{x+1}{\sqrt{2}} = a \qquad = \arcsin\left(\frac{x+1}{\sqrt{2}}\right) + C$

$\frac{dx}{\sqrt{2}} = da$
$dx = \sqrt{2}\,da$

40. $\int \frac{1-x+x^2}{\sqrt{1+x-x^2}}dx = \int \frac{(x-1/2)^2+3/4}{\sqrt{\frac{5}{4}-(x-1/2)^2}}dx = \int \frac{a^2+3/4}{\sqrt{\frac{3}{4}-a^2}}da =$

$x-1/2=a$
$dx=da$

$= \int \frac{\frac{5}{4}\cancel{-t^2}}{\frac{5}{4}\cancel{-t^2}} \cdot t\,dt + \frac{3}{\cancel{4}_2} \cdot \frac{\cancel{2}^1}{\sqrt{5}} \int \frac{da}{\sqrt{1-\left(\frac{2a}{\sqrt{5}}\right)^2}} = da =$

$\frac{5}{4}-a^2=t^2 \quad a^2=\frac{5}{4}-t^2$
$-2a\,da = 2t\,dt$
$da = -\frac{t}{a}dt \quad a=\sqrt{\frac{5}{4}-t^2}$

$= -\frac{(5/4-(x-1/2)^2)}{2} + 3\arcsin\left(\frac{2(x-1/2)}{\sqrt{5}}\right) + C$

$$\int \frac{x^2}{\sqrt{2-2x-x^2}}dx = \int \frac{(x+1)^2-2x-1}{\sqrt{2-(x+1)^2}}dx = \int \frac{(x+1)^2}{\sqrt{2-(x+1)^2}}dx + \int \frac{-2x-2\,dx}{\sqrt{1-2x-x^2}} + \int \frac{dx}{\sqrt{1-2x-x^2}} =$$

$$\int \frac{a^2}{\sqrt{2-a^2}}da = \int \frac{\sqrt{2-t^2}\,(-t)}{|t|\,\sqrt{2-t^2}}dt = \pm$$

t^2 $I\ \Delta \neq 0$

$a = 2t\,dt$

$\frac{t}{a}dt$

$a^2 = t^2-2$

$a^2 = 2-t^2$

$a = \sqrt{2-t^2}$

$$\int \frac{db}{\sqrt{b}} = \int b^{-1/2}db = 2\sqrt{1-2x-x^2} + C_2$$

$2x-x^2 = b$

$(-2-2x)dx = db$

$$\int \frac{dx}{\sqrt{2-(x+1)^2}} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{1-\left(\frac{x+1}{\sqrt{2}}\right)^2}} = \int \frac{dc}{\sqrt{1-c^2}} = \arcsin\left(\frac{x+1}{\sqrt{2}}\right) + C_3$$

$\frac{x+1}{\sqrt{2}} = c$

$\frac{1\,dx}{\sqrt{2}} = dc$

$dx = \sqrt{2}\,dc$

42. $$\int \frac{2x^2-x-5}{\sqrt{x^2-2x}}dx = \pm\int \frac{2x^2-x-5}{x-1}\cdot dt = \pm\int \left(2x+3-\frac{2}{x-1}\right)dt =$$

$x^2-2x = t^2$

$(2x-2)dx = 2t\,dt$

$dx = \frac{t\,dt}{x-1}$

$(x-1)dx = t\,dt$

$\frac{x-1}{\sqrt{x^2-2x}}dx = dt$

$(2x^2-x-5):(x-1) = 2x+3$

$-\ 2x^2-2x$

$3x-5$

$-\ 3x-3$

-2

$$= \pm\int \left(2x+3-\frac{2}{x-1}\right)\frac{x-1}{\sqrt{x^2-2x}}dx =$$

$$= \pm\int$$

$$\int \frac{|x|\cancel{x}\left(2-\frac{1}{x}-\frac{5}{x^2}\right)}{\cancel{|x|}\sqrt{1-\frac{2}{x}}}dx = \pm\int \frac{2x-1-\frac{5}{x}}{\sqrt{1-\frac{2}{x}}} = \pm\int \frac{\frac{2}{a}-1-5a}{\sqrt{1-2a}}\cdot\frac{da}{a^2} =$$

