

V) ИЗРАЧУНАТИ ИНТЕГРАЛ:

$$1. \int \frac{dx}{(x+1)(x-2)} = -\frac{1}{3} \ln|x+1| + \frac{1}{3} \ln|x-2| + C$$

$$\frac{1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} \quad | \cdot (x+1)(x-2)$$

$$1 = Ax + 2A + Bx + B$$

$$-2A + B = 1$$

$$A + B = 0$$

$$-3A = 1$$

$$A = -\frac{1}{3} \quad B = \frac{1}{3}$$

$$2. \int \frac{x dx}{2x^2 - 3x - 2} = \int \frac{x dx}{(2x+1)(x-2)} = \frac{1}{5} \ln|2x+1| + \frac{2}{5} \ln|x-2| + C$$

$$\frac{x}{(2x+1)(x-2)} = \frac{A}{2x+1} + \frac{B}{x-2} \quad | \cdot (2x+1)(x-2)$$

$$x = Ax - 2A + 2Bx + B$$

$$A + 2B = 1 \quad B - 2A = 0$$

$$A + 4A = 1 \quad B = 2A$$

$$5A = 1 \quad A = 1/5$$

$$B = 2/5$$

$$3. \int \frac{3x+1}{x^2-4x+4} dx = \int \frac{3x+1}{(x-2)^2} dx = 3 \ln|x-2| - \frac{1}{x-2} + C$$

$$\frac{3x+1}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} \quad | \cdot (x-2)^2$$

$$3x+1 = Ax - 2A + B$$

$$A = 3 \quad -2A + B = 1$$

$$-6 + B = 1$$

$$B = 7$$

$$4. \int \frac{2-x}{x^2-x-2} dx = \int \frac{2-x}{(x-2)(x+1)} dx = -\int \frac{dx}{x+1} = -\ln|x+1| + C$$

$$5. \int \frac{2x^2+41x-91}{(x-1)(x-4)(x+3)} dx = 4 \ln|x-1| + 5 \ln|x-4| - 7 \ln|x+3| + C$$

$$\frac{2x^2+41x-91}{(x-1)(x-4)(x+3)} = \frac{A}{x-1} + \frac{B}{x-4} + \frac{C}{x+3} \quad | \cdot (x-1)(x-4)(x+3)$$

$$2x^2+41x-91 = Ax^2 - Ax - 12A + Bx^2 + 2Bx - 3B + Cx^2 - 5Cx + 4C$$

$$A+B+C=2 \quad -A+2B-3C=41 \quad -12A-3B+4C=-91$$

$$3B-4C=43$$

$$9B+16C=-67$$

$$28C = -196$$

$$C = -7$$

$$B = 5$$

$$A = 4$$

$$6. \int \frac{x^3 + 5x^2 - 1}{x^2 - 6x + 8} dx = \frac{x^2}{2} + 11x + \int \frac{58x - 89}{(x-2)(x-4)} dx = \frac{x^2}{2} + 11x + \frac{27}{2} \ln|x-2| + \frac{143}{2} \ln|x-4|$$

$$(x^3 + 5x^2 - 1) : (x^2 - 6x + 8) = x + 11$$

$$\begin{array}{r} x^3 + 5x^2 - 1 \\ - (x^2 - 6x + 8) \\ \hline \end{array}$$

$$11x^2 - 8x + 1$$

$$\frac{58x - 89}{(x-2)(x-4)} = \frac{A}{x-2} + \frac{B}{x-4} \quad | \cdot (x-2)(x-4)$$

$$11x^2 - 6x + 88$$

$$58x - 89 = Ax - 4A + Bx - 2B$$

$$58x - 89$$

$$A + B = 58$$

$$-4A - 2B = -89$$

$$2A + 2B = 116$$

$$-2A = 27$$

$$A = -\frac{27}{2} \quad B = \frac{143}{2}$$

$$0111310 \quad 29 \quad 22$$

$$x(x^2 - 4) - (x^2 - 4)$$

$$7. \int \frac{x dx}{(x+1)(x+2)(x-3)} = \frac{5}{20} \ln|x+1| - \frac{2}{5} \ln|x+2| + \frac{3}{20} \ln|x-3| + c$$

$$\frac{x}{(x+1)(x+2)(x-3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3} \quad | \cdot (x+1)(x+2)(x-3)$$

$$x = Ax^2 - Ax - 6A + Bx^2 - 2Bx - 3B + Cx^2 + 3Cx + 2C$$

$$A + B + C = 0 \quad -A - 2B + 3C = 1 \quad -6A - 3B + 2C = 0$$

$$A - \frac{2}{5} + \frac{3}{20} = 0 \quad A + B + C = 0 \quad -6A + 6B + 6C = 0$$

$$A + \frac{3-8}{20} = 0$$

$$-B + 4C = 1$$

$$3B + 8C = 0$$

$$-B = 1 - 4C$$

$$-B + 4C = 1 \quad | \cdot 3$$

$$A = \frac{5}{20}$$

$$B = 4C - 1$$

$$20C = 3$$

$$B = -\frac{2}{5}$$

$$C = \frac{3}{20}$$

$$8. \int \frac{5x-3}{(x-2)(3x^2+2x-1)} dx =$$



$$\frac{5x-3}{(x-2)(3x^2+2x-1)} = \frac{A}{x-2} + \frac{Bx+C}{3x^2+2x-1} \quad | \cdot (x-2)(3x^2+2x-1)$$

$$5x - 3 = 3Ax^2 + 2Ax - A + Bx^2 - 2Bx + Cx - 2C$$

$$3A + B = 0$$

$$2A - 2B + C = 5$$

$$-A - 2C = -3 \quad | \cdot 2$$

$$-2A - 4C = -6$$

$$3A + B = 0$$

$$-2B - 3C = -1$$

$$2A + B - 2C = -3$$

$$3A + B = 0$$

$$3A + B = 0 \rightarrow B = -3A$$

$$2A - 2B + C = 0 \rightarrow$$

$$-A - 2C = -3 \rightarrow -2C = -3 + A \quad C = -\frac{A}{2} + \frac{3}{2}$$

$$2A + 6A = \frac{A}{2} + \frac{3}{2} = 0$$

$$\frac{15A}{2} = -\frac{3}{2}$$

$$18 \quad A = -\frac{1}{5}$$

$$B = +\frac{3}{5}$$

$$C = \frac{16}{10}$$

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$$9) \int \frac{5x-14}{x^3-x^2-4x+4} dx = \int \frac{5x-14}{(x-1)(x-2)(x+2)} dx$$

$$\frac{5x-14}{(x-1)(x-2)(x+2)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+2} \quad | \cdot (x-1)(x-2)(x+2)$$

$$5x-14 = \frac{Ax^2-4A}{x-1} + \frac{Bx^2+Bx-2B}{x-2} + \frac{Cx^2-3Cx+2C}{x+2}$$

$$\begin{aligned} A+B+C &= 0 & B-3C &= 5 & -4A-2B+2C &= -14 \quad | :(-2) \\ & & B &= 3C+5 & 2A+B-C &= 7 \\ & & & & 2A+3C+5-C &= 7 \\ & & & & 2A+2C &= 2 \quad | :2 \\ & & & & A+C &= 1 \end{aligned}$$

$$\begin{aligned} A+4C &= -5 \\ -A-C &= -1 \\ \hline 3C &= -6 \\ C &= -2 \end{aligned}$$

$$C = -2 \quad A = 3 \quad B = 14$$

$$= 3 \ln|x-1| + 14 \ln|x-2| - 2 \ln|x+2| + C$$

$$10) \int \frac{x^2+1}{(x^2-1)(x^2+4)} dx = -\frac{1}{6} \ln|x-1| + \frac{1}{6} \ln|x+1| + \frac{1}{12} \ln|x-2| - \frac{1}{12} \ln|x+2| + C$$

$$\frac{x^2+1}{(x^2-1)(x^2+4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2} + \frac{D}{x+2} \quad | \cdot (x^2-1)(x^2+4)$$

