а) $V = 4\cdot\iint\limits_D z\,dxdy$, $z = \frac{1}{2}(x^2+y^2)$

1) СЛИКА

2) $y = \frac{x}{4}$: ?

3) Отуде видно да је нетривијално

$V = 2\iint\limits_D (x^2+y^2)\,dxdy$

Прелазимо на поларне координате:

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

$dxdy = \rho\,d\rho\,d\varphi$

$V = 2\iint\limits_{D'} \rho^2\cdot\rho\,d\rho\,d\varphi = 2\iint\limits_{D'} \rho^3\,d\rho\,d\varphi$

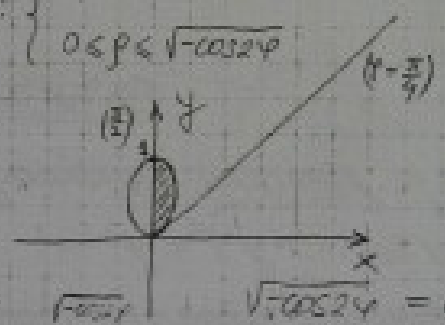
D: $(x^2+y^2)^2 = y^2 - x^2$

$(\rho^2)^2 = \rho^2\sin^2\varphi - \rho^2\cos^2\varphi$

$\rho^2 = \sin^2\varphi - \cos^2\varphi = -\cos 2\varphi$

$\rho = \sqrt{-\cos 2\varphi}$

$\Rightarrow$ D': $\begin{cases} \frac{\pi}{4} \le \varphi \le \frac{\pi}{2} \\ 0 \le \rho \le \sqrt{-\cos 2\varphi} \end{cases}$



$\sqrt{-\cos 2\varphi} = 0$

$-\cos 2\varphi = 0$

$\cos 2\varphi = 0$

$\cos\frac{\pi}{2} = 0 \Rightarrow 2\varphi = \frac{\pi}{2}$

$\varphi = \frac{\pi}{4}$

$V = 2\iint\limits_{D'} \rho^3\,d\rho\,d\varphi = 2\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi\cdot\int\limits_0^{\sqrt{-\cos 2\varphi}} \rho^3\,d\rho = 2\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\rho^4}{4}\Big|_0^{\sqrt{-\cos 2\varphi}}\,d\varphi$

$= \frac{1}{2}\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 2\varphi\,d\varphi = \frac{1}{4}\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1+\cos 4\varphi)\,d\varphi = \frac{1}{4}\left(\varphi + \frac{\sin 4\varphi}{4}\right)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{4}\left[\frac{\pi}{2} - \frac{\pi}{4} - 0\right]$

$\boxed{V = \frac{\pi}{16}}$

б) $S = 4\iint\limits_D \sqrt{1+p^2+q^2}\,dxdy$, $z = \frac{1}{2}(x^2+y^2)$

$p = \frac{\partial z}{\partial x} = x$, $q = \frac{\partial z}{\partial y} = y$

$S = 4\iint\limits_D \sqrt{1+x^2+y^2} = \begin{bmatrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \\ dxdy = \rho\,d\rho\,d\varphi \end{bmatrix} = 4\iint\limits_{D'} \sqrt{1+\rho^2}\,\rho\,d\rho\,d\varphi$, D': $\begin{cases} \frac{\pi}{4} \le \varphi \le \frac{\pi}{2} \\ 0 \le \rho \le \sqrt{-\cos 2\varphi} \end{cases}$

$S = 4\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi\cdot\int\limits_0^{\sqrt{-\cos 2\varphi}} \sqrt{1+\rho^2}\,\rho\,d\rho = \ldots = \frac{20}{9} - \frac{\pi}{3}$

$\boxed{S = \frac{20}{9} - \frac{\pi}{3}}$