III) Израчунати интеграле:

1. $\int (e^{2x}+e^{-x})\,dx = \int e^{2x}dx + \int e^{-x}dx = \frac{e^{2x}}{2} - e^{-x} + C$

$2x = a \qquad -x = b$
$2dx = da \qquad -dx = db$
$dx = \frac{da}{2} \qquad dx = -db$

2. $\int (e^x-1)^3 dx = \int e^{3x}dx - \int 3e^{2x}dx + \int 3e^x dx - \int dx = \frac{e^{3x}}{3} - \frac{3}{2}e^{2x} + 3e^x - x + C$

3. $\int 4x^2 e^{-x^3}dx = -\frac{4}{3}\int e^t dt = -\frac{4}{3}e^{-x^3} + c$

$-x^3 = t$
$-3x^2dx = dt$
$x^2dx = -dt/3$

4. $\int x e^{-x^2}dx = -\frac{1}{2}\int e^t dt = -\frac{1}{2}e^{-x^2} + c$

$-x^2 = t$
$-2xdx = dt$
$xdx = -dt/2$

5. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int 2e^t dt = 2e^t + c = 2e^{\sqrt{x}} + c$

$\sqrt{x} = t$
$\frac{dx}{2\sqrt{x}} = dt$
$\frac{dx}{\sqrt{x}} = 2dt$

6. $\int \frac{e^x}{e^{2x}+1}dx = \int \frac{dt}{t^2+1} = \operatorname{arctg} t + C = \operatorname{arctg} e^x + C$

$e^x = t$
$e^x dx = dt$

7. $\int \frac{e^x}{e^x+3}dx = \int \frac{dt}{t} = \ln|t| + c = \ln|e^x+3| + c$

$e^x + 3 = t$
$e^x dx = dt$

8. $\int \frac{e^x}{\sqrt{e^{2x}+1}}dx = \int \frac{dt}{\sqrt{t^2+1}} = \ln|t+\sqrt{t^2+1}| + c = \ln|e^x + \sqrt{e^{2x}+1}| + c$

$e^x = t$
$e^x dx = dt$

9. $\int \frac{dx}{1+e^{3x}} = \int \frac{dx}{(1+e^x)\cdot(1-e^x+e^{2x})} = \int \frac{dt}{t(1+t)\cdot(1-t+t^2)} = \int \frac{dt}{t+t^4}$

$e^x = t$
$e^x dx = dt$
$dx = dt/e^x$
$dx = dt/t$

$$\frac{1}{\sqrt2}\int\frac{\sqrt{17}/2t-11/2}{\sqrt{t^2-1}}\,dt=\frac{\sqrt{17}}{2\sqrt2}\sqrt{\left(\frac{x-3/4}{\sqrt{17}/4}\right)^2-1}+\frac{11}{2\sqrt2}\ln\left|t+\sqrt{t^2-1}\right|+C$$

$t^2-1=a$
$2t\,dt=da$
$t\,dt=da/2$

102. $$\int\frac{x\,dx}{\sqrt{1-3x-2x^2}}=\frac{1}{\sqrt2}\int\frac{x\,dx}{\sqrt{\frac12-\left(\frac32x+x^2\right)}}=\frac{1}{\sqrt2}\int\frac{x\,dx}{\sqrt{\frac{17}{16}-\left(\frac34+x\right)^2}}=$$

$$=\frac{4}{\sqrt2\cdot\sqrt{17}}\int\frac{x\,dx}{\sqrt{1-\left(\frac{3/4+x}{\sqrt{17}/4}\right)^2}}=\frac{1}{4\sqrt2}\int\frac{\sqrt{17}t+3}{\sqrt{1-t^2}}\,dt=\frac{\sqrt{17}}{4\sqrt2}\int\frac{t}{\sqrt{1-t^2}}\,dt+\frac{3}{4\sqrt2}\cdot$$

$$\frac{3/4+x}{\sqrt{17}/4}=t\qquad \frac34+x=\frac{\sqrt{17}}{4}t\qquad \cdot\int\frac{dt}{\sqrt{1-t^2}}=-\frac{\sqrt{17}}{4\sqrt2}\cdot\sqrt{1-t^2}+$$

$$\frac{dx}{\sqrt{17}/4}=dt\qquad x=\frac{\sqrt{17}}{4}t+\frac34\qquad \frac{3}{4\sqrt2}\cdot\arcsin t+C$$

$$dx=\frac{\sqrt{17}}{4}dt\qquad -2t\,dt=da$$

$$t\,dt=-\frac{da}{2}$$

2

10. $\int \frac{dx}{\sqrt{e^x+e^x}} = \int \frac{dt}{2t(t+t^2)} = \int \frac{dt}{2t^2+2t^3} = \frac{1}{2}\int \frac{dt}{t^2+t^3}$

