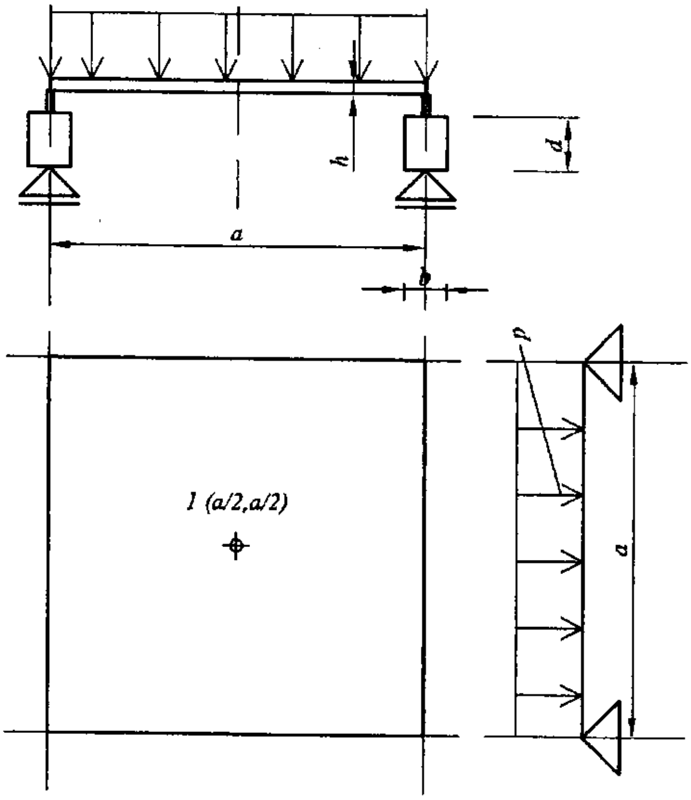
УНИВЕРЗИТЕТ У БЕОГРАДУ
ГРАЂЕВИНСКИ ФАКУЛТЕТ

КАТЕДРА ЗА ТЕХНИЧКУ МЕХАНИКУ
И ТЕОРИЈУ КОНСТРУКЦИЈА
ТЕОРИЈА ПЛОЧА И ЉУСКИ
Школска 2008/2009.

Кандидат:

ПРВИ ГРАФИЧКИ РАД

1. За квадратну плочу приказану на слици:
   а) Одредити решење за угиб и пресечне силе у произвољној тачки плоче
   б) Користећи решење под а) одредити померање плоче и моменте савијања у тачки 1.
   Користити први члан реда усвојеног решења.



E=33 GPa

$\nu = 0.3$

h = 0,2 m

a = 8 m

p = 6 kN/m^2

греда:

b/d= 30/40 cm

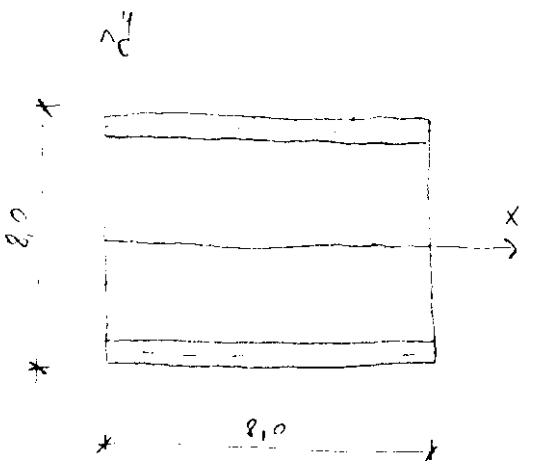
I_t = 0,002 m^4

Задатак задала: [signature]

Датум: 24.03.2009.

