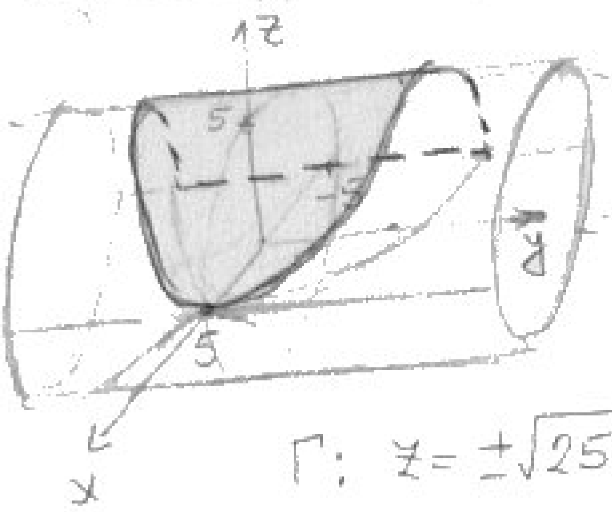
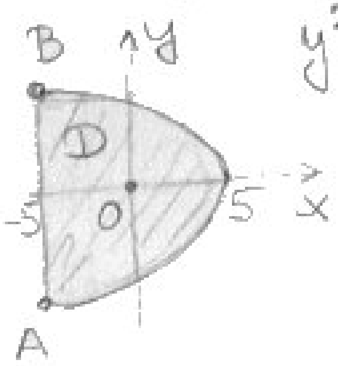
# ЧЕТВРТИ ЗАДАТАК

**6.5B(21)** $\Gamma: x^2+z^2=25$ кружни цилиндар, паралелан $y$-оси.




$$\left.\begin{matrix} y^2=25-5x \\ x=-5 \end{matrix}\right\} \quad \begin{matrix} y^2=50 \\ y=5\sqrt{2} \end{matrix}$$

$A(-5,-5\sqrt{2})$, $B(-5,5\sqrt{2})$

горњи део цилиндра

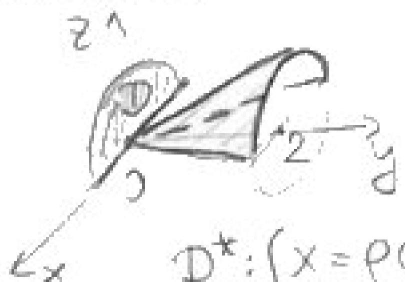
$$\Gamma:\ z=\pm\sqrt{25-x^2}\ ;\quad z=\sqrt{25-x^2} \qquad p=z'_x=\frac{-x}{\sqrt{25-x^2}} \qquad q=z'_y=0$$

$$P=2\iint_D \sqrt{1+p^2+q^2}\,dx\,dy=2\iint_D \sqrt{1+\frac{x^2}{25-x^2}}\,dx\,dy=2\iint_D \frac{5\,dx\,dy}{\sqrt{25-x^2}}=$$

$5-\frac{1}{5}y^2$

$$=10\int_{-5}^{5}dx\,\frac{1}{\sqrt{25-x^2}}\int_{-\sqrt{25-5x}}^{\sqrt{25-5x}}dy=10\int_{-5}^{5}\frac{dx}{\sqrt{25-x^2}}\cdot 2\sqrt{25-x^2}=20\int_{-5}^{5}dx=20\cdot 10=\underline{200}$$