$\sqrt{e^x}=t$

$\frac{1}{2\sqrt{e^x}} \cdot e^x\,dx = dt$

$\frac{\sqrt{e^x}}{2}\,dx = dt$

$dx = \frac{dt}{2\sqrt{e^x}}$

$dx = \frac{dt}{2t}$

$\frac{1}{t^2(t+1)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t+1} \,/\cdot(t^2(t+1))$

$1 = At^2 + At + Bt + B + Ct^2$

$A+C=0 \quad A+B=0 \quad B=1$

$A=-C \quad A=-B$

$C=-A \quad A=-1 \quad t^{-2} \quad t^{+1}$

$C=1$

$= \frac{1}{2}\left[-\int \frac{dt}{t} + \int \frac{dt}{t^2} + \int \frac{dt}{t+1}\right] =$

$= -\frac{1}{2}\ln\sqrt{e^x} - \frac{1}{2}\frac{1}{\sqrt{e^x}} + \frac{1}{2}\ln|\sqrt{e^x}+1| + C$

11. $\int \frac{dx}{\sqrt{e^x-1}} = \int \frac{2t\,dt}{t(t^2+1)} = 2\int \frac{dt}{1+t^2} = 2\,\mathrm{arctg}\,t + C = 2\,\mathrm{arctg}\sqrt{e^x-1} + C$

$\sqrt{e^x-1}=t \quad e^x-1=t^2 \quad e^x=t^2+1$

$\frac{1}{2\sqrt{e^x-1}} \cdot e^x\,dx = dt$

$dx = \frac{2\sqrt{e^x-1}}{e^x}\,dt$

$dx = \frac{2t}{t^2+1}\,dt$

12. $\int \frac{e^{2x}}{\sqrt{e^x-1}}\,dx = \int \frac{2t\cdot(t^2+1)}{(t^2+1)\cdot t}\,dt = 2\int (t^2+1)\,dt = \frac{2t^3}{3} + 2t + C = \frac{2(\sqrt{e^x-1})^3}{3} + 2\sqrt{e^x-1} + C$

иста смена као у
11. задатку

13. $\int \frac{2^x}{\sqrt{1-4^x}}\,dx = \int \frac{2^x}{\sqrt{1-(2^x)^2}}\,dx = \left[\int \frac{dt}{\sqrt{1-t^2}}\right]\ln 2 = \ln 2 \arcsin t + C =$

$2^x = t$

$2^x \ln 2\,dx = dt$

$2^x\,dx = \frac{dt}{\ln 2}$

$= \ln 2 \cdot \arcsin 2^x + C$

14. $\int \frac{5^x}{\sqrt{25^x+1}}\,dx = \int \frac{5^x}{\sqrt{1+(5^x)^2}}\,dx = \frac{1}{\ln 5}\int \frac{dt}{\sqrt{1+t^2}} = \frac{1}{\ln 5}\ln|5^x + \sqrt{25^x+1}| + C$

$5^x = t$

$5^x \ln 5\,dx = dt$

$5^x\,dx = dt/\ln 5$

15. $\int \frac{dx}{\sqrt{2^x-1}} = 2\int \frac{dt}{1+t^2} = 2\,\mathrm{arctg}\,t + C = 2\,\mathrm{arctg}\sqrt{2^x-1} + C$

$\sqrt{2^x-1} = t \quad 2^x = t^2+1$

$\frac{1\cdot 2^x}{2\sqrt{2^x-1}}\,dx = dt$

$\frac{t^2+1}{2t}\,dx = dt$

$dx = \frac{2t}{t^2+1}\,dt$

16. $\int \frac{e^x}{\sqrt{4-e^{2x}}}dx = \frac{1}{2}\int \frac{e^x}{\sqrt{1-\left(\frac{e^x}{2}\right)^2}}dx = \int \frac{dt}{\sqrt{1-t^2}} = \arcsin t + C = \arcsin \frac{e^x}{2} + C$

