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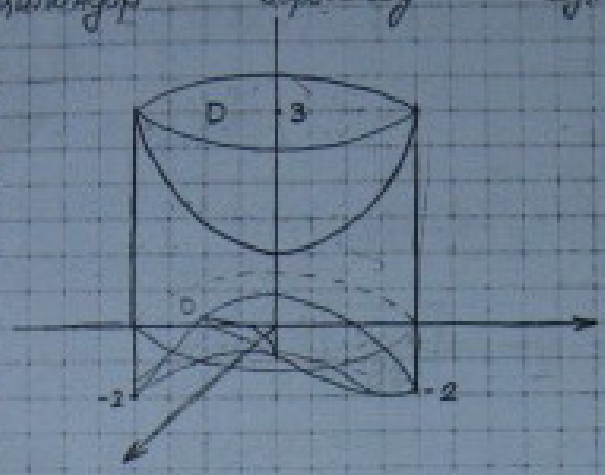
5 Изчислете повърхнината и затворения обем на Т, ограничено повърхнини: $\overset{\Gamma_1}{x^2+y^2=4}$, $\overset{\Gamma_2}{x^2+y^2=2z-2}$, $\overset{\Gamma_3}{2z=x^2-y^2}$

цилиндър, параболоид, седло



$\Gamma_2 \cap \Gamma_3$

$x^2+y^2+2 = x^2-y^2$

$2y^2=-2$ ≠

$\Rightarrow \Gamma_2 \cap \Gamma_3 = \emptyset$

$\Gamma_2: z-1 = \frac{x^2}{2}+\frac{y^2}{2}$

за $z=1 \Rightarrow x^2+y^2=0$ (ТОЧКА)

за $z=2 \Rightarrow x^2+y^2=2$

за $z=3 \Rightarrow \boxed{x^2+y^2=4}$ пресек Γ_1 и Γ_2

Γ_3: $x=0, y=2 \Rightarrow z=-2$

$$V=\iint_D (z_2-z_1)\,dx\,dy$$

$z_1=\frac{x^2-y^2}{2}$

$z_2=\frac{x^2+y^2+2}{2}$

$\begin{cases} x=\rho\cdot\cos\varphi \\ y=\rho\cdot\sin\varphi \end{cases}$, $dx\,dy=\rho\,d\varphi\,d\rho$

$$V=\iint_D\left(\frac{x^2+y^2+2-x^2+y^2}{2}\right)=\iint_D (y^2+1)\,dx\,dy$$

$D: x^2+y^2=4$, $0\le\varphi\le 2\pi$, $0\le\rho\le 2$

$$V=\int_0^{2\pi} d\varphi\cdot\int_0^2(\rho^2\sin^2\varphi+1)\cdot\rho\,d\rho=\int_0^{2\pi} I_1$$

$$I_1=\int_0^2(\rho^3\sin^2\varphi+\rho)\,d\rho=\sin^2\varphi\int_0^2\rho^3\,d\rho+\int_0^2\rho\,d\rho=$$

$$=\sin^2\varphi\cdot\frac{\rho^4}{4}\Big|_0^2+\frac{\rho^2}{2}\Big|_0^2=4\sin^2\varphi+2$$

$$V=\int_0^{2\pi}(4\sin^2\varphi+2)\,d\varphi=4\int_0^{2\pi}\sin^2\varphi\,d\varphi+2\int_0^{2\pi}d\varphi=I_1+4\pi$$

$$I_1=4\int_0^{2\pi}\sin^2\varphi\,d\varphi=2\int_0^{2\pi}(1-\cos 2\varphi)\,d\varphi=2\int_0^{2\pi}d\varphi-\int_0^{2\pi}\cos 2\varphi\,d2\varphi=$$

$$=2\varphi\Big|_0^{2\pi}-\sin 2\varphi\Big|_0^{2\pi}=4\pi$$

$\Rightarrow \boxed{V=8\pi}$