Оцена: 7(седам) 8

Први графички рад



$E = 33\ GPa$

$\nu = 0,3$

$l_1 = 0,2\ m$

$a = 8\ m$

$p = 6\ kN/m^2$

Греда:

$b/d = 30/50\ cm$

$I_t = 0,002\ m^4$

$I_G = \frac{0,3\cdot 0,5^3}{12} = 0,0016\ m^4$

$$K = \frac{E l_1^3}{12(1-\nu^2)} = \frac{33\cdot 10^6 \cdot 0,2^3}{12(1-0,3^2)} = 24175,82$$

$$y = \pm\frac{a}{2} \quad \begin{cases} w = 0 \\ \frac{\partial w^p}{\partial y} = \varphi^G \end{cases}$$

a) из услова симетрије $B_1 = C_1 = 0$

$$w_h = \left(A_1 ch\frac{\pi y}{a} + D_1 \frac{\pi y}{a} sh\frac{\pi y}{a}\right)\sin\frac{\pi x}{a}$$

$$Z_1 = \frac{2}{a}\int_{x_1}^{x_2} z(x)\sin\frac{\pi x}{a}dx$$

$$= \frac{2}{a}\int_0^a p\sin\frac{\pi x}{a}dx = -\frac{2p}{a}\cdot\frac{a}{\pi}\cos\frac{\pi x}{a}\Big|_0^a = -\frac{2p}{\pi}(\cos\pi - \cos 0) = \frac{4p}{\pi}$$

$$w_p = \left(\frac{a}{\pi}\right)^4\frac{Z_1}{K}\sin\frac{\pi x}{a} = \left(\frac{a}{\pi}\right)^4\cdot\frac{4p}{K\pi}$$

$$w = w_h + w_p = \left[A_1 ch\frac{\pi y}{a} + D_1\frac{\pi y}{a} sh\frac{\pi y}{a} + \left(\frac{a}{\pi}\right)^4\cdot\frac{4p}{K\pi}\right]\sin\frac{\pi x}{a}$$

(1) $w = 0$ за $y = \pm\frac{a}{2}$

$$A_1 ch\frac{\pi}{2} + D_1\frac{\pi}{2} sh\frac{\pi}{2} + \left(\frac{a}{\pi}\right)^4\frac{4p}{K\pi} = 0$$

(2) $\frac{\partial w^p}{\partial y} = \varphi^G \ / \ \frac{\partial}{\partial x}$

$$\frac{\partial^2 w^p}{\partial x\partial y} = \frac{\partial\varphi^G}{\partial x}, \quad \frac{\partial\varphi^G}{\partial x} = \theta = \frac{M_t}{GI_t}$$

$$\frac{\partial^2 w^p}{\partial x\partial y} = \frac{M_t}{GI_t} \ / \ \frac{\partial}{\partial x}$$

$$\frac{\partial^3 w^p}{\partial x^2\partial y} = \frac{1}{GI_t}\frac{dM_t}{dx}, \quad \frac{dM_t}{dx} = m_y$$

$$\frac{\partial^3 w}{\partial x^2\partial y} = \frac{1}{GI_t}m_y$$

$$-\left(\frac{\pi}{a}\right)^3\left[D_1\frac{\pi}{2}\operatorname{ch}\frac{\pi}{2}+(A_1+D_1)\operatorname{sh}\frac{\pi}{2}\right]=-\frac{k}{GI_t}\left(\frac{\pi}{a}\right)^2\left[\left((1-\nu)A_1+2D_1\right)\operatorname{ch}\frac{\pi}{2}+(1-\nu)D_1\frac{\pi}{2}\operatorname{sh}\frac{\pi}{2}-\nu\left(\frac{a}{\pi}\right)^4\frac{4p}{k\pi}\right]$$

$$\frac{GI_t}{k}\cdot\frac{\pi}{a}\left[D_1\frac{\pi}{2}\operatorname{ch}\frac{\pi}{2}+(A_1+D_1)\operatorname{sh}\frac{\pi}{2}\right]=\left((1-\nu)A_1+2D_1\right)\operatorname{ch}\frac{\pi}{2}+(1-\nu)D_1\frac{\pi}{2}\operatorname{sh}\frac{\pi}{2}-\nu\left(\frac{a}{\pi}\right)^4\frac{4p}{k\pi}$$