$\frac{e^x}{2} = t$

$\frac{e^x dx}{2} = dt$

$e^x dx = 2dt$

17. $\int \frac{e^{2x}}{\sqrt{e^{4x}+1}}dx = \int \frac{e^{2x}}{\sqrt{(e^{2x})^2+1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t^2+1}} = \frac{1}{2}\ln\left|e^{2x}+\sqrt{e^{4x}+1}\right|+C$

$e^{2x}=t$

$2e^{2x}dx=dt$

$e^{2x}dx = dt/2$

18. $\int \frac{e^x(e^x+1)}{\sqrt{1-2e^x-e^{2x}}}dx = \int \frac{e^x(e^x+1)}{\sqrt{2-(e^x+1)^2}}dx = \int \frac{t\,dt}{\sqrt{2-t^2}} = -\frac{1}{2}\int a^{-1/2}da =$

$e^x+1=t$ $\quad$ $2-t^2=a$

$e^x dx = dt$ $\quad$ $-2t\,dt = da$ $\quad$ $t\,dt = -da/2$

$= -a^{1/2}+C = -\sqrt{2-(e^x+1)^2}+C$

19. $\int (1+2e^{2x})^{-1}dx = \frac{1}{2}\int \frac{dt}{t^2-t} = \frac{1}{2}\int -\frac{dt}{t} + \frac{1}{2}\int \frac{dt}{t-1} =$

$1+2e^{2x}=t$ $\quad$ $2e^{2x}=t-1$

$4e^{2x}dx=dt$

$dx = dt/4e^{2x}$

$dx = dt/2(t-1)$

$\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \; / \cdot t(t-1)$

$1 = At - A + Bt$

$A+B=0 \quad -A=1$

$A=-B \quad A=-1$

$B=1$

$= -\frac{1}{2}\ln|1+2e^{2x}| + \frac{1}{2}\ln|2e^{2x}+2|+C$

20. $\int e^{2x}\sqrt{e^x+2}\,dx = \int \frac{2\sqrt{e^x+2}}{\cancel{e^x}}\cdot e^{\cancel{2x}}\, dt = \int 2\cdot t\cdot(t^2-2)\,dt =$

$\sqrt{e^x+2}=t$ $\quad$ $e^x+2=t^2$ $\quad$ $e^x=t^2-2$ $\quad$ $= 2\int t^3 dt - 4\int t\,dt = \frac{t^4}{2} - 2t^2+C =$

$\frac{e^x}{2\sqrt{e^x+2}}dx = dt$ $\quad$ $= \frac{(e^x+2)^2}{2} - 2(e^x+2)+C$

$dx = \frac{2\sqrt{e^x+2}}{e^x}dt$

21. $\int e^x\sqrt{e^x-1}\,dx = 2\int t^2 dt = 2\frac{t^3}{3}+C = 2\frac{(\sqrt{e^x-1})^3}{3}+C$

$\sqrt{e^x-1}=t$

$\frac{e^x}{2\sqrt{e^x-1}}dx = dt$

$e^x dx = 2\sqrt{e^x-1}\,dt$

$e^x dx = 2t\,dt$

22. $\int \frac{e^{2x}}{3+4e^{2x}}\,dx = \frac{1}{8}\int\frac{dt}{t} = \frac{1}{8}\ln|t|+C = \frac{1}{8}\ln(3+4e^{2x})+C$

$3+4e^{2x}=t$
$8e^{2x}dx=dt$
$e^{2x}dx=dt/8$

23. $\int\frac{dx}{\operatorname{sh}x} = \int\frac{2dx}{e^x-e^{-x}} = \int\frac{2e^x dx}{e^{2x}-1} = \int\frac{2dt}{t^2-1} = 2\int\frac{dt}{t^2-1} =$

$e^x=t$
$e^x dx = dt$

$\frac{2}{(t+1)(t-1)} = \frac{A}{t+1} + \frac{B}{t-1} \,/\cdot(t+1)\cdot(t-1)$

$2 = At - A + Bt + B$ $\qquad = -2\ln(e^x+1) + 2\ln|e^x-1| + C$
$A+B=0 \qquad -A+B=2$
$B=-A \qquad A+B=0$
$2B=2$
$B=1$
$A=-1$

24. $\int\frac{dx}{\operatorname{ch}x} = \int\frac{2e^x dx}{e^{2x}+1} = \int\frac{2dt}{t^2+1} = \int\frac{2\cdot 2}{t^2+1}dt = 4\operatorname{arctg}t+C =$

$e^x=t$
$e^x dx = dt$

$\frac{2}{t^2+1} = \frac{At+B}{t^2+1}\,/\cdot(t^2+1)$ $\qquad = 4\operatorname{arctg}e^x+C$

