D: → Од слике се види: $0 \le \varphi \le 2\pi$

D- пресечна површина $0 \le \rho \le 2$

$$V = \int_0^{2\pi} d\varphi \cdot \int_0^2 (\rho^3 + \rho^2 - 6\rho)\, d\rho = \int_0^{2\pi} d\varphi \cdot I_1$$

$$I_1 = \int_0^2 \rho^3 d\rho + \int_0^2 \rho^2 d\rho - 6\int_0^2 \rho\, d\rho = \frac{\rho^4}{4}\Big|_0^2 + \frac{\rho^3}{3}\Big|_0^2 - 3\rho^2\Big|_0^2 = 4 + \frac{8}{3} - 12 = -\frac{16}{3}$$

$$V = -\frac{16}{3}\int_0^{2\pi} d\varphi = -\frac{16}{3}\varphi\Big|_0^{2\pi} = \left|-\frac{32\pi}{3}\right| \Rightarrow \boxed{V = \frac{32\pi}{3}}$$

$$\boxed{P = \iint_D \sqrt{1 + p^2 + q^2}\, dx\, dy}$$

$P = P_{en} + P_{cos}$

$$P_{en} = \iint_D \sqrt{1 + p_1^2 + q_1^2}$$

$$p_1 = \frac{\partial z}{\partial x} = 0 - 2x = -2x$$

$$q_1 = \frac{\partial z}{\partial y} = 0 + 0 - 2y = -2y$$

$$P_{en} = \iint_D \sqrt{1 + 4x^2 + 4y^2}\, dx\, dy$$ смена: $x = \rho\cos\varphi$

$y = \rho\sin\varphi$

$dx\,dy = \rho\, d\rho\, d\varphi$

$$P_{en} = \iint_D \sqrt{1 + 4\rho^2}\, \rho\, d\rho\, d\varphi = \int_0^{2\pi} d\varphi \cdot \int_0^2 \sqrt{1 + 4\rho^2} \cdot \rho\, d\rho = \int_0^{2\pi} d\varphi \cdot I_1$$

$$I_1 = \int_0^2 \sqrt{1 + 4\rho^2} \cdot \rho\, d\rho \Rightarrow \begin{cases} t^2 = 1 + 4\rho^2 \Rightarrow 4\rho^2 = t^2 - 1 \Rightarrow \rho^2 = \frac{t^2 - 1}{4} \\ \rho = \frac{1}{2}\sqrt{t^2 - 1} \Rightarrow d\rho = \frac{1}{2} \cdot \frac{t\, dt}{\sqrt{t^2 - 1}} \end{cases}$$, граничне $0 = 1$, $2 = \sqrt{17}$

$$= \int_1^{\sqrt{17}} t \cdot \frac{1}{2}\sqrt{t^2 - 1} \cdot \frac{1}{2} \cdot \frac{t\, dt}{\sqrt{t^2 - 1}} = \frac{1}{4}\int_1^{\sqrt{17}} t^2 dt = \frac{1}{4} \cdot \frac{t^3}{3} = \frac{1}{12}\left(\sqrt{17}^3 - 1\right) = \frac{17\sqrt{17} - 1}{12}$$

$$P_{en} = \frac{17\sqrt{17} - 1}{12}\int_0^{2\pi} d\varphi = \frac{(17\sqrt{17} - 1)}{6}\pi \Rightarrow \boxed{P_{en} = \frac{\pi}{6}(17\sqrt{17} - 1)}$$

$$P_{cos} = \iint_D \sqrt{1 + q_2^2 + q_2^2}\, dx\, dy$$

$$p_2 = \frac{x}{\sqrt{x^2 + y^2}}$$

$$q_2 = \frac{y}{\sqrt{x^2 + y^2}}$$

$$P_{cos} = \iint_D \sqrt{1 + \frac{x^2}{x^2 + y^2} + \frac{y^2}{x^2 + y^2}} = \iint_D \sqrt{\frac{x^2 + y^2 + x^2 + y^2}{x^2 + y^2}} = \iint_D \sqrt{2}\, dx\, dy = \sqrt{2} \cdot P(D)$$

P(D) - површина пресека два круга

D: $(x-2)^2 + (y-2)^2 = 4$

$P(D) = \pi r^2 = 4\pi \Rightarrow \boxed{P_{cos} = 4\sqrt{2}\pi}$

$$\boxed{P = P_{en} + P_{cos} = \frac{\pi}{6}(17\sqrt{17} - 1) + 4\sqrt{2}\pi}$$