$$\left(\frac{GI_t}{k}\cdot\frac{\pi}{a}\operatorname{sh}\frac{\pi}{2}-(1-\nu)\operatorname{ch}\frac{\pi}{2}\right)A_1+\left(\frac{GI_t}{k}\cdot\frac{\pi}{a}\cdot\left(\frac{\pi}{2}\operatorname{ch}\frac{\pi}{2}+\operatorname{sh}\frac{\pi}{2}\right)-(1-\nu)\frac{\pi}{2}\operatorname{sh}\frac{\pi}{2}\right)D_1=-\nu\left(\frac{a}{\pi}\right)^4\cdot\frac{4p}{k\pi}$$

$$2{,}5092A_1+3{,}6149\,D_1=-13{,}2874\cdot10^{-3}$$

$$-0{,}8075A_1+0{,}0437\,D_1=-3{,}9862\cdot10^{-3}$$

$$A_1=4{,}5661\cdot10^{-3}$$

$$D_1=-6{,}8453\cdot10^{-3}$$

$$M_x=-k\left(\frac{\partial^2w}{\partial x^2}+\nu\frac{\partial^2w}{\partial y^2}\right)$$

$$=k\left(\frac{\pi}{a}\right)^2\left[\left((1-\nu)A_1-2\nu D_1\right)\operatorname{ch}\frac{\pi y}{a}+(1-\nu)D_1\frac{\pi y}{a}\operatorname{sh}\frac{\pi y}{a}+\frac{4pa^4}{k\pi^5}\right]\sin\frac{\pi x}{a}$$

$$M_y=-k\left(\frac{\partial^2w}{\partial y^2}+\nu\frac{\partial^2w}{\partial x^2}\right)$$

$$=-k\left(\frac{\pi}{a}\right)^2\left[\left((1-\nu)A_1+2D_1\right)\operatorname{ch}\frac{\pi y}{a}+(1-\nu)D_1\frac{\pi y}{a}\operatorname{sh}\frac{\pi y}{a}-\nu\frac{4pa^4}{k\pi^5}\right]\sin\frac{\pi x}{a}$$

$$M_{xy}=-k(1-\nu)\frac{\partial^2w}{\partial x\partial y}$$

$$=-k(1-\nu)\cdot\left(\frac{\pi}{a}\right)^2\left[D_1\cdot\frac{\pi y}{a}\operatorname{ch}\frac{\pi y}{a}+(A_1+D_1)\operatorname{sh}\frac{\pi y}{a}\right]\cos\frac{\pi x}{a}$$

$$T_x=-k\left(\frac{\partial^3w}{\partial x^3}+\frac{\partial^3w}{\partial x\partial y^2}\right)$$

$$=-k\left(\frac{\pi}{a}\right)^3\cdot\left(2D_1\operatorname{ch}\frac{\pi y}{a}-\frac{4pa^4}{k\pi^5}\right)\cos\frac{\pi x}{a}$$

$$T_y=-k\left(\frac{\partial^3w}{\partial y^3}+\frac{\partial^3w}{\partial x^2\partial y}\right)$$

$$=-k\left(\frac{\pi}{a}\right)^3\cdot2D_1\operatorname{sh}\frac{\pi y}{a}\sin\frac{\pi x}{a}$$

$$w=10^{-3}\cdot\left(4{,}5661\operatorname{ch}\frac{\pi y}{8}-2{,}6881\,y\operatorname{sh}\frac{\pi y}{8}+13{,}2874\right)\sin\frac{\pi x}{8}$$

$$M_x=\left(27{,}2288\operatorname{ch}\frac{\pi y}{8}-7{,}0154\,y\operatorname{sh}\frac{\pi y}{8}+49{,}5384\right)\sin\frac{\pi x}{8}$$

