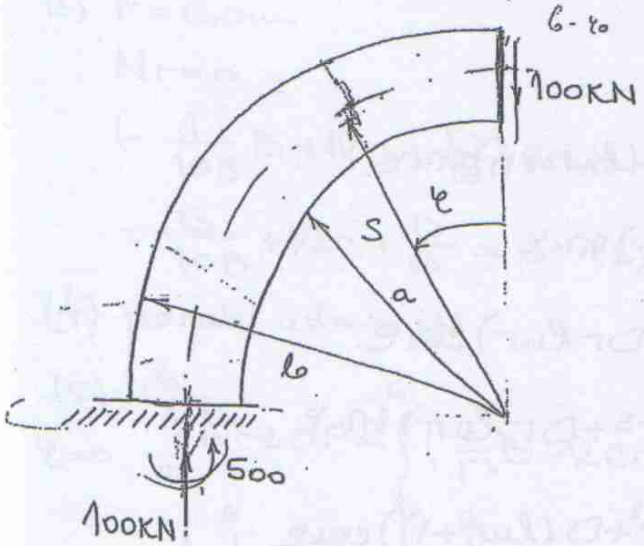
(13)

05.07.1997.

2. Ispitati da li naponska f-ja

$F = (A \cdot r + B\frac{1}{r} + Cr^3 + Dr\ln r)\sin\varphi$

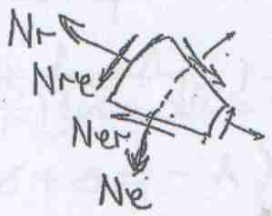
odgovara naprezanju kružnog lučnog nosača.



$a = 4{,}0\,m$

$b = 6{,}0\,m$

$P = 100\,KN$



Granični uslovi:

$r = a = 4{,}0\,m \begin{cases} N_r = 0 & (1) \\ N_{\varphi r} = 0 & (2) \end{cases}$

$r = b = 6{,}0\,m \begin{cases} N_r = 0 & (3) \\ N_{\varphi r} = 0 & (4) \end{cases}$

$\varphi = 0 \begin{cases} \int_a^b N_{r\varphi}\,dr = 100 & (5) \\ \int_a^b N_\varphi (r - r_0)\,dr = 0 & (6) \\ \int_a^b N_\varphi\,dr = 0 & (7) \end{cases}$

$\varphi = \bar{u}/2 \begin{cases} \int_a^b N_\varphi\,dr = -100 & (8) \\ \int_a^b N_\varphi (r - r_0)\,dr = 500 & (9) \\ \int_a^b N_{r\varphi}\,dr = 0 & (10) \end{cases}$

$r_0 = 5{,}10\,m$

$$F = (Ar + B\frac{1}{r} + Cr^3 + Dr\ln r)\sin\varphi$$

$$N_r = \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} + \frac{1}{r}\frac{\partial F}{\partial r}$$

$$N_\varphi = \frac{\partial^2 F}{\partial r^2}$$

$$N_{r\varphi} = \frac{1}{r^2}\frac{\partial F}{\partial \varphi} - \frac{1}{r}\frac{\partial^2 F}{\partial r \partial \varphi}$$

$$\frac{\partial F}{\partial r} = (A - B\frac{1}{r^2} + 3Cr^2 + D(\ln r + 1))\sin\varphi$$

$$\frac{\partial^2 F}{\partial r^2} = (+\frac{2}{r^3}B + 6Cr + D\frac{1}{r})\sin\varphi$$

$$\frac{\partial F}{\partial \varphi} = (Ar + B\frac{1}{r} + Cr^3 + Dr\ln r)\cos\varphi$$

$$\frac{\partial^2 F}{\partial \varphi^2} = -(Ar + B\frac{1}{r} + Cr^3 + Dr\ln r)\sin\varphi$$

$$\frac{\partial^2 F}{\partial r \partial \varphi} = (A - \frac{1}{r^2}B + 3Cr^2 + D(\ln r + 1))\cos\varphi$$

$$N_r = \frac{1}{r}\left(\frac{A}{r} + \frac{1}{r^3}B + Cr + \frac{\ln r}{r}D\right)\sin\varphi + \left(\frac{A}{r} - \frac{1}{r^3}B + 3Cr + D\left(\frac{\ln r}{r} + \frac{1}{r}\right)\right)\sin\varphi$$

$$\boxed{N_r = \left(-\frac{2}{r^3}B + 2Cr + \frac{D}{r}\right)\sin\varphi}$$

$$\boxed{N_\varphi = \left(+\frac{2}{r^3}B + 6C\cdot r + \frac{1}{r}\cdot D\right)\sin\varphi}$$

$$N_{r\varphi} = \left(\frac{A}{r} + \frac{B}{r^3} + Cr + \frac{\ln r}{r}D\right)\cos\varphi - \left(\frac{A}{r} - \frac{1}{r^3}B + 3Cr + D\left(\frac{\ln r}{r} + \right.\right.$$

$$\boxed{N_{r\varphi} = \left(\frac{2}{r^3}B - 2Cr - \frac{1}{r}D\right)\cos\varphi}$$

(1): $a = 4{,}10\,m$

$$N_r = 0$$

$$\left(-\frac{2}{4^3}\cdot B + 8C + \frac{D}{4}\right)\sin\varphi = 0$$

$$-\frac{1}{32}B + 8C + \frac{D}{4} = 0 \quad (1)$$

(2): $N_{\varphi r} = 0$

$$\left(\frac{1}{32}B - 8C - \frac{1}{4}D\right)\cos\varphi = 0$$

$$\frac{1}{32}B - 8C - \frac{1}{4}D = 0 \quad \rightarrow \text{uslov koji je identički jednak uslovu (1)}$$

(3) $r = 6{,}0\,m$

$N_r = 0$

$$\left(-\frac{1}{108}B + 12C + \frac{D}{6}\right)\sin\varphi = 0$$

$$-\frac{B}{108} + 12C + \frac{D}{6} = 0 \quad (3)$$

(4) jednak uslovu (3)

(5): $$\int_{\varphi=0}^{\varphi_0} N_{r\varphi}\, dr = \int_4^6 \left(\frac{2}{r^3}B - 2Cr - \frac{D}{r}\right)\cos\varphi\, d\varphi = 100$$

$$-\frac{1}{r^2}\Big|_4^6 \cdot B - r^2 C\Big|_4^6 - \ln r \cdot D\Big|_4^6 = 100$$

$$0{,}03472\,B - 20C - 0{,}4055\cdot D = 100 \quad (5)$$

(6): $$\int_{\varphi=0}^{6} \left(+\frac{2}{r^3}B + 6Cr + \frac{1}{r}D\right)\sin\varphi\,(r-5) = 0$$

$$0 = 0 \ \checkmark \quad (6)$$

(7): $$\int_4^6 \left(+\frac{2}{r^3}B + 6Cr + \frac{1}{r}D\right)\sin\varphi\, dr = 0 \quad (7)$$

$$0 = 0 \ \checkmark$$

(8): $$\int_{\varphi=\pi/2}^{6} \left(+\frac{2}{r^3}B + 6Cr + \frac{D}{r}\right)dr = -100$$

$$\left(-\frac{1}{r^2}B + 3r^2 C + \ln r \cdot D\right)\Big|_4^6 \sin\varphi\Big|_{\pi/2} = -100$$

$$+0{,}03472\cdot B + 60C + 0{,}4055\cdot D = -100 \quad (8)$$

(9): $$\int_4^6 \left(+\frac{2}{r^3}B + 6Cr + \frac{D}{r}\right)(r-5)\,dr = 500$$

$$\left(-\frac{2}{r}B + \frac{5}{r^2}B + 2Cr^3 - 15r^2 C + r\cdot D - 5\ln r\cdot D\right)\Big|_4^6 = 500$$

$$-6{,}94\cdot10^{-3}\cdot B + 4\cdot C - 0{,}0273\cdot D = 500 \quad (9)$$

(10):

$$\psi = \bar{u}/2 \int_4^6 \left(\frac{2}{r^3}B - 2Cr - \frac{D}{r}\right)\cancel{\cos\varepsilon}^{\,0} = 0$$

$$0 = 0$$

Iz (1), (3), (5)

$$\Rightarrow B = -26563{,}7170$$

$$C = 46{,}1175$$

$$D = -4796{,}2267$$

$B, C, D \rightarrow (8)$

$$\Rightarrow \quad -100 = -100 \checkmark$$

$\rightarrow (9)$

$$\Rightarrow 500 = 500 \checkmark$$

Koeficijent A nema uticaja na naprezanje.

F-ja F odgovara naprezanju ležišnog lučnog nosača

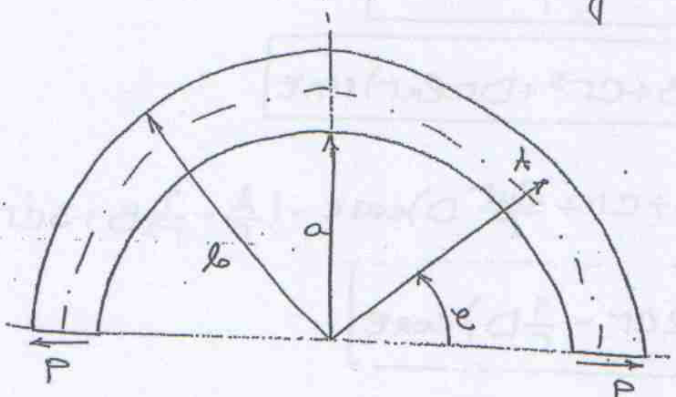
(14)

06.05.1995.

2. Ispitati da li naponska f-ja:

$$F = (Ar + B\frac{1}{r} + Cr^3 + Dr\ln r)\cdot \sin\varphi$$

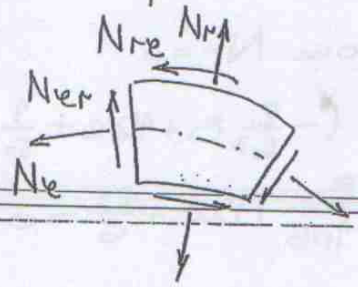
odgovara naprezanju poluokruženog luka na slici.



$a = 6{,}0\,m$

$b = 8{,}0\,m$

$P = 300\,kN$



$$N_r = \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} + \frac{1}{r}\frac{\partial F}{\partial r}$$

$$N_\varphi = \frac{\partial^2 F}{\partial r^2}$$

$$N_{r\varphi} = \frac{1}{r^2}\frac{\partial F}{\partial \varphi} - \frac{1}{r}\frac{\partial^2 F}{\partial r\partial \varphi}$$

Granični uslovi:

$$r = a \left\{ N_r = 0 \quad (1) \right.$$

$$r = b \left\{ N_r = 0 \quad (2) \right.$$

$$\varphi = 0 \left\{ \int_a^b N_{r\varphi}\,dr = -300 \quad (3) \right.$$

$$\varphi = \bar{u} \left\{ \int_a^b N_{r\varphi}\,dr = 300 \quad (4) \right.$$

$$\frac{\partial F}{\partial r} = \left(A - \frac{1}{r^2}B + 3Cr^2 + D(\ln r + 1)\right)\sin\varphi$$

$$\frac{\partial^2 F}{\partial r^2} = \left(\frac{2}{r^3}B + 6Cr + \frac{1}{r}D\right)\sin\varphi$$

$$\frac{\partial F}{\partial \varphi} = (Ar + B\frac{1}{r} + Cr^3 + Dr\ln r)\cos\varphi$$

$$\frac{\partial^2 F}{\partial \varphi^2} = -\left(Ar + \frac{1}{r}B + Cr^3 + Dr\ln r\right)\sin\varphi$$

$$\frac{\partial^2 F}{\partial r\,\partial \varphi} = \left(A - \frac{1}{r^2}B + 3Cr^2 + D(\ln r + 1)\right)\cos\varphi$$

$$N_r = -\left(\frac{A}{r} + \frac{B}{r^3} + Cr + \frac{\ln r}{r}D\right)\sin\varphi + \left(\frac{A}{r} - \frac{B}{r^3} + 3Cr^2 + \frac{\ln r}{r}D + \frac{1}{r}D\right)\sin$$

$$\boxed{N_r = \left(-\frac{2}{r^3}B + 2Cr + \frac{1}{r}D\right)\sin\varphi}$$

$$\boxed{N_\varphi = -\left(Ar + \frac{1}{r}B + Cr^3 + Dr\ln r\right)\sin\varphi}$$

$$N_{r\varphi} = \left(\frac{A}{r} + \frac{1}{r^3}B + Cr + \frac{\ln r}{r}D\right)\cos\varphi - \left(\frac{A}{r} - \frac{1}{r^3}B + 3Cr + \frac{\ln r}{r}D + \frac{1}{r}D\right)\cdot\cos$$

$$\boxed{N_{r\varphi} = \left(\frac{2}{r^3}B - 2Cr - \frac{1}{r}D\right)\cos\varphi}$$

(1): $r = 6{,}0\,m$ $N_r = 0$

$$N_r = \left(-\frac{2}{6^3}B + 12C + \frac{1}{6}D\right)\sin\varphi = 0$$

$$-\frac{B}{108} + 12C + \frac{D}{6} = 0 \quad (1)$$

(2): $r = 8{,}0\,m$ $N_r = 0$

$$-\frac{B}{256} + 16C + \frac{1}{8}D = 0 \quad (2)$$

(3): $\varphi = 0$

$$\int_6^8 \left(\frac{2}{r^3}B - 2Cr - \frac{1}{r}D\right)\cos^1\varphi\, dr = -300$$

$$\left(-\frac{B}{r^2} - Cr^2 - \ln r \cdot D\right)\Big|_6^8 = -300$$

$$0{,}0121527\,B - 28C - 0{,}2877\cdot D = -300 \quad (3)$$

(4) $$\int_6^8 \left(\frac{2}{r^3}B - 2Cr - \frac{1}{r}D\right)\cos^{-1}\varphi\, dr = 300$$

$$-0{,}0121527\,B + 28C + 0{,}2877\cdot D = 300 \quad (4)$$

B=449878,6002

C=-195,2598

D= 39051,9618

Da, odgovara.

