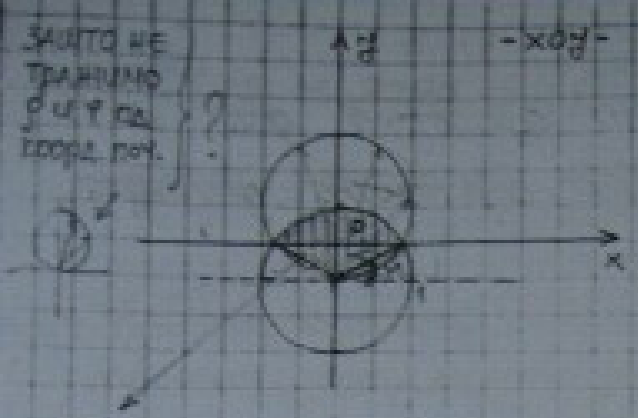
ЗАШТО НЕ ТРАЖИМО ρ И φ ПО ПОРЕД ПОЧ.



-xoy- +ПРОЈЕКЦИЈА ПРЕСЕКА:

① $z = x^2 + y^2$

$z = \frac{3}{4} - y$

$\frac{3}{4} - y = x^2 + y^2$

$\boxed{x^2 + (y + \frac{1}{2})^2 = 1}$

② $z = x^2 + y^2$

$z = y + \frac{3}{4}$

$x^2 + y^2 - y - 3/4 = 0$

$x^2 + (y - \frac{1}{2})^2 = 1$

Пошто су тражене области испод и изнад x-ose једнаке довољно је израчунати запремину за један део па помножити са 2

$$\frac{V}{2} = \iint_D [\underbrace{z_2}_{z_{pos}} - \underbrace{z_1}_{z_{en}}]\, dx\,dy$$

D: $\rho = ?$ ЗА $y = 0 \Rightarrow 0 + \frac{1}{2} = \rho \sin\varphi \Rightarrow \rho = \frac{1}{2\sin\varphi}$

$\boxed{\frac{1}{2\sin\varphi} \leq \rho \leq 1}$

Поларне координате:

$x = \rho\cos\varphi$

$y + \frac{1}{2} = \rho\sin\varphi$

$J = \rho$

$\varphi = ?$

$\frac{y}{x} = tg\varphi$, $y = \frac{1}{2}$, $x = \sqrt{1 - \frac{1}{4}} \Rightarrow x = \frac{\sqrt{3}}{2}$

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = tg\varphi \Rightarrow tg\varphi = \frac{1}{\sqrt{3}} \Rightarrow \boxed{\varphi_1 = \frac{\pi}{6}}, \boxed{\varphi_2 = \frac{5\pi}{6}}$

$\boxed{\frac{\pi}{6} \leq \varphi \leq \frac{5\pi}{6}}$

Може и овако:

$\Rightarrow \rho \cdot \sin\varphi = \frac{1}{2} \Rightarrow \rho = \frac{1}{2\sin\varphi}$

$z_{en} = \rho^2\cos^2\varphi + \rho^2\sin^2\varphi = \rho^2$

$z_{pos} = \frac{3}{4} - \rho\sin\varphi + \frac{1}{2}$

?

$$\frac{V}{2} = \iint_D \left(\frac{3}{4} - \rho\sin\varphi + \frac{1}{2} - \rho^2\right) dx\,dy = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} d\varphi \cdot \int_{\frac{1}{2\sin\varphi}}^{1} \left(\frac{5}{4} - \rho\sin\varphi - \rho^2\right) \cdot \rho\, d\rho =$$

$$= \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} d\varphi \int_{\frac{1}{2\sin\varphi}}^{1} \left(\frac{5}{4}\rho - \rho^2\sin\varphi - \rho^3\right) d\rho = \ldots \frac{\pi}{6} - \frac{\sqrt{3}}{4}$$

$\frac{V}{2} = \frac{\pi}{6} - \frac{\sqrt{3}}{4} \Rightarrow$

$\Rightarrow \boxed{V = \frac{\pi}{3} - \frac{\sqrt{3}}{2}}$