- Криволинијски интеграл I врсте

$$I=\oint_C f(x,y)\,ds=\int_{t_1}^{t_2} f(x(t),y(t))\cdot\underbrace{\sqrt{x'^2+y'^2}\,dt}_{ds}$$

$$C=\begin{cases}x=x(t)\\ y=y(t)\end{cases}$$

или:

$$I=\int_a^b f(x,y(x))\cdot\sqrt{1+y'^2}\,dx\ ,\quad y=y(x)$$

12 $\oint_C (x^2+y^2+z^2)\,ds$ - прва формула

C: обвојница: $x=a\cdot\sin t$ $\qquad x'=a\cos t$

$y=a\cdot\cos t$ $\qquad y'=-a\sin t$

$z=b\cdot t$ $\qquad z'=b$

$t=0,2\pi$ $\qquad t=\sqrt{x'^2+y'^2+z'^2}\,dt$

$$t=\sqrt{a^2\cos^2 t+a^2\sin^2 t+b^2}=\sqrt{a^2+b^2}$$

$$ds=\sqrt{a^2+b^2}\,dt$$

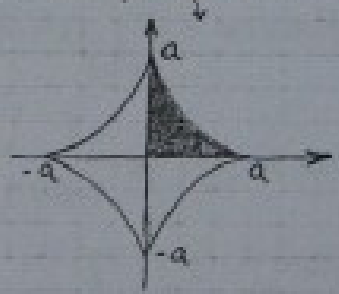
$$I=\int_0^{2\pi}(a^2\sin^2 t+a^2\cos^2 t+b^2t^2)\sqrt{a^2+b^2}\,dt=\sqrt{a^2+b^2}\int_0^{2\pi}(a^2+b^2t^2)\,dt=$$

$$=\sqrt{a^2+b^2}\left(a^2t+b^2\frac{t^3}{3}\right)\Big|_0^{2\pi}=\sqrt{a^2+b^2}\cdot\left(2a^2\pi+\frac{8}{3}b^2\pi^3\right)$$

13 $I=\oint_C (x^{\frac{2}{3}}+y^{\frac{2}{3}})\,ds$; C: $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$; $x,y>0$ - (астероида)

$y^{\frac{2}{3}}=a^{\frac{2}{3}}-x^{\frac{2}{3}}$ - друга формула

$y=(a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{3}{2}}$



$y'=-\frac{3}{2}\cdot\frac{2}{3}\cdot x^{-\frac{1}{3}}\cdot(a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{1}{2}}$

$$I=\int_0^a\left(x^{\frac{2}{3}}+\left((a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{3}{2}}\right)^{\frac{2}{3}}\right)\cdot\sqrt{1+x^{-\frac{2}{3}}\cdot(a^{\frac{2}{3}}-x^{\frac{2}{3}})}\,dx=$$

$$=\int_0^a\left(x^{\frac{2}{3}}+(a^{\frac{2}{3}}-x^{\frac{2}{3}})^2\right)\cdot\sqrt{1+a^{\frac{2}{3}}x^{-\frac{2}{3}}-x}\,dx=$$

$$=\int_0^a(x^{\frac{2}{3}}+a^{\frac{4}{3}}-2a^{\frac{2}{3}}x^{\frac{2}{3}}+x^{\frac{4}{3}})\cdot a^{\frac{1}{3}}\cdot x^{-\frac{1}{3}}\,dx=$$

$$=\int_0^a(2x^{\frac{2}{3}}+a^{\frac{4}{3}}-2a^{\frac{2}{3}}x^{\frac{2}{3}})\,a^{\frac{1}{3}}\cdot x^{-\frac{1}{3}}\,dx$$

$$=\int_0^a(2x\cdot a^{\frac{1}{3}}+a^{\frac{5}{3}}x^{-\frac{1}{3}}-2ax^{-\frac{1}{3}})\,dx=a^{\frac{1}{3}}\int_0^a(2x+a^{\frac{4}{3}}x^{-\frac{1}{3}}-2a^{\frac{2}{3}}x^{-\frac{1}{3}})\,dx=$$