6) $\boxed{\Pi = \iint\limits_{x^2+y^2+z^2=1} \vec{A}\cdot\vec{N}\,dS = \iint\limits_{x^2+y^2+z^2=1} \left[\left(\frac{1}{x^2yz}+\frac{1}{x}\right)\cos\alpha + \left(\frac{1}{xy^2z}+\frac{1}{y}\right)\cos\beta + \left(\frac{1}{xyz^2}+\frac{1}{z}\right)\cos\gamma\right] dS}$

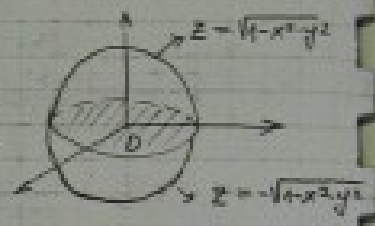
$\Sigma_1: z = +\sqrt{1-x^2-y^2}$, $p = \frac{\partial z}{\partial x} = \frac{-x}{\sqrt{1-x^2-y^2}}$, $q = \frac{-y}{\sqrt{1-x^2-y^2}}$, $\sqrt{1+p^2+q^2} = \frac{1}{\sqrt{1-x^2-y^2}}$ ОСВА 9-10

$\cos\alpha = \frac{-p}{+\sqrt{1+p^2+q^2}} = x$, $\cos\beta = \frac{-q}{+\sqrt{1+p^2+q^2}} = y$, $\cos\gamma = \frac{1}{+\sqrt{1+p^2+q^2}} = z$

$\Sigma_2: z = -\sqrt{1-x^2-y^2}$, $p = \frac{\partial z}{\partial x} = \frac{x}{\sqrt{1-x^2-y^2}}$, $q = \frac{\partial z}{\partial y} = \frac{y}{\sqrt{1-x^2-y^2}}$, $\sqrt{1+p^2+q^2} = \frac{1}{\sqrt{1-x^2-y^2}}$

$\cos\alpha = \frac{-p}{-\sqrt{1+p^2+q^2}} = x$, $\cos\beta = \frac{-q}{-\sqrt{1+p^2+q^2}} = y$, $\cos\gamma = \frac{1}{-\sqrt{1+p^2+q^2}} = -\sqrt{1-x^2-y^2} = z$

$$\Pi = \iint\limits_{x^2+y^2+z^2=1} \left(\frac{1}{x^2yz}+\frac{1}{x}\right)x + \left(\frac{1}{xy^2z}+\frac{1}{y}\right)\cdot y + \left(\frac{1}{xyz^2}+\frac{1}{z}\right)z\Big]\cdot dS =$$



$$= \iint\limits_{x^2+y^2+z^2=1} \left(\frac{3}{xyz}+3\right) dS = 3\iint\limits_{\Sigma_1}\left(\frac{1}{xyz}+1\right) dS + 3\iint\limits_{\Sigma_2}\left(\frac{1}{xyz}+1\right) dS \ominus$$

$\Sigma_1: dS = \sqrt{1+p^2+q^2}\,dx\,dy = \Sigma_2: dS = \frac{dx\,dy}{\sqrt{1-x^2-y^2}}$,

$$= 3\iint\limits_{x^2+y^2\le 1}\left(\frac{1}{xy\sqrt{1-x^2-y^2}}+1\right)\frac{dx\,dy}{\sqrt{1-x^2-y^2}} + 3\iint\limits_{x^2+y^2\le 1}\left(\frac{-1}{xy\sqrt{1-x^2-y^2}}+1\right)\frac{dx\,dy}{\sqrt{1-x^2-y^2}} = 6\iint\limits_{x^2+y^2\le 1}\frac{dx\,dy}{\sqrt{1-x^2-y^2}}$$

$$= 6\iint\limits_{\substack{0\le\varphi\le 2\pi\\ 0\le\rho\le 1}}\frac{\rho\,d\rho\,d\varphi}{\sqrt{1-\rho^2}} = 6\int_0^{2\pi}d\varphi\int_0^1\frac{\cancel{\rho}\,d\rho}{\cancel{\rho}\sqrt{1-\rho^2}} = 6\cdot 2\pi\left[-\sqrt{1-\rho^2}\right]_{\rho=0}^{\rho=1} = 12\pi$$

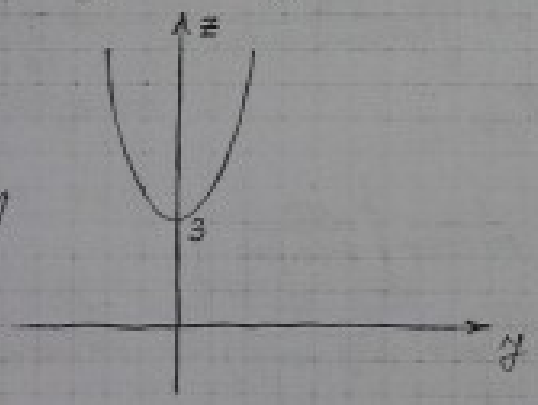
27.09.1991.

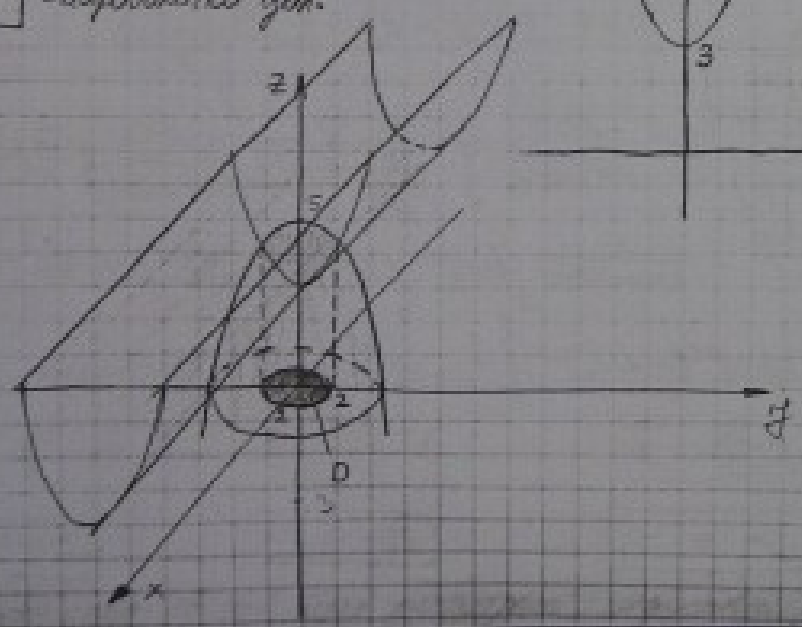
31 Израчунати запремину тела ограничену елиптичким параболоидом $z = 5-y^2-2x^2$ и параболичким цилиндром $z = y^2-3$.

$z-5 = -2x^2-y^2$

$\boxed{5-z = \frac{x^2}{\frac{1}{2}} + y^2}$ - елиптички параболоид

$\boxed{z+3 = y^2}$ - параболички цил.





$\frac{x^2}{\frac{1}{2}} - y^2 = 5$

$\frac{x^2}{\frac{5}{2}} + \frac{y^2}{5} = 1$