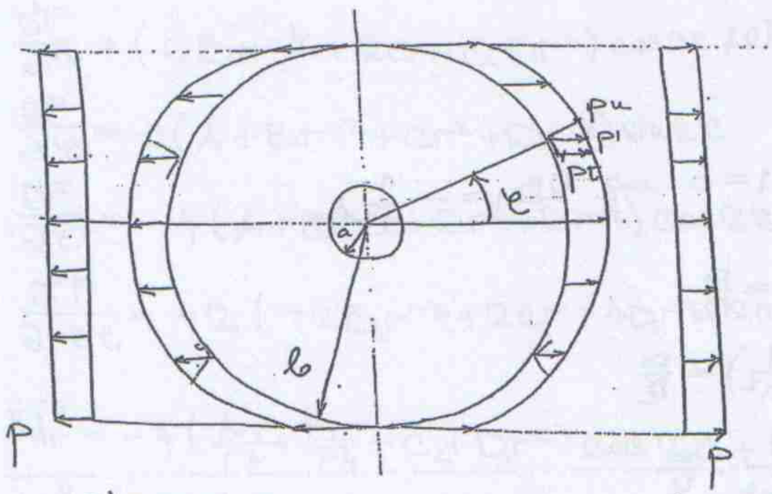
(15)

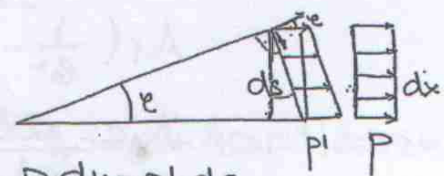
28.02.2004.

2. $b \gg a$

Za $\varphi = 0$ i $\varphi = \pi/2$ nacrtati dijagram sila u presecima.

$$F(r,\varphi) = (A + Br^{-2} + Cr^{2} + Dr^{4})\cos 2\varphi$$





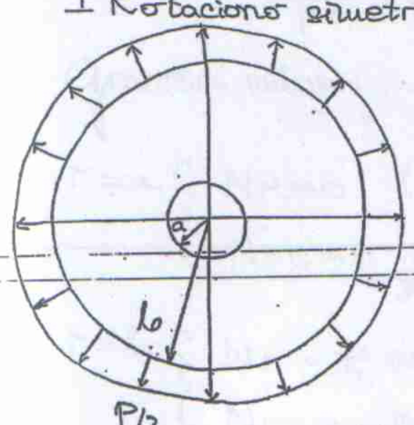
$$p' = p\cdot\cos\varphi \qquad p\cdot dx = p'\cdot ds$$

$$p_n = p'\cdot\cos\varphi = p\cos^{2}\varphi \qquad \frac{dx}{ds} = \cos\varphi$$

$$p_t = p'\cdot\sin\varphi = p\sin\varphi\cos\varphi = \frac{p}{2}\sin 2\varphi$$

$$p_n = p'\cos^{2}\varphi = p\cdot\frac{1+\cos 2\varphi}{2} = \frac{p}{2} + \frac{p}{2}\cos 2\varphi$$

I Rotaciono simetrično



nema uticaja na naprezanje, pa se usvaja $D = 0$

$$F_I = \cancel{D_1}^{0} + A_1 \ln r + B_1 r^{2} + \cancel{C_1}^{0} r^{2}\ln r$$

→ važi za kružne ploče napregnute u svojoj ravni opterećene rotaciono simetričnim opterećenjem po konturi

$$F_I = A_1 \ln r + B_1 r^{2}$$

$$N'_r = \frac{1}{r}\frac{dF_I}{dr} = \frac{1}{r^2}A_1 + 2B_1$$

$$N'_\theta = \frac{d^2F_I}{dr^2} = 2B_1$$

$$N'_{r\theta} = 0$$

Granični uslovi:

$$r=a \begin{cases} N_r = 0 & (1) \\ \vdots & \end{cases}$$

$$r=b \begin{cases} N_r = \frac{p}{2} & (2) \end{cases}$$

(1): $\frac{1}{a^2}A_1 + 2B_1 = 0 \Rightarrow 2B_1 = -\frac{1}{a^2}A_1$

(2): $\frac{1}{b^2}A_1 + 2B_1 = \frac{p}{2}$

$$A_1\left(\frac{1}{b^2} - \frac{1}{a^2}\right) = \frac{p}{2}$$

$$A_1 = \frac{a^2b^2}{a^2-b^2}\cdot\frac{p}{2}$$

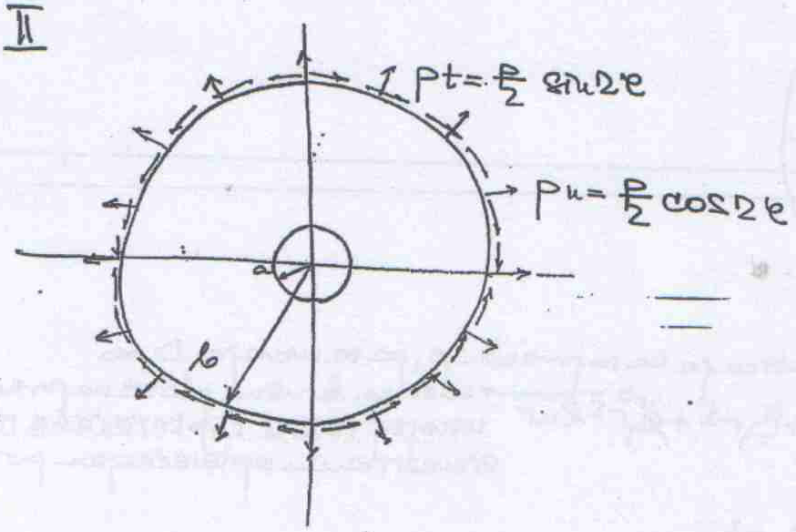
$$B_1 = -\frac{b^2}{2(a^2-b^2)}\cdot\frac{p}{2}$$

$$N'_r = \frac{1}{r^2}\frac{a^2b^2}{a^2-b^2}\frac{p}{2} - \frac{b^2}{a^2-b^2}\frac{p}{2}$$

$$N'_\theta = -\frac{b^2}{a^2-b^2}\frac{p}{2}$$

$$N'_{r\theta} = 0$$

II



$$F_{II} = (A_2 + B_2 r^{-2} + C_2 r^2 + D_2 r^4)\cos 2\varphi$$

$$N_r'' = \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} + \frac{1}{r}\frac{\partial F}{\partial r}$$

$$N_\varphi'' = \frac{\partial^2 F}{\partial r^2}$$

$$N_{r\varphi}'' = \frac{1}{r^2}\frac{\partial F}{\partial \varphi} - \frac{1}{r}\frac{\partial^2 F}{\partial r \partial \varphi}$$

$$\frac{\partial F}{\partial r} = (-2Br^{-3} + 2Cr + 4Dr^3)\cos 2\varphi$$

$$\frac{\partial^2 F}{\partial r^2} = (6Br^{-4} + 2C + 12Dr^2)\cos 2\varphi$$

$$\frac{\partial F}{\partial \varphi} = -2(A + Br^{-2} + Cr^2 + Dr^4)\sin 2\varphi$$

$$\frac{\partial^2 F}{\partial \varphi^2} = -4(A + Br^{-2} + Cr^2 + Dr^4)\cos 2\varphi$$

$$\frac{\partial^2 F}{\partial r \partial \varphi} = -2(-2B_2 r^{-3} + 2C_2 r + 4D_2 r^3)\sin 2\varphi$$

$$N_r'' = -4\left(\frac{A_2}{r^2} + \frac{B_2}{r^4} + C_2 + D_2 r^2\right)\cos 2\varphi + \left(\frac{2B}{r^4} + 2C + 4Dr^2\right)\cos 2\varphi$$

$$N_r'' = \left(-4\frac{A_2}{r^2} - 2\frac{B_2}{r^4} - 2C_2\right)\cos 2\varphi$$

$$N_\varphi'' = \left(6\frac{B_2}{r^4} + 2C_2 + 12D_2 r^2\right)\cos 2\varphi$$

$$N_{r\varphi}'' = \left(-\frac{2A_2}{r^2} - \frac{2B_2}{r^4} - 2C_2 - 2D_2 r^2\right)\sin 2\varphi + \left(-\frac{4B_2}{r^4} + 4C_2 + 8D_2 r^2\right)\sin 2\varphi$$

$$N_{r\varphi}'' = \left(-\frac{2A_2}{r^2} - \frac{6B_2}{r^4} + 2C_2 + 6D_2 r^2\right)\sin 2\varphi$$

Granični uslovi:

$r = a$ $\begin{cases} N_r = 0 & (1) \\ N_{r\varphi} = 0 & (2) \end{cases}$

$r = b$ $\begin{cases} N_r = \frac{P}{2}\cos 2\varphi & (3) \\ N_{r\varphi} = -\frac{P}{2}\sin 2\varphi & (4) \end{cases}$

$\Rightarrow N_r'', N_\varphi'', N_{r\varphi}''$

$N_r = N_r' + N_r''$

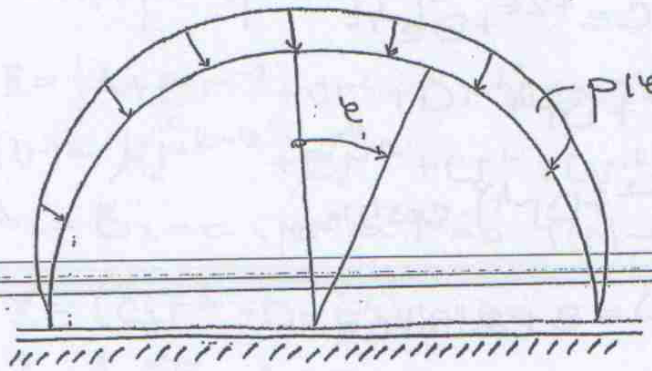
(16)

24.05.2003.

2. Za ploču koja je opterećena u svojoj ravni odrediti komponentalne napone. Uslovi oslanjanja duž kontura $\varphi=-\pi/2$ i $\varphi=\pi/2$ sprečavaju pomeranja u pravcu upravnom na konturu.

Naponsku f-ju $F(r,\varphi)$ pretpostaviti u obliku proizvoda f-je $Y(r)$ i odgovarajuće trigonometrijske f-je. Nepoznatu f-ju $Y(r)$ odrediti iz diferencijalne j-ne:

$$\frac{\partial^4 Y}{\partial r^4}+\frac{2}{r}\frac{\partial^3 Y}{\partial r^3}-\frac{9}{r^2}\frac{\partial^2 Y}{\partial r^2}+\frac{9}{r^3}\frac{\partial Y}{\partial r}=0$$



$E=30\ GPa$

$\nu=0,15$

$h=0,10\ m$

$p=8(1+\cos 2\varphi)$

$$\frac{d^4 Y}{dr^4}+\frac{2}{r}\frac{d^3 Y}{dr^3}-\frac{9}{r^2}\frac{d^2 Y}{dr^2}+\frac{9}{r^3}\frac{dY}{dr}=0$$

$r=e^t$

$$\rightarrow \frac{dY}{dr}=\frac{dY}{dt}\frac{dt}{dr}=\frac{1}{e^t}\frac{dY}{dt}$$

$$\frac{d^2 Y}{dr^2}=\frac{d}{dt}\left(e^{-t}\frac{dY}{dt}\right)\cdot\frac{dt}{dr}=\left(-e^{-t}\frac{dY}{dt}+e^{-t}\frac{d^2 Y}{dt^2}\right)e^{-t}$$

$$\rightarrow \frac{d^2 Y}{dr^2}=\frac{d^2 Y}{dt^2}e^{-2t}-\frac{dY}{dt}e^{-2t}$$

$$\frac{d^3 Y}{dr^3}=\frac{d}{dt}\left(\frac{d^2 Y}{dt^2}e^{-2t}-\frac{dY}{dt}e^{-2t}\right)\frac{dt}{dr}=\left(\frac{d^3 Y}{dt^3}e^{-2t}-\right.$$

$$\left.-2\frac{d^2 Y}{dt^2}e^{-2t}-\frac{d^2 Y}{dt^2}e^{-2t}+2\frac{dY}{dt}e^{-2t}\right)\cdot e^{-t}=$$

$$\rightarrow \frac{d^3 Y}{dr^3}=\frac{d^3 Y}{dt^3}\cdot e^{-3t}-3\frac{d^2 Y}{dt^2}e^{-3t}+2\frac{dY}{dt}e^{-3t}$$

$$\rightarrow \frac{d^4 Y}{dr^4}=\frac{dY'''}{dt}\cdot\frac{dt}{dr}=\frac{d^4 Y}{dt^4}\cdot e^{-4t}-6\frac{d^3 Y}{dt^3}\cdot e^{-4t}+11\frac{d^2 Y}{dt^2}e^{-4t}-$$

$$-6\frac{dY}{dt}e^{-4t}+$$

$$\frac{d^4Y}{dt^4}e^{-4t} - 6\frac{d^3Y}{dt^3}e^{-4t} + 11\frac{d^2Y}{dt^2}e^{-4t} - 6\frac{dY}{dt}e^{-4t} +$$
$$+2\frac{d^3Y}{dt^3}e^{-4t} - 6\frac{d^2Y}{dt^2}e^{-4t} + 4\frac{dY}{dt}e^{-4t} - 9\frac{d^2Y}{dt^2}e^{-4t} + 9\frac{dY}{dt}e$$
$$+9\frac{dY}{dt}e^{-4t} = 0$$

$$\frac{d^4Y}{dt^4} - 4\frac{d^3Y}{dt^3} - 4\frac{d^2Y}{dt^2} + 16\frac{dY}{dt} = 0$$