$$M_y=\left(39{,}1251\operatorname{ch}\frac{\pi y}{8}+7{,}0154\,y\operatorname{sh}\frac{\pi y}{8}+14{,}8615\right)\sin\frac{\pi x}{8}$$

$$M_{xy}=\left(7{,}0154\,y\operatorname{ch}\frac{\pi y}{8}+5{,}9481\operatorname{sh}\frac{\pi y}{8}\right)\cos\frac{\pi x}{8}$$

$$T_x=\left(20{,}0439\operatorname{ch}\frac{\pi y}{8}+19{,}4537\right)\cos\frac{\pi x}{8}$$

$$T_y=20{,}0439\operatorname{sh}\frac{\pi y}{8}\sin\frac{\pi x}{8}$$

в) Точка (4;0)

$w = 17{,}8535 \cdot 10^{-3}$

$M_x = 76{,}7672$ kNm

$M_y = 53{,}9866$ kNm

$M_{xy} = 0$

$T_x = 0$

$T_y = 0$

✓

7 (седам)

02.06.2009.

Н Понаревић

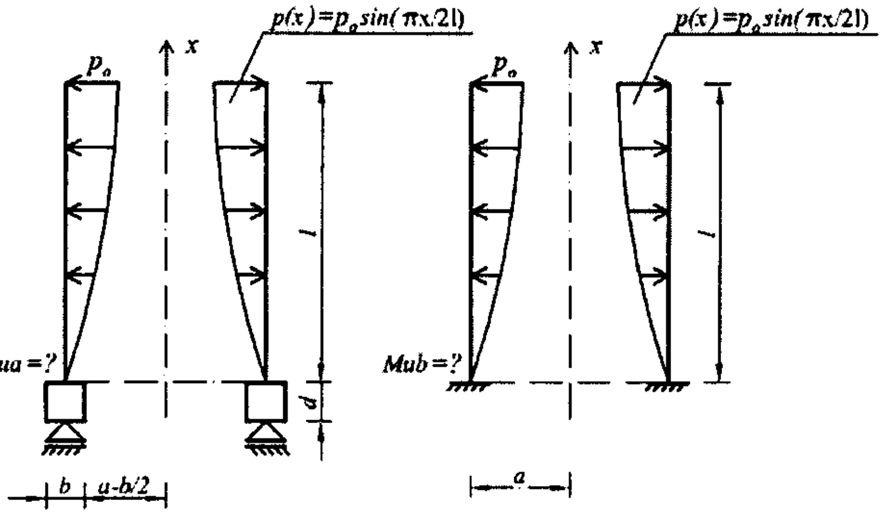
УНИВЕРЗИТЕТ У БЕОГРАДУ
ГРАЂЕВИНСКИ ФАКУЛТЕТ

КАТЕДРА ЗА ТЕХНИЧКУ МЕХАНИКУ
И ТЕОРИЈУ КОНСТРУКЦИЈА
ТЕОРИЈА ПЛОЧА И ЉУСКИ
Школска 2008/2009.

Кандидат:

ТРЕЋИ ГРАФИЧКИ РАД

1. За ротационо симетричне конструкције и оптерећење приказане на слици:
   а) Одредити степен укљештења љуске у кружни прстен
   б) Одредити максимални напон у кружном прстену.
   Напомена: степен укљештења је однос M_{ua}/M_{ab}.



