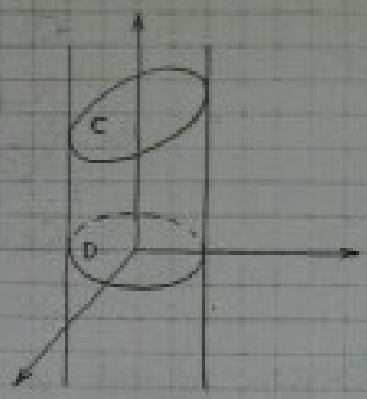
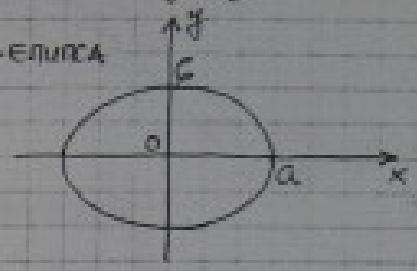
18 Израчунати: $I = \oint_C (x-y)dx + (x-z)dy + (y-z)dz$

$$C: \begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ x + z = a \end{cases}$$



- Пројекција криве C на xoy осу:

D: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ - ЕЛИПСА



- параметризација криве: $x = a\cos t$ $\quad dx = -a\sin t\,dt$

$y = b\sin t$ $\quad dy = b\cos t\,dt$

$z = a - x = a - a\cos t$ $\quad dz = a\sin t\,dt$

$$I = \int_{-\pi}^{\pi} [(a\cos t - b\sin t)\cdot(-a\sin t) + (a\cos t - a + a\cos t)\cdot b\cos t + (b\sin t - a + a\cos t)\,a\sin t]\,dt$$

$$= \int_{-\pi}^{\pi} [-a^2 \underbrace{\sin t\cos t}_{\text{непарна}} + ab\sin^2 t + 2ab\cos^2 t - ab\cos t + ab\sin^2 t -$$

$$- a^2\sin t + a^2 \underbrace{\sin t\cos t}_{-''-=0}]\,dt =$$

$$= \int_{-\pi}^{\pi} [2ab - ab\cos t - a^2\sin t]\,dt = (4abt - 2ab\sin t + a^2\cos t)\Big|_{0}^{\pi} = 4ab\pi$$

$\Rightarrow \boxed{I = 4ab\pi}$

- ПОВРШИНСКИ ИНТЕГРАЛИ I-ВРСТЕ -

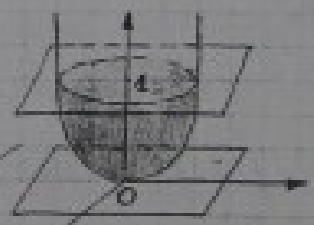
$$\boxed{I = \iint_S f(x,y,z)\,dS = \iint f(x,y,\varphi(x,y))\cdot\sqrt{1+\varphi_x'^2+\varphi_y'^2}\,dx\,dy}$$

$S: z = \varphi(x,y)$

19 Израчунати површину тела ограничене:

$\Gamma_1: x^2+y^2 \le 2z$ - еилиптичан параболоид

$\Gamma_2: 0 \le z \le 1$ - равни



ЗАШТО НИСМО МОГЛИ ДА РАДИМО КЛАСИЧНО (КАО $\boxed{11}$ ЗАД.)!?