$$\lambda^4 - 4\lambda^3 - 4\lambda^2 + 16\lambda = 0$$

$$\lambda^3(\lambda - 4) - 4\lambda(\lambda - 4) = 0$$

$$\lambda(\lambda - 4)(\lambda^2 - 4) = 0$$

$$\lambda_1 = 0 \quad \lambda_2 = -2 \quad \lambda_3 = +2 \quad \lambda_4 = 4$$

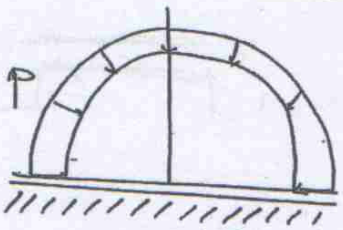
$$Y = A + Be^{-2t} + Ce^{+2t} + De^{4t}$$

$$Y = A + \frac{B}{r^2} + Cr^2 + Dr^4$$

$$F = \left(A + \frac{B}{r^2} + Cr^2 + Dr^4\right) \cdot \cos 2\varphi$$

$$P = 8(1 + \cos 2\varphi) = 8 + 8\cos 2\varphi$$

I Rotacioni simetrični opterećenje



$F_I = D_1 + K_1 \ln r + B_1 r^2 + C_1 r^2 \ln r$ nema ubrzanja na napregnuće

$A_1 = C_1 = 0$, jer za $r = 0$ $\ln r \to \infty$

$F_I = B_1 r^2$

$N_r = \frac{1}{r}\frac{dF}{dr} = 2B_1$

$N_\varphi = \frac{d^2F}{dr^2} = 2B_1$

$N_{r\varphi} = 0$

Granični uslovi:

$$r = a = 3{,}10\text{m} \begin{cases} N_r = -p = -8 \quad (1) \end{cases}$$

(1):

$$2B_1 = -8 \Rightarrow B_1 = -4$$

$$F_I = -4r^2$$

$$N_r' = -8$$

$$N_\varphi' = -8$$

$$N_{r\varphi} = 0$$

II Proizvoljno opterećenje

$$F_{II} = (A + Br^{-2} + Cr^2 + Dr^4)\cos 2\varphi$$

$$Y(r) = \cancel{A_2 r^{k-2}}^{\,=0} + \cancel{B_2 r^{-k}}^{\,=0} + C_2 r^k + D_2 r^{k+2}, \quad k = 2$$

$A_2 = B_2 = 0$, jer za $r = 0$ $Y(r) \to \infty$

$$F_{II} = (C_2 r^2 + D_2 r^4)\cos 2\varphi$$

$$N_r'' = \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2} + \frac{1}{r}\frac{\partial F}{\partial r}$$

$$N_\varphi'' = \frac{\partial^2 F}{\partial r^2}$$

$$N_{r\varphi}'' = \frac{1}{r^2}\frac{\partial F}{\partial \varphi} - \frac{1}{r}\frac{\partial^2 F}{\partial r \partial \varphi}$$

$$\frac{\partial F}{\partial r} = (2C_2 r + 4D_2 r^3)\cos 2\varphi$$

$$\frac{\partial^2 F}{\partial r^2} = (2C_2 + 12D_2 r^2)\cos 2\varphi$$

$$\frac{\partial F}{\partial \varphi} = -2(C_2 r^2 + D_2 r^4)\sin 2\varphi$$

$$\frac{\partial^2 F}{\partial \varphi^2} = -4(C_2 r^2 + D_2 r^4)\cos 2\varphi$$

$$\frac{\partial^2 F}{\partial r \partial \varphi} = -2(2C_2 r + 4D_2 r^3)\sin 2\varphi$$

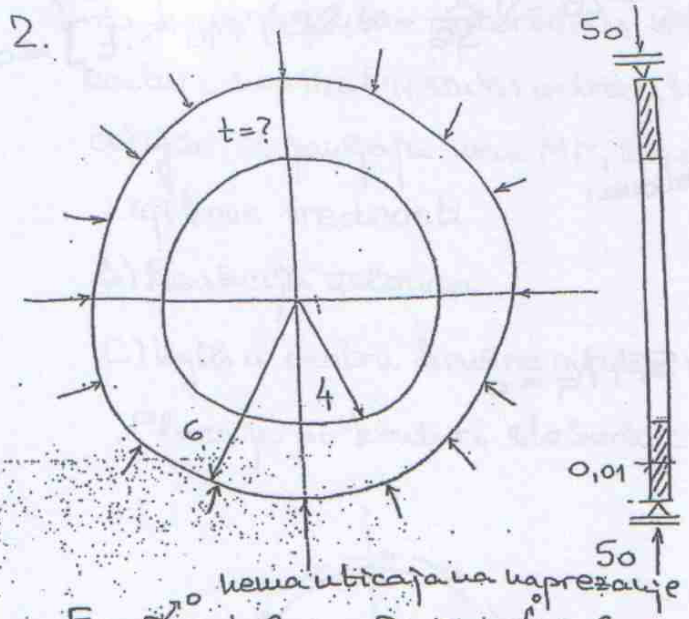
$$N_r'' = (-4C_2 - 4D_2 r^2)\cos 2\varphi + (2C_2 + 4D_2 r^2)\cos 2\varphi$$

$$N_r'' = -2C_2 \cdot \cos 2\varphi$$

(18)

26.10.2002.

2.



$E = 210\ GPa$

$\nu = 0{,}3$

$h = 0{,}01\ m$

shema utjecajna naprezanja

$$F = \cancel{B}^{0} + A \ln r + Br^2 + \cancel{C}^{0} r^2 \ln r$$

Vrijedi za kružne ploče napregnute u svojoj ravni i opterećene rotacijski simetričnim opterećenjem po svojoj konturi

$$F = A \ln r + Br^2$$

Granični uvjeti:

$r = 4{,}0\ m \left\{ N_r = 0 \quad (1) \right.$

$r = 6{,}0\ m \left\{ \begin{array}{ll} N_r = -50 & (2) \\ u = 0 & (3) \end{array} \right.$

$$N_r = \frac{1}{r}\frac{dF}{dr} = \frac{A}{r^2} + 2B$$

$$N_\varphi = \frac{d^2F}{dr^2} = -\frac{A}{r^2} + 2B$$

$$N_{r\varphi} = 0$$

(1): $\frac{A}{16} + 2B = 0 \quad (1)$

(2): $\frac{A}{36} + 2B = -50 \quad (2)$

$\frac{A}{36} - \frac{A}{16} = -50 \Rightarrow A = 1440$

$B = -1c$

(3): $u = 0$

$$u = r \cdot \varepsilon_\varphi = r\left(\frac{1}{E_w}(N_\varphi - \nu N_r) + \alpha_t \cdot t\right)$$

$$6\left[\frac{1}{210\cdot 10^6 \cdot 0{,}01}\left(-\frac{A}{36} + 2B - \nu\frac{A}{36} - \nu \cdot 2B\right) + 10^{-5} \cdot t\right] = 0$$

$$t = 5{,}476\,^\circ C$$

Kontrola graničnih uslova:

$$\frac{A}{16} + 2B = 0$$

$$-\frac{A}{36} \cdot 1{,}3 + 2B \cdot 0{,}7 + 21 \cdot 5{,}476 = 0$$

$$A = 1440$$

$$B = -45 \quad \checkmark$$

(19)

29.03.2008.

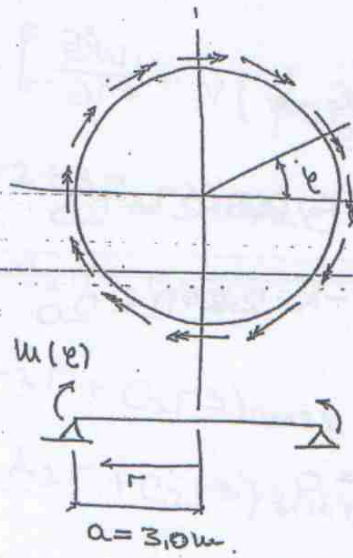
1. Za kružnu ploču opterećenu kružnim momentima po konturi $m = m_0(1+\sin\varphi)$ odrediti:

a) gde se javljaju $\max M_r$, $\max M_\varphi$ i $\max M_{r\varphi}$ i koje su njihove vrednosti.

b) Reakciju oslonca

c) Ugib u centru kružne ploče.

Ploča je po konturi slobodno oslonjena.



$E = 30\ GPa$

$\nu = 0,25$

$h = 25\ cm$

$m_0 = 20\ KNm/m$

$m = m_0(1+\sin\varphi) = m_0 + m_0 \cdot \sin\varphi$

I Rotaciono simetrično opterećenje

$m = m_0 = 20\ KNm/m$

$W^I = A_1 + \cancel{B_1 \ln r}^{\to 0} + C_1 r^2 + \cancel{D_1 r^2 \ln r}^{\to 0}$

$B_1 = D_1 = 0$, jer za $r = 0$, $\ln r \to \infty$

$W^I = A_1 + C_1 r^2$

Granični uslovi:

$r = a = 3,0m$ $\begin{cases} W = 0 & (1) \\ M_r = m_0 & (2) \end{cases}$

(1): $r = 3{,}0\,m$

$$W^{I} = A_1 + 9C_1 = 0$$

(2):

$$M_r = -K\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) = m_0$$

$$\frac{dw}{dr} = 2rC_1 \qquad \frac{d^2w}{dr^2} = 2C_1$$

$$-K(2C_1 + 0{,}5C_1) = m_0$$

$$K = \frac{E\cdot h^3}{12(1-\nu^2)} = \frac{30\cdot 10^6 \cdot 0{,}25^3}{12(1-0{,}25^2)} = 41666{,}6$$

$$C_1 = -1{,}92\cdot 10^{-4}$$

$$A_1 = 1{,}728\cdot 10^{-3}$$

$$W^{I} = 1{,}728\cdot 10^{-3} - 1{,}92\cdot 10^{-4}\cdot r^2$$

$$M'_r = -K\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) = -K\cdot 2{,}5\cdot C = 20$$

$$M'_\varphi = -K\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2w}{dr^2}\right) = -K\cdot 2{,}5\cdot C = 20$$

$$M'_{r\varphi} = 0$$

$$T'_r = -K\frac{d}{dr}\left(\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) = 0$$

$$W'(r=0) = 1{,}728\cdot 10^{-3}\,m$$

II Proizvoljno opterećenje

$$m = 20\cdot \sin\varphi$$

$$W^{II} = \sum_{n=1}^{\infty} Y_n \cdot \sin n\varphi \,, \quad n = 1$$

$$Y_n = A_2 r^n + \cancel{B_2 r^{-n}} + C_2 r^{n+2} + \cancel{D_2 r^{n-2}}$$

$B_2 = D_2 = 0$ jer za $r = 0$ $Y_n \to \infty$

$$Y_1 = A_2 r + C_2 r^3$$

$W'' = (A_2 r + C_2 r^3) \sin\varphi$

Granični uslovi:

$r = a = 3{,}0\,m$ $\begin{cases} W = 0 & (1) \\ M_r = 20 \sin\varphi & (2) \end{cases}$

(1): $r = 3{,}0$

$(3A_2 + 27C_2)\sin\varphi = 0$

$A_2 + 9C_2 = 0$

(2): $r = 3{,}0$

$M_r = -K\left[\frac{\partial^2 W}{\partial r^2} + \nu\left(\frac{1}{r^2}\frac{\partial^2 W}{\partial \varphi^2} + \frac{1}{r}\frac{\partial W}{\partial r}\right)\right]$

$\frac{\partial W}{\partial r} = (A_2 + 3C_2 r^2)\sin\varphi$

$\frac{\partial^2 W}{\partial r^2} = 6C_2 r \sin\varphi$

$\frac{\partial W}{\partial \varphi} = (A_2 r + C_2 r^3)\cos\varphi$

$\frac{\partial^2 W}{\partial \varphi^2} = -(A_2 r + C_2 r^3)\sin\varphi$

$M_r = -K\left[6 \cdot C_2 \cdot 3 + 0{,}25 \cdot \left(-\frac{A_2}{3} \div 3C_2 \div \frac{A_2}{3} + 9C_2\right)\right]\sin\varphi =$

$= 20 \sin\varphi$

$-K \cdot 27{,}0\, C_2 = 20 \Rightarrow C_2 = -1{,}7 \cdot 10^{-5}$

$A_2 = 1{,}6 \cdot 10^{-4}$

$W'' = (1{,}6 \cdot 10^{-4} r - 1{,}7 \cdot 10^{-5} r^3)\sin\varphi$

$\varphi'' = -K\left[\frac{1}{r^2}\frac{\partial^2 W}{\partial \varphi^2} + \frac{1}{r}\frac{\partial W}{\partial r} + \nu\frac{\partial^2 W}{\partial r^2}\right]$

$\varphi = -K(1-\nu)\left[\frac{1}{r}\frac{\partial^2 W}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial W}{\partial \varphi}\right]$

$W'' = (A_2 r + C_2 r^3) \sin\varphi$

Granični uslovi:

$r = a = 3{,}0\,m$ $\begin{cases} W = 0 & (1) \\ M_r = 20 \sin\varphi & (2) \end{cases}$

(1): $r = 3{,}0$

$(3A_2 + 27 C_2) \sin\varphi = 0$

$A_2 + 9 C_2 = 0$

(2): $r = 3{,}0$

$M_r = -K \left[\frac{\partial^2 W}{\partial r^2} + \nu \left(\frac{1}{r^2} \frac{\partial^2 W}{\partial \varphi^2} + \frac{1}{r} \frac{\partial W}{\partial r} \right) \right]$

$\frac{\partial W}{\partial r} = (A_2 + 3 C_2 r^2) \sin\varphi$

$\frac{\partial^2 W}{\partial r^2} = 6 C_2 r \sin\varphi$

$\frac{\partial W}{\partial \varphi} = (A_2 r + C_2 r^3) \cos\varphi$

$\frac{\partial^2 W}{\partial \varphi^2} = -(A_2 r + C_2 r^3) \sin\varphi$

$M_r = -K \left[6 \cdot C_2 \cdot 3 + 0{,}25 \cdot \left(-\frac{A_2}{3} + 3 C_2 + \frac{A_2}{3} + 9 C_2 \right) \right] \sin\varphi =$