E = 30 GPa

ν = 0.2

p_0 = 10 kN/m^2

h_c = 0,2 m

a = 3 m

l = 6 m

кружни прстен:

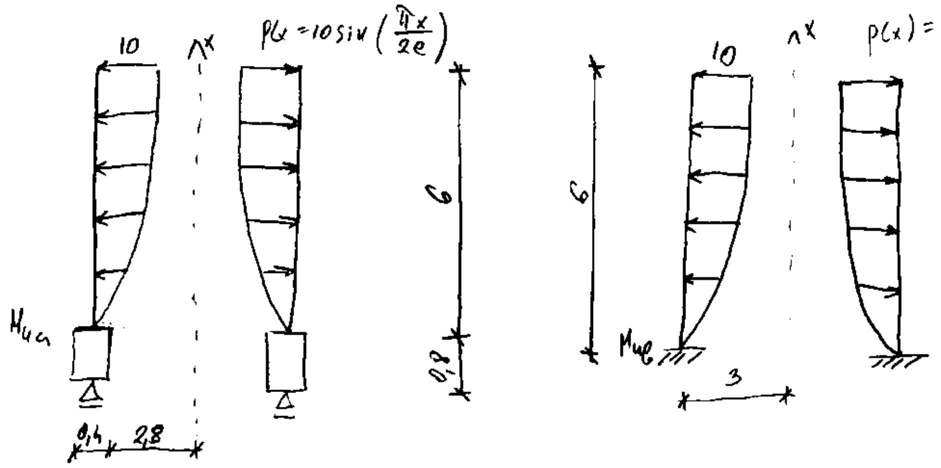
b/d = 0,4/0,8 m

Задатак задала: Жлонарević

Датум: 26.05.2009.

Оцена: 10 (десет)

Трећи графички рад



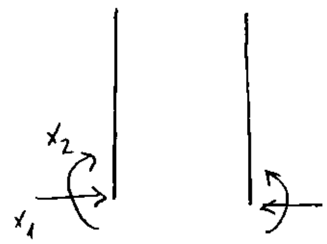
$E = 30 GPa$

$\nu = 0,2$

$h_c = 0,2 m$

$b/d = 0,4/0,8 m$

Декомпозиција:



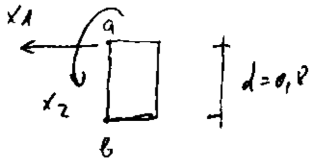
$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{a^2 h^2}} = \sqrt[4]{\frac{3(1-0,2^2)}{3^2 \cdot 0,2^2}} = 1,68$$

$$K = \frac{E h^3}{12(1-\nu^2)} = \frac{30 \cdot 10^6 \cdot 0,2^3}{12(1-0,2^2)} = 20833,33$$

$$\delta_{11} = \frac{1}{2\beta^3 K} = 5,061 \cdot 10^{-6}$$

$$\delta_{21} = -\frac{1}{2\beta^2 K} = -8,503 \cdot 10^{-6}$$

$$\delta_{22} = \frac{1}{\beta K} = 2,857 \cdot 10^{-5}$$



$N = 1 \cdot a$

$X_1 = 1$:

$$E\delta_{11} = u_a = E\varepsilon_0 = \ell\varepsilon_0 = \left(\frac{N}{F} \pm \frac{M}{I} \cdot z\right) \cdot a$$

$$= \left(\frac{1 \cdot 3}{0,8 \cdot 0,4} + \frac{1 \cdot 3 \cdot 0,4 \cdot 12}{0,8^3 \cdot 0,4} \cdot 0,4\right) \cdot 3 = 112,5$$

$$\delta_{11} = 3,75 \cdot 10^{-6}$$

$$E\delta_{21} = \varphi = + \frac{u_a + u_b}{d}$$

$$E u_b = \left(\frac{1 \cdot 3}{0,8 \cdot 0,4} - \frac{1 \cdot 3 \cdot 0,4 \cdot 12}{0,8^3 \cdot 0,4} \cdot 0,4\right) \cdot 3 = -56,25$$

$$E\delta_{21} = \frac{112,5 - 56,25}{0,8} = 70,31 \Rightarrow \delta_{21} = 2,344 \cdot 10^{-6}$$