$2=At+B$
$A=0 \qquad B=2$

25. $\int\frac{\ln^2 x}{x}dx = \int t^2 dt = \frac{t^3}{3}+C = \frac{\ln^3 x}{3}+C$

$\ln x = t$
$\frac{dx}{x}=dt$

26. $\int\frac{\sqrt{\ln x}}{x}\,dx = \int t^{1/2}dt = \frac{2}{3}t^{3/2}+C = \frac{2}{3}(\ln x)^{3/2}+C$

$\ln x=t$
$\frac{dx}{x}=dt$

27. $\int\frac{1+\ln x}{x}\,dx = \int t\,dt = \frac{t^2}{2}+C = \frac{(1+\ln x)^2}{2}+C$

$1+\ln x = t$
$\frac{dx}{x}=dt$

28. $\int\frac{dx}{x\ln^2 x} = \int t^{-2}dt = -\frac{1}{t}+C = \frac{-1}{\ln x}+C$

$\ln x=t$
$\frac{dx}{x}=dt$

29. $\int \frac{dx}{x\sqrt[3]{\ln x}} = \int t^{-1/3}dt = \frac{3}{2}t^{2/3}+C = \frac{3}{2}(\ln x)^{2/3}+C$

$\ln x = t$

$\frac{dx}{x} = dt$

30. $\int \frac{dx}{x(2+\ln x)} = \int \frac{dt}{t} = \ln|t|+C = \ln|2+\ln x|+C$

$2+\ln x = t$

$\frac{dx}{x} = dt$

31. $\int \frac{dx}{x\ln x \ln\ln x} = \int \frac{dt}{t\ln t} = \int \frac{da}{a} = \ln|a|+C = \ln|\ln t|+C = t+C$

$\ln x = t \qquad \ln t = a$

$\frac{dx}{x} = dt \qquad \frac{dt}{t} = da$

$\ln|\ln|\ln x||+C$

32. $\int \frac{dx}{x\ln x(\ln(\ln x))^3} = \int \frac{dt}{t(\ln t)^3} = \int \frac{da}{a^3} = \int a^{-3}da = -\frac{1}{2}a^{-2}+C =$

$\ln x = t \qquad \ln t = a$

$\frac{dx}{x} = dt \qquad \frac{dt}{t} = da$

$= -\frac{1}{2\ln|\ln x|}+C$

33. $\int \frac{\ln\ln x}{x\ln x}dx = \int \frac{\ln t}{t}dt = \int a\,da = \frac{a^2}{2}+C = \frac{\ln^2|\ln x|}{2}+C$

$\ln x = t \qquad \ln t = a$

$\frac{dx}{x} = dt \qquad \frac{dt}{t} = da$

34. $\int \frac{\ln x}{x\sqrt{1+\ln x}}dx = \int \frac{t-1}{\sqrt{t}}dt = \int \sqrt{t}\,dt - \int t^{-1/2}dt =$

$1+\ln x = t$

$\frac{dx}{x} = dt$

$= \frac{2}{3}(1+\ln x)^{3/2} - \frac{2}{1}(1+\ln x)^{1/2}+C$

35. $\int \frac{\ln x}{x\sqrt{1+\ln^2 x}}dx = \frac{1}{2}\int t^{-1/2}dt = t^{1/2}+C = \sqrt{1+\ln^2 x}+C$

$1+\ln^2 x = t$

$\frac{2\ln x\,dx}{x} = dt$

$\frac{\ln x}{x}dx = \frac{dt}{2}$

36. $\int \frac{\sqrt[3]{1+\ln x}}{x}dx = \int t^{1/3}dt = \frac{3}{4}t^{4/3}+C = \frac{3}{4}(1+\ln x)^{4/3}+C$

$1+\ln x = t$

$\frac{dx}{x} = dt$

37. $\int \frac{\ln x-3}{x\sqrt{\ln x}}dx = \int \frac{t-3}{\sqrt{t}}dt = \int t^{1/2}dt - 3\int t^{-1/2}dt+C =$

$\ln x = t$

$\frac{dx}{x}\,dt$

$= \frac{2}{3}t^{3/2} - 6t^{1/2}+C = \frac{2}{3}(\ln x)^{3/2} - 6(\ln x)^{1/2}+C$