$= 20 \sin\varphi$

$-K \cdot 27{,}0\, C_2 = 20 \Rightarrow C_2 = -1{,}7 \cdot 10^{-5}$

$A_2 = 1{,}6 \cdot 10^{-4}$

$W = (1{,}6 \cdot 10^{-4} r - 1{,}7 \cdot 10^{-5} r^3) \sin\varphi$

$= -K \left[\frac{1}{r^2} \frac{\partial^2 W}{\partial \varphi^2} + \frac{1}{r} \frac{\partial W}{\partial r} + \nu \frac{\partial^2 W}{\partial r^2} \right]$

$= -K(1-\nu) \left[\frac{1}{r} \frac{\partial^2 W}{\partial r \partial \varphi} - \frac{1}{r^2} \frac{\partial W}{\partial \varphi} \right]$

$$M''_r = -K\left[6 \cdot C_2 \cdot r - 0{,}25\frac{A_2}{r} - 0{,}25 \cdot C_2 r + 0{,}25 \cdot \frac{A_2}{r} + 0{,}75 \cdot C_2 r\right] \cdot \operatorname{sen}\varphi$$

$$M''_r = -K \cdot 6{,}5 \cdot C_2 \cdot r \cdot \operatorname{sen}\varphi = 4{,}8148 \cdot r \cdot \operatorname{sen}\varphi \leftarrow$$

$$M''_\varphi = -K\left[-\frac{A_2}{r} + C_2 r + \frac{A_2}{r} + 3C_2 r + 0{,}25 \cdot 6 \cdot C_2 r\right] \operatorname{sen}\varphi$$

$$M_\varphi = -K \cdot 5{,}5 \cdot C_2 \cdot r \cdot \operatorname{sen}\varphi = 4{,}0741 \cdot r \cdot \operatorname{sen}\varphi \leftarrow$$

$$M_{r\varphi} = -K(1-0{,}25)\left[\frac{1}{r}(A_2 + 3C_2 r^2)\cos\varphi - \frac{1}{r^2}(A_2 r + C_2 r^3)\right]$$

$$M_{r\varphi} = -0{,}75 \cdot K\left[\frac{A_2}{r} + 3C_2 r - \frac{A_2}{r} - C_2 r\right]\cos\varphi$$

$$M''_{r\varphi} = 1{,}8 \cdot r \cdot \cos\varphi \leftarrow$$

$$M_r = M'_r + M''_r = 20 + 4{,}8148 \cdot r \cdot \operatorname{sen}\varphi$$

Za $\operatorname{sen}\varphi = 1 \Rightarrow \varphi = \frac{\pi}{2} \wedge r = a = 3{,}0\,m$

$\longrightarrow$ max $M_r = 34{,}4$ KNm/m

$$M_\varphi = M'_\varphi + M''_\varphi = 20 + 4{,}0741 \cdot r \cdot \operatorname{sen}\varphi$$

Za $\operatorname{sen}\varphi = 1 \Rightarrow \varphi = \frac{\pi}{2} \wedge r = a = 3{,}0\,m$

$\longrightarrow$ max $M_\varphi = 32{,}2$ KNm/m

$$M_{r\varphi} = M'_{r\varphi} + M''_{r\varphi} = 0 + 0{,}8 \cdot r \cdot \cos\varphi$$

Za $\cos\varphi = 1 \Rightarrow \varphi = 0 \wedge r = a = 3{,}0\,m$

$\longrightarrow$ max $M_{r\varphi} = 1{,}6$ KNm/m

b) $T'_r = 0$

$$T''_r = -K\frac{\partial}{\partial r}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right]$$

$$T''_r = -K\frac{\partial}{\partial r}\left[6 \cdot C_2 r + \frac{A_2}{r} + C_2 r + \frac{A_2}{r} + 3C_2 r\right]\operatorname{sen}\varphi$$

$$T''_r = -K\frac{\partial}{\partial r}(8\,C_2 r)$$

$$T_r'' = -K \cdot 8 \cdot C_2 = 5{,}9259 \cdot \text{sin}\varepsilon$$

$$T_r = T_r' + T_r'' = 0 + 5{,}9259 \cdot \text{sin}\varepsilon = 5{,}9259 \cdot \text{sin}\varepsilon$$

c)

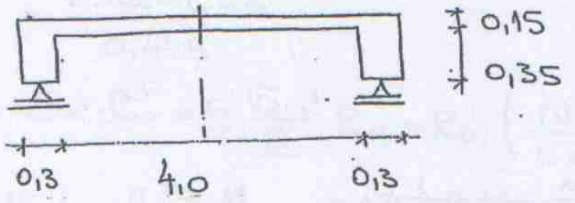
$$W''(r=0) = 0$$

$$W = W' + W'' = 1{,}728 \cdot 10^{-3}\ m$$

(20)

20.04.2006. isti kao 31.05.2008. samo druge vrednosti

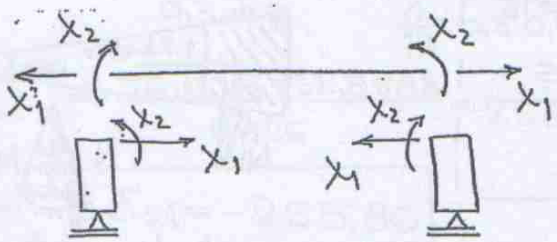
2. Usled zagrevanja kružnog prstena za $t=30°$, sračunati momente savijanja i ugib u centru kružne ploče.



$E = 30\ GPa$

$\nu = 0,2$

$\alpha t = 10^{-5}\ 1/°C$



$\delta_{11} X_1 + \delta_{12} X_2 + \delta_{10} = 0$

$\delta_{21} X_1 + \delta_{22} X_2 + \delta_{20} = 0$

Stanje $X_1 = 1$

Ploča:

$\delta_{11}^{pl} = u = r \cdot \varepsilon_\varphi = r \frac{1}{E h} (N_\varphi - \nu N_r)$

$F = \cancel{D^0} + \cancel{A^0 \ln r} + Br^2 + \cancel{Cr^2 \ln r}$

D nema uticaja na napregnuće ploče usled X_1: $D = 0$

~~Za $r = 0$, $\ln r \to \infty$, pa je $A = C = 0$~~

$F = Br^2$

granični uslovi:

$r = 2,0\ m \left\{ N_r = 1,0 \quad (1) \right.$

$N_r = \frac{1}{r} \frac{dF}{dr} = \frac{1}{r} 2rB = 2B$

$N_\varphi = \frac{d^2F}{dr^2} = 2B$

$N_{r\varphi} = 0$

(1): $r = 2{,}0\,m$

$N_r = 2B = 1 \Rightarrow B = 0{,}5$

$N_r = 1{,}0$

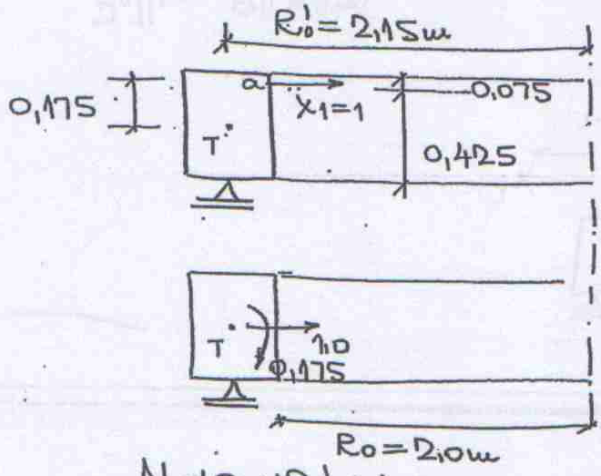
$N_\varphi = 1{,}0$

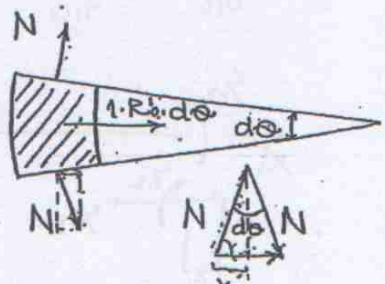
$N_{r\varphi} = 0$

$$E\,\delta_{11}^{pl} = 2{,}0\,\frac{1}{0{,}15}\,(1-0{,}2) = 10{,}\dot{6}$$

$$E\,\delta_{12}^{pl} = 0$$

Prsten:

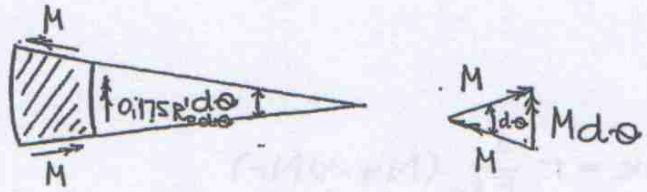




$$x = N \cdot \cos\left(\frac{\pi}{2} - \frac{d\Theta}{2}\right) = N \sin\frac{d\Theta}{2} \cong \frac{d\Theta}{2}$$

$$2x = N\,d\Theta$$

$$N\,d\Theta + 1 R_0' \cdot d\Theta = 0 \Rightarrow N = -1 \cdot R_0'$$



$$0{,}175 \cdot R_0\,d\Theta + M\,d\Theta = 0 \Rightarrow M = -0{,}175\,R_0'$$

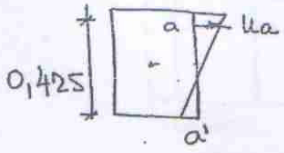
$$\varepsilon_a = \frac{\sigma_a}{E}$$

$$\sigma_a = \frac{N}{F} + \frac{M}{I} \cdot \frac{a}{2} = -\frac{R_0'}{6 \cdot a} + \frac{-0{,}175 \cdot R_0'}{\frac{1}{12}\,6 \cdot a^3} \cdot \frac{a}{2}$$

$$E \cdot \varepsilon_a = -\frac{2{,}15}{0{,}3 \cdot 0{,}5} - \frac{0{,}175 \cdot 2{,}15}{\frac{1}{12}\,0{,}3 \cdot 0{,}5^3} \cdot 0{,}175 = -35{,}40\dot{3}$$

$$E\,\delta_{11}^{pr} = -E \cdot \varepsilon_a \cdot R_0' = 35{,}40\dot{3} \cdot 2{,}15 = 76{,}1171\dot{6}$$

$$E\delta_{11} = E\delta_{11}^{pl} + E\delta_{11}^{pr} = 86{,}783$$



$$E\delta_{21}^{pr} = \frac{E\cdot ua - E\cdot ua'}{0{,}425}$$

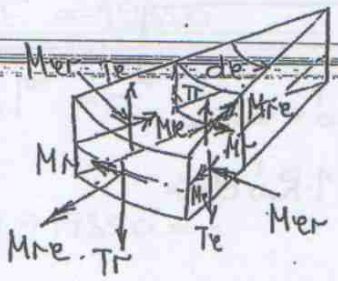
$$E\cdot ua' = E\cdot \varepsilon a' \cdot R_0' = E\frac{\sigma a'}{E}\cdot R_0' = R_0'\cdot\left(\frac{N}{b\cdot d} - \frac{M}{\frac{1}{12}bd^3}\cdot\frac{d}{2}\right) =$$

$$E\cdot ua' = 2{,}15\left(-\frac{2{,}15}{0{,}3\cdot 0{,}5} + \frac{0{,}175\cdot 2{,}15}{\frac{1}{12}\cdot 0{,}3\cdot 0{,}5^3}\cdot 0{,}25\right) = 33{,}898\dot{3}$$

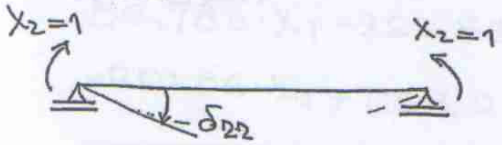
$$E\delta_{21}^{pr} = -\frac{-76{,}117\dot{1}6 + 33{,}898\dot{3}}{0{,}425} = -258{,}86$$

$$E\delta_{12} = E\delta_{21} = -258{,}86$$

Stanje $X_2 = 1$



Ploča:



$$E\delta_{22} = E\frac{dw}{dr}$$

nema površinskog opterećenja

$$W = W_0 + W_1 = \cancel{W_0}^{\,0} + A + \cancel{B\ln r}^{\,0} + Cr^2 + \cancel{Dr^2\ln r}^{\,0}$$

za $r = 0$ $\ln r \to \infty$

$$W = A + Cr^2$$

Granični uvjeti:

$$r = 2{,}0 \begin{cases} W = 0 & (1) \\ M_r = 1 & (2) \end{cases}$$

(1):

$$W = A + 4C = 0$$

(2):

$$M_r = -K\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right)$$

$$K = \frac{E h^3}{12(1-\nu^2)} = \frac{30\cdot10^6 \cdot 0{,}15^3}{12(1-0{,}2^2)} = 8789{,}0625$$

$$\frac{dw}{dr} = 2Cr$$

$$\frac{d^2w}{dr^2} = 2C$$

$$M_r = -8789{,}0625(2C + 0{,}2\cdot 2C) = 1$$

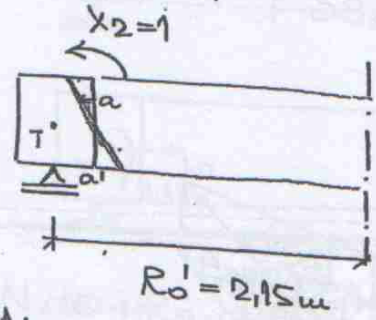
$$C = -4{,}\overline{740}\cdot10^{-5}$$

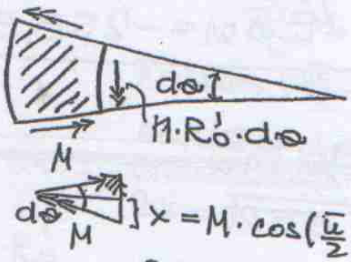
$$A = 1{,}8\overline{962}\cdot10^{-5}$$

$$w = 1{,}8963\cdot10^{-5} - 4{,}741\cdot10^{-5}\cdot r^2$$

$$E\delta_{22}^{pl} = 30\cdot10^6(+2\cdot4{,}741\cdot10^{-5}\cdot2{,}0) = 5688{,}8$$