$X_2 = 1$ $(N \equiv 0)$

$$E\delta_{22} = \varphi = \ell \cdot \varepsilon_0 = \frac{M}{I} \cdot z \cdot a = \frac{1 \cdot 3 \cdot 0,4 \cdot 12}{0,8^3 \cdot 0,4} \cdot 0,4 \cdot 3 = 84,375 \Rightarrow \delta_{22} = 2,8125 \cdot 10^{-6}$$

$$\delta_{11} = \delta_{11}^{I} + \delta_{11}^{II} = (5,061 + 3,75) \cdot 10^{-6} = 8,811 \cdot 10^{-6}$$

$$\delta_{12} = \delta_{21} = \delta_{12}^{I} + \delta_{21}^{II} = (-8,503 + 2,344) \cdot 10^{-6} = -6,159 \cdot 10^{-6}$$

$$\delta_{22} = \delta_{22}^{I} + \delta_{21}^{II} = (28,57 + 2,8125) \cdot 10^{-6} = 31,3825 \cdot 10^{-6}$$

$$\delta + X + \delta_0 = 0$$

$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \delta_{10} \\ \delta_{20} \end{bmatrix} = 0$$

$$\delta_{10} = u\ (x=0) \qquad \delta_{20} = \frac{\partial u}{\partial x}\ (x=0)$$

$$u = \varepsilon_\varphi \cdot R_0 = \frac{1}{Eh} \cdot N_\varphi \cdot R_0 = \frac{1}{Eh} \cdot (\cdot z R_0) \cdot R_0 =$$

$$= \frac{1}{30 \cdot 10^6 \cdot 0{,}2} \cdot \left(-10 \sin\left(\frac{\pi x}{12}\right) \cdot 3\right) \cdot 3$$

$$= -1{,}5 \cdot 10^{-5} \sin\left(\frac{\pi x}{12}\right)$$

$$\delta_{10} = u(x=0) = 0$$

$$\delta_{20} = \frac{\partial u}{\partial x}\ (x=0) = -\frac{\pi}{12} \cdot 1{,}5 \cdot 10^{-5} \cdot \cancel{\cos\left(\frac{\pi x}{12}\right)}^{1} = -3{,}927 \cdot 10^{-6}$$

$$8{,}811 \cdot 10^{-6} \cdot X_1 - 6{,}159 \cdot 10^{-6} \cdot X_2 = 0$$

$$-6{,}159 \cdot 10^{-6} X_1 + 31{,}3825 \cdot 10^{-6} \cdot X_2 = 3{,}927 \cdot 10^{-6}$$

$$X_1 = 0{,}1014$$

$$X_2 = 0{,}145 \text{ kNm} = M_{un}$$

$$M_{ub} = ?$$

$$M_{ub} = -K \cdot \frac{d^2 w}{dx^2}\ (x=0) \qquad \beta = 1{,}68$$

$1{,}68 \cdot 6 > 5 \rightarrow$ ДУГА ЛУСКА!

$$w = w_0 + e^{-\beta x}(C_3 \cos\beta x + C_4 \sin\beta x)$$

$$w_0 = ? \qquad w_0 = \varepsilon_\varphi \cdot a = \frac{N_\varphi}{Eh} \cdot a = \frac{-10 \sin\left(\frac{\pi x}{2l}\right) \cdot a^2}{Eh} =$$

$$= -1{,}5 \cdot 10^{-5} \cdot \sin\left(\frac{\pi x}{2l}\right)$$

Гранични услови:

$x = 0 \Rightarrow w = 0$

$$0 + e^{0}(C_3 \cdot \cancel{\cos 0}^{1} + C_4 \sin 0^{0}) = 0 \rightarrow C_3 = 0$$