$$\begin{aligned} x^2+1 &= \frac{Ax^3+Ax^2-4Ax-4A}{(x-1)(x^2+4)} + \frac{Bx^3+Bx^2-4Bx+4B}{(x+1)(x^2+4)} + \frac{Cx^3-Cx^2+2Cx-2C}{(x-2)(x^2+4)} + \frac{Dx^3+Dx^2-2Dx+2D}{(x+2)(x^2+4)} \\ &= \frac{Ax^3+Ax^2-4Ax-4A}{(x-1)(x^2+4)} + \frac{Bx^3+Bx^2-4Bx+4B}{(x+1)(x^2+4)} + \frac{Cx^3-Cx^2+2Cx-2C}{(x-2)(x^2+4)} + \frac{Dx^3+Dx^2-2Dx+2D}{(x+2)(x^2+4)} \end{aligned}$$

$$\begin{aligned} A+B+C+D &= 0 & A-B+2C-2D &= 1 \\ -4A-4B-C-D &= 0 & -4A+4B-2C+2D &= 1 \end{aligned}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & -2 \\ -4 & -4 & -1 & -1 \\ -4 & 4 & -2 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 2 & -2 \\ -4 & -1 & -1 \end{vmatrix}$$

$$\begin{aligned} A+B+C+D &= 0 \\ A-B+2C-2D &= 1 \\ -4A-4B-C-D &= 0 \\ -4A+4B-2C+2D &= 1 \end{aligned}$$

$$A+B+C+D=0$$

$$-2B+C-3D=6$$

$$3C+3D=0$$

$$8B+2C+6D=1$$

$$6C-6D=7$$

$$3C+3D=0$$

$$6C-6D=1$$

$$6C+6D=0 \quad | :6 \quad C+D=0$$

$$12C=1$$

$$C = 1/12$$

$$D = -1/12$$

$$-2B + 1/12 + 3/12 = 0$$

$$-2B = -4/12$$

$$B = -1/6$$

$$B = -1/6$$

$$A = -1/6$$

$$11. \int \frac{-x+0}{x^2-4} dx = \frac{1}{2}x^2 + \int \frac{11+0}{x^2-4} dx =$$

$$(3x^3 - 5x + 8) : (x^2 - 4) = 3x \quad = \frac{3}{2}x^2 + \frac{3}{2} \ln|x+2| + \frac{11}{2} \ln|x-2| + C$$

$$\begin{array}{r} 3x^3 - 12x \\ - \quad + \\ \hline 7x + 8 \end{array}$$

$$\frac{7x+8}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2} \quad / \cdot (x^2-4)$$

$$7x+8 = Ax - 2A + Bx + 2B$$

$$\begin{array}{rcl} A+B & = & 7 \\ -2A+2B & = & 8 \quad / : (-2) \\ A-B & = & -4 \\ 2A & = & 3 \\ A & = & 3/2 \end{array} \quad B = \frac{14-3}{2} = \frac{11}{2}$$

$$12. \int \frac{x^3+1}{x^2-3x+2} dx = \frac{x^2}{2} + 3x + \int \frac{7x-5}{(x-1)(x-2)} dx =$$

$$(x^3+1) : (x^2-3x+2) = x+3 \quad = \frac{x^2}{2} + 3x + 2 \ln|x-1| + 9 \ln|x-2| + C$$

$$\begin{array}{r} x^3 - 3x^2 + 2x \\ - \quad - \\ \hline 3x^2 - 2x + 1 \end{array}$$

$$\frac{7x-5}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \quad / \cdot (x-1)(x-2)$$

$$3x^2 - 2x + 1$$

$$3x^2 - 9x + 6$$

$$7x - 5$$

$$7x-5 = Ax - 2A + Bx - B$$

$$\begin{array}{rcl} A+B & = & 7 \\ -2A-B & = & -5 \quad / : (-1) \end{array}$$

$$\begin{array}{rcl} -2A+B & = & -5 \\ +4-B & = & -5 \quad -B = -9 \end{array}$$

$$-A = 2$$

$$A = -2$$

$$B = 9$$

$$13. \int \frac{x^5+x^4-8}{x^3-4x} dx = \frac{x^3}{3} + \frac{x^2}{2} + 4x + 4 \left[\frac{1}{2} \ln|x| + \frac{5}{4} \ln|x-2| - \frac{3}{4} \ln|x+2| \right] + C$$

$$(x^5+x^4-8) : (x^3-4x) = x^2+x+4$$

$$x^5 - 4x^3$$

$$x^4 + 4x^3 - 8$$

$$x^4 - 4x^2$$

$$4x^3 + 4x^2 - 8$$

$$4x^3 - 16x$$

$$4x^2 + 16x - 8$$

$$4x^2 + 4x - 2$$

$$4x^2 + 16x - 8$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$4x^2 + 4x - 2$$

$$\frac{x^2+4x-2}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} \quad / \cdot x(x^2-4)$$

$$x^2+4x-2 = Ax^2-4A+Bx^2+2B+Cx^2-2Cx$$

$$\begin{array}{rcl} A+B+C & = & 1 \\ 2B-2C & = & 4 \\ -4A & = & -2 \end{array}$$

$$B+C = 1/2 \quad A = 1/2$$

$$2B+2C = 1$$

$$4B = 5$$

$$B = 5/4$$

$$\frac{2+5}{4} + C = \frac{4}{4}$$

$$C = -\frac{3}{4}$$

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$$15. \int \frac{5-x^3}{x^3-7x^2+10x} dx = x + \frac{1}{2} \ln|x| + \frac{1}{2} \ln|x-2| - \frac{3}{2} \ln|x-5| + C$$

$$\begin{array}{r} (-x^3+5):(x^3-7x^2+10x) = -1 \\ \underline{-x^3+7x^2-10x} \\ 10x-5 \end{array}$$

$$\frac{-7x^2+10x+5}{x(x-2)(x-5)}$$

$$\frac{-7x^2+10x+5}{x(x-2)(x-5)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-5}$$

$$-7x^2+10x+5 = \underline{Ax^2-7Ax+10A+Bx^2-5Bx+Cx^2-2Cx}$$

$$A+B+C = -7$$

$$\frac{1}{2}+B+C = -7$$

$$-7A-5B-2C = 10$$

$$-\frac{7}{2}-5B-2C = 10$$

$$10A = 5$$

$$A = 1/2$$

$$2B+2C = -15/2$$

$$-10B+2C = 27$$

$$C = -7 - A = -8$$

$$-8B = 27$$

$$B = 1/2$$

$$16. \int \frac{x(x^2+1)}{(x+1)(x^2+2x+2)} dx = \int \frac{x^3+x}{x^3+3x^2+4x+2} = x - 2 \ln|x+1| + \int \frac{2-x}{x^2+2x+2} dx + C$$

$$\begin{array}{r} (x^3+x):(x^3+3x^2+4x+2) = 1 \\ \underline{x^3+3x^2+4x+2} \\ -3x^2-3x-2 \end{array}$$

$$\frac{-3x^2-3x-2}{(x+1)(x^2+2x+2)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2x+2}$$

$$-3x^2-3x-2 = \underline{Ax^2+2Ax+2A+Bx^2+Bx+Cx+C}$$

$$A+B = -3 \rightarrow B = -A-3$$

$$2A+B+C = -3 \rightarrow 2A-A-3-2A-2 = -3$$

$$2A+C = -2 \rightarrow C = -2A-2$$

$$B = -1$$

$$A = -2$$

$$C = 2$$

$$17. \int \frac{2x^4-2x^3-x^2+2}{2x^3-4x^2+3x-1} dx = \frac{x^2}{2} + x + \int \frac{-2x+3}{(x-1)(2x^2-2x+1)} dx$$

$$\begin{array}{r} (2x^4-2x^3-x^2+2):(2x^3-4x^2+3x-1) = x+1 \\ \underline{2x^4-4x^3+3x^2-x} \\ 2x^3-4x^2+3x^2-x+2 \end{array}$$

