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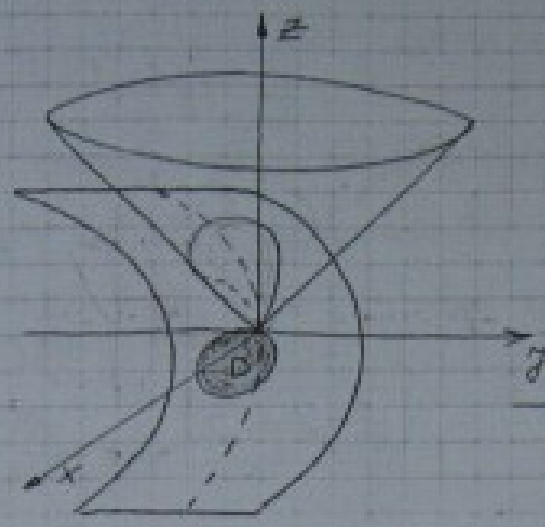
11 Израчунати површину и запремину тела ограничене делом ротационог конуса $x^2+y^2=z^2$ $(z\geq 0)$ и делом параболичног цилиндра $z^2=2x$ $(z\geq 0)$.

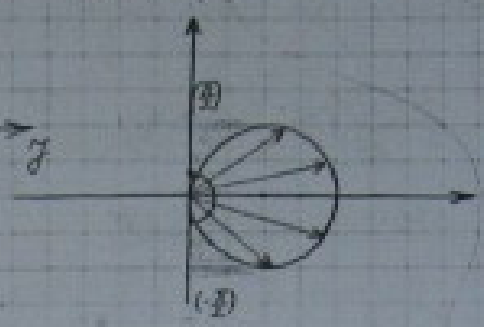
$x=\frac{1}{2}z^2$



- пројекција пресека на xy:

$x^2+y^2=2x$

$(x-1)^2+y^2=1$



$0\leq\rho\leq 2\cos\varphi$

$-\frac{\pi}{2}\leq\varphi\leq\frac{\pi}{2}$

Поларне координате:

$x=\rho\cos\varphi$

$y=\rho\sin\varphi$

$$V=\iint_D (z_{гл}-z_{кон})\,dx\,dy$$

$$\left.\begin{array}{l} z_{кон}=\rho \\ z_{гл}=\sqrt{2\cdot\rho\cos\varphi}\end{array}\right\}\Rightarrow V=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\cdot\int_0^{2\cos\varphi}(\sqrt{2\cdot\rho\cos\varphi}-\rho)\cdot\rho\,d\rho=$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\cdot\int_0^{2\cos\varphi}(\sqrt{2\cos\varphi}\cdot\rho^{\frac{1}{2}}-\rho^2)\,d\rho=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\cdot I_1$$

$$I_1=\sqrt{2\cos\varphi}\int_0^{2\cos\varphi}\rho^{\frac{1}{2}}d\rho-\int_0^{2\cos\varphi}\rho^2 d\rho=\sqrt{2\cos\varphi}\cdot\frac{2}{3}\rho^{\frac{3}{2}}\Big|_0^{2\cos\varphi}-\frac{1}{3}\rho^3\Big|_0^{2\cos\varphi}=$$

$$=\sqrt{2\cos\varphi}\cdot\frac{2}{3}\sqrt{2\cos\varphi}^3-\frac{1}{3}8\cos^3\varphi=\frac{2}{3}\cdot 2\cos^2\varphi-\frac{8}{3}\cos^3\varphi=\frac{2}{15}\cos^3\varphi$$

$$V=\frac{2}{15}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^3\varphi\,d\varphi=\frac{2}{15}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1-\sin^2\varphi)\,d(\sin\varphi)=\frac{2}{15}\left[\sin\varphi-\frac{1}{3}\sin^3\varphi\right]\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\frac{28}{45}$$

- Површина дела конуса $z=\sqrt{x^2+y^2}$ унутар параболичног цилиндра $z=\sqrt{2x}$ је:

$z=\sqrt{x^2+y^2}$, $p=\frac{x}{\sqrt{x^2+y^2}}$, $q=\frac{y}{\sqrt{x^2+y^2}}$