38. $\int \ln\frac{1+x}{1-x}\cdot\frac{dx}{x^2-1} = -\frac{1}{2}\int t\,dt = -\frac{t^2}{4}+C = -\frac{1}{4}\left(\ln\frac{1+x}{1+x}\right)^2+C$

$\ln\frac{1+x}{1-x} = t$

$\frac{1-x}{1+x}\cdot\frac{1-x+1+x}{(1-x)^2} = \frac{-2}{x^2-1}dx = dt$

$\frac{dx}{x^2-1} = -dt/2$

39. $\int\frac{\ln(\sqrt{x}+1)}{x+\sqrt{x}}dx = \int 2t\,dt = 2\frac{t^2}{2}+C = t^2+C = (\ln(\sqrt{x}+1))^2+C$

$\ln(\sqrt{x}+1) = t$

$\frac{1}{\sqrt{x}+1}\cdot\frac{1}{2\sqrt{x}}dx = dt$

$\frac{1}{x+\sqrt{x}}\cdot\frac{1}{2}dx = dt$

$\frac{dx}{x+\sqrt{x}} = 2dt$

40. $\int(\sin5x-\cos4x)dx = \int(\sin(x+4x)-\cos4x)dx = \int(\sin x\cos4x+\sin4x\cos x-\cos4x)dx =$

$= \int((\sin x-1)\cos4x+\sin4x\cos x)dx =$

$\cos4x = t$

$-4\sin4x\,dx = dt$

41. $\int\sin^2x\,dx = \int\left(\sqrt{\frac{1-\cos2x}{2}}\right)^2dx = \int\frac{1-\cos2x}{2}dx = \frac{1}{2}\int(1-\cos2x)dx = \frac{1}{2}\int dx-\frac{1}{2}\int\cos2x\,dx =$

$= \frac{1}{2}x-\frac{1}{4}\int\cos t\,dt = \frac{1}{2}x-\frac{1}{4}\sin t+C = \frac{1}{2}x-\frac{1}{4}\sin2x+C$

$2x = t$

$2dx = dt$

$dx = dt/2$

42. $\int\cos^2x\,dx = \int\left(\sqrt{\frac{1+\cos2x}{2}}\right)^2dx = \int\frac{1+\cos2x}{2}dx = \int\frac{1}{2}dx+\int\frac{1}{2}\cos2x\,dx =$

иста смена као у 42. задатку

$= \frac{x}{2}+\frac{1}{4}\sin2x+C$

43. $\int\frac{\sin x}{1+\cos x}dx = -\int\frac{dt}{t} = -\ln|1+\cos x|+C$

$1+\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

44. $\int\frac{\sin2x}{\sin^2x+3}dx = \int\frac{2\sin x\cos x}{\sin^2x+3}dx = \int\frac{dt}{t} = \ln(\sin^2x+3)+C$

$\sin^2x+3 = t$

$2\sin x\cos x\,dx = dt$

45. $\int \frac{1}{x^2}\cos\frac{1}{x}\,dx = -\int \cos t\,dt = -\sin\frac{1}{x} + C$

$\frac{1}{x} = t$

$\frac{-1}{x^2}\,dx = dt$

$\frac{dx}{x^2} = -dt$

46. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int 2\sin t\,dt = -2\cos\sqrt{x} + C$

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}\,dx = dt$

$\frac{dx}{\sqrt{x}} = 2dt$

47. $\int \frac{\cos x}{\sqrt{1-3\sin x}}\,dx = \frac{1}{3}\int \frac{dt}{\sqrt{t}} = \frac{2}{3}\sqrt{1+3\sin x} + C$

$1+3\sin x = t$

$3\cos x\,dx = dt$

$\cos x\,dx = dt/3$

48. $\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx = \int \frac{\sin x \cdot \sin^2 x}{\sqrt{\cos x}}\,dx = \int \frac{\sin x(1-\cos^2 x)}{\sqrt{\cos x}}\,dx = -\int \frac{(1-t^2)}{\sqrt{t}}\,dt =$

$\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

$= -\int t^{-1/2}\,dt + \int t^{3/2}\,dt = -2\sqrt{\cos x} + \frac{2}{5}\sqrt{\cos^5 x} + C$

49. $\int \frac{\cos \ln x}{x}\,dx = \int \cos t\,dt = \sin(\ln x) + C$

$\ln x = t$

$\frac{dx}{x} = dt$

50. $\int \frac{\sin x}{\sqrt{\cos 2x}}\,dx = \int \frac{\sin x}{\sqrt{\cos^2 x - \sin^2 x}}\,dx = \int \frac{\sin x}{\sqrt{\cos^2 x - (1-\cos^2 x)}}\,dx =$