$$= \frac{x^2}{2} + x + 1 \cdot \ln|x-1| +$$

$$+ \int \frac{-2x+3}{2x^2-2x+1} dx + C$$

$$\begin{array}{c|c|c|c|c} 1 & 2 & -4 & 3 & -1 \\ \hline & 2 & -2 & 1 & 0 \\ \hline & 2x^2-2x+1 & & & \end{array}$$

$$-2x+3 = \underline{2Ax^2-2Ax+A+Bx^2-Bx+Cx-C}$$

$$2A+B=0$$

$$-2A-B+C=-2 \quad A-C=3$$

$$B = -2A$$

$$-2A+2A+A-3=-2 \quad -C=3-A$$

$$A = 2$$

$$C = 1$$

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$$18. \int \frac{x^3+3}{(x+1)(x^2+1)} dx = \int \frac{x^3+3}{x^3+x^2+x+1} dx = X + 2 \ln|x+1| - \int \frac{3x dx}{x^2+1}$$

$$(x^3+3):(x^3+x^2+x+1)=1$$

$$\begin{aligned} x^2+1 &= t \\ 2x dx &= dt \\ x dx &= dt/2 \end{aligned}$$

$$-\frac{3}{2} \ln(x^2+1) + C$$

$$-x^2-x+2$$

$$-x^2-x+2 = Ax^2+A+Bx^2+Bx+Cx+C$$

$$A+B=-1 \quad B+C=-1 \quad A+C=2$$

$$B=-A-1$$

$$C=2-A$$

$$-A-1+2-A=-1$$

$$-A=-2$$

$$A=2 \quad B=-3 \quad C=0$$

$$19. \int \frac{2x^4+5x^2-2}{2x^3-x-1} dx = \frac{x^2}{2} + 1 \cdot \ln|x-1| + \int \frac{4x+3}{2x^3-x-1} dx + C$$

$$(2x^4+5x^2-2):(2x^3-x-1) = x-1 \quad \begin{array}{r|rrrr} 2 & 0 & -1 & -1 \\ \hline 2 & 2 & -1 & 0 \end{array}$$

$$\begin{aligned} & 2x^2+2x+1 \\ & \quad x-1 \\ \hline 6x^2+x-2 &= 2Ax^2+2Ax+A+Bx^2+Bx+Cx-C \end{aligned}$$

$$2A+B=6 \quad 2A-B+C=1 \quad A-C=-2$$

$$B=6-2A \quad 2A-6+2A+A-2=-C=-2-A \quad C=A+2$$

$$5A=5$$

$$A=1$$

$$B=4$$

$$C=3$$

$$20. \int \frac{x^4+1}{x^3-x^2+x-1} dx = \frac{x^2}{2} + x + \ln|x-1| + \int \frac{x+1}{x^2+1} dx =$$

$$(x^4+1):(x^3-x^2+x-1)=x+1$$

$$\begin{array}{r} x^4-x^3-x^2-x \\ \hline \end{array}$$

$$2 =$$

$$-\frac{1}{2} \ln(x^2+1) - \arctan x + C$$

$$\begin{array}{r} x^3-x^2+x-1 \\ \hline \end{array}$$

$$2$$

$$x^2(x-1) + (x-1)$$

$$(x-1)(x^2+1)$$

$$B=-A$$

$$-B=-C$$

$$B=C$$

$$B=-1$$

$$C=-1$$

$$2 = Ax^2+A+Bx^2+Bx+Cx-C$$

$$C=-A$$

$$A+B=0 \quad -B+C=0 \quad A-C=2$$

$$A+A=2$$

$$2A=2 \quad A=1$$

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$$21. \int \frac{x}{x^3-1} dx = \frac{1}{3} \ln|x-1| + \frac{1}{3} \int \frac{1}{x^2+x+1} dx$$

$$\frac{x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$x = \underline{Ax^2 + Ax + A} + \underline{Bx^2 + Bx + C} - \underline{C}$$

$$A+B=0 \quad A-B+C=1 \quad A-C=0$$

$$B=-A \quad A+A+A=1 \quad -C=-A$$

$$B=-\frac{1}{3} \quad 3A=1 \quad C=A$$

$$22. \int \frac{x^3+x+1}{x^4-1} dx =$$

$$\frac{x^3+x+1}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C+D}{x^2+1}$$

$$x^3+x+1 = \underline{Ax^3 + Ax^2 + Ax + A} + \underline{Bx^3 + Bx^2 + Bx - B} + \underline{Cx^3 - Cx + Dx^2 - D}$$

$$A+B+C=1$$

$$A-B+D=0$$

$$A+B-C=1$$

$$A-B-D=1$$

$$A+B+C=1$$

$$-2B-C-D=-1$$

$$-2C=0 \Rightarrow C=0$$

$$-2B-C-D=0$$

$$-2B-D=-1$$

$$-2B-D=0$$

$$23. \int \frac{x^3+4x^2-2x+1}{x^4+x} dx = -\ln|x| + 2\ln|x+1| - \int \frac{2x}{x^2-x+1} dx + C$$

$$x^3+4x^2-2x+1 = \underline{Ax^3 + A} + \underline{Bx^2 + Bx + B} + \underline{Cx^3 + Cx^2 + Dx^2 + Dx}$$

$$A+C=1$$

$$C=1-A$$

$$\boxed{C=2}$$

$$B+C+D=4$$

$$1-A+1-A-A=4$$

$$-3A=3$$

$$\boxed{A=-1}$$

$$-B+D=-2$$

$$D=-2+B$$

$$D=-2+1-A$$

$$D=-A-1$$

$$\boxed{D=0}$$

$$A+B=1$$

$$B=1-A$$

$$24. \int \frac{dx}{x^3+1} = \frac{1}{3} \ln|x+1| - \frac{1}{3} \int \frac{2-x}{x^2-x+1} dx$$

$$1 = \underline{Ax^2 - Ax + A} + \underline{Bx^2 + Bx + C} + \underline{C}$$

$$A+B=0 \quad -A+B+C=0 \quad A+C=1$$

$$B=-A \quad -A-A+1-A=0 \quad C=1-A$$

$$-3A=-1$$

$$A=-1/3$$

$$B=-1/3 \quad C=2/3$$

$$25. \int \frac{7x^2-1}{x^4+4x^2-5} dx = \frac{1}{3} \ln|x-1| + \int \frac{6x+6}{x^2+5} dx \rightarrow \text{quadratic denominator}$$

$$\frac{7x^2-1}{(x^2+5)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+5}$$

$$7x^2-1 = \underline{Ax^2 + 5A} + \underline{Bx^2 - Bx - C} - \underline{C}$$

$$A+B=7$$

$$-B+C=0$$

$$5A-C=-1$$

$$C=B$$

$$5A-B=-1$$

$$A+B=7$$

$$\boxed{B=8}$$

$$1/2=C$$

$$\boxed{C=6}$$

$$1/2=C$$

$$26. \int \frac{x^2 - 2x - 5}{x^3 - x^2 + 2x - 2} dx = -2 \ln|x-1| + \int \frac{3x+1}{x^2+2} dx$$

$$x^2(x-1) + 2(x-1)$$

$$x^2 - 2x - 5 = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+2}$$

$$\begin{aligned} A+B &= 1 \rightarrow B = 1-A \\ -B+C &= -2 \quad -1+A+2A+5 = -2 \quad 3A = -6 \\ 2A-C &= -5 \rightarrow -C = -5-2A \rightarrow C = 2A+5 \end{aligned}$$

$$A = -2 \quad B = 3 \quad C = 1$$

$$27. \int \frac{3-5x}{(2x-1)(x^2+4)} dx = \frac{2}{17} \ln|2x-1| + \int \frac{1}{x^2+4} dx$$

$$-5x+3 = \frac{A}{2x-1} + \frac{B}{x^2+4} + \frac{C}{x^2+4}$$

$$\begin{aligned} A+2B &= 0 \\ -B+2C &= -5 \quad / \cdot 2 \quad \begin{aligned} A+4C &= -10 \quad / \cdot 7 \\ 4A-5C &= 3 \quad / \cdot 4 \end{aligned} \\ 4A-C &= 3 \end{aligned}$$