$x=0 \Rightarrow \frac{\partial w}{\partial x}=0$

$$-\frac{\pi}{2l}\cdot 1{,}5\cdot 10^{-5}\cos\left(\frac{\pi x}{2l}\right)^{\to 1}+e^{-\beta x}{}^{\to 1}\left(C_3(-\beta\sin\beta x)^{\to 0}+C_4\cdot\beta\cos\beta x\right)^{\to 1}+$$

$$+\left(C_3\cdot\cos\beta x^{\to 1}+C_4\cdot\sin\beta x\right)^{\to 0}\cdot\left(-\beta\cdot e^{-\beta x}\right)^{\to 1}=0$$

$$-\frac{\pi}{2l}\cdot 1{,}5\cdot 10^{-5}+C_4\beta-C_3\beta^{\to 0}=0$$

$$C_4\beta=\frac{\pi}{2l}\cdot 1{,}5\cdot 10^{-5} \Rightarrow C_4=1{,}963\cdot 10^{-6}$$

$$\frac{d}{dx}\frac{dw}{dx}=\frac{d}{dx}\Big(-\frac{\pi}{2l}\cdot 1{,}5\cdot 10^{-5}\cdot\cos\left(\frac{\pi x}{2l}\right)+e^{-\beta x}\left(-\beta\cdot C_3\sin\beta x+\beta\cdot C_4\cos\beta x\right)$$

$$-\beta e^{-\beta x}\left(C_3\cos\beta x+C_4\sin\beta x\right)\Big)$$

$$e^{-\beta x}\left(-\beta^2C_3\cos\beta x-\beta^2C_4\sin\beta x\right)-\beta e^{-\beta x}\left(-\beta\cdot C_3\sin\beta x+\beta C_4\cos\beta x\right)$$

$$-\beta\cdot e^{-\beta x}\left(-C_3\beta\sin\beta x+C_4\beta\cos\beta x\right)+\beta^2e^{-\beta x}\left(C_3\cos\beta x+C_4\sin\beta x\right)$$

$$e^{-\beta x}\beta^2\left(-C_3\cos\beta x-C_4\sin\beta x+C_3\sin\beta x-C_4\cos\beta x+C_3\sin\beta x-C_4\cos\beta x+C_3\cos\beta x+C_4\sin\beta x\right)$$

$$\frac{d^2w}{dx^2}=2e^{-\beta x}\beta^2\left(-C_4\cos\beta x\right)=2\cdot e^{-1{,}68x}\,1{,}68^2\left(-1{,}963\cdot 10^{-6}\cos(1{,}68x)\right)=$$

$$=-11{,}081\cdot 10^{-6}\cdot e^{-1{,}68x}\cdot\cos(1{,}68x)$$

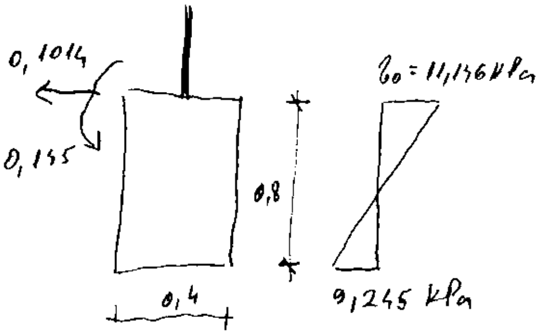
$$M_x=-K\frac{d^2w}{dx^2}=-20833{,}33\cdot\left(-11{,}081\cdot 10^{-6}\cdot e^{-1{,}68x}\cdot\cos(1{,}68x)\right)=0{,}231\cdot e^{-1{,}68x}\cos(1{,}68x)$$

$$M_{ub}=M_x(x=0)=0{,}231\cdot 1\cdot 1=0{,}231\ kNm$$

степен укљештења:

$$\frac{M_{ua}}{M_{ub}}=\frac{0{,}145}{0{,}231}=0{,}628$$

максимални напон у нужном пресеку



$$\sigma_{u,0}=\frac{N}{F}\pm\frac{M}{W}=\frac{0{,}1014\cdot 3}{0{,}8\cdot 0{,}4}\pm\frac{0{,}145\cdot 3}{\frac{0{,}4\cdot 0{,}8^2}{6}}$$

$$\sigma_0=11{,}146\ kN/m^2=11{,}146\ kPa$$

10 (десет)

09.06.2005.