Сферне координате

$x = a\rho\cos\varphi\cos\theta$

$y = b\rho\sin\varphi\cos\theta$

$z = c\rho\sin\theta$

$J = abc\rho^2\cos\theta$

$T_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 9$

$\frac{a^2\rho^2\cos^2\varphi\cos^2\theta}{a^2} + \frac{b^2\rho^2\sin^2\varphi\cos^2\theta}{b^2} + \frac{c^2\rho^2\sin^2\theta}{c^2} = 9$

$\rho^2\cos^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\theta = 9$

$\rho^2\cos^2\theta\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1} + \rho^2\sin^2\theta = 9$

$\rho^2 = 9 \Rightarrow \boxed{\rho = 3}$

$T_2: \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

$\rho^2\cos^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\cos^2\theta - \rho^2\sin^2\theta = 1$

$\rho^2\cos^2\theta\cdot\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1} - \rho^2\sin^2\theta = 1$

$\rho^2\cos^2\theta - \rho^2\sin^2\theta = 1$

$\rho^2 = \frac{1}{\cos^2\theta - \sin^2\theta} \Rightarrow \rho = \frac{1}{\sqrt{\cos 2\theta}} \Rightarrow \frac{1}{\sqrt{\cos 2\theta}} = 3 \Rightarrow$

$\cos 2\theta = \pm\frac{1}{9} \Rightarrow \boxed{\theta = \pm\frac{1}{2}\arccos\frac{1}{9}}$

Тражене тело (горња половина):

$0 \le \varphi \le 2\pi \quad \begin{cases} 0 < \theta \le \frac{1}{2}\arccos\frac{1}{9}, & 0 \le \rho \le \frac{1}{\sqrt{\cos 2\theta}} \\ \frac{1}{2}\arccos\frac{1}{9} \le \theta \le \frac{\pi}{2}, & 0 < \rho \le 3 \end{cases}$

$\frac{1}{2}V = \iiint_\Omega dx\,dy\,dz = \int_0^{2\pi} d\varphi \int_0^{\frac{1}{2}\arccos\frac{1}{9}} d\theta \int_0^{\frac{1}{\sqrt{\cos 2\theta}}} abc\rho^2\cos\theta\,d\rho + \int_0^{2\pi} d\varphi \cdot \int_0^{\frac{\pi}{2}} d\theta \cdot \int_0^{3} abc\rho^2\cos\theta\,d\rho =$

$= \frac{22\pi}{3}abc \Rightarrow \boxed{V = \frac{44\pi}{3}abc}$

II Начин: $V = V_1 - V_2$

$\frac{1}{2}V_1 = \iint_D c\sqrt{9 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}\,dx\,dy$, $D: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 5 \Rightarrow$ $\left[\begin{array}{l} x = a\rho\cos\varphi \\ y = b\rho\sin\varphi \\ J = ab\rho \\ 0 \le \rho \le \sqrt{5} \\ (0 \le \varphi \le 2\pi) \end{array}\right]$

$= \int_0^{2\pi} d\varphi \cdot \int_0^{\sqrt{5}} \sqrt{9 - \rho^2}\,ab\rho\,d\rho = \ldots = \frac{28\pi}{3}abc \Rightarrow \boxed{V_1 = \frac{28\pi}{3}abc}$

$\frac{1}{2}V_2 = \iint_D c\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1}\,dx\,dy$ $D: 1 \le \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 5 =$ $\left[\begin{array}{l} x = a\rho\cos\varphi \\ y = b\rho\sin\varphi \\ J = ab\rho \\ 0 \le \varphi \le 2\pi \\ 1 \le \rho \le \sqrt{5} \end{array}\right]$

$= \int_0^{2\pi} d\varphi \cdot \int_1^{\sqrt{5}} \sqrt{\rho^2 - 1}\,ab\rho\,d\rho = \ldots = \frac{16\pi}{3}abc \Rightarrow \boxed{V_2 = \frac{16\pi}{3}abc}$