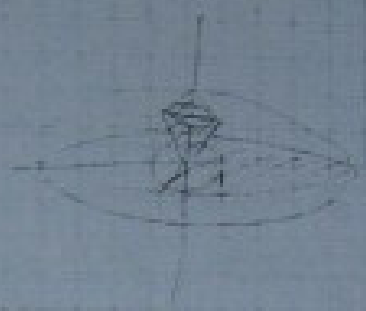
$$V=\int_0^{2\pi} d\varphi \cdot \int_0^{\pi/4} d\theta \cdot \int_0^{\sqrt{2}} J\, d\rho = \int_0^{2\pi} d\varphi \int_0^{\pi/4} d\theta \cdot \int_0^{\sqrt{2}} \rho^2 \cdot \cos\theta \, d\rho =$$

$$= \int_0^{2\pi} d\varphi \cdot \int_0^{\pi/4} d\theta \cdot \cos\theta \cdot \frac{1}{3} \cdot 2\sqrt{2} =$$

$$= \int_0^{2\pi} d\varphi \cdot \frac{2\sqrt{2}}{3} \cdot \sin\theta \Big|_0^{\pi/4} = \int_0^{2\pi} d\varphi \cdot \frac{2\sqrt{2}}{3} \cdot \frac{\sqrt{2}}{2} =$$

$$= \int_0^{2\pi} \frac{2}{3}\, d\varphi = \frac{2}{3}\varphi \Big|_0^{2\pi} = \frac{4\pi}{3}$$



$$\boxed{V=\frac{4\pi}{3}}$$

8 Израчунати V тела ограниченог једначинама:

$$1 \le \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} \le 4$$

$$\rho_1^2 \qquad \rho_2^2$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} \le \frac{z^2}{c^2}, \quad z \ge 0$$

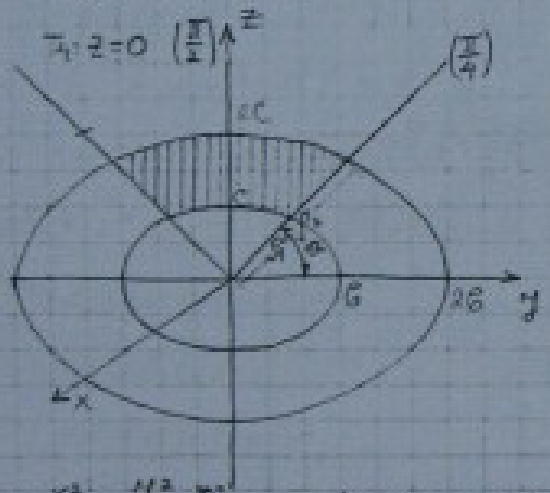
→ ОБЛАСТ УНУТАР КОНУСА!

$\Rightarrow T_1: \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ - елипсоид са осама a, b, c

$T_2: \frac{x^2}{(2a)^2}+\frac{y^2}{(2b)^2}+\frac{z^2}{(2c)^2}=1$ - елипсоид са осама $2a, 2b, 2c$

$T_3: \frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$ - конус

$T_4: z=0$



Сферне координате:

$x = a\rho\cos\varphi\cos\theta$

$y = b\rho\cdot\sin\varphi\cos\theta$

$z = c\cdot\rho\cdot\sin\theta$

$J = abc\cdot\rho^2\cdot\cos\theta$

$T_1: \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (нађи елипсоид):

$$\frac{a^2\rho^2\cos^2\varphi\cos^2\theta}{a^2}+\frac{b^2\rho^2\sin^2\varphi\cos^2\theta}{b^2}+\frac{c^2\rho^2\sin^2\theta}{c^2}=1$$

$$\rho^2\cos^2\theta\cdot(\cos^2\varphi+\sin^2\varphi)+\rho^2\sin^2\theta=1$$

$$\rho^2(\cos^2\theta+\sin^2\theta)=1$$

$$\rho^2=1$$

$$\boxed{\rho_1=1}$$