$$\begin{aligned} 17A &= 2 \\ A &= 2/17 \quad C = \end{aligned}$$

$$\begin{aligned} \frac{8}{17} - C &= 3 \\ -C &= \frac{51-8}{17} \end{aligned}$$

$$28. \int \frac{x^3+x^2+x+3}{(x+3)(x^2+x+1)} dx = \int \frac{x^3+x^2+x+3}{x^3+4x^2+4x+3} dx = x$$

$$x^3+x^2+x+3 = x^3+4x^2+4x+3$$

$$(x^3+x^2+x+3) - (x^3+4x^2+4x+3) = 1$$

$$-3x^2-3x$$

$$-3x^2-3x = \frac{A}{x+3} + \frac{B}{x^2+x+1} + \frac{C}{x^2+x+1}$$

$$\begin{aligned} A+B &= -3 \\ A+3B+C &= -3 \\ A+3C &= 0 \end{aligned}$$

$$\begin{aligned} A+B &= -3 \\ 2B+C &= 0 \\ -B+3C &= 3 \end{aligned}$$

$$\begin{aligned} 7C &= 6 \\ C &= 6/7 \\ B &= -\frac{3}{7} \\ A &= \end{aligned}$$

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$$29. \int \frac{x^2 - 5x}{(x-1)(x^2+x+2)} dx = 1 \cdot \ln|x-1| - \int \frac{2}{x^2+x+2} dx$$

$$\begin{aligned} x^2 - 5x &= Ax^2 + A + 2A + Bx^2 - Bx + Cx - C \\ A+B &= 1 & B &= 1-A \\ A-B+C &= -5 & A-1+A+2A &= -5 \\ 2A-C &= 0 & 4A &= -4 & B &= 0 \\ -C &= -2A & A &= -1 & C &= 2 \\ C &= 2A \end{aligned}$$

$$30. \int \frac{dx}{x(x^2-2x+8)} = \frac{1}{8} \ln|x| + \int \frac{-x/8 + 1/4}{x^2-2x+8} dx$$

$$1 = Ax^2 - 2Ax + 8A + Bx^2 + Cx$$

$$\begin{aligned} A+B &= 0 & -2A+C &= 0 & 8A &= 1 \\ B &= -1/8 & -1/4+C &= 0 & A &= 1/8 \\ C &= 1/4 \end{aligned}$$

Omheer

$$31. \int \frac{2x^3 + x^2 + 3x + 1}{(x^2+3)(x^2-x+1)} dx = \int \frac{1}{x^2+3} dx + \int \frac{2x}{x^2-x+1} dx =$$

$$2x^3 + x^2 + 3x + 1 = Ax^3 - Ax^2 + Ax + Bx^2 - Bx + B + Cx^3 + 3Cx + Dx^2 + 3D$$

$$A+C=2 \rightarrow C=2-A$$

$$-A+B+D=1$$

$$A-B+3C=5$$

$$B+3D=1 \rightarrow 31$$

$$A+C=2$$

$$B+C+D=3$$

$$A-B+3C=5$$

$$31$$

$$-A+A-2A+D=1$$

$$-3A+D=0$$

$$D=3A$$

$$-2A-B=-1$$

$$-B=-1+2A$$

$$B=1-2A$$

$$1-2A+3A=1$$

$$A=0$$

$$A=0$$

$$B=1$$

$$B=1$$

$$C=2$$

$$C=2$$

$$D=0$$

Arctg

$$32. \int \frac{0x}{x^2(x^2+4)} = \cancel{\ln|x|} + \int \frac{1}{4x^2} dx + \int \frac{-1/4}{x^2+4} dx =$$

$$1 = \underline{Ax^3 + Ax} + \underline{Bx^2 + 4B} + \underline{Cx^3 + Dx^2} - \frac{1}{4} \frac{1}{x} + \text{crewm}$$

$$A+C=0 \rightarrow C=0$$

$$B+D=0 \rightarrow D=-1/4$$

$$A=0 \rightarrow A=0$$

$$4B=1 \rightarrow B=1/4$$

$$33. \int \frac{3x-5}{x^2(x^2-2x+2)} dx = -\ln|x| + \frac{5}{2} \cdot \frac{1}{x} + \int \frac{x+1/2}{x^2-2x+2} dx \quad \text{yavryha yo 1. usboja}$$

$$3x-5 = \underline{Ax^3 - 2Ax^2 + 2Ax + Bx^2} + \underline{2Bx + 2B} + \underline{Cx^3 + Dx^2}$$

$$A+C=0$$

$$-2A+B+D=0$$

$$2A-2B=3$$

$$2B=-5$$

$$C=1$$

$$A=-1$$

$$D=1/2$$

$$B=-5/2$$

$$34. \int \frac{dx}{x(x+1)(x^2+x+1)} = \ln|x| - \ln|x+1| + \int \frac{-1}{x^2+x+1} dx$$

$$1 = \underline{Ax^3 + Ax^2 + Ax} + \underline{Ax^2 + Ax + A} + \underline{Bx^3 + Bx^2 + Bx} + \underline{Cx^3 + Cx^2 + Dx^2 + Dx}$$

$$A+B+C=0$$

$$B+C=-1$$

$$D=-1$$

$$2A+B+C+D=0$$

$$B+C+D=-2$$

$$2A+B+D=0$$

$$B+D=-2$$

$$B=-1$$

$$A=1$$

$$C=0$$

$$35. \int \frac{3x^2+5x+12}{(x^2+1)(x^2+3)} dx = \int \frac{5/2x+9/2}{x^2+1} dx + \int \frac{-5/2x+3/2}{x^2+3} dx$$

$$3x^2+5x+12 = \underline{Ax^3 + 3Ax} + \underline{Bx^2 + 3B} + \underline{Cx^3 + Cx} + \underline{Dx^2 + D}$$

$$A+C=0$$

$$B+D=3$$

$$3A+C=5$$

$$3B+D=12$$

$$C=-A$$

$$3B+D=12$$

$$3A-A=5$$

$$2A=5$$

$$A=5/2$$

$$C=-5/2$$

$$B=9/2$$

$$D=-3/2$$

$$36. \int \frac{x^2+2}{(x-1)(x+1)^2} dx \quad x^2+2x+1$$

$$x^2+2 = \underline{Ax^2 + 2Ax + A} + \underline{Bx^2 - B} + \underline{Cx - C}$$

$$A+B=1$$

$$2A+C=0$$

$$A-C=2$$

$$-C=2-A$$

$$C=A-2$$

$$2A+A-2=0$$

$$3A=2$$

$$A=\frac{2}{3}$$

$$B=\frac{1}{3}$$

$$C=-\frac{4}{3}$$

$$27. \int \frac{dx}{x^3(x-1)^2} = 3 \ln|x| + 2 \int \frac{dx}{x^2} + \int \frac{dx}{x^3} - 3 \int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2}$$

$$1 = \frac{Ax^4 - 2Ax^3 + Ax^2 + Bx^3 - 2Bx^2 + Bx + Cx^2 - 2Cx + C + Dx^4 - Dx^3 + Ex^3}{x^2 - 2x + 1}$$

$$A + D = 0 \rightarrow D = -A$$

$$-2A + B - D + E = 0$$

$$A - 2B + C = 0 \quad A - 4C + C = 0 \quad -3C = -A \quad C = A/3$$

$$B - 2C = 0 \quad B = 2C$$

$$B = 2A/3$$

$$C = 1$$

$$A = 3$$

$$B = 2$$

$$D = -3$$

$$E = 1$$

$$-6 + 2 + 3 + E = 0$$

$$= 3 \ln|x| + 2 \frac{1}{x} - \frac{1}{2} \frac{1}{x^2} - 3 \ln|x-1| - \frac{1}{x-1} + C$$

