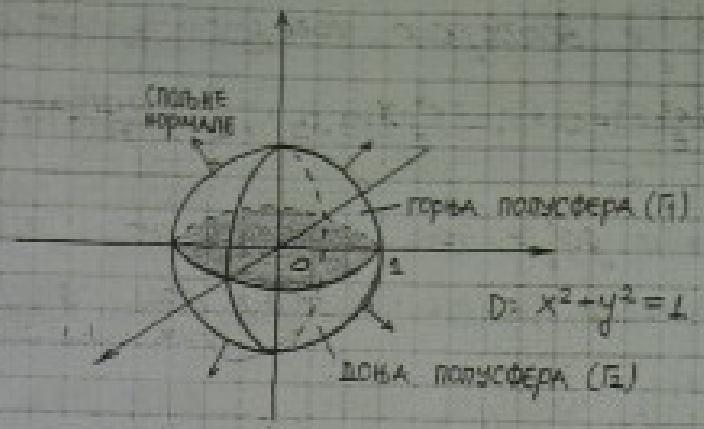
б) Проток дола П:

$\Gamma: x^2+y^2+z^2=1$



- Пошто P, Q, и R нису непрекидно-диференцијабилне фје не можемо применити формулу Гаус Остроградског!

$$\Pi=\oiint_{\Gamma}\left(\frac{3x^2y^2}{z}-2x^3\right)dydz+\left(\frac{2x^3y}{z}+3y^3\right)dxdz+\left(z^3-\frac{x^3y^2}{z^2}\right)dxdy$$

Γ_1 - горња полусфера $\rightarrow$ $z=\sqrt{1-x^2-y^2}$ (изражавамо z јер ако хоћемо да пројектујемо на xy)

Γ_2 - доња полусфера $\rightarrow$ $z=-\sqrt{1-x^2-y^2}$

↳ + ГЛЕДАМО ДАЛИ НОРМАЛЕ ИДУ У СМЕРУ ИЛИ НЕ ОД z-ОСЕ

$$\oiint_{\Gamma}=\oiint_{\Gamma_1}+\oiint_{\Gamma_2}\quad,\ \Pi=\Pi_{\Gamma}+\Pi_{\Delta}$$

$$\Pi_{\Gamma}=\oiint_{\Gamma_1}\left(\frac{3x^2y^2}{z}-2x^3\right)dydz+\left(\frac{2x^3y}{z}+3y^3\right)dxdz+\left(z^3-\frac{x^3y^2}{z^2}\right)dxdy=$$

$$=\iint_D\left(\frac{3x^2y^2}{\sqrt{1-x^2-y^2}}-2x^3,\ \frac{2x^3y}{\sqrt{1-x^2-y^2}}+3y^3,\ \sqrt{(1-x^2-y^2)^3}-\frac{x^3y^2}{1-x^2-y^2}\right)\cdot\left(\frac{-x}{\sqrt{1-x^2-y^2}},\ \frac{-y}{\sqrt{1-x^2-y^2}},\ 1\right)dxdy=$$

$$=\iint_D\left(\frac{3x^2y^2}{1-x^2-y^2}-\frac{2x^4}{\sqrt{1-x^2-y^2}}+\frac{2x^3y^2}{1-x^2-y^2}+\frac{3y^4}{\sqrt{1-x^2-y^2}}+\sqrt{(1-x^2-y^2)^3}-\frac{x^3y^2}{1-x^2-y^2}\right)dxdy=$$

$$=\iint_D\left(\frac{4x^3y^2}{1-x^2-y^2}+\frac{3y^4-2x^4}{\sqrt{1-x^2-y^2}}+\sqrt{(1-x^2-y^2)^3}\right)dxdy=\left(\begin{array}{ll}x=\rho\cos\varphi & 0\le\varphi\le2\pi\\ y=\rho\sin\varphi & \\ z=\rho & 0\le\rho\le1\end{array}\right)=$$

НЕПОТРЕБНО

$$=\int_0^{2\pi}d\varphi\cdot\int_0^1\left(\frac{4\rho^6\cos^3\varphi\cdot\sin^2\varphi}{1-\rho^2}+\frac{3\rho^5\sin^4\varphi-2\rho^5\cos^4\varphi}{\sqrt{1-\rho^2}}+\sqrt{(1-\rho^2)^3}\cdot\rho\right)d\rho=$$

$$=\int_0^{2\pi}d\varphi\cdot\left[4\sin^2\varphi\cos^3\varphi\cdot\underbrace{\int_0^1\frac{\rho^6}{1-\rho^2}d\rho}_{I_1}+(3\sin^4\varphi-2\cos^4\varphi)\underbrace{\int_0^1\frac{\rho^5}{\sqrt{1-\rho^2}}d\rho}_{I_2}+\underbrace{\int_0^1\sqrt{1-\rho^2}\cdot\rho\,d\rho}_{I_3}\right]$$

$$I_1=\int_0^1\frac{\rho^6}{1-\rho^2}d\rho=\left(\begin{array}{l}\rho^6=(1-\rho^2)\cdot(-\rho^4-\rho^2-1)+1\\ -\rho^6-\rho^4\\ \hline 0+\rho^4\\ -\rho^4-\rho^2\\ \hline 0+\rho^2\\ -\rho^2-1\\ \hline 0+1\end{array}\right.$$

$$I_1=\int_0^1\frac{(1-\rho^2)\cdot(-\rho^4-\rho^2-1)}{1-\rho^2}d\rho+\int_0^1\frac{d\rho}{1-\rho^2}=-\left(\frac{\rho^5}{5}+\frac{\rho^3}{3}+\rho\right)\Big|_0^1+\frac{1}{2}\ln\left|\frac{1+\rho}{1-\rho}\right|\Big|_0^1$$

$$=-\frac{1}{5}+\frac{1}{3}+1+\frac{1}{2}+\frac{1}{2}\ln\left|\frac{2}{0}\right|\ ?$$