$\frac{1}{x} = a \quad x = \frac{1}{a}$

$u = \frac{1}{\sqrt{x^2-2x}} = (x^2-2x)^{-1/2} \quad du = -\frac{1}{2}(x^2-2x)^{3/2}$

$\frac{1}{x^2}dx = da$

$dx = x^2\,da$

$dx = \frac{1}{a^2}da$

52. $$\int \sqrt{3-4x+4x^2}\,dx = \int \sqrt{(2x-1)^2+2}\,dx = \sqrt{2}\int \sqrt{\left(\frac{2x-1}{\sqrt{2}}\right)^2+1}\,dx =$$

$\frac{2x-1}{\sqrt{2}} = a$

$$= \int \sqrt{a^2+1}\,da = \int b^2\frac{db}{\sqrt{b^2-1}}$$

$\frac{2}{\sqrt{2}}dx = da$

$dx = \frac{\sqrt{2}}{2}da$

$a^2+1 = b^2 \quad b = \sqrt{a^2+1}$

$2a\,da = 2b\,db$

$da = \frac{b\,db}{a}$

$a = \sqrt{b^2-1}$

$u = (a^2+1)^{1/2} \quad du = \frac{1}{2}\cdot 2a\cdot(a^2+1)^{-1/2}$

$dv = da \quad v = a$

$$= a\sqrt{a^2+1} - \int \frac{a^2\,da}{\sqrt{a^2+1}}$$

53. $\int \sqrt{3x^2+10x+9}\,dx = \int \sqrt{(x^2+\frac{10}{3}x+3)\cdot 3}\,dx = \sqrt{3}\int \sqrt{(x+\frac{5}{3})^2+\frac{2}{9}}\,dx =$

$= \frac{\sqrt{6}}{3}\int \sqrt{\left(\frac{x+5/3}{\sqrt{2}/3}\right)^2+1}\,dx = \frac{2\sqrt{3}}{9}\int \sqrt{a^2+1}\,da$

$\frac{x+5/3}{\sqrt{2}/3} = a$

$\frac{dx}{\sqrt{2}/3} = da$

$dx = \frac{\sqrt{2}}{3}\,da$

54. $\int x\sqrt{x^2+2x+2}\,dx = \int x\sqrt{(x+1)^2+1}\,dx = \int (a-1)\sqrt{a^2+1}\,da =$

$x+1=a \quad x=a-1 \qquad = \int a\sqrt{a^2+1}\,da - \int \sqrt{a^2+1}\,da =$

$dx = da$

$a^2+1=b$

$2a\,da = db \qquad = \frac{1}{2}\int b^{1/2}\,db - \int \sqrt{a^2+1}\,da =$

$a\,da = db/2$

$= \frac{1}{3}\sqrt{((x+1)^2+1)^3} - \int \sqrt{a^2+1}\,da$

55. $\int (2x-5)\sqrt{2+3x-x^2}\,dx - \int (3-2x+2)\sqrt{2+3x-x^2}\,dx =$

$2+3x-x^2 = a$

$(3-2x)\,dx = da \qquad = -\int (3-2x)\sqrt{2+3x-x^2}\,dx - 2\int \sqrt{2+3x-x^2}\,dx =$

$= -\int a^{1/2}\,da - 2\int \sqrt{\frac{17}{4}-(\cancel{x}-\frac{3}{2})^2}\,dx - \boxed{\frac{2}{3}(2+3x-x^2)^{3/2}}$ готово

$x-\frac{9}{4}=2 \qquad -\sqrt{17}\int \sqrt{1-\left(\frac{x-3/2}{\sqrt{17}/2}\right)^2}\,dx$

$x=\frac{17}{4}$

$\frac{x-3/2}{\sqrt{17}/2} = b \qquad -\frac{17}{2}\int \sqrt{1-b^2}\,db$

$\frac{dx}{\sqrt{17}/2} = db \qquad \ldots$

$dx = \frac{\sqrt{17}}{2}\,db$

57. $\int x^2\sqrt{x^2+4}\,dx = \int \sqrt{(t^2-4)}\cdot t^2\frac{dt}{\sqrt{t^2-4}}$

$x^2+4=t^2 \qquad t=\sqrt{x^2+4}$

$2x\,dx = 2t\,dt$

$dx = \frac{t}{x}\,dt \qquad x=\sqrt{t^2-4}$