Prstenu:





$$x = M\cdot\cos\left(\frac{\pi}{2} - \frac{d\Theta}{2}\right) = M$$

$$2x = M\,d\Theta$$

$$M\,d\Theta = 1\cdot R'_o\,d\Theta$$

$$M = R'_o$$

$$E\delta_{22}^{pr} = \frac{Eu_a - Eu_{a'}}{0{,}425}$$

$$Eu_a = E\varepsilon_a\cdot R'_o = E\frac{\sigma_a}{E}R'_o = -R'_o\cdot\frac{M}{I}\cdot z = -R_o'^2\frac{1}{\frac{1}{12}\cdot0{,}3\cdot0{,}15^3}$$

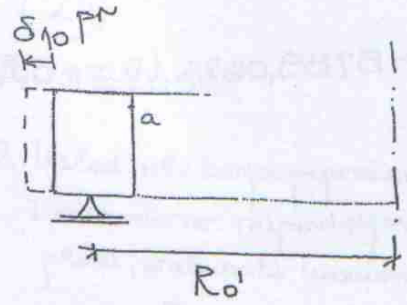
$$EI\delta_{12} = Eu_a = -258{,}86 \cdot \square T$$

$$Eu_{a'} = +\frac{2{,}15^2}{\frac{1}{12}\cdot0{,}3\cdot0{,}15^3}\cdot0{,}25 = 369{,}8$$

$$E\delta_{22}^{pr} = \frac{+258{,}86 + 369{,}8}{0{,}425} = 1479{,}2$$

$$\boxed{E\delta_{22} = E\delta_{22}^{pl} + E\delta_{22}^{pr} = 7168{,}08}$$

Stanje $X_i=0$ $t=30°C$:



$$E\delta_{10}{}^{pr} = E\cdot u_a = E\,\varepsilon_a \cdot R_0'$$

$$E\varepsilon_a R_0' = E\,\alpha t\cdot t\cdot R_0' = 19350$$

$$E\delta_{10}{}^{pr} = -19350$$

$$E\delta_{20}{}^{pr} = 0$$

$$E\delta_{10}{}^{pl} = 0$$

$$E\delta_{20}{}^{pl} = 0$$

$$E\delta_{10} = E\delta_{10}{}^{pl} + E\delta_{10}{}^{pr} = -19350$$

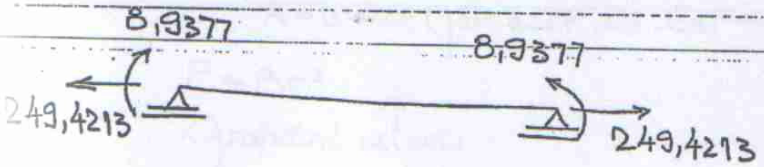
$$E\delta_{20} = E\delta_{20}{}^{pl} + E\delta_{20}{}^{pr} = 0$$

$$86,78\dot{3}\cdot X_1 - 256,86\cdot X_2 + 19350 = 0$$

$$-256,86\cdot X_1 + 7168,0\dot{8}\,X_2 = 0$$

$$X_1 = 249,4213$$

$$X_2 = 8,9377$$



$$W = W_0 + W_1 = A + Cr^2$$

Granični uslovi:

$r=20$ { $W=0$ (1)

$M_r = 8,9377$ (2)

(1): $A + 4C = 0$

(2):

$$M_r = -K\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) = -8789{,}0625\,(2C + 0{,}4C) = 8$$

$$C = -4{,}2371 \cdot 10^{-4}$$

$$A = 1{,}6949 \cdot 10^{-3}$$

$$w = 1{,}6949 \cdot 10^{-3} - 4{,}2371 \cdot 10^{-4} \cdot r^2$$

$$w(r=0) = 1{,}6949 \cdot 10^{-3}\ m$$

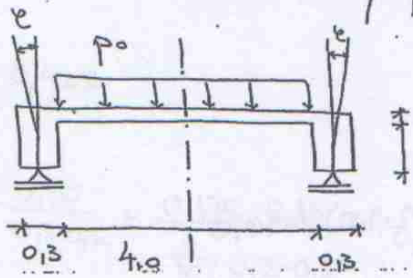
$$M_r(r=0) = -K \cdot 2{,}4C = 8{,}9377$$

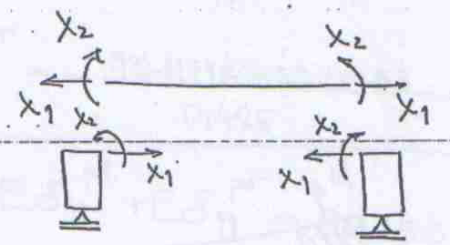
$$M_\varphi = -K\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2w}{dr^2}\right) = -K(2C + 0{,}4C) = 8{,}9377$$

(21)

27.05.2006.

2. Usled obrtanja poprečnog preseka kružnog prstena za $\varphi = 0{,}1$ rad i ravnomerno raspodeljenog opterećenja $p_0 = 20\,kN/m^2$ na kružnoj ploči, sračunati momente savijanja i ugib u centru kružne ploče.



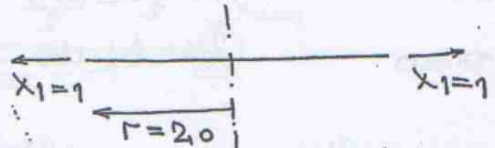


$\delta_{11} X_1 + \delta_{12} X_2 + \delta_{10} = 0$

$\delta_{21} X_1 + \delta_{22} X_2 + \delta_{20} = 0$

1. Stanje $X_1 = 1$

Ploča:



$E\delta_{11}^{pl} = Eu = E \cdot r \cdot \varepsilon = E \cdot r \frac{1}{E h} (N_r - \nu N_\varphi) = r \frac{1}{h} (N_r - \nu N_\varphi)$

$F = \cancel{D}^{0} + \cancel{A \ln r}^{0} + Br^2 + \cancel{C}^{0} r^2 \ln r$ (jer nema uticaja na napregnutje)

$A = C = 0$ (jer za $r = 0$ $\ln r \to \infty$)

$F = Br^2$

Granični uslovi:

$r = 2{,}0\,m \Rightarrow N_r = 1{,}0$

$N_r = \frac{N}{r} \frac{dF}{dr}$ $N_{r\varphi} = 0$ (rotaciono simetrično opterećenje)

$N_\varphi = \frac{d^2F}{dr^2}$

$$\frac{dF}{dr} = 2Br$$

$$\frac{d^2F}{dr^2} = 2B$$

(1): $r = 2{,}0\,m$

$$N_r = \frac{1}{2} \cdot 2B \cdot 2 = 1{,}0 \Rightarrow B = +0{,}5$$

$$F = +0{,}5\,r^2$$

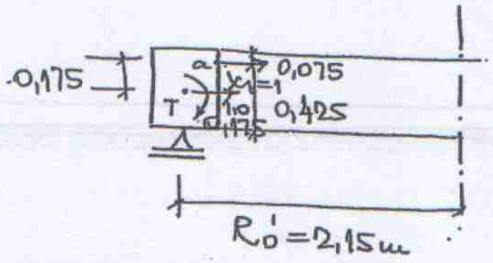
$$N_r = +1{,}0$$

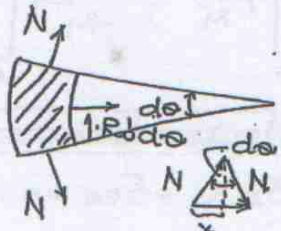
$$N_\varphi = +1{,}0$$

$$E\delta_{11}^{pl} = 2{,}0 \frac{1}{0{,}15}(+1{,}0 - 0{,}2 \cdot 1{,}0) = 10{,}6$$

$$E\delta_{21}^{pl} = 0$$

Prsten:





$$x = N \cdot \cos\left(\frac{\pi}{2} - \frac{d\theta}{2}\right) = N \cdot \sin\frac{d\theta}{2}$$

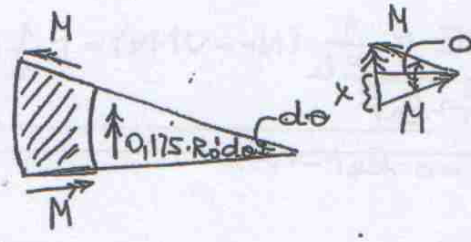
$$\sin\frac{d\theta}{2} = \frac{d\theta}{2}$$

$$2x = N \cdot d\theta$$

Iz uslova ravnoteže:

$$\Rightarrow N\,d\theta + 1 \cdot R_0' d\theta = 0$$

$$N = -R_0'$$



$$x = M \cdot \cos\left(\frac{\pi}{2} - \frac{d\theta}{2}\right) = M \cdot \frac{d\theta}{2}$$

$$2x = M \cdot d\theta$$

$$M\,d\theta + 0{,}175 R_0' d\theta = 0 \Rightarrow M = -0{,}175 R_0'$$

$$\sigma_a = \frac{N}{F} + \frac{M}{I} \cdot z = \frac{-R_0'}{6 \cdot d} + \frac{-0{,}175 \cdot R_0'}{\frac{1}{12} 6 \cdot d^3} \cdot 0{,}175$$

$$\sigma_a = \frac{-2{,}15}{0{,}3 \cdot 0{,}5} - \frac{0{,}175 \cdot 2{,}15}{\frac{1}{12} \cdot 0{,}3 \cdot 0{,}5^3} \cdot 0{,}175 = -35{,}4\,\sigma$$

$$\frac{dF}{dr} = 2Br$$

$$\frac{d^2F}{dr^2} = 2B$$

(1): $r = 2{,}0\,m$

$$N_r = \frac{1}{2} \cdot 2B \cdot 2 = 1{,}0 \Rightarrow B = +0{,}5$$

$$F = +0{,}5\,r^2$$

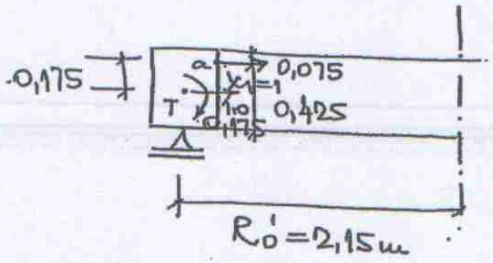
$$N_r = +1{,}0$$

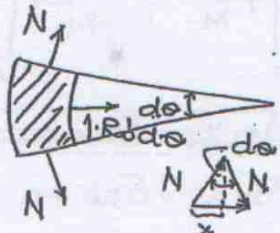
$$N_\varphi = +1{,}0$$

$$E\delta_{11}^{pl} = 2{,}0 \frac{1}{0{,}15}(+1{,}0 - 0{,}2 \cdot 1{,}0) = 10{,}6$$

$$E\delta_{21}^{pl} = 0$$

Prsten:





$$x = N \cdot \cos\left(\frac{\pi}{2} - \frac{d\Theta}{2}\right) = N \cdot \sin\frac{d\Theta}{2}$$

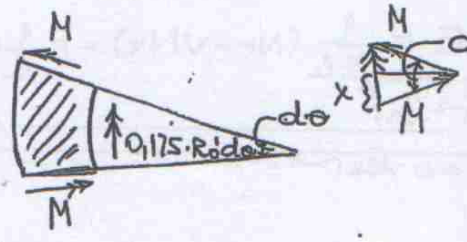
$$\sin\frac{d\Theta}{2} = \frac{d\Theta}{2}$$

$$2x = N \cdot d\Theta$$

Iz uslova ravnoteže:

$$\Rightarrow N\,d\Theta + 1 \cdot R_0' d\Theta = 0$$

$$N = -R_0'$$



$$x = M \cdot \cos\left(\frac{\pi}{2} - \frac{d\Theta}{2}\right) = M \cdot \frac{d\Theta}{2}$$

$$2x = M \cdot d\Theta$$

$$M\,d\Theta + 0{,}175 R_0' d\Theta = 0 \Rightarrow M = -0{,}175 R_0'$$

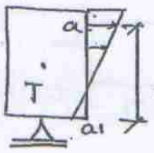
$$\sigma_a = \frac{N}{F} + \frac{M}{I} \cdot z = \frac{-R_0'}{6 \cdot d} + \frac{-0{,}175 \cdot R_0'}{\frac{1}{12} 6 \cdot d^3} \cdot 0{,}175$$

$$\sigma_a = \frac{-2{,}15}{0{,}3 \cdot 0{,}5} - \frac{0{,}175 \cdot 2{,}15}{\frac{1}{12} \cdot 0{,}3 \cdot 0{,}5^3} \cdot 0{,}175 = -35{,}4\,\dot{3}$$

$$E\delta_{11}^{pr} = E\kappa_a = E\,\varepsilon_a \cdot R_0' = E\,\frac{G_a}{E} R_0'$$

$$E\delta_{11}^{pr} = +35,403 \cdot R_0' = 76,11716$$

$$E\delta_{21}^{pr} = \frac{E\kappa_a - E\kappa_a'}{0,425}$$



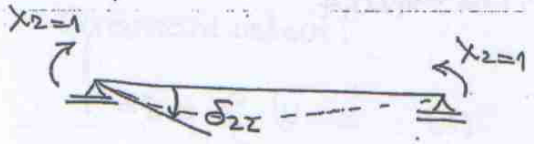
$$E\kappa_a' = \left(-\frac{2,15}{0,3 \cdot 0,5} + \frac{0,175 \cdot 2,15}{\frac{1}{12} \cdot 0,3 \cdot 0,5^3} \cdot 0,25\right) \cdot 2,15 = 33,898\dot{3}$$

$$E\delta_{21}^{pr} = -\frac{76,11716 + 33,898\dot{3}}{0,425} = -258,86$$

$$E\delta_{11} = E\delta_{11}^{pr} + E\delta_{11}^{pr} = 86,78\dot{3}$$

$$E\delta_{12} = E\delta_{21} = -258,86$$

2. Stanje $X_2 = 1$



$$E\delta_{22}^{pr} = E\,\frac{dw}{dr}$$

$$w = w_a + w_1 = A + B\ln r + Cr^2 + Dr^2 \ln r$$

$$\frac{dw}{dr} = 2Cr$$

$$\frac{d^2w}{dr^2} = 2C$$

$$M_r = -K\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right)$$

$$K = \frac{Eh^3}{12(1-\nu^2)} = \frac{30 \cdot 10^6 \cdot 0,15}{12(1-0,2^2)} = 8789,0625$$