$= \int \frac{\sin x\,dx}{\sqrt{2\cos^2 x - 1}} = \int \frac{\sin x\,dx}{\sqrt{(\sqrt{2}\cos x)^2 - 1}} = -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{t^2-1}} =$

$\sqrt{2}\cos x = t$

$-\sqrt{2}\sin x\,dx = dt$

$\sin x\,dx = -\frac{dt}{\sqrt{2}}$

$= -\frac{1}{\sqrt{2}}\ln\left|\sqrt{2}\cos x + \sqrt{2\cos^2 x - 1}\right| + C$

51. $\int \frac{\cos x}{\sqrt{\cos 2x}}\,dx = \int \frac{\cos x}{\sqrt{\cos^2 x - \sin^2 x}}\,dx = \int \frac{\cos x\,dx}{\sqrt{(1-2\sin^2 x)}} = \int \frac{\cos x\,dx}{\sqrt{1-(\sqrt{2}\sin x)^2}}$

$\sqrt{2}\sin x = t$

$\sqrt{2}\cos x\,dx = dt$

$\cos x\,dx = \frac{dt}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{\sqrt{2}}\arcsin(\sqrt{2}\sin x) + C$

52. $\int tg x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{dt}{t} = -\ln|\cos x| + C$

$\cos x = t$
$-\sin x\,dx = dt$
$\sin x\,dx = -dt$

53. $\int ctg x\,dx = \int \frac{\cos x}{\sin x}\,dx = \int \frac{dt}{t} = \ln|\sin x| + C$

$\sin x = t$
$\cos x\,dx = dt$

54. $\int \sin^3 x\,dx = \int \sin x(1-\cos^2 x)\,dx = \int (t^2-1)\,dt = \int t^2\,dt - \int dt = \frac{t^3}{3} - t + C =$
$\cos x = t$
$-\sin x\,dx = dt$ $\quad = \frac{\cos^3 x}{3} - \cos x + C$
$\sin x\,dx = -dt$

55. $\int \cos^3 x\,dx = \int \cos x(1-\sin^2 x)\,dx = \int (1-t^2)\,dt = \int dt - \int t^2\,dt = \sin x - \frac{\sin^3 x}{3} + C$
$\sin x = t$
$\cos x\,dx = dt$

56. $\int \sin^4 x\,dx = \int \left(\sqrt{\frac{1-\cos 2x}{2}}\right)^4 dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{2}\int \left(\frac{1-\cos t}{2}\right)^2 dt =$
$2x = t$
$2dx = dt$ $\quad = \frac{1}{2}\int \frac{1-2\cos t+\cos^2 t}{4}\,dt = \frac{1}{8}\int dt - \frac{1}{4}\int \cos t\,dt + \frac{1}{8}\int \left(\sqrt{\frac{1-\cos 2t}{2}}\right)^2 dt =$
$dx = dt/2$

$= \frac{1}{8}t - \frac{1}{4}\sin t + \frac{1}{8}\int \frac{1-\cos 2t}{2}\,dt = \frac{1}{8}\cdot 2x - \frac{1}{4}\sin 2x + \frac{1}{16}\cdot 2x - \frac{1}{32}\sin 4x + C =$

$2t = a$
$2dt = da$ $\quad = -\frac{\sin 2x}{4} + \frac{3x}{8} - \frac{\sin 4x}{32} + C \qquad \left(\frac{1+\cos 4x}{2}\right)^2$
$dt = da/2$

57. $\int \cos^4 x\,dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\left[\int dx + 2\int \cos 2x\,dx + \int \cos^2 2x\,dx\right] =$
$\frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{16}x$

58. $\int \sin^3 x \cos^4 x\,dx = \int \sin x(1-\cos^2 x)\cos^4 x\,dx = -\int (1-t^2)t^4\,dt =$
$\cos x = t$
$-\sin x\,dx = dt$ $\quad = -\int t^4\,dt + \int t^6\,dt = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$
$\sin x\,dx = -dt$

59. $\int \sin^4 x \cos^3 x\,dx = \int \sin^4 x(1-\sin^2 x)\cos x\,dx = \int t^5(1-t^2)\,dt = \int t^5\,dt - \int t^7\,dt =$
$\sin x = t$
$\cos x\,dx = dt$ $\quad = \frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + C$

