Поларне координате:

$$\begin{cases} x=\rho\cos\varphi \\ y=\rho\sin\varphi \\ J=\rho \end{cases} \Rightarrow z=4-(x^2+y^2) \Rightarrow z=4-\rho^2$$

$$\boxed{V=\iint\limits_{D_1} z\,dx\,dy+\iint\limits_{D_2} z\,dx\,dy} =$$

$$=\int_0^{\pi/6} d\varphi\cdot\underbrace{\int_0^{2\cos\varphi}(4-\rho^2)\rho\,d\rho}_{I_1}+\int_{\pi/6}^{\pi/2} d\varphi\cdot\underbrace{\int_0^{2\sin\varphi}(4-\rho^2)\cdot\rho\,d\rho}_{I_2}$$

$$I_1=\int_0^{\pi/6} d\varphi\cdot\int_0^{2\cos\varphi}(4\rho-\rho^3)\,d\rho=\int_0^{\pi/6} d\varphi\cdot\left(2\rho^2-\frac{1}{4}\rho^4\right)\Big|_0^{2\cos\varphi}=\int_0^{\pi/6} d\varphi\left(24\cos^2\varphi-\frac{9}{4}\cos^4\varphi\right)$$

$$=24\underbrace{\int_0^{\pi/6}\cos^2\varphi\,d\varphi}_{I_3}-\frac{9}{4}\underbrace{\int_0^{\pi/6}\cos^4\varphi\,d\varphi}_{I_4}$$

$$I_3=12\int_0^{\pi/6}(1-\cos 2\varphi)\,d\varphi=12\int_0^{\pi/6} d\varphi-6\int_0^{\pi/6}\cos 2\varphi\,d\varphi=12\varphi\Big|_0^{\pi/6}-6\cdot\sin 2\varphi\Big|_0^{\pi/6}=2\pi-3\sqrt{3}\pi$$

$$I_4=\frac{9}{32}\int_0^{\pi/6}(3-4\cos 2\varphi+\cos 4\varphi)\,d\varphi=\frac{27}{32}\int_0^{\pi/6} d\varphi-\frac{18}{32}\int_0^{\pi/6}\cos 2\varphi\,d\varphi+\frac{9}{128}\int_0^{\pi/6}\cos 4\varphi\,d\varphi=$$

$$=\frac{27}{36}\varphi\Big|_0^{\pi/6}-\frac{9}{16}\sin 2\varphi\Big|_0^{\pi/6}+\frac{9}{128}\sin 4\varphi\Big|_0^{\pi/6}=\frac{27\pi}{216}-\frac{9\sqrt{3}\pi}{32}+\frac{9\sqrt{3}\pi}{256}=\frac{27\pi}{216}-\frac{72\sqrt{3}\pi}{256}+\frac{9\sqrt{3}\pi}{256}=$$

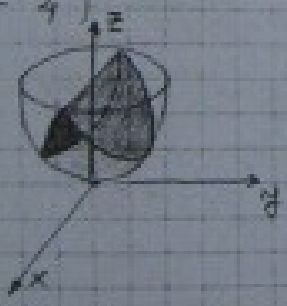
$$=\frac{27\pi}{32}-\frac{63\sqrt{3}\pi}{256} \Rightarrow I_1=\frac{64\pi}{32}-\frac{27\pi}{32}-\frac{709\sqrt{3}\pi}{256}+\frac{63\sqrt{3}\pi}{256}=\frac{37}{32}\pi-\frac{646\sqrt{3}\pi}{256}$$

На исти начин налазимо и I_2 $\Rightarrow$ $\boxed{V=I_1+I_2}$

10 Зоне су одређене $z=x^2+y^2$, $|y|=\frac{3}{4}-z$. Одредити V тела ограниченог овим површинама $\left(z\geq\frac{3}{4}\right)$

$z=x^2+y^2$ - елиптички параболоид

$$\left.\begin{aligned} y&=\tfrac{3}{4}-z \\ y&=z-\tfrac{3}{4} \end{aligned}\right\}\text{равни}$$



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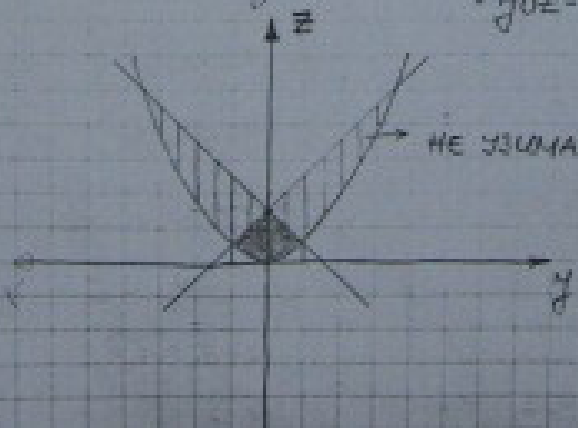

НЕ УЗИМАМО ЗАТО ШТО ЈЕ $z\geq\frac{3}{4}$