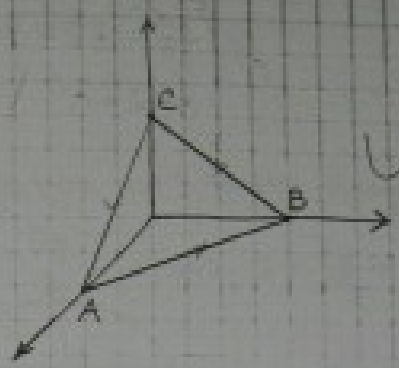
17 Израчунати: $\oint\limits_{C} x^2dx + xydy + xyzdz = \oint\limits_{AB} + \oint\limits_{BC} + \oint\limits_{CA}$

C: ΔABC

$A(a,0,0)$

$B(0,b,0)$

$C(0,0,c)$



- Тражимо решење страних; AB, AC, BC у параметарском облику:

$AB = ?$

Вектор: $\vec{AB}(-a,b,0)$ $A(a,0,0)$ => $AB: \begin{cases} x = a - at \\ y = bt \\ z = 0 \end{cases}$ $\begin{matrix} dx = -adt \\ dy = bdt \\ dz = 0 \end{matrix}$

$CA = ?$

$\vec{AC}(-a,0,c)$, $A(a,0,0)$ => $AC: \begin{cases} x = a - at \\ y = 0 \\ z = ct \end{cases}$ $\begin{matrix} dx = -adt \\ dy = 0 \\ dz = cdt \end{matrix}$

$BC = ?$

$\vec{BC}(0,-b,c)$, $B(0,b,0)$ => $BC: \begin{cases} x = 0 \\ y = b - bt \\ z = ct \end{cases}$ $\begin{matrix} dx = 0 \\ dy = -bdt \\ dz = cdt \end{matrix}$

Дужи AB:

$\oint\limits_{AB} x^2dx + xydy + xyzdz = \int\limits_{\text{пресек?}}$ => $AB: \begin{cases} x = a - at \\ y = bt \\ z = 0 \end{cases}$

пресек прамс CA коорд. осама

$A = (a,0,0)$ => $a = a - at$ => $t_A = 0$

$0 = bt$ => $t_A = 0$

$0 = 0$

$B = (0,b,0)$ => $0 + at = a$, $t_B = 1$

$b = bt$ => $t_B = 1$

$0 = 0$

$$\oint\limits_{AB} x^2dx + xydy + xyzdz = \int_0^1 [(a-at)^2(-a) + (a-at)\cdot bt\cdot b + (a-at)\cdot bt\cdot 0\cdot 0]\,dt$$

$$= \int_0^1 [-a^3\cdot(1-2t+t^2) + abt - abt^2]\,dt =$$

$$= -a^3\cdot\left(t - t^2 + \frac{t^3}{3}\right)\Big|_0^1 + ab\frac{t^2}{2}\Big|_0^1 - ab\frac{t^3}{3}\Big|_0^1 = -\frac{a^3}{3} + \frac{ab^2}{2} - \frac{ab^2}{3} = -\frac{a^3}{3} + \frac{ab^2}{6}$$

Дужи CA:

$$\oint\limits_{CA} x^2dx + xydy + xyzdz = \int_1^0 (a-at)^2\cdot(-a)\,dt + 0 + 0 =$$

$$= -a^3\cdot\int_1^0 (1-2t+t^2)\,dt = -a^3\cdot\left(t - t^2 + \frac{t^3}{3}\right)\Big|_1^0 = a^3\cdot\left(1 - 1 + \frac{1}{3}\right) = \frac{a^3}{3}$$

Дужи BC:

$$\oint\limits_{BC} x^2dx + xydy + xyzdz = \int_0^1 0 + 0 + 0 = 0$$

$$I = \oint\limits_{AB} + \oint\limits_{BC} + \oint\limits_{CA} = \left(-\frac{a^3}{3} + \frac{ab^2}{6}\right) + 0 + \frac{a^3}{3} = \frac{ab^2}{6} \Rightarrow \boxed{I = \frac{ab^2}{6}}$$