60. $\int \sin^3 x \cos^3 x\,dx = \int \sin x \cos x\,(1-\sin^2 x)\cdot(1-\cos^2 x)\,dx = \int \sin x\cos x\,dx\cdot(1-\sin^2 x-\cos^2 x+$
$+\sin^2 x\cos^2 x) =$

$$\int \frac{\ldots}{\cos^5 x}\,dx = \int \frac{\sin x(1-\cos^2 x)}{\cos^5 x}\,dx = -\int \frac{1-t^2}{t^5}\,dt = -\int t^{-5}dt + \int t^{-3}dt =$$

$\cos x = t$
$-\sin x\,dx = dt$
$\sin x\,dx = -dt$

$$= \frac{1}{4}\frac{1}{\cos^4 x} - \frac{1}{2}\frac{1}{\cos^2 x} + C$$

62. $$\int \frac{\cos^3 x}{\sin^6 x}\,dx = \int \frac{\cos x(1-\sin^2 x)}{\sin^6 x}\,dx = \int \frac{1-t^2}{t^6}\,dt = \int t^{-6}dt - \int t^{-4}dt =$$

$\sin x = t$
$\cos x\,dx = dt$

$$= -\frac{1}{5}(\sin x)^{-5} + \frac{1}{3}(\sin x)^{-3} + C$$

63. $$\int \frac{\sin x}{\sqrt{\cos^5 x}}\,dx = \int -\frac{dt}{t^{5/2}} = -\int t^{-5/2}dt = +\frac{2}{3}t^{-3/2} + C =$$

$\cos x = t$
$-\sin x\,dx = dt$
$\sin x\,dx = -dt$

$$= \frac{2}{3}(\cos x)^{-3/2} + C$$

64. $$\int \frac{\sin^5 x}{\cos^2 x\sqrt[4]{\cos^3 x}}\,dx = \int \frac{\sin x(1-\cos^2 x)^2}{\cos^2 x\sqrt[4]{\cos^3 x}}\,dx = \int -\frac{(1-t^2)^2}{t^2\cdot t^{3/4}}\cdot dt =$$

$\cos x = t$
$-\sin x\,dx = dt$
$\sin x\,dx = -dt$

$$= -\int \frac{1-2t^2+t^4}{t^{11/4}}\,dt = -\int t^{-11/4}dt + 2\int t^{-3/4}dt + \int t^{5/4}dt =$$

$$= \frac{4}{7}(\cos x)^{-7/4} + 8(\cos x)^{1/4} - \frac{4}{9}(\cos x)^{9/4} + C$$

65. $$\int \frac{\sqrt[3]{\operatorname{tg} x}}{\cos^2 x}\,dx = \int t^{1/3}dt = \frac{3}{4}(\operatorname{tg} x)^{4/3} + C$$

$\operatorname{tg} x = t$
$\frac{dx}{\cos^2 x} = dt$

66. $$\int \frac{\cos x}{\sqrt{e^{\sin x}-1}}\,dx = \int \frac{dt}{\sqrt{e^t-1}} = \int \frac{da}{(a+1)\cdot a^{1/2}} = \int \frac{da}{(a+1)\cdot\sqrt{a}}$$

$\sin x = t$
$\cos x\,dx = dt$

$e^t - 1 = a$
$e^t dt = da$
$dt = da/e^t$
$dt = da/(a+1)$

$$\frac{1}{(a+1)\cdot\sqrt{a}} = \frac{A}{a+1} + \frac{B}{\sqrt{a}} \quad /\cdot(a+1)\cdot\sqrt{a}$$

$$1 = \sqrt{a}A + Ba + B$$

$A = 0 \quad B = 0 \quad B = 1$

67. $$\int \frac{dx}{(1+x^2)\operatorname{arctg} x} = \int \frac{dt}{t} = \ln|\operatorname{arctg} x| + C$$

$\operatorname{arctg} x = t$
$\frac{dx}{1+x^2} = dt$

68. $$\int \operatorname{arctg}\frac{1}{x}\cdot\frac{dx}{1+x^2} = \int \operatorname{arctg} t\cdot\frac{-dt}{t^2\cdot(1+\frac{1}{t^2})} = -\int \operatorname{arctg} t\cdot\frac{dt}{t^2+1} =$$

$\frac{1}{x} = t \quad x = \frac{1}{t}$
$-\frac{1}{x^2}dx = dt \quad x^2 = \frac{1}{t^2}$
$dx = -x^2 dt \quad x^2+1 = 1+\frac{1}{t^2} \quad dx = -\frac{dt}{t^2}$