$$38. \int \left(\frac{x+2}{x-1} \right)^2 \frac{dx}{x} = \int \frac{x^2 + 4x + 4}{x^2(x-1)^2} dx = 4 \ln|x| + 3 \ln|x-1| - 15 \frac{1}{x-1} + C$$

$$x^2 + 4x + 4 = \frac{Ax^2 - 2Ax + A + Bx^2 - Bx + Cx}{x^2 - 2x + 1}$$

$$A + B = 1$$

$$-2A - B + C = 4$$

$$A = 4 \quad B = 3 \quad C = 15$$

$$39. \int \frac{x^2 - 2x + 3}{(x-1)(x^2 - 4x + 3)} dx = \int \frac{x^2 - 2x + 3}{(x-1)^2 \cdot x(x-3)} dx = \ln|x| + \frac{1}{2} \ln|x-3| + \frac{1}{2} \ln|x-1| + \frac{1}{x-1} + C$$

$$\begin{array}{c|c|c|c|c} 1 & 1 & -4 & -3 & 0 \\ \hline 1 & -3 & 0 & 0 & 0 \end{array}$$

$$\frac{x^2 - 3x}{x(x^2 - 4x + 3)}$$

$$B + C = 1$$

$$-2B - 4C + D = -4$$

$$B + 3C - 3D = 5$$

$$C = 1 - B$$

$$\begin{array}{l} -2B - 4(1-B) + D = -4 \\ B + 3 - 3(1-B) - 3D = 5 \end{array}$$

$$\begin{array}{l} 2B + D = 0 \\ -2B - 3D = 2 \\ -2D = 2 \end{array}$$

$$D = -1$$

$$B = 1/2$$

$$C = 1/2$$

$$40. \int \frac{x^3 - 2x^2 + 4}{x^3(x-2)^2} dx = \frac{1}{4} \ln|x| - \frac{1}{x} - \frac{1}{2x^2} - \frac{1}{4} \ln|x-2| - \frac{1}{2} \frac{1}{x-2} + C$$

$$x^3 - 2x^2 + 4 = \frac{Ax^4 - 4Ax^3 + 4Ax^2 + Bx^3 - 4Bx^2 + 4Bx + Cx^2 - 4Cx + 4C + Dx^4 - 2Dx^3 + Ex^3}{x^2(x^2 - 4x + 4)}$$

$$A + D = 0$$

$$-4A + B - 2D + E = 1$$

$$4A - 4B + C = -2$$

$$4B - 4C = 0$$

$$4 = 4C$$

$$-1/4 + 1/2 + E = 1$$

$$E = 1/2$$

$$C = 1 \quad B = 1$$

$$A = 1/4 \quad D = -1/4$$

$$41. \int \frac{3x+2}{x(x+1)^3} dx = 2 \ln|x| - 2 \ln|x+1| + 2 \frac{1}{x+1} + \frac{1}{2} \frac{1}{(x+1)^2} + C$$

$$3x+2 = \frac{Ax^3+3Ax^2+3Ax+A}{x^2+2x+1} + \frac{Bx^3+2Bx^2+Bx+Cx^2+Cx+D}{x^2+2x+1}$$

$$\begin{aligned} A+B &= C & 3A+2B+C &= 0 & 3A+B+C+D &= 3 & A &= 2 & B &= -2 & C &= -2 & D &= 1 \\ 6-4+C &= 0 & 6-2-2+D &= 3 \end{aligned}$$

$$42. \int \frac{1-x}{(x+2)(2x-3)^2} dx = \frac{3}{37} \ln|x+2| - \ln|2x-3| + \frac{1}{2} - \frac{1}{2x-3}$$

$$1-x = \frac{4Ax^2-12Ax+9A}{4x^2-12x+9} + \frac{2Bx^2+4Bx-3B}{4x^2-12x+9} + \frac{Cx+2C}{4x^2-12x+9}$$

$$\begin{aligned} 4A+2B &= 0 & -12A+4B-3B+C &= -1 & 9A-6B+2C &= 1 \\ 2B &= -4A & -12A-2A+3A+C &= -1 & 9A+12A+2C &= 1 \\ B &= -2A & 9A+12A+2C &= 1 \end{aligned}$$

$$\begin{aligned} -8A+C &= -1 \quad / (-2) \\ 21A+2C &= 1 \end{aligned}$$

$$37A = 3 \quad A = 3/37$$

$$43. \int \frac{x-1}{(x-2)(x^2+x)^2} dx = \int \frac{x-1}{(x-2)(x+1)^2 x^2} dx =$$

$$x-1 = \frac{Ax^4+2Ax^3+Ax^2}{x^2+2x+1} + \frac{Bx^4+2Bx^3+Bx^2-2Bx^3-4Bx^2-2Bx}{x^2+2x+1} + \frac{Cx^3-2Cx^2+2Cx^2-4Cx+C}{x^2+2x+1} + \frac{Dx^4-2Dx^3+Dx^2-2Dx^2+E}{x^2+2x+1}$$

$$\begin{aligned} \checkmark A+B+D &= 0 \quad \checkmark \\ \checkmark 2A+2B-2B+C-2D+D+E &= 0 \\ A+B-4B-2C+2C-2D-2E &= 0 \\ \checkmark -2B-4C+C &= 1 \quad -2B-3C=1 \\ \checkmark -2C &= -1 \quad -2B=1+3C \end{aligned}$$

$$C = 1/2$$

$$\frac{15}{4}$$

$$B = -\frac{1}{2}(1+\frac{3}{2})$$

$$B = -\frac{5}{4}$$

$$\begin{aligned} A+D &= 5/4 & D &= 5/4-A \\ 2A-D+E &= -1/2 \\ A-2D-2E &= -15/4 \end{aligned}$$

$$2A-5/4+A+E = -1/2$$

$$A-5/2+2A-2E = -15/4$$

$$3A+E = +\frac{3}{4} \quad / \cdot 2$$

$$3A-2E = -\frac{5}{4}$$

$$3A = \frac{1}{4}$$

$$A = \frac{1}{36}$$

$$E =$$

$$44. \int \frac{3x}{x(x-3)^2(x+1)} dx = -\frac{3}{16} \ln|x-3| - \frac{3}{4} \cdot \frac{1}{x-3} + \frac{3}{16} \ln|x+1| + C$$

$$3x = \frac{Ax^3 - 6Ax^2 + 9Ax + A}{x^2 - 6x + 9} + \frac{Bx^3 - 2Bx^2 - 3Bx + Cx^2 + Cx + D}{x^2 - 2x + 3} - \frac{6Dx^2 + 9Dx}{x^2 - 2x + 3}$$

$$\begin{aligned} A+B+D &= 0 \\ -6A+A-2B+C-6D &= 0 \\ 9A-6A-3B+C+9D &= 3 \\ 9A &= 0 \quad A=0 \end{aligned}$$

$$\begin{aligned} B+D &= 0 \quad D=-B \\ -2B+C-6D &= 0 \quad -2B+C+6B=0 \\ -3B+C+9D &= 3 \quad -3B+C-9B=3 \\ 4B+C &= 0 \quad (-1) \quad 3/8 + 3/4 = 6D \quad 9/8 = 6D \\ -12B+C &= 3 \quad -16B=3 \end{aligned}$$

