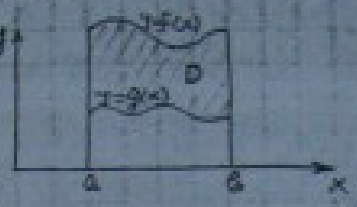
- Двојни интеграли -

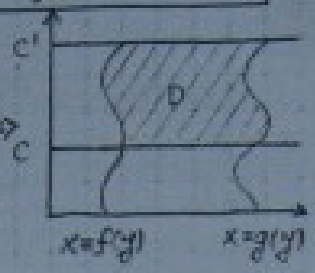
$$\boxed{I=\iint_D F(x,y)\,dx\,dy=\int_a^b dx\cdot\int_{g(x)}^{f(x)} F(x,y)\,dy}$$

$\Rightarrow$



$$\boxed{I=\int_c^{c'} dy\int_{f(y)}^{g(y)} F(x,y)\,dx}$$

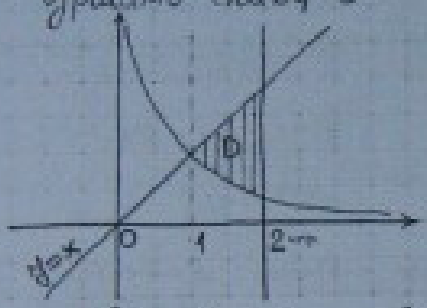
$\Rightarrow$



ПРИМЕР 1:

$$I=\iint_D \frac{x^2}{y^2}\,dx\,dy \qquad D:\ y=x\ ,\ y=\frac{1}{x}\ ,\ x=2\ ,\ x>0$$

- Цртамо слику D:



Са слике се види да је:

$a=1$ $\qquad g(x)=\frac{1}{x}$

$b=2$ $\qquad f(x)=x$

$$I=\int_1^2 dx\cdot\int_{\frac{1}{x}}^{x}\frac{x^2}{y^2}\,dy=\int_1^2 dx\cdot I_1$$

$$I_1=\int_{\frac{1}{x}}^{x}\frac{x^2}{y^2}\,dy=-\frac{x^2}{y}\Big|_{\frac{1}{x}}^{x}=-\frac{x^2}{x}+\frac{x^2}{\frac{1}{x}}=-x+x^3$$

$$I=\int_1^2(-x+x^3)\,dx=-\int_1^2 x\,dx+\int_1^2 x^3\,dx=\left(-\frac{x^2}{2}+\frac{x^4}{4}\right)\Big|_1^2=2-\frac{1}{4}=\frac{9}{4}\Rightarrow\boxed{I=\frac{9}{4}}$$

- Поларне координате:

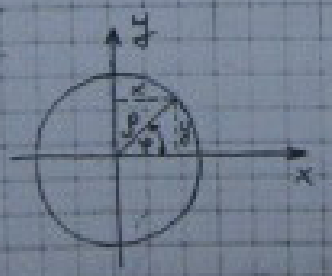
$x=\rho\cdot\cos\varphi$

$y=\rho\cdot\sin\varphi$

$J=\rho$ - Јакобијан

$$\boxed{I=\iint_D f(x,y)\,dx\,dy=\iint f(\rho\cdot\cos\varphi;\ \rho\cdot\sin\varphi)\cdot|J|\cdot d\varphi\,d\rho}$$

$$\boxed{I=\int_\alpha^\beta d\varphi\cdot\int_0^R f(\rho\cdot\cos\varphi;\ \rho\cdot\sin\varphi)\cdot|J|\cdot d\varphi}$$



$x^2+y^2=R^2 \Leftrightarrow \rho^2=R^2$

$0\leq\rho\leq R$

$0\leq\varphi\leq 2\pi$

$J=\rho$ < поларна / цилиндрична

$J=\rho\cdot\sin\varphi$ - сферна

$$J=\begin{vmatrix}\frac{\partial x}{\partial\rho} & \frac{\partial x}{\partial\varphi}\\ \frac{\partial y}{\partial\rho} & \frac{\partial y}{\partial\varphi}\end{vmatrix}=\begin{vmatrix}\cos\varphi & -\rho\sin\varphi\\ \sin\varphi & \rho\cos\varphi\end{vmatrix}=\rho$$