$\operatorname{arctg} t = a$
$\frac{1}{1+t^2}dt = da$

$$= -\int a\cdot da = -\frac{a^2}{2} + C = -\frac{(\operatorname{arctg}\frac{1}{x})^2}{2} + C$$

69. $\int \frac{e^x dx}{(e^{2x}+1)\operatorname{arctg} e^x} = \int \frac{dt}{(t^2+1)\operatorname{arctg} t} = \int \frac{da}{a} = \ln|\operatorname{arctg} e^x| + c$

$e^x = t \qquad \operatorname{arctg} t = a$

$e^x dx = dt \qquad \frac{dt}{t^2+1} = da$

70. $\int \frac{\sqrt{\operatorname{arctg} x^3}}{1+x^2} dx = \int a^{3/2} da = \frac{2}{5} a^{5/2} + c = \frac{2}{5} (\operatorname{arctg} x)^{5/2} + c$

$\operatorname{arctg} x = a$

$\frac{1}{1+x^2} dx = da$

71. $\int \frac{\operatorname{arctg}\sqrt{x}}{1+x} \cdot \frac{dx}{\sqrt{x}} = 2\int a\,da = 2 \cdot \frac{a^2}{2} + c = a^2 + c =$

$\operatorname{arctg}\sqrt{x} = a \qquad = (\operatorname{arctg}\sqrt{x})^2 + c$

$\frac{dx}{1+x} \cdot \frac{1}{2\sqrt{x}} = da$

$\frac{dx}{(1+x)\sqrt{x}} = 2da$

72. $\int \frac{dx}{\sqrt{1-x^2}\arccos x} = \int \frac{da}{a} = \ln|a| + c = \ln|\arcsin x| + c$

$\arccos x = a$

$\frac{dx}{\sqrt{1-x^2}} = da$

73. $\int \frac{dx}{\sqrt{1-x^2}\arcsin^2 x} = \int t^{-2} dt = -\frac{1}{t} + c = -\frac{1}{\arcsin x} + c$

$\arcsin x = t$

$\frac{dx}{\sqrt{1-x^2}} = dt$

74. $\int \sqrt{\frac{\arcsin x}{1-x^2}} dx = \int \frac{(\arcsin x)^{1/2}}{\sqrt{1-x^2}} dx = \int a^{1/2} da = \frac{2}{3} a^{3/2} + c =$

$\arcsin x = a$

$\frac{dx}{\sqrt{1-x^2}} = da \qquad = \frac{2}{3} (\arcsin x)^{3/2} + c$

75. $\int \frac{\arcsin x + x}{\sqrt{1-x^2}} dx = \int \frac{\arcsin x}{\sqrt{1-x^2}} dx + \int \frac{x dx}{\sqrt{1-x^2}} = \int a\,da - \frac{1}{2} \int b^{-1/2} db =$

$\arcsin x = a \qquad 1-x^2 = b$

$\frac{dx}{\sqrt{1-x^2}} = da \qquad -2x dx = db \qquad = \frac{a^2}{2} + 1 \cdot b^{1/2} + c = \frac{(\arcsin x)^2}{2} + \sqrt{1-x^2} + c$

$x dx = -db/2$

76. $\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}} dx = \int e^t dt = e^t + c = e^{\arcsin x} + c$

$\arcsin x = t$

$\frac{dx}{\sqrt{1-x^2}} = dt$

77. $$\int \frac{\arccos^2 2x}{\sqrt{1-4x^2}}\,dx = -\frac{1}{2}\int a^2\,da = -\frac{1}{2}\cdot\frac{a^3}{3}+C = -\frac{1}{6}\cdot(\arccos 2x)^3+C$$

$$\arccos 2x = a$$

$$\frac{-dx}{\sqrt{1-4x^2}}\cdot 2 = da$$

$$\frac{dx}{\sqrt{1-4x^2}} = -da/2$$

78. $$\int \frac{(\arcsin\sqrt{x})^3}{\sqrt{x-x^2}}\,dx = 2\int a^3\,da = 2\cdot\frac{a^4}{4}+C = \frac{a^4}{2}+C =$$

$$\arcsin\sqrt{x} = a$$

$$= \frac{1}{2}(\arcsin\sqrt{x})^4+C$$

$$\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}\,dx = da$$

$$\frac{dx}{\sqrt{x-x^2}} = 2da$$