Granični uslovi:

$$r = 2{,}0\,m \begin{cases} M_r = 1 & (1) \\ W = 0 & (2) \end{cases}$$

$$M_r = -K\left(\frac{d^2W}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) = -8789{,}0625\,(2C + 0{,}4C) = 1$$

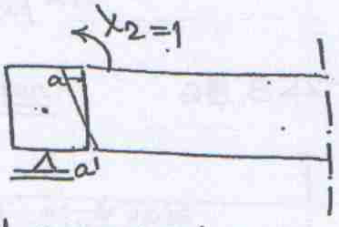
$$C = -4{,}7407 \cdot 10^{-5}$$

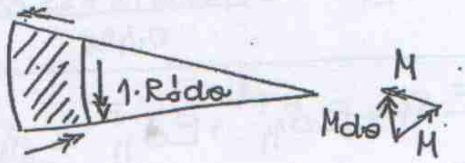
$$(2): A = 1{,}8963 \cdot 10^{-4}$$

$$W = -4{,}7407 \cdot 10^{-5} \cdot r^2 + 1{,}8963 \cdot 10^{-4}$$

$$E\delta_{22}^{pe} = +30 \cdot 10^6 \cdot 4{,}7407 \cdot 10^{-5} \cdot 2{,}0^2 = 5688{,}8$$

Prsten:





$$M d\alpha + 1 \cdot R_0' d\alpha = 0$$

$$M = R_0'$$

$$\sigma_a = +\frac{M}{I} \cdot z = +\frac{2{,}15}{\frac{1}{12} \cdot 0{,}3 \cdot 0{,}5^3} \cdot 0{,}175 = +120{,}4$$

$$\varepsilon_a = \frac{\sigma_a}{E}$$

$$E u_a = -E\,\varepsilon_a \cdot R_0' = -E\,\frac{\sigma_a}{E} R_0' = -120{,}4 \cdot 2{,}15 = -258{,}86$$

$$E\delta_{12}^{pr} = E u_{a'} = -258{,}86$$

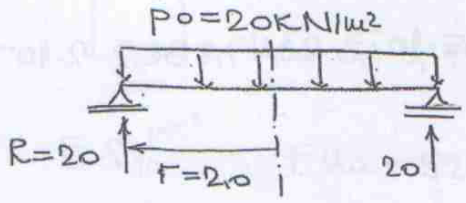
$$E\delta_{22}^{pr} = \frac{E u_a - E u_{a'}}{0{,}425}$$

$$E u_{a'} = -2{,}15 \cdot \frac{2{,}15}{\frac{1}{12} 0{,}3 \cdot 0{,}5^3} \cdot 0{,}125 = +369{,}8$$

$$E\delta_{22}^{pr} = +\frac{258{,}86 + 369{,}8}{0{,}425} = 1479{,}2$$

$$E\delta_{22} = E\delta_{22}^{pl} + E\delta_{22}^{pr} = 7168{,}8$$

3. Stanje $X_i = 0$, $p \neq 0$



$$R = \frac{2^2 \cdot \pi \cdot 20}{2 \cdot 2 \cdot \pi} = 20\,KN/m$$

$$W = W_0 + W_1$$

$$W_0 = -\frac{1}{K} \int \frac{dr}{r} \int r M dr$$

$$M = -\int \frac{dr}{r} \int r\, z(r) dr = -\int \frac{dr}{r} \int r \cdot 20 dr = -\int 10 r dr = -5r^2$$

$$W_0 = -\frac{1}{K} \int \frac{dr}{r} \int -5r^3 dr = \frac{5}{K} \frac{1}{4} \int r^3 dr = \frac{5}{16K} r^4$$

$$W = \frac{5}{16K} r^4 + A + B \ln r + Cr^2 + Dr^2 \ln r$$

$$W = \frac{5}{16K} r^4 + A + Cr^2$$

Granični uslovi:

$r = 2{,}0$ $\begin{cases} W = 0 & (1) \\ M_r = 0 & (2) \end{cases}$

(1): $\frac{5}{16K} \cdot 2^4 + A + C \cdot 4 = 0$

(2) $M_r = -K\left(\frac{d^2W}{dr^2} + \frac{\nu}{r}\frac{dW}{dr}\right) = 0$

$$\frac{dW}{dr} = 2Cr + \frac{20}{16K} r^3$$

$$\frac{d^2W}{dr^2} = 2C + \frac{60}{16K} r^2$$

$$-K\left(2C + \frac{60}{16K} \cdot 4 + 0{,}2\left(2C \cdot + \frac{20}{16K} \cdot 4\right)\right) = 0$$

$C = -7{,}5852 \cdot 10^{-4}$

$A = 2{,}4652 \cdot 10^{-3}$

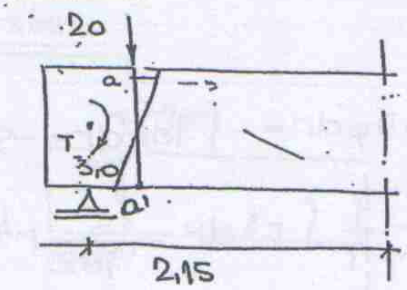
$W = 2{,}4652 \cdot 10^{-3} + 3{,}5 \cdot 10^{-5} r^4 - 7{,}5852 \cdot 10^{-4} r^2$

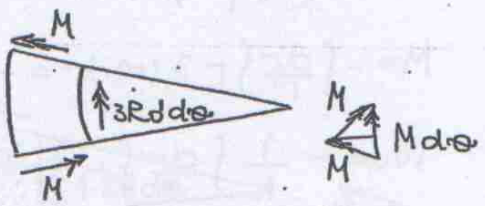
$E\delta_{20}^{pl'} = -E\left(\frac{dw}{dr}\right)_{r=2} = (4 \cdot 3{,}5 \cdot 10^{-5} \cdot 2^3 - 7{,}5852 \cdot 2 \cdot 10^{-4} \cdot 2) \cdot E$

$E\delta_{20}^{pl'} = 56888{,}8$

$E\delta_{10}^{pl'} = 0$

Prsten:





$Md\theta = -3R_0' d\theta$

$M = -3R_0' = -6{,}45$

$E\delta_{10}^{pr'} = E\,l\!l_a = E \cdot R_0' \cdot \varepsilon_a = E\frac{\sigma_a}{E} \cdot R_0' = \frac{M}{I} \cdot z \cdot R_0'$

$E\delta_{10}^{pr'} = \frac{+6{,}45}{\frac{1}{12} \cdot 0{,}3 \cdot 0{,}5^3} \cdot 0{,}175 \cdot 2{,}15 = 776{,}58$

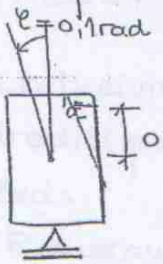
$E\delta_{10}^{pr'} = 776{,}58$

$E\delta_{20}^{pr'} = \frac{E\,l\!l_a - E\,l\!l_{a'}}{0{,}425}$

$E\,l\!l_{a'} = -\frac{6{,}45}{\frac{1}{12} \cdot 0{,}3 \cdot 0{,}5^3} \cdot 0{,}25 \cdot 2{,}15 = 1109{,}4$

$E\delta_{20}^{pr'} = -\frac{776{,}58 + 1109{,}4}{0{,}425} = 4437{,}6$

4. Starea $X_i = 0$, $p = 0$ $\varphi \neq 0$



$u_a = 0{,}1 \cdot 0{,}175 = 0{,}0175$

$E\delta_{10}^{pr''} = -Eu_a = -525000$

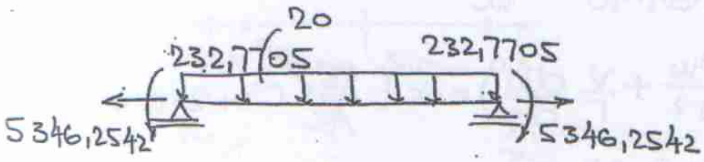
$E \cdot \delta_{20}^{pr''} = E \cdot \varphi = 30 \cdot 10^6 \cdot 0{,}1 = 30 \cdot 10^5$

$E\delta_{10} = E\delta_{10}^{pl'} + E\delta_{10}^{pr'} + E\delta_{10}^{pl''} + E\delta_{10}^{pr''} = -524223{,}42$

$E\delta_{20} = E\delta_{20}^{pl'} + E\delta_{20}^{pr'} + E\delta_{20}^{pl''} + E\delta_{20}^{pr''} = 3052451{,}28$

$X_1 = 5346{,}2542$

$X_2 = -232{,}7705$



$W = W_0 + W_1 = W_0 + A + B\ln r + Cr^2 + D r^2 \ln r$

$W_0 = -\frac{1}{K} \int \frac{dr}{r} \int r M dr$

$M = -\int \frac{dr}{r} \int r \cdot 2(r) dr = -\int 10 r dr = -5r^2$

$W_0 = \frac{5}{K} \int \frac{1}{4} r^3 dr = \frac{5}{16K} r^4$

$W = \frac{5}{16K} r^4 + A + Cr^2$

Condiții limită:

$r = 2.0$ $\begin{cases} W = 0 & (1) \\ M_r = -232{,}7705 & (2) \end{cases}$

$$\frac{dw}{dr} = \frac{5}{4K} r^3 + 2Cr$$

$$\frac{d^2w}{dr^2} = \frac{15}{4K} r^2 + 2C$$

(1): $\frac{5}{16K} \cdot 2^4 + A + 4C = 0$

(2)

$$M_r = -K\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) = -K\left(\frac{15}{4K} 2^2 + 2C + \right.$$

$$\left. + \nu \frac{5}{4K} 2^2 + \nu 2C\right) = -232{,}7705$$

$$K \cdot 2{,}4C = +14{,}5482$$

$$C = +6{,}8969 \cdot 10^{-4}$$

$$A = -1{,}1861 \cdot 10^{-2}$$

$$W = \frac{5}{16K} r^4 - 1{,}1861 \cdot 10^{-2} + 6{,}8969 \cdot 10^{-4} r^2$$

$$W(r=0) = -1{,}1861 \cdot 10^{-2}\ m$$

$$M_{r,r=0} = -K\left(\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) = -K\left(\frac{15}{4K}\cdot r^2 + 2 \cdot C + \right.$$

$$\left. + \nu \frac{5}{4K} \cdot r^2 + 2 \cdot 0{,}2 \cdot C\right)$$

$$M_{r,(r=0)} = -14{,}5482$$

$$M_e = -K\left(\frac{1}{r}\frac{dw}{dr} + \nu \frac{d^2w}{dr^2}\right) = -K\left(\frac{5}{4K} \cdot r^2 + 2C + \nu \frac{15}{4K} \cdot r^2 + 0{,}4C\right)$$

$$M_e(r=0) = -14{,}5482$$

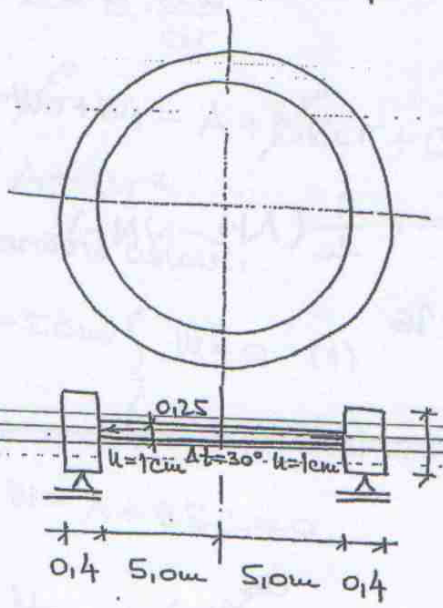
(22)

24.11.2007.

1. Za rotaciono simetričnu konstrukciju prikazanu na slici odrediti pomeranja i presečne sile u kružnoj ploči i prstenu usled:

a) Pomeranja spoja grede i prstena za $u=1cm$

b) Razlike zagrevanja donje i gornje strane ploče za $\Delta t=30°C$.



