

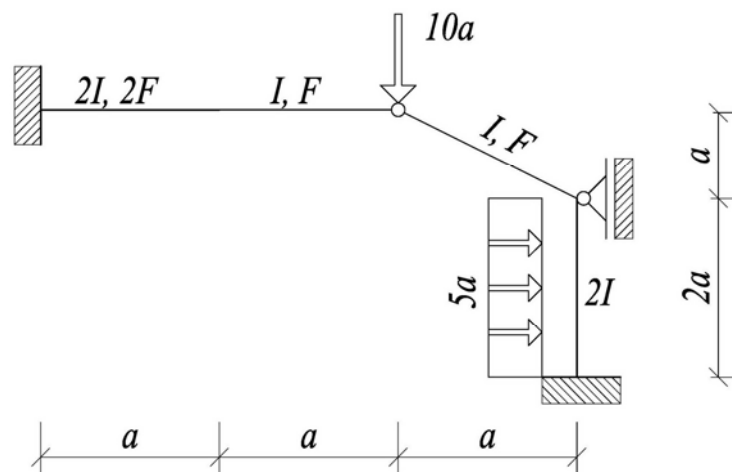
Ispitni rok: Septembar 2, 2008.

Za nosač prikazan na slici:

- Odrediti matricu krutosti uz nepoznata pomeranja K_{nn}^* i vektor slobodnih članova S_n^* .
- Za $a=2$ m sračunati sile na krajevima štapova.

Napomena: Zanemariti uticaj normalnih sila na deformaciju samo na vertikalnom štapu.

$$F/I=10/a^2$$

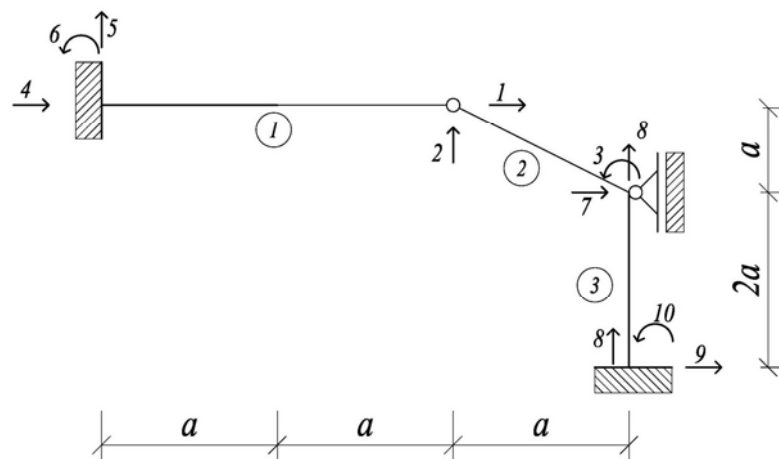


Rešenje:

- Odrediti matricu krutosti uz nepoznata pomeranja K_{nn}^* i vektor slobodnih članova S_n^* .

Usvojeno $I_c = I$.

Oznake štapova i pomeranja:

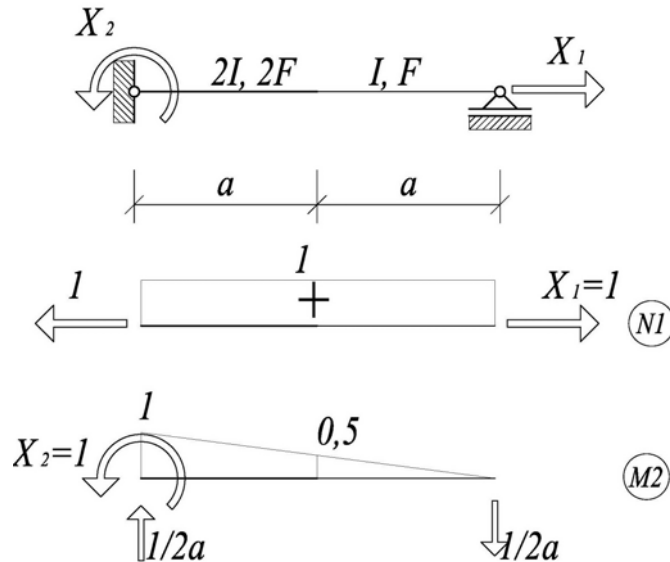


Nepoznata pomeranja: 1,2,3.

