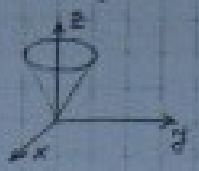
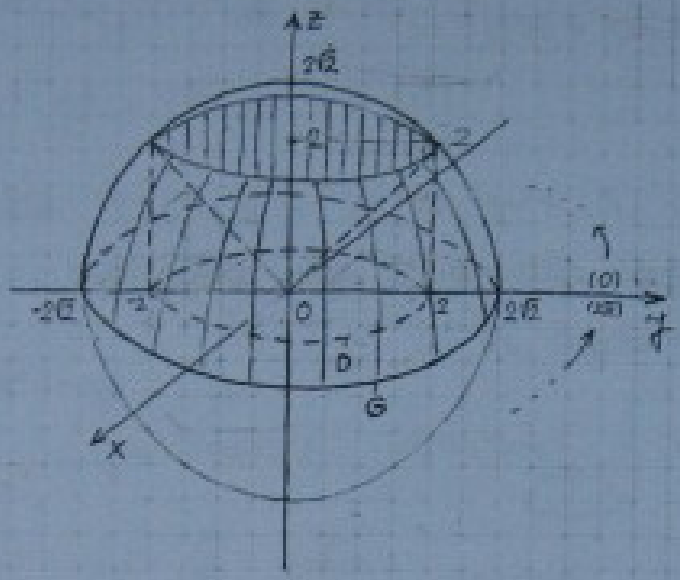
04.09.2002.

3. Дате су површи $\Sigma_1: z=\sqrt{x^2+y^2}$, $\Sigma_2: x^2+y^2+z^2=8$.
Израчунати P и V тела T које је ограничено датим површинама и равни $z=0$.

$\Sigma_1: z=\sqrt{x^2+y^2}$ - конус



$\Sigma_2: x^2+y^2+z^2=8$ - сфера са центром у $O(0,0,0)$ и полупр. $r=2\sqrt{2}$



Пресек две површи:

$z=\sqrt{x^2+y^2}$

$z=\sqrt{8-x^2-y^2}$

$x^2+y^2=8-x^2-y^2$

$2\cdot(x^2+y^2)=8$

$\boxed{x^2+y^2=4}$

- круг: $C(0,0,0)$, $r=2$

$V=V_1+V_2$

$V_1=\iint_D z_{кон}\,dxdy \qquad V_2=\iint_{G-D} z_{сфера}\,dxdy$

$V_1=\iint_D \sqrt{x^2+y^2}\,dxdy = \left(\text{СМЕНА: } \begin{array}{l} x=\rho\cos\varphi \\ y=\rho\sin\varphi \\ dxdy=\rho\,d\varphi\,d\rho \end{array}\right) =$

$=\iint_D \rho\cdot\rho\,d\varphi\,d\rho = \int_0^{2\pi} d\varphi\cdot\int_0^2 \rho^2 d\rho = \int_0^{2\pi}\frac{8}{3}d\varphi = \frac{16\pi}{3}$

$V_2=\iint_{G-D} z_{сф}\,dxdy = \iint_{G-D}\sqrt{8-x^2-y^2}\,dxdy = \iint_{G-D}\sqrt{8-\rho^2}\cdot\rho\,d\varphi\,d\rho = \int_0^{2\pi}d\varphi\int_2^{2\sqrt{2}}\sqrt{8-\rho^2}\,\rho\,d\rho =$

$\ldots \Rightarrow V_2=\frac{16\pi}{3} \Rightarrow V=V_1+V_2 \Rightarrow \boxed{V=\frac{32\pi}{3}}$

$\boxed{P=P_1+P_2+P_3}$

P_1 - површина дела сфере:

$P_1=\iint_{G-D}\sqrt{1+p^2+q^2}$, $z=\sqrt{8-x^2-y^2}$, $p=\frac{\partial z}{\partial x}=-\frac{x}{\sqrt{8-x^2-y^2}}$, $q=\frac{-y}{\sqrt{8-x^2-y^2}}$

$P_1=\iint_{G-D}\sqrt{1+\frac{x^2}{8-x^2-y^2}+\frac{y^2}{8-x^2-y^2}}\,dxdy=\iint_{G-D}\sqrt{\frac{8-x^2-y^2+x^2+y^2}{8-x^2-y^2}}\,dxdy=\iint_{G-D}\sqrt{\frac{8}{8-x^2-y^2}}\,dxdy$

$P_1=\int_0^{2\pi}d\varphi\cdot\int_2^{2\sqrt{2}}\frac{\sqrt{8}}{\sqrt{8-\rho^2}}\rho\,d\rho=\int_0^{2\pi}2\sqrt{8}\,d\varphi=4\sqrt{8}\pi \Rightarrow \boxed{P_1=4\sqrt{8}\pi}$