$B = -3/16$	$C = 3/4 \checkmark$
$A = 0 \checkmark$	$D = 3/16 \checkmark$

$$D = \frac{9/8}{3/16} = 6D$$

$$45. \int \frac{dx}{(x-3)^2(x^2+1)} =$$

$$1 = \frac{Ax^3 + Ax - 3Ax^2 - 3A}{x^2 - 6x + 9} + \frac{Bx^2 + B}{x^2 + 1} + \frac{Cx^3 - 6Cx^2 + 9Cx + Dx^2 - 6Dx + 3D}{x^2 + 1}$$

$$\begin{aligned} A+C &= 0 \quad C=-A \\ -3A+B-6C+D &= 0 \quad -3A+B+6A+D=0 \quad 3A+B+D=0 \\ A+9C-6D &= 0 \quad A-9A-6D=0 \\ -3A+9D &= 1 \quad (-8/3)/9 \\ -8A-6D &= 0 \\ -30D &= -8/3 \\ D &= 8/160 \end{aligned}$$

$$x^4(x+1) - x^2(x+1) \quad x^2(x^2-1)(x+1) \quad x^2(x+1)^2(x-1)$$

$$46. \int \frac{x^4+1}{x^5+x^4-x^3-x^2} dx = \int \frac{x^4+1}{(x-1) \cdot x^2(x+1)^2} dx = \cancel{0 \ln|x-1|} - \ln|x| - \frac{1}{x} +$$

$$+ 2\ln|x+1| - \frac{x^2}{x+1} + C$$

$$x^4+1 = \frac{Ax^4+2Ax^3+Ax^2+Bx^4-Bx^3+2Bx^2-2Bx^2+Bx^2-Bx+Cx^3-Cx^2+2Cx^2-2Cx+}{x^5+x^4-x^3-x^2} + \frac{Cx-C}{x^5+x^4-x^3-x^2} + \frac{Dx^4-Dx^2+E}{x^5+x^4-x^3-x^2} + \frac{Ex^3-Ex^2}{x^5+x^4-x^3-x^2}$$

$$\begin{aligned} A+B+D &= 1 \quad 2A-B+2B+C+E=0 \quad A-2B+B-C+2C-D-E=0 \\ -B-2C+C &= 0 \quad -C=1 \quad B+C \quad A+B=2 \quad -B+C \quad D=2-A \\ -B-C &= 0 \quad B=-C \quad 2A+E=0 \quad A-D-E=-2 \quad E=-2A \\ B &= -C \end{aligned}$$

$$\boxed{C=1} \checkmark$$

$$\boxed{B=-1} \checkmark$$

$$\boxed{A=0} \checkmark$$

$$\boxed{E=0} \checkmark$$

$$\boxed{D=-2} \checkmark$$

$$\begin{aligned} A-2A+2A &= -2 \\ 0 &= -2 \end{aligned}$$

$$47. \int \frac{(3x^2+1)dx}{x^4-6x^3+13x^2-12x+4} = \int \frac{(3x^2+1)dx}{(x-1)^2(x-2)^2} = 14\ln|x-1| + \frac{14}{x-1} - 14\ln|x-2| -$$

$$\begin{array}{c|c|c|c|c|c} 1 & 1 & -6 & 13 & -12 & 4 \\ \hline & 1 & -5 & 8 & -4 & 0 \end{array} \quad - \frac{13}{x-2} + C$$

$$x^3 - 5x^2 + 8x - 4 = 0$$

$$\begin{array}{c|c|c|c|c} 1 & 1 & -5 & 8 & -4 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$$x^2 - 4x + 4 = 0 \quad x-1 \quad x^2 - 2x + 1 \quad (x-2)$$

$$3x^2+1 = \frac{Ax^3-4Ax^2+1Ax-Ax^2+4Ax-4A}{x^4-6x^3+13x^2-12x+4} + \frac{Bx^2-4Bx+4B}{x^4-6x^3+13x^2-12x+4} + \frac{Cx^3-2Cx^2+Cx-2Cx^2+4Cx-2C}{x^4-6x^3+13x^2-12x+4} + \frac{Dx^2-2Dx+D}{x^4-6x^3+13x^2-12x+4}$$

$$A+C=0 \rightarrow C=-A$$

$$-4A-A+B-2C-2C+D=9$$

$$-5A+B-4C+D=9$$

$$-A+B+D=9$$

$$8A-4B+5A-2D=0$$

$$-4A-4B+2A+D=1$$

$$4A+4A-4B+C+4C-2D=0$$

$$-4A+4B-2C+D=1$$

$$-A+B+D=9 \quad / -3 \quad / -(21)$$

$$3A-4B-2D=0$$

$$-2A+4B+D=1$$

$$-B+D=9$$

$$2B-D=-5$$

$$B=4$$

$$D=13$$

$$A=14 \quad C=-14$$

$$48. \int \frac{x^5 dx}{x^4-2x^3+2x-1} = \frac{1}{10} \ln|x+1| - \frac{39}{10} \ln|x-1| - \frac{28/10}{x-1} - \frac{18/10}{(x-1)^2} + C$$

$$x^4-1 = (x-1)(x+1)(x^2+1)$$

$$-2(x^2-1) = -2(x-1)$$

$$\begin{array}{c|c|c|c|c|c} 1 & 1 & -2 & 0 & 2 & -1 \\ \hline & 1 & -1 & -1 & 1 & 0 \end{array}$$

$$x^3 - x^2 - x + 1$$

$$\begin{array}{c|c|c|c|c} 1 & 1 & -1 & -1 & 1 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$$x^2 - 1$$

$$(x-1)^3(x+1)$$

$$x^2-2x+1 \quad x^2-1$$

$$A+B=4 \rightarrow B=4-A$$

$$-3A-2B+B+C=-2$$

$$-3A-B+C=-2$$

$$-3A-4+A+C=-2$$

$$-2A+C=2$$

$$C=2+2A$$

$$3A+B-2B+D=-3$$

$$3A-B+D=-3$$

$$3A-4+A+D=-3$$

$$D=1-4A$$

$$A=11/10$$

$$14$$

$$49. \int \frac{1-x^2}{x^4+2x^3} dx = \ln|x+2| - \frac{3}{8} \ln|x| + \frac{1/4}{x} - \frac{1/4}{x^2} + C$$

$$1-x^2 = \frac{A}{x^3(x+2)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{x+2}$$

$$A+B=0 \rightarrow B=-A$$

$$2B+C=-1 \quad C=-1-2B \rightarrow C=2A-1$$

$$2C+D=0 \quad D=-2C \rightarrow D=2-4A$$

$$2D=1 \quad \boxed{D=1/2} \quad \boxed{C=-1/4} \quad A=$$

$$B=-\frac{3}{8}$$

$$50. \int \frac{x^3+1}{x(x-1)^3} dx = -\ln|x| + 2\ln|x-1| - \frac{1}{x-1} - \frac{21}{2(x-1)^2} + C$$

$$x^3+1 = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$$

$$A+B=1 \quad -3A+2B+C=0 \quad 3A+B-C+D=0$$

$$-1+B=1 \quad -A=1 \quad A=-1 \quad -3+2-1+D=0$$

$$B=2 \quad 3-4+C=0 \quad C=1 \quad D=2$$

$$51. \int \frac{dx}{x(x^2+1)^2} = \ln|x| + \int \frac{-x}{x^2+1} dx + \int \frac{-x}{(x^2+1)^2} dx =$$

$$1 = \frac{A}{x} + \frac{B}{x^2+1} + \frac{C}{(x^2+1)^2}$$

$$A=0 \quad A+B=0 \rightarrow B=-A \quad B=-1$$

$$C=0$$

$$A=1$$

$$2A+B+D=0$$

$$2-1+D=0$$

$$C=0 \quad C+E=0$$

$$D=-1$$

$$= \ln|x| - \frac{1}{2} \ln(x^2+1) + \frac{1}{x^2+1} + C$$

$$52. \int \frac{x^3+3}{(x+1)(x^2+1)^2} dx = \frac{1}{2} \ln|x+1| + \int \frac{-1/2x+3/2}{x^2+1} dx + \int \frac{-2x+1}{(x^2+1)^2} dx$$

$$x^3+3 = \frac{A}{x+1} + \frac{B}{x^2+1} + \frac{C}{(x^2+1)^2}$$

$$A+B=0 \quad B=-A$$

$$B+C=0 \quad C=1-B$$

$$C=1+A$$

$$2A+B+C+D=0$$

$$2A-A+1+A+D=0$$

$$D=-2A-1$$

$$B+C+D+E=0$$

$$-A+1+A-2A-1+E=0$$

$$E=2A$$

$$A+C+E=3$$

$$A+1+A+2A=3$$

$$4A=2$$

$$A=1/2$$

$$53. \int \frac{x^3+x^2-4x+1}{(x^2+1)^2} dx = \int \frac{x+1}{x^2+1} dx + \int \frac{-3x+0}{(x^2+1)^2} dx$$