Matrice krutosti štapova:

Štap 1:

Štap nije konstantnog poprečnog preseka, pa se matrica krutosti mora odrediti preko bazne matrice krutosti štap.



$$EI_c \delta_{11} = \int N_1^2 \frac{I_c}{F} ds = \frac{I}{2F} (1 \cdot a \cdot 1) + \frac{I}{F} (1 \cdot a \cdot 1) = \frac{a^2}{20} \cdot a + \frac{a^2}{10} \cdot a = \frac{a^3}{20} + \frac{a^3}{10} = \frac{a^3 + 2a^3}{20} = \frac{3a^3}{20} = 0,15a^3$$

$$EI_c \delta_{22} = \int M_1^2 \frac{I_c}{I} ds = \frac{1}{2} \cdot \frac{2}{3} (1 + 1 \cdot 0,5 + 0,5^2) + \frac{a}{3} \cdot 0,5^2 = 0,2916a + 0,083a = 0,375a$$

Bazna matrica fleksibilnosti:

$$F_0 = \frac{1}{EI_c} \begin{bmatrix} 0,15a^3 & 0 \\ 0 & 0,375a \end{bmatrix}$$

Bazna matrica krutosti:

$$K_0 = F_0^{-1} = EI_c \begin{bmatrix} \frac{6,666}{a^3} & 0 \\ 0 & \frac{2,666}{a} \end{bmatrix}$$

$$C^T = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2a} \\ 0 & 1 \\ 1 & 0 \\ 0 & -\frac{1}{2a} \end{bmatrix} \quad C^T \cdot K_0 = EI_c \begin{bmatrix} -\frac{6,666}{a^3} & 0 \\ 0 & \frac{1,333}{a^2} \\ 0 & \frac{2,666}{a} \\ \frac{6,666}{a^3} & 0 \\ 0 & -\frac{1,333}{a^2} \end{bmatrix}$$

$$K_1 = K_1^* = C^T \cdot K_0 \cdot C = EI_c \begin{bmatrix} 6,666/a^3 & 0 & 0 & -6,666/a^3 & 0 \\ & 0,666/a^3 & 1,333/a^2 & 0 & -0,666/a^3 \\ & & 2,666/a & 0 & -1,333/a^2 \\ & & & 6,666/a^3 & 0 \\ & & & & 0,666/a^3 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \\ 1 \\ 2 \end{matrix}$$

Štap 2:

$$l = a\sqrt{2}$$

Geometrijske karakteristike: $\cos \alpha = -\frac{1}{\sqrt{2}}$

$$\sin \alpha = \frac{1}{\sqrt{2}}$$

$$K_2 = EI_c \begin{bmatrix} 7,0711/a^3 & 0 & 0 & -7,0711/a^3 & 0 \\ & 1,0607/a^3 & 1,5/a^2 & 0 & -1,0607/a^3 \\ & & 2,1213/a & 0 & -1,5/a^2 \\ & & & 7,0711/a^3 & 0 \\ & & & & 1,0607/a^3 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ & & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ & & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$T_2^T \cdot K_2 = EI_c \begin{bmatrix} \frac{7,0711}{a^3 \sqrt{2}} & -\frac{1,0607}{a^3 \sqrt{2}} & -\frac{1,5}{a^2 \sqrt{2}} & \frac{7,0711}{a^3 \sqrt{2}} & \frac{1,0607}{a^3 \sqrt{2}} \\ \frac{7,0711}{a^3 \sqrt{2}} & -\frac{1,0607}{a^3 \sqrt{2}} & -\frac{1,5}{a^2 \sqrt{2}} & -\frac{7,0711}{a^3 \sqrt{2}} & \frac{1,0607}{a^3 \sqrt{2}} \\ 0 & \frac{1,5}{a^2} & \frac{2,1213}{a} & 0 & -\frac{1,5}{a^2} \\ \frac{7,0711}{a^3 \sqrt{2}} & \frac{1,0607}{a^3 \sqrt{2}} & -\frac{1,5}{a^2 \sqrt{2}} & -\frac{7,0711}{a^3 \sqrt{2}} & -\frac{1,0607}{a^3 \sqrt{2}} \\ -\frac{7,0711}{a^3 \sqrt{2}} & \frac{1,0607}{a^3 \sqrt{2}} & -\frac{1,5}{a^2 \sqrt{2}} & \frac{7,0711}{a^3 \sqrt{2}} & -\frac{1,0607}{a^3 \sqrt{2}} \end{bmatrix}$$

$$K_2^* = T_2^T \cdot K_2 \cdot T_2 = EI_c \begin{matrix} & \begin{matrix} 7 & 8 & 3 & 1 & 2 \end{matrix} \\ \begin{matrix} 7 \\ 8 \\ 3 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} \frac{4,0659}{a^3} & -\frac{3,0052}{a^3} & -\frac{1,0607}{a^2} & -\frac{4,0659}{a^3} & \frac{3,0052}{a^3} \\ & \frac{4,0659}{a^3} & -\frac{1,5}{a^2 \sqrt{2}} & \frac{3,0052}{a^3} & -\frac{4,0659}{a^3} \\ & & \frac{2,1213}{a} & \frac{1,5}{a^2 \sqrt{2}} & \frac{1,5}{a^2 \sqrt{2}} \\ & & & \frac{4,0659}{a^3} & -\frac{3,0052}{a^3} \\ & & & & \frac{4,0659}{a^3} \end{bmatrix} \end{matrix}$$

Štap 3:

