$P = P_{пар} + P_{сед} + P_{цил}$

$$P_{пар} = \iint_D \sqrt{1+p^2+q^2}\,dx\,dy$$

$$z = \frac{x^2+y^2+2}{2}, \quad p = \frac{\partial z}{\partial x} = x, \quad q = \frac{\partial z}{\partial y} = y$$

$$P_{пар} = \iint_D \sqrt{1+x^2+y^2}\,dx\,dy = \int_0^{2\pi} d\varphi \cdot \int_0^2 \sqrt{1+\rho^2}\cdot\rho\,d\rho = \int_0^{2\pi} d\varphi \cdot I_1$$

$$I_1 = \int_0^2 \sqrt{1+\rho^2}\cdot\rho\,d\rho = \left(\begin{array}{l} \sqrt{1+\rho^2} = t \Rightarrow 1+\rho^2 = t^2 \Rightarrow \rho = \sqrt{t^2-1} \\ d\rho = \frac{t\,dt}{\sqrt{t^2-1}} \quad \begin{cases} 2 = \sqrt{5} \\ 0 = 1 \end{cases} \end{array} \right.$$

$$= \int_1^{\sqrt{5}} t\cdot\sqrt{t^2-1}\cdot\frac{t\,dt}{\sqrt{t^2-1}} = \int_1^{\sqrt{5}} t^2\,dt = \frac{t^3}{3}\Big|_1^{\sqrt{5}} = \frac{\sqrt{5}^3}{3} - \frac{1}{3} = \frac{1}{3}(5\sqrt{5}-1)$$

$$\boxed{P_{пар}} = \frac{1}{3}(5\sqrt{5}-1)\int_0^{2\pi} d\varphi = \boxed{\frac{2\pi}{3}(5\sqrt{5}-1)}$$

$$P_{сед} = \iint_D \sqrt{1+p^2+q^2}\,dx\,dy$$

$$z = \frac{1}{2}(x^2-y^2), \quad p = x, \quad q = -y$$

$$P_{сед} = \iint_D \sqrt{1+x^2+y^2}\,dx\,dy = \boxed{P_{пар} = P_{сед} = \frac{2\pi}{3}(5\sqrt{5}-1)}$$

$$P_{цил} = 2\pi r\cdot H = 2\pi\cdot 2\cdot \overset{?}{3} = 12\pi$$

$$\boxed{P = 12\pi + \frac{4\pi}{3}(5\sqrt{5}-1)}$$

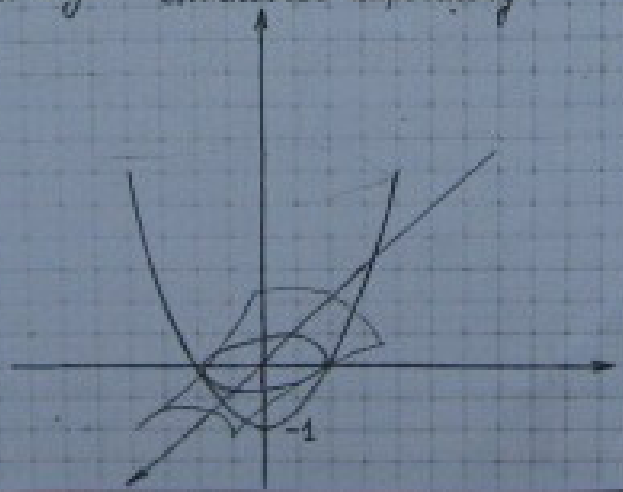
20.09.2001.

6 Дате су површи: $\Gamma_1: z = \frac{1}{2}(x^2-y^2)$, $\Gamma_2: z = x^2+y^2+1$.

а) Израчунати запремину тела које је ограничено тим површима.

Γ_1: $2z = x^2-y^2$ - седло

Γ_2: $z+1 = x^2+y^2$ - елиптички параболоид



$\Gamma_1 \cap \Gamma_2$:

$$\frac{1}{2}(x^2-y^2) - x^2 - y^2 + 1 = 0 \;/\cdot 2$$

$$x^2 - y^2 - 2x^2 - 2y^2 + 2 = 0$$

$$-x^2 - 3y^2 + 2 = 0$$

$$x^2 + 3y^2 = 2 \quad /:2$$

D: $\boxed{\frac{x^2}{2} + \frac{y^2}{\frac{2}{3}} = 1}$ - елипса