**6.5B.(29)** $\Gamma: y=\sqrt{x^2+z^2} \qquad p=y'_x=\frac{x}{\sqrt{x^2+z^2}} \qquad q=y'_z=\frac{z}{\sqrt{x^2+z^2}}$

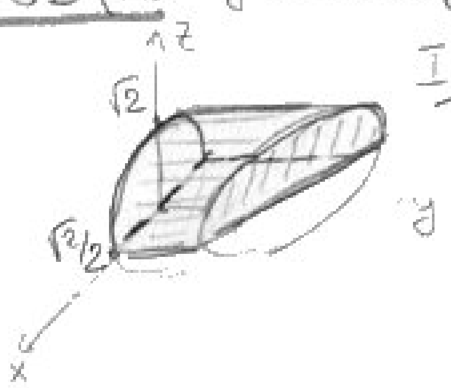



$$P=\iint_D\sqrt{1+p^2+q^2}\,dx\,dz=\iint_D\sqrt{2}\,dx\,dz=\sqrt{2}\iint_{D^*}\rho\,d\rho\,d\varphi=\sqrt{2}\int_0^{\pi}d\varphi\int_0^{2}\rho\,d\rho=$$

$$=\frac{\sqrt{2}}{2}\,4\pi=2\sqrt{2}\,\pi$$

$$D^*:\begin{cases} x=\rho\cos\varphi \\ z=\rho\sin\varphi \end{cases}$$

**6.6B(13)** у тексту треба додати $\underline{y=0\ (z\geq 0)}$. Дата су два начина.



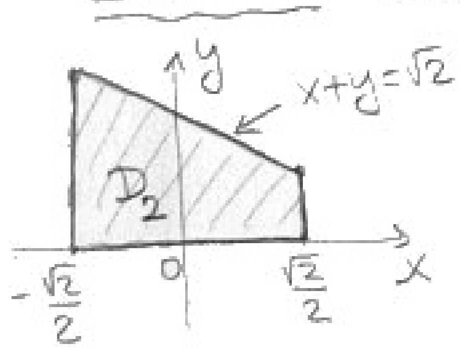

<u>I начин</u>: пројектујемо на $xOz$:


$$V=\iint_{D_1}(\sqrt{2}-x)\,dx\,dz=\int_0^{\pi}d\varphi\int_0^{1}\left(\sqrt{2}-\frac{\sqrt{2}}{2}\rho\cos\varphi\right)\cdot\rho\,d\rho$$

$$=\int_0^{\pi}d\varphi\cdot\sqrt{2}\left(\frac{1}{2}\rho^2-\frac{1}{6}\rho^3\cos\varphi\right)\Big|_0^1=\sqrt{2}\int_0^{\pi}\left(\frac{1}{2}-\frac{1}{6}\cos\varphi\right)d\varphi=$$

$$\left.\begin{matrix} x=\frac{\sqrt{2}}{2}\rho\cos\varphi \\ y=\sqrt{2}\rho\sin\varphi \end{matrix}\right\}\ J=\rho \qquad V=\sqrt{2}\cdot\frac{1}{2}\pi$$

<u>II начин</u> ако пројектујемо на $xOy$:




$$V=\iint_{D_2}\sqrt{2-4x^2}\,dx\,dy=\int_{-\sqrt{2}/2}^{\sqrt{2}/2}dx\int_0^{\sqrt{2}-x}\sqrt{2-4x^2}\,dy=$$

$$=\int_{-\sqrt{2}/2}^{\sqrt{2}/2}dx\,\sqrt{2-4x^2}\,(\sqrt{2}-x)=\int_{-\sqrt{2}/2}^{\sqrt{2}/2}\sqrt{2}\sqrt{2-4x^2}\,dx+\underbrace{\int_{-\sqrt{2}/2}^{\sqrt{2}/2}-x\sqrt{2-4x^2}\,dx}_{\text{непарно}}=$$

$$=2\int_0^{\sqrt{2}/2}2\sqrt{1-2x^2}\,dx=4\int_{\pi/2}^{0}\sin t\cdot\left(-\frac{1}{\sqrt{2}}\sin t\right)dt=\frac{4}{\sqrt{2}}\int_0^{\pi/2}\sin^2 t\,dt=\frac{4}{\sqrt{2}}\int_0^{\pi/2}\frac{1-\cos 2t}{2}\,dt=\frac{\pi}{\sqrt{2}}$$

$\sqrt{2}x=\cos t$