$E=3,3\cdot10^7\ KN/m^2$

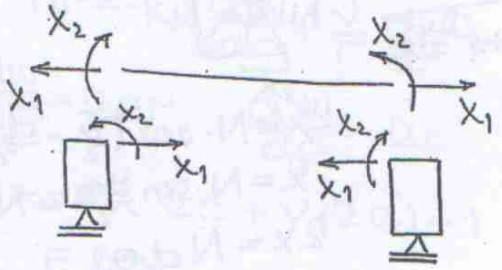
$\nu=0,2$

$d_{pl}=0,25m$

$\alpha_t=1,2\cdot10^{-5}$

Greda:

$b/d=0,4/0,16m$

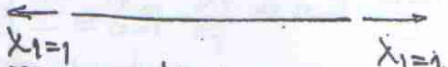


$\delta_{11}X_1+\delta_{12}X_2+\delta_{10}=0$

$\delta_{21}X_1+\delta_{22}X_2+\delta_{20}=0$

1. Stanje $X_1=1$

Ploča:



$F=\cancel{D}+\cancel{A^0}\ln r+Br^2+\cancel{C}r^2\ln r$ ← nepovoljan uticaj na napregnuje

$A=C=0$, jer za $r=0$ $\ln r\to\infty$

$F=Br^2$

Okrajični uslovi:

$r = 5{,}0\,m \;\Big\}\; N_r = 1{,}0$

$N_r = \frac{1}{r}\frac{dF}{dr} = \frac{1}{r}\cdot 2B\cdot r = 2B = 1{,}0$

$B = 0{,}5$

$N_r = 2B = 1$

$N_\varepsilon = \frac{d^2F}{dr^2} = 2B = 1$

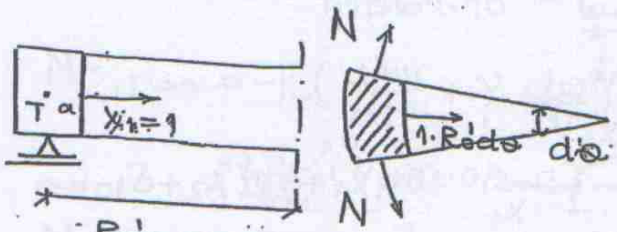
$\varepsilon_\varepsilon = \frac{1}{Eh}(N_\varepsilon - \nu N_r)$

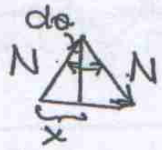
$E\delta_{11}^{pl} = E\cdot u = E\cdot\varepsilon_\varepsilon\cdot r = r\,\frac{1}{h}(N_\varepsilon - \nu N_r)$

$\rightarrow E\delta_{11}^{pl} = 5\,\frac{1}{0{,}25}(1-0{,}2) = 16$

$E\delta_{21}^{pl} = 0$

Prsten:





$\chi = N\cdot\cos\left(\frac{\pi}{2} - \frac{d\Theta}{2}\right)$

$\chi = N\cdot\sin\frac{d\Theta}{2} = N\frac{d\Theta}{2}$

$2\chi = N\,d\Theta$

$N\,d\Theta + 1\cdot R'_o\,d\Theta = 0$

$N = -R'_o$

$E u_a = E\varepsilon_a\cdot R'_o = E\frac{\sigma_a}{E}\cdot R'_o = \frac{N}{F}\cdot R'_o = -\frac{R_o'^2}{b\cdot d} = -\frac{5{,}2^2}{0{,}4\cdot 0{,}6}$

$E u_a = -112{,}6$

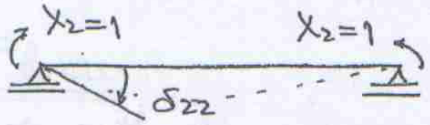
$E\delta_{11}^{pr} = -E u_a = 112{,}6$

$E\delta_{21}^{pr} = 0$

$$E\delta_{11} = E\delta_{11}^{pl} + E\delta_{11}^{pr} = 128{,}6$$

$$E\delta_{21} = E\delta_{12} = 0$$

2. Stanje $X_2 = 1$



$$E\delta_{22}^{pl} = E \cdot \frac{dw}{dr}$$

$$W = W_0^{\nearrow 0} + W_1 = A + B\ln r + Cr^2 + D^{\nearrow 0} r^2 \ln r$$

$$W = A + Cr^2$$

Granični uslovi:

$r = 5{,}0\,m$ $\begin{cases} W = 0 & (1) \\ M_r = 1 & (2) \end{cases}$

(1): $W = A + 25C = 0$

(2):

$$M_r = -K\left(\frac{d^2W}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right)$$

$$\frac{dw}{dr} = 2Cr \qquad \frac{d^2W}{dr^2} = 2C$$

$$M_r = -K(2C + \nu \cdot 2C) = 1 \Rightarrow C = -9{,}3091 \cdot 10^{-6}$$

$$K = \frac{E h^3}{12(1-\nu^2)} = 44759{,}1146$$

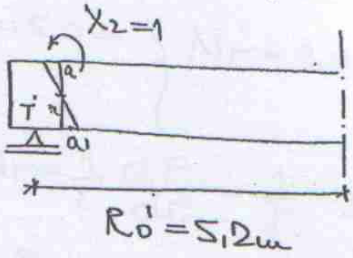
$$A = 2{,}3273 \cdot 10^{-4}$$

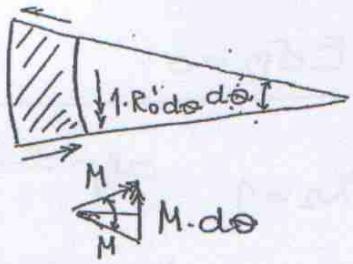
$$W = 2{,}3273 \cdot 10^{-4} - 9{,}3091 \cdot 10^{-6} \cdot r^2$$

$$E\delta_{22}^{pl} = -E \cdot (-2 \cdot 9{,}3091 \cdot 10^{-6} \cdot 5{,}0) = +3072$$

$$E\delta_{12}^{pl} = 0$$

Prsten:





$M d\Theta - 1\cdot R_0' d\Theta = 0$

$M = R_0'$

$E\delta_{22}^{pr} = E\cdot\chi = E\frac{u_a - u_a'}{-a} = E\frac{\varepsilon_a\cdot R_0' - \varepsilon_a'\cdot R_0'}{a} = \frac{\frac{M}{I}\cdot\frac{d}{2} + \frac{M}{I}\cdot\frac{d}{2}}{a}\cdot R_0'$

$E\delta_{22}^{pr} = \left[\frac{2R_0'}{\frac{1}{12}\cdot 0,4\cdot 0,6^3\cdot 0,3}\right]\cdot 5,12 = 3755,\dot{5}$

$E\delta_{12}^{pr} = 0$

$$E\delta_{22} = E\delta_{22}^{pl} + E\delta_{22}^{pr} = 6827,\dot{5}$$
$$E\delta_{12} = 0$$

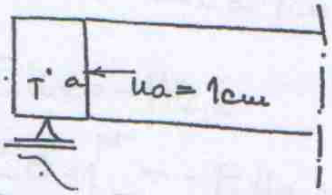
a) $u = 1cm$

Ploča:

$E\delta_{10}^{pl} = E\cdot u = 3,3\cdot 10^7\cdot 0,01 = 330000$

$E\delta_{20}^{pl} = 0$

Prsten:



$E\delta_{10}^{pr} = E u_a = 330000$

$E\delta_{20}^{pr} = 0$

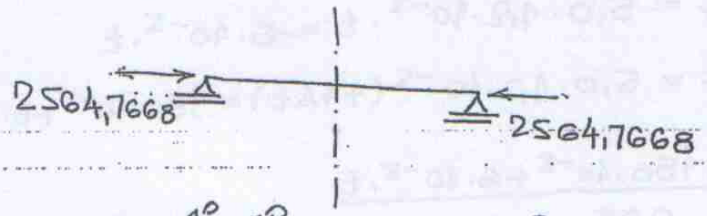
$$128{,}6\,X_1 + 0\cdot X_2 + 830000 = 0$$

$$0\cdot X_1 + 6827{,}5\,X_2 + 0 = 0$$

$$X_1 = -2564{,}7668$$

$$X_2 = 0$$

Ploča:



$$F = \cancel{D}^{r^0} + \cancel{A}^{r^0}\ln r + Br^2 + \cancel{C}^{r^0} r^2 \ln r$$

$$F = Br^2$$

Granični uslovi:

$$r = 5{,}0 \;\Big\{\; N_r = -2564{,}7668 \quad (1)$$

(1).:

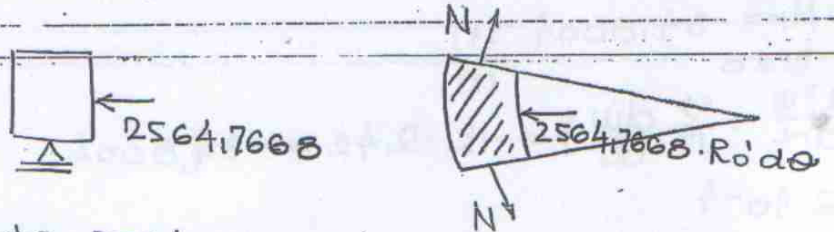
$$N_r = \frac{1}{r}\frac{dF}{dr} = \frac{1}{r}\cdot 2Br = 2B = -2564{,}7668$$

$$B = -1282{,}3834$$

$$N_r = N_\theta = -2564{,}7668 \text{ KN/m}$$

$$N_{r\theta} = 0$$

Prsten:



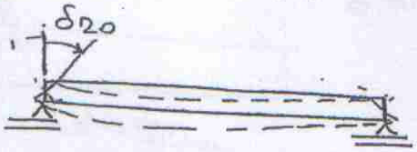
$$N\,d\theta = 2564{,}7668\cdot R_0'\,d\theta$$

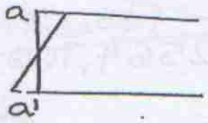
$$N = 13336{,}7876 \text{ KN}$$

Oni su radili odvojeno ploču, a odvojeno prsten?

b) $\Delta t = 30°$

Ploča:





$$E\delta_{20} = E \cdot \chi = E \frac{u_{a'} + u_a}{h}$$

$$u_a = r \cdot \varepsilon_e = r \cdot \alpha_t \cdot t = 5{,}0 \cdot 1{,}2 \cdot 10^{-5} \cdot t = -6 \cdot 10^{-5} \cdot t$$

$$u_{a'} = r \cdot \varepsilon_{e'} = r \cdot \alpha_t \cdot t = 5{,}0 \cdot 1{,}2 \cdot 10^{-5} (t + \Delta t) = 180 \cdot 10^{-5} + 6 \cdot$$

$$E\delta_{20} = E \cdot \frac{-6 \cdot 10^{-5} t + 180 \cdot 10^{-5} + 6 \cdot 10^{-5} \cdot t}{0{,}25}$$

$$E\delta_{20}^{M} = 237600$$

$$E\delta_{10}^{M} = 0$$

$$6827{,}5 \cdot X_2 + 237600 = 0$$

$$X_2 = -34{,}8004$$

$$W = W_0 + W_1 = A + Cr^2$$

$$\frac{dw}{dr} = 2Cr \qquad \frac{d^2w}{dr^2} = 2C$$

Granični uslovi:

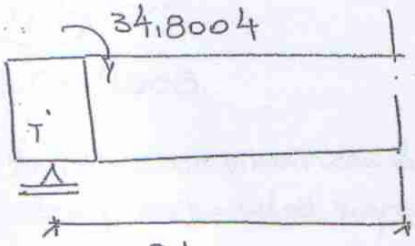
$r = 5{,}0$ $\begin{cases} M_r = -34{,}8004 & (1) \\ W = 0 \end{cases}$

$$M_r = +K \left(\frac{d^2w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right) = -K \cdot 2{,}4C = -34{,}8004 \qquad (2)$$

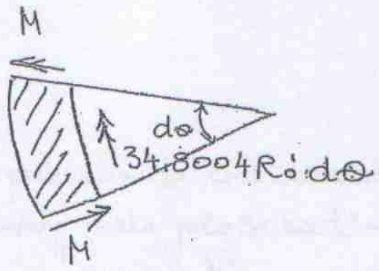
$$C = 3{,}2396 \cdot 10^{-4}$$

(2): $A = -8{,}0990 \cdot 10^{-3}$

$$M_r = M_e = -34{,}8004 \text{ KNm/m'}$$



$R_0' = 5,2$



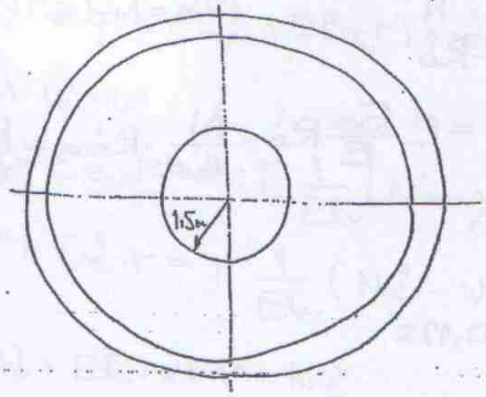
$Mdθ = -34,8004 \cdot R_0' dθ$

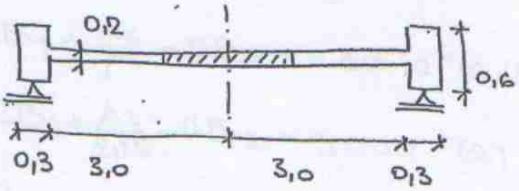
$M = -180,9623\ KNm$

(23)

22.04.2008.

2. Za rotaciono simetričnu konstrukciju prikazanu na slici odrediti pomeranja i sile u preseku usled zagrevanja označenog dela ploče za $t° = 50°C$.