Geometrijske karakteristike: $l = 2a$

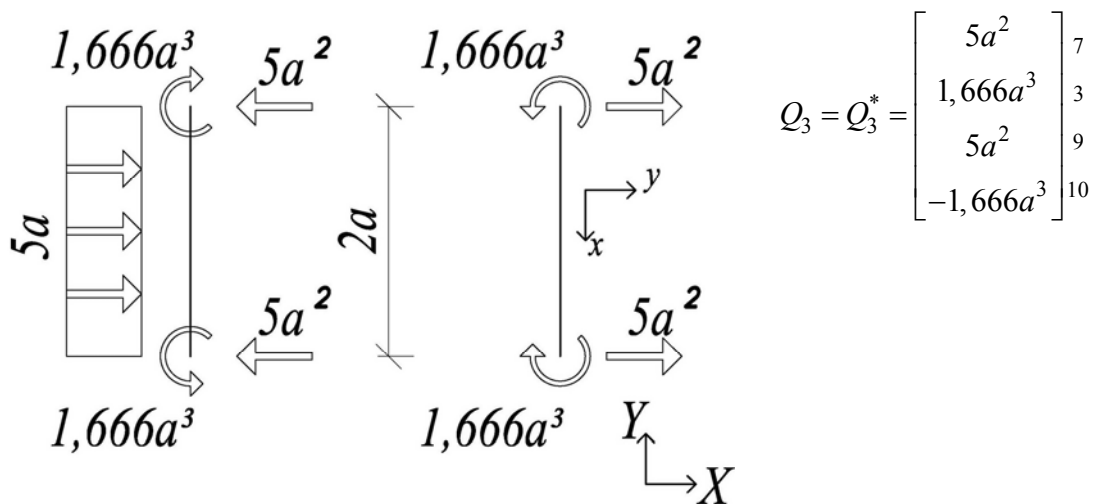
$$K_3 = K_3^* = EI_c \begin{bmatrix} 3/a^3 & 3/a^2 & -3/a^3 & 3/a^2 \\ & 4/a & -3/a^2 & 2/a \\ & & 3/a^3 & -3/a^2 \\ & & & 4/a \end{bmatrix} \begin{matrix} 7 \\ 3 \\ 9 \\ 10 \end{matrix}$$

Submatrica sistema uz nepoznata pomeranja:

$$K_{nn}^* = EI_c \begin{bmatrix} 10,7326/a^3 & -3,0052/a^3 & 1,0607/a^2 \\ & 4,7326/a^3 & 1,0607/a^2 \\ & & 6,1213/a \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Vektor ekvivalentnog opterećenja:

Štap 3:



$$Q_n^* = \begin{bmatrix} 0 \\ 0 \\ 1,666a^3 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \quad P_n^* = \begin{bmatrix} 0 \\ -10a \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$$\text{Vektor slobodnih članova: } S_n^* = P_n^* + Q_n^* = \begin{bmatrix} 0 \\ -10a \\ 1,666a^3 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

b) Za $a=2$ m sračunati sile na krajevima štapova.

Submatrica sistema uz nepoznata pomeranja:

$$K_{nn}^* = EI_c \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1,341575 & -0,37565 & 0,265175 \\ & 0,591575 & 0,265175 \\ & & 3,06065 \end{bmatrix} \end{matrix}$$

Vektor slobodnih članova:

$$S_n^* = \begin{bmatrix} 0 \\ -20 \\ 13,333 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Vektor nepoznatih pomeranja sistema:

$$q_n^* = K_{nn}^{*-1} \cdot S_n^* = \frac{1}{EI_c} \begin{bmatrix} -15,3842 \\ -47,9911 \\ 9,8472 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Sile na krajevima štapova:

$$q_1 = q_1^* = \frac{1}{EI_c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -15,3842 \\ -47,9911 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \\ 1 \\ 2 \end{matrix} \quad R_1 = K_1 \cdot q_1 = \begin{bmatrix} 12,82 \\ 4 \\ 16 \\ -12,82 \\ -4 \end{bmatrix}$$

$$q_2^* = \frac{1}{EI_c} \begin{bmatrix} 0 \\ 0 \\ 9,8472 \\ -15,3842 \\ -47,9911 \end{bmatrix} \begin{matrix} 7 \\ 8 \\ 3 \\ 1 \\ 2 \end{matrix} \quad q_2 = T_2 \cdot q_2^* = \frac{1}{EI_c} \begin{bmatrix} 0 \\ 0 \\ 9,8472 \\ -23,06 \\ 44,82 \end{bmatrix} \quad R_2 = K_2 \cdot q_2 = \begin{bmatrix} 20,382 \\ -2,25 \\ -6,314 \\ -20,382 \\ 2,25 \end{bmatrix}$$

$$q_3 = q_3^* = \frac{1}{EI_c} \begin{bmatrix} 0 \\ 9,8472 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 7 \\ 3 \\ 9 \\ 10 \end{matrix} \quad R_3 = K_3 \cdot q_3 - Q_3 = \begin{bmatrix} -12,615 \\ 6,361 \\ -27,385 \\ 23,181 \end{bmatrix}$$