$$S = \iint_{S_1} + \iint_{S_2}, \text{ где } \underline{S_1: z=1} \Rightarrow z'_x = 0$$

$$z'_y = 0$$

$$d\sigma = \sqrt{1+z'^2_x+z'^2_y}\,dx\,dy \Rightarrow d\sigma = dx\,dy$$

$$\iint_{S_1} = \iint (x^2+y^2)\,d\sigma = \iint_D (x^2+y^2)\,dx\,dy$$

D – проекция S_1 на xoy oy

$$\begin{cases} z=1 \\ x^2+y^2=2z \end{cases} \Rightarrow D: x^2+y^2=2 \text{ – круг; } r=\sqrt{2},\ C(0,0)$$

Смена $x=\rho\cos\varphi$ $\quad 0\le\rho\le\sqrt{2}$

$y=\rho\sin\varphi$ $\quad 0\le\varphi\le 2\pi$

$$= \int_0^{2\pi} d\varphi \cdot \int_0^{\sqrt{2}} \rho^2\cdot\rho\,d\rho = \int_0^{2\pi} d\varphi \cdot \frac{\rho^4}{4}\Big|_0^{\sqrt{2}} = \int_0^{2\pi} d\varphi = 2\pi \Rightarrow \boxed{S_1 = 2\pi}$$

$$S_2: z=\frac{1}{2}(x^2+y^2) \Rightarrow z'_x = x$$

$$z'_y = y$$

$$d\sigma = \sqrt{1+x+y}\,dx\,dy$$

$$\iint_{S_2} = \iint_D (x^2+y^2)\cdot\sqrt{1+x+y}\,dx\,dy = \int_0^{2\pi} d\varphi \cdot \int_0^{\sqrt{2}} \rho^3\cdot\sqrt{1+\rho^2}\,d\rho = \int_0^{2\pi} d\varphi \cdot I_1$$

$$I_1 = \int_0^{\sqrt{2}} \rho^3\sqrt{1+\rho^2}\,d\rho = \begin{pmatrix} \sqrt{1+\rho^2}=t \Rightarrow \rho=\sqrt{t^2-1} \\ d\rho = \frac{t\,dt}{\sqrt{t^2-1}} \end{pmatrix} =$$

$$= \int_1^{\sqrt{3}} (t^2-1)^{\frac{3}{2}}\cdot t\cdot\frac{t\,dt}{\sqrt{t^2-1}} = \int_1^{\sqrt{3}} t^2\cdot(t^2-1)\,dt = \int_1^{\sqrt{3}} (t^4-t^2)\,dt = \frac{t^5}{5}\Big|_1^{\sqrt{3}} - \frac{t^3}{3}\Big|_1^{\sqrt{3}} =$$

$$= \frac{9\sqrt{3}}{5} - \frac{1}{5} - \frac{3\sqrt{3}}{3} + \frac{1}{3} = \frac{27\sqrt{3}-3-15\sqrt{3}+5}{15} = \frac{12\sqrt{3}+2}{15}$$

$$S_2 = \frac{12\sqrt{3}+2}{15}\int_0^{2\pi} d\varphi = \frac{12\sqrt{3}+2}{15}\cdot 2\pi = \frac{8\sqrt{3}\pi}{5} + \frac{4\pi}{15}$$

$$\boxed{S = 2\pi + \frac{8\sqrt{3}\pi}{5} + \frac{4\pi}{15}}$$