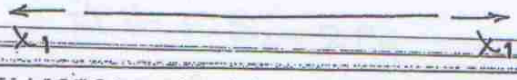




$\alpha_t = 10^{-5}\ 1/°C$

$E = 30\ GPa$

$\nu = 0$



Stanje $X_1 = 1$

Ploča:

$$E\delta_{11} = Eu = E\varepsilon_\varphi \cdot r = E\frac{1}{Eh}(N_r - \nu N_\varphi)\cdot r = r\frac{1}{h}\cdot N_r$$

$F = \overset{0}{D} + \overset{0}{C}\ln r + Br^2 + \overset{0}{C_1} r^2 \ln r$

$F = Br^2$

Granični uslovi:

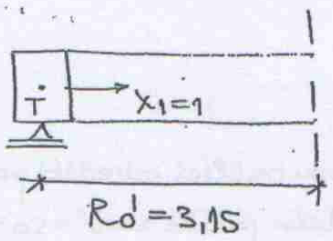
$r = a = 3{,}0\,m \Rightarrow N_r = 1{,}0$

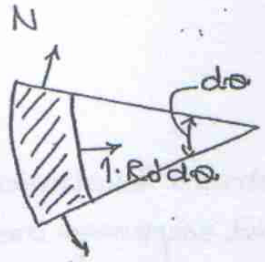
$$N_r = \frac{1}{r}\frac{dF}{dr} = 2B = 1{,}0 \Rightarrow B = 0{,}5$$

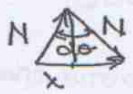
$N_r = 1{,}0$

$$E\delta_{11}^{P} = 3\cdot\frac{1}{0{,}2}\cdot 1 = 15$$

Prsten:







$X = N \cdot \cos\left(\frac{\pi}{2} - \frac{d\vartheta}{2}\right) = N$

$2X = N\,d\vartheta$

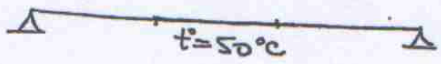
$N\,d\vartheta + 1 \cdot R_0'\,d\vartheta = 0 \Rightarrow N = -R_0'$

$E\delta_{11}^{pr} = -E \cdot \varepsilon_a = E \cdot \varepsilon_a \cdot R_0' = E\,\frac{\sigma_a}{E} R_0' = \frac{N}{b \cdot d} \cdot R_0' = +\frac{R_0'^2}{b \cdot d}$

$E\delta_{11}^{pr} = \frac{3{,}15^2}{0{,}3 \cdot 0{,}6} = 55{,}125$

$E\delta_{11} = E\delta_{11}^{pl} + E\delta_{11}^{pr} = 70{,}125$

Stanje $X_1 = 0$, $t = 50°C$



I

II $\overline{t = 50°C}$

$F' = \cancel{D_1^{=0}} + \cancel{A_1^{=0}} \ln r + B_1 r^2 + \cancel{C_1^{=0}} r^2 \ln r$

$F' = B_1 r^2$

$F'' = \cancel{D_2^{=0}} + A_2 \ln r + B_2 r^2 + \cancel{C_2^{=0}} r^2 \ln r$ ← 0 važi za kružne ploče napregnute u svojoj ravni i rotaciono simetričnim opterećenjem

$F'' = A_2 \ln r + B_2 r^2$

$\varepsilon_r = \frac{du}{dr}$, $u = \varepsilon_\theta \cdot r \Rightarrow \frac{du}{dr} = \varepsilon_\theta$

$\varepsilon_r = \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr} \Rightarrow C = 0$

Granični uslovi:

$r = 3{,}0\,m \;\Big\{\; N_r'' = 0 \quad (1)$

Prelazni uslovi:

$r = 1{,}5\,m \;\Big\{\; u' = u'' \quad (2)$

$N_r' = N_r'' \quad (3)$

$$N_r = \frac{1}{r}\frac{dF}{dr}$$

$$N_e = \frac{d^2F}{dr^2}$$

(1) $r = 3{,}0\,m$

$$N_r = \frac{1}{r}\left(A_2\frac{1}{r} + 2B_2 r\right) = \frac{1}{r^2}A_2 + 2B_2 = \frac{1}{9}A_2 + 2B_2 = 0 \quad (1)$$

(2) $u' = u''$

$$u' = \varepsilon'_e \cdot r = r\left[\frac{1}{Eh}\left(N'_e - \nu N'_r\right) + \alpha_t \cdot t^\circ\right]$$

$$u'' = \varepsilon''_e \cdot r = r\,\frac{1}{Eh}\left(N''_e - \nu N''_r\right)$$

$$N'_e + Eh \cdot \alpha_t \cdot t^\circ = N''_e$$

$$2B_1 + \frac{A_2}{r^2} - 2B_2 = -30\cdot 10^6 \cdot 0{,}2\cdot 10^{-5}\cdot 50$$

$$2B_1 + \frac{A_2}{2{,}25} - 2B_2 = -3000 \quad (2)$$

(3) $r = 1{,}5$

$$\frac{1}{r^2}A_2 + 2B_2 = 2B_1$$

$$2B_1 - \frac{1}{2{,}25}A_2 - 2B_2 = 0 \quad (3)$$

$$B_1 = -562{,}5$$

$$A_2 = -3375$$

$$B_2 = 187{,}5$$

$$F' = -562{,}5\cdot r^2$$

$$F'' = -3375\ln r + 187{,}5\cdot r^2$$

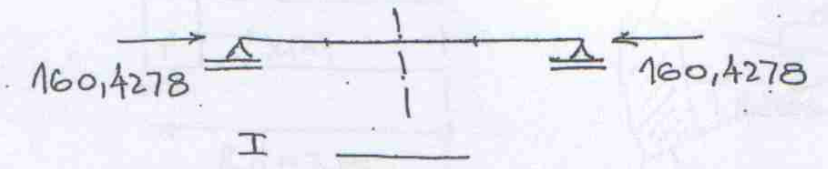
$$E\delta_{10} = E u''_{(r=3{,}0\,m)} = E\cdot\varepsilon''_e\cdot r = E\,\frac{1}{Eh}\left(N_x - \nu N_r\right)\cdot r$$

$$E\delta_{10} = \frac{1}{0{,}2}\cdot\left(-\frac{1}{9}\cdot A_2 + 2B_2\right)\cdot 3{,}0 = 11250$$

$$\delta_{11}X_1 + \delta_{10} = 0$$

$$70{,}125X_1 + 11250 = 0 \Rightarrow X_1 = -160{,}4278$$

Ploča:



Granični uslovi:

$$r=3{,}0\,m \;\Big\{\; N_r'' = -160{,}4278 \quad (1)$$

Prelazni uslovi:

$$r=1{,}5\,m \;\Big\{\; u_r' = u_r'' \quad (2)$$

$$N_r' = N_r'' \quad (3)$$

$$F' = B_1 r^2$$

$$F'' = A_2 \ln r + B_2 r^2$$

(1): $\frac{1}{9} A_2 + 2B_2 = -160{,}4278$ (1)

(2): $2B_1 + \frac{A_2}{2{,}25} - 2B_2 = -3000$ (2)

(3): $2B_1 - \frac{A_2}{2{,}25} - 2B_2 = 0$ (3)

$B_1 = -642{,}7139$

$A_2 = -3375$

$B_2 = 107{,}2861$

$N_r' = -642{,}7139$

$N_\theta' = -642{,}7139$

$N_{r\theta}' = 0$

$u' = \varepsilon_\theta' \cdot r = r\left[\frac{1}{Eh}\left(N_\theta' - \nu N_r'\right) + \text{[illegible]}\right] = 2{,}8576 \cdot 10^{-4} \cdot r$

$u'(r=1{,}5) = 4{,}2864 \cdot 10^{-4}$

$$N_r'' = \frac{1}{r}\left(\frac{A_2}{r} + 2B_2 r\right) = \frac{A_2}{r^2} + 2B_2 = -\frac{3375}{r^2} + 214{,}5722$$

$$N_r''(r=3{,}0) = -160{,}4278$$

$$N_e'' = \frac{d^2 r}{dr^2} = -\frac{A_2}{r^2} + 2B_2 = \frac{3375}{r^2} + 214{,}5722$$

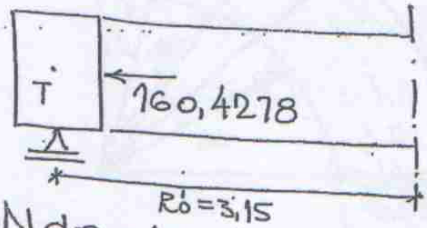
$$N_e''(r=3{,}0) = 589{,}5722$$

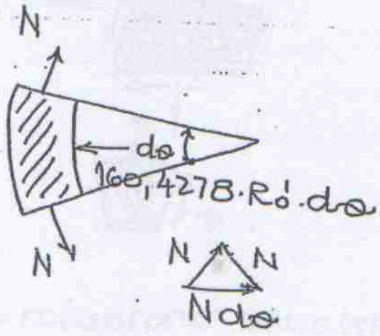
$$N_{re}'' = 0$$

$$u'' = \varepsilon_e'' \cdot r = r \cdot \frac{1}{Eh}\left(N_e'' - \cancel{\nu} N_r''\right) = \frac{1}{Eh} \cdot \left(\frac{3375}{r} + 214{,}5722\, r\right)$$

$$u''(r=3{,}0) = 2{,}9479 \cdot 10^{-4}$$

Prsten:





$$N d\alpha = 160{,}4278 \cdot R_0' d\alpha$$

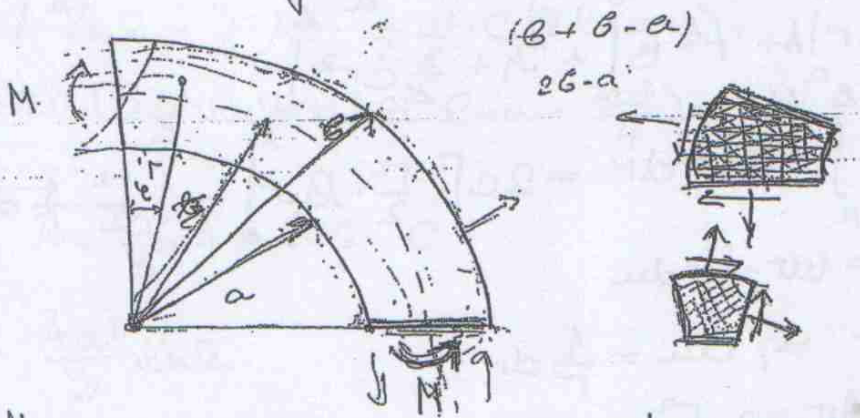
$$N = 160{,}4728 \cdot R_0' = 505{,}3476$$

$$u = \varepsilon_a \cdot R_0' = \frac{\sigma_a}{E} \cdot R_0' = \frac{N}{EF} \cdot R_0' = \frac{160{,}4728 \cdot R_0'^2}{30 \cdot 10^6 \cdot 0{,}3 \cdot 0{,}6} = 2{,}9479 \cdot 10^{-4}$$

(24)

22.12.2007.

2. Za kružni luk prikazan na slici, koji je na krajevima opterećen momentima M odrediti izraze za presečne sile. Za $b = 2a$ nacrtati dijagrame presečnih sila. Uporediti rešenje za silu N_φ sa rešenjem dobijenim ako se dati luk tretira kao gredni nosač.



Može da se primeni rešenje za rotaciono simetrično opterećenje jer ubrzanja koja će se javiti ne zavise od φ

$$F = \cancel{D} + A \ln r + B r^2 + C r^2 \ln r$$

Granični uslovi:

$$r = a \begin{cases} N_r = 0 \ (1) \\ N_{r\varphi} = 0 \end{cases} \qquad r = b \begin{cases} N_r = 0 \ (2) \\ N_{r\varphi} = 0 \end{cases}$$

$$\varphi = 0 \\ \varphi = \frac{\pi}{4} \begin{cases} \int_a^b N_\varphi \, dr = 0 \\ \int_a^b N_\varphi \cdot r \, dr = -M \ (3) \end{cases}$$

$$N_r = \frac{1}{r}\frac{dF}{dr} = \frac{A}{r} + 2Br + 2Cr \ln r + Cr = \frac{A}{r} + 2Br + Cr(2\ln r +$$

$$N_\varphi = \frac{d^2F}{dr^2} = -\frac{A}{r^2} + 2B + C(2\ln r + 3)$$

(1): $r = a$

$$\frac{A}{a} + 2Ba + C \cdot a(2\ln a + 1) = 0$$

(2) $r=b$

$$\frac{A}{b}+2B\cdot b+C\cdot b(2\ln b+1)=0$$

(3) $$\int_a^b\left(-\frac{A}{r^2}+2B+C(2\ln r+3)\right)\cdot r\,dr=$$

$$=\int_a^b-\frac{A}{r}dr+\int_a^b 2Br\,dr+2C\int_a^b r\ln r\,dr+3C\int_a^b r\,dr=$$

$$=-\ln r\Big|_a^b A+r^2\cdot B\Big|_a^b+J_1+\frac{3}{2}Cr^2\Big|_a^b$$

$$J_1=2C\int_a^b r\cdot\ln r\,dr=2C\left[\frac{r^2}{2}\cdot\ln r\Big|_a^b-\int_a^b\frac{r^2}{2}\cdot\frac{1}{r}dr\right]$$

$\int u\cdot dv=uv-\int v\,du$

$u=\ln r \Rightarrow du=\frac{1}{r}dr$

$r\,dr=dv\Rightarrow \frac{r^2}{2}=v$

$$J_1=2C\left[\frac{b^2}{2}\ln b-\frac{a^2}{2}\ln a-\frac{r^2}{4}\Big|_a^b\right]$$

$$J=-\ln\frac{b}{a}\cdot A+B(b^2-a^2)+2C\left(\frac{b^2}{2}\ln b-\frac{a^2}{2}\ln a-\frac{b^2}{4}+\frac{a^2}{4}\right)$$
$$+\frac{3}{2}C(b^2-a^2)=-M$$

$b=2a$:

$$\frac{A}{a}+2Ba+Ca(2\ln a+1)=0 \quad (1)$$

$$\frac{A}{2a}+4Ba+2Ca(2\ln 2a+1)=0 \quad (2)$$

$$-\ln 2\cdot A+B\cdot 3a^2+2C\left(2a^2\ln 2a-\frac{a^2}{2}\ln a-a^2+\frac{a^2}{4}\right)=$$

$(1)\Rightarrow 2B\cdot a=-Ca(2\ln a+1)-\frac{A}{a}$

$$\frac{A}{2a}-\frac{2A}{a}+2Ca(2\ln 2a+1)-2Ca(2\ln a+1)=0$$

$$-\frac{3A}{a} + 2C \cdot a\left(2\ln 2 + \cancel{2\ln a} \cancel{+1 - 2\ln a} \cancel{-1}\right) = 0$$

$$-\frac{3A}{a} + 4a\ln 2 \cdot C = 0$$

$$-\ln 2 \cdot A - 3a^2\left(\frac{1}{2}C(2\ln a + 1) + \frac{1}{2a^2}A\right) +$$

$$+2C\left(2a^2\ln 2 + 2a^2\ln a - \frac{a^2}{2}\ln a - \frac{3a^2}{4}\right) = -M$$

$$-A\cdot\ln 2 - \frac{3}{2}\cdot A - \frac{3}{2}a^2 \cdot C(2\ln a + 1) +$$

$$+2C\left(2a^2\ln 2 + \frac{3a^2}{2}\ln a - \frac{3a^2}{4}\right) = -M$$

$$A = \frac{a}{3} \cdot 4a\ln 2 \cdot C$$

$$-\frac{4a^2}{3}\ln 2$$