$$x^3+x^2-4x+1 = \frac{A}{x} + \frac{B}{x^2+1} + \frac{C}{(x^2+1)^2}$$

$$A=1 \quad B=1 \quad B+D=1$$

$$A+C=-4$$

$$C=-5$$

$$54. \int \frac{3x^2-2x+1}{(x-2)(x^3-8)} dx = \int \frac{3x^2-2x+1}{(x-2)^2(x^2+2x+4)} dx =$$

$$3x^2-2x+1 = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x^2+2x+4}$$

$$A+C=0$$

$$C=-A$$

$$B-4C+D=3$$

$$-8A+4B+4D=1$$

$$2A-2A$$

$$4A$$

$$2B+4C-4D=-2$$

$$B+4A+D=3$$

$$-8A+4B+4D=1$$

$$2B-4A-4D=-2$$

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$$55. \int \frac{x^2 dx}{(x+1)(x^2+1)} = \int \frac{x^2 dx}{(x+1)^2(x^2-x+1)} = -\frac{1}{3} \ln|x+1| + \frac{1}{3} \int \frac{x+1}{(x+1)^2} dx + \int \frac{1}{x^2-x+1} dx$$

$$x^2 = \underline{Ax^3 + Ax^2 - Ax - A} + \underline{Bx^2 - Bx + B} + \underline{Cx^3 + 2Cx^2 + Cx + Dx^2 + 2Dx + D}$$

$$A + C = 0 \rightarrow C = -A$$

$$C = 1/3$$

$$B + 2C + D = 1 \quad B - 2A + D = 1 \quad -3A - 2D = 1$$

$$-B + C + 2D = 0 \quad A + B + D = 0$$

$$A = -1/3$$

$$A + B + D = 0 \quad -B - A + 2D = 0$$

$$3D = 0 \quad \boxed{D = 0} \quad B = 1/3$$

$$56. \int \frac{dx}{x^4 - x^3 - x + 1} = -\frac{1}{3} \ln|x-1| - \frac{1/3}{x-1} + \frac{1}{3} \int \frac{x+1}{x^2+x+1} dx$$

$$\begin{array}{c|c|c|c|c|c} 1 & 1 & -1 & 0 & -1 & 1 \\ \hline & 1 & 0 & 0 & -1 & 0 \end{array}$$

$$\frac{x^3 - 1}{(x-1)^2(x^2+x+1)} \quad x^2 - 2x + 1$$

$$1 = \underline{Ax^3 + Ax^2 + Ax - A} + \underline{Bx^2 + Bx + B} + \underline{Cx^3 - 2Cx^2 + Cx} + \underline{Dx^2 - 2Dx + D}$$

$$A + C = 0 \quad C = -A$$

$$B = 1/3$$

$$C = 1/3$$

$$B - 2C + D = 0$$

$$B + 2A + D = 0$$

$$A = -1/3$$

$$B + C - 2D = 0$$

$$B - A - 2D = 0$$

$$3A = -1$$

$$-A + B + D = 1$$

$$-3D = -1$$

$$\boxed{D = 1/3}$$

$$57. \int \frac{3x^2 - 2x + 1}{(x^2 + x + 1)^2} dx = \int \frac{3}{x^2 + x + 1} dx + \int \frac{-5x - 2}{(x^2 + x + 1)^2} dx$$

$$3x^2 - 2x + 1 = \underline{Ax^3 + Ax^2 + Ax + Bx^2 + Bx + B} + \underline{Cx + D}$$

$$A = 0$$

$$A + B = 3$$

$$B = 3$$

$$A + B + C = 2$$

$$0 + 3 + C = 2$$

$$C = -5$$

$$B + D = 1$$

$$3 + D = 1$$

$$D = -2$$

$$58. \int \frac{3x^2 + x + 3}{(x-1)^3(x^2+1)} dx = -\frac{1}{4} \ln|x-1| - \frac{1}{x-1} - \frac{7}{4(x-1)^2} + \int \frac{1/4 x}{x^2+1} dx$$

$$3x^2 + x + 3 = \underline{Ax^4 + Ax^3 - 2Ax^2 - 2Ax + A} + \underline{Bx^3 - Bx^2 + Bx - B} + \underline{Cx^2 + Cx + C} + \underline{Dx^2 + Dx + D} + \underline{Ex^2 + Ex + E}$$

$$A + D = 0 \quad -2A + B - 3D + E = 0$$

$$A + A - B + C + 3D - 3E = 3$$

$$-2A + B - D + 3E = 1$$

$$A - B + C - E = 3$$

$$D = -A$$

$$A + B + E = 0$$

$$-A - B + C - 3E = 3$$

$$-A - B + 3E = 1$$

$$A - B + C - E = 3$$

$$2B + 4E = 1$$

$$C + 12E = 4$$

$$E = B$$

$$E = -A - B$$

$$2A + 2B + C - 2A - 2B = 3$$

$$2A + 2B + C = 3$$

$$-4A - 2B = 1$$

$$2D + C = 3$$

$$C = 3 - 2A$$

$$-A - B + C + 3A + 3B = 3$$

$$-A - B - 3A - 3B = 1$$

$$A - B + C + A - B = 3$$

$$\int \frac{3x^2+2x+1}{x(x+1)^2(x^2+1)} dx = \frac{1}{4} \ln|x| - \ln|x+1| - \frac{1}{x+1} + \int \frac{1}{x^2+1} dx$$

$$3x^2+2x+1 = \frac{A}{x} + \frac{4A}{x+1} + \frac{2A}{x^2+1} + \frac{8A}{x^2+1} + \frac{B}{x^2+1} + \frac{4B}{x^2+1} + \frac{B}{x^2+1} + \frac{4B}{x^2+1} + \frac{C}{x^2+1} + \frac{4C}{x^2+1} + \frac{E}{x^2+1}$$

$$A+B+D=0$$

$$2A+B+C+2D+E=0$$

$$4A+A+4B+D+2E=3$$

$$8A+4B+4C+E=2$$

$$4A=1 \quad A=1/4$$

$$B+D=1/4 \quad B=-1/4-D$$

$$B+C+2D+E=-1/2$$

$$4B+D+2E=7/4$$

$$4B+4C+E=0$$

$$-8D-3E=2 \quad /: (-3, 2, 1)$$

$$-3D+2E=11/4$$

$$\frac{9}{8} + \frac{16}{8} E = \frac{4}{11}$$

$$60. \int \frac{x^3+9x^2+16x+6}{(x+3)(x^2+1)^2} dx = \ln|x+3| + \int \frac{1}{x^2+1} dx + \int \frac{1}{(x^2+1)^2} dx$$

$$x^3+9x^2+16x+6 = \frac{A}{x+3} + \frac{2A}{x^2+1} + \frac{A}{x^2+1} + \frac{B}{x^2+1} + \frac{3B}{x^2+1} + \frac{B}{x^2+1} + \frac{3B}{x^2+1} + \frac{C}{x^2+1} + \frac{3C}{x^2+1} + \frac{C}{x^2+1} + \frac{3C}{x^2+1} + \frac{E}{x^2+1}$$

$$A+B=0 \quad B=-A$$

$$3B+C=1 \quad C=1+3A$$

$$2A+B+3C+D=9 \quad 2A-A+3+9A+D=9 \quad D=-10A+6$$

$$3B+C+3D+E=16 \quad -3A+1+3A-30A+18+E=16 \quad E=30A-3$$

$$A+3C+3E=6$$

$$A+3+9A+90A-9=6$$

$$900E=12 \quad E=1/75$$

$$61. \int \frac{4x^2-8x}{(x-1)^2(x^2+1)^2} dx = \ln|x-1| - \frac{1}{x-1} + \int \frac{1}{x^2+1} dx + \int \frac{1}{(x^2+1)^2} dx$$

$$4x^2-8x = \frac{A}{x-1} + \frac{A}{x-1} + \frac{2A}{x^2+1} + \frac{2A}{x^2+1} + \frac{B}{x^2+1} + \frac{2B}{x^2+1} + \frac{B}{x^2+1} + \frac{2B}{x^2+1} + \frac{C}{x^2+1} + \frac{2C}{x^2+1} + \frac{C}{x^2+1} + \frac{2C}{x^2+1} + \frac{E}{x^2+1}$$

$$A+C=0$$

$$-A+B-2C+D=0$$

$$62. \int \frac{dx}{x^6 + 2x^4 + x^2} = \ln|x| - \frac{1}{x} + \int \frac{1}{x^2+1} dx + \int \frac{1}{(x^2+1)^2} dx$$

$$\frac{x^2(x^4+2x^2+1)}{x^2(x^2+1)^2} = \frac{x^4+2x^2+1}{x^2(x^2+1)^2}$$

$$1 = \frac{Ax^5+2Ax^3+Ax}{x^2(x^2+1)^2} + \frac{Bx^4+2Bx^2+B}{x^2(x^2+1)^2} + \frac{Cx^5+Cx^3+Dx^4+Dx^2+Ex^2+Fx^2}{x^2(x^2+1)^2}$$

$$A+C=0 \rightarrow C=-A$$

$$\checkmark B+D=0 \rightarrow D=-B$$

$$2A+C+E=0 \rightarrow E=-A$$

$$\checkmark 2B+D+F=0 \rightarrow F=-B$$

$$\underline{B=1} \quad \underline{F=-1} \quad \underline{D=-1}$$

$$63. \int \frac{x^4-4}{(x^2+1)(x^3+1)} dx = -\frac{1}{2} \ln|x+1| + \int \frac{-2x^2-2x-2}{x^2+1} dx + \int \frac{3x-2}{x^2+1} dx$$

$$x^4-4 = \frac{Ax^4+Ax^2-Ax^3-Ax+Ax^2+A}{x^3+x^2+x+1} + \frac{Bx^4+Bx^2+B}{x^3+x^2+x+1} + \frac{Cx^3+C}{x^3+x^2+x+1} + \frac{Dx^4+Dx^3+Dx^2+Dx}{x^3+x^2+x+1} + \frac{Ex^3+Ex^2+Ex+E}{x^3+x^2+x+1}$$

$$A+B+D=1 \quad -A+C+D+E=0$$

$$A+A+D+E=0 \quad -A+B+D+E=0$$

$$A+C+E=-4$$

$$\underline{A+B+D=1}$$

$$-A+C+D+E=0$$

$$2A+D+E=0$$

$$-A+B+D+E=0$$

$$\underline{A+C+E=-4}$$

$$C+B+2D+E=1$$

$$-2B-D+E=-2$$

$$2B+2D+E=1$$

$$-B+C-D+E=-5$$

$$-3/2 + 6 - 2 - 1 = -B$$

$$\underline{\underline{B = -3/2}}$$

$$2C+3D+3E=0$$

$$\underline{-2C-2D-E=-1}$$

$$2C+D+2E=-4$$

$$-4C-3D=-3$$

$$-2C-3D=-6$$

$$2C=-3$$

$$\underline{C = -3/2}$$

$$\underline{\underline{E = -2}}$$

$$\underline{\underline{D = 3}}$$

$$A = 1/2$$

$$64. \int \frac{dx}{(x+1)(x^2+1)(x^3+1)} = \int \frac{dx}{(x+1)^2(x^2+1)(x^2-x+1)} = \frac{\ln|x+1|}{x+1} + \int \frac{dx}{x^2+1} + \int \frac{dx}{x^2-x+1} + C$$

$$1 = \frac{Ax^5 + Ax^3 + Ax^2 + A}{x^4 + x^3 + x^2 + x + 1} + \frac{Bx^4 + 2Bx^2 - Bx^3 - Bx + B}{x^4 + 2x^3 + x^2 + x^2 + 2x + 1} + \frac{Cx^5 + Cx^4 + Cx^2 + Cx}{x^4 + 2x^3 + 2x^2 + 2x + 1} + \frac{Dx^4 + Dx^3 + Dx + D}{x^4 + 2x^3 + 2x^2 + 2x + 1}$$

$$A + C + E = 0 \quad B + C + D + 2E + F = 0$$

$$A - B + D + 2E + 2F = 0 \quad -B + C + D + E + 2F = 0$$

$$A + 2B + C + 2E + 2F = 0 \quad A + B + D + F = 1$$

$$65. \int \frac{dx}{x^4+1} = \int \frac{1}{x^4+1} dx =$$

$$1 = Ax + B \quad (x^2)^2$$

$$66. \int \frac{3-x}{x^4+1} dx$$

$$3-x = Ax + B$$

$$A = -1$$

$$67. \int \frac{dx}{(x^3+1)^2} = \frac{\ln|x+1|}{x+1} + \int \frac{dx}{x^2-x+1} + \int \frac{dx}{(x^2+1)^2}$$

$$1 =$$

$$68. \int \frac{x^3}{(x^2+1)^3} dx = \int \frac{x}{(x^2+1)^2} dx + \int \frac{x}{(x^2+1)^3} dx =$$

$$x^3 = Ax^5 + 2Ax^3 + Ax + Bx^4 + 2Bx^2 + B + Cx^3 - Cx + Dx^2 + D + Ex + F$$

$$A = 0 \quad B = 0 \quad 2A + C = 1 \quad A + F = 0$$

$$2B + D = 0$$

$$C = 1$$

$$B + F = 0$$

$$x^2+1 = a$$

$$2x dx = da$$

$$x dx = da/2$$

$$\frac{1}{2} \int \frac{da}{a^2} = -\frac{1}{2a} = -\frac{1}{2(x^2+1)}$$

$$69. \int \frac{dx}{x^4(x^3+1)^2} = \int \frac{1}{x^4} dx + \int \frac{1}{x^2} dx + \int \frac{1}{x^3} dx + \int \frac{1}{x^4} dx + \int \frac{1}{x+1} dx + \int \frac{1}{(x+1)^2} dx + \int \frac{1}{x^2-x+1} dx + \int \frac{1}{(x^2-x+1)^2} dx$$

$$1 = Ax^5 + 2Ax^4 + Ax^3 + Bx^2 - 2Bx^5 - Bx^2 + Cx^2 + 2Cx^4 + Cx + Dx^6 + 2Dx^3 + D + \dots$$

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$$70. \int \frac{x^4+x^3+4x^2+3x+4}{(x^2+2)^5} dx = \int \frac{1}{x^2+2} dx + \int \frac{x}{(x^2+2)^2} dx + \int \frac{x}{(x^2+2)^3} dx$$

$$\frac{x^4+x^3+4x^2+3x+4}{(x^2+2)^5} = \frac{Ax^5+4Ax^3+4A}{x^4+4x^2+4} + \frac{Bx^4+4Bx^2+4B}{x^4+4x^2+4} + \frac{Cx^3+2Cx+Dx^2+2D+Ex+F}{(x^2+2)^3}$$

$$\begin{aligned} A=0 & \checkmark & 4A+C=1 & \checkmark & 4B+D=4 & \checkmark & D=0 & \checkmark & F=0 & \checkmark \\ B=1 & \checkmark & C=1 & \checkmark & 4+D=4 & \checkmark & & & & \checkmark \\ 4A+2C+E=3 & & E=1 & \checkmark & & & & & & \checkmark \\ 4B+2D+F=4 & & & & & & & & & \checkmark \\ 4+F=4 & & & & & & & & & \checkmark \end{aligned}$$

$$= \frac{1}{2} \int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1} dx +$$

$$+ \frac{1}{2} \int \frac{da}{a^2} + \frac{1}{2} \int \frac{da}{a^3} = \frac{1}{\sqrt{2}} \arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{2} \frac{1}{x^2+2} - \frac{1}{4} \frac{1}{(x^2+2)^2} + C$$