KOLOKVIJUMI TM2:

1. 7. nedelja
2. 13. nedelja

$\Big\}$ >75 ukupno

3. >50

KINEMATIKA TAČKE

① Date su konačne jedn. kretanja

$x(t) = 4\cos t + 3$

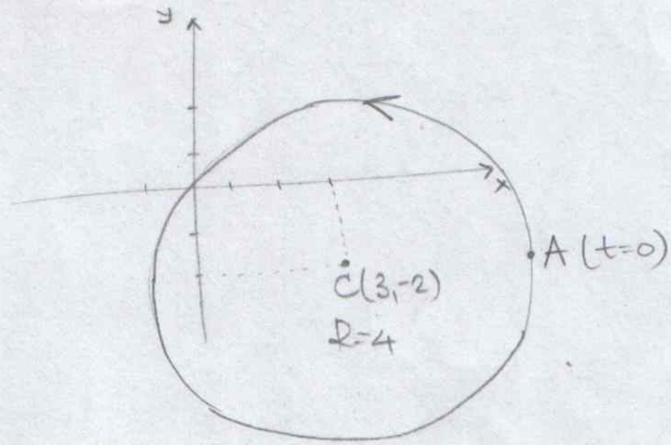
$y(t) = 4\sin t - 2$. Odredi: 1) trajektoriju i zakon puta uz uslov $s(0)=0$; 2) brzinu i ubrzanje tačke.

$x(t) = 4\cos t + 3$

$y(t) = 4\sin t - 2$

1) $\left.\begin{array}{l} x-3 = 4\cos t \;/^2 \\ y+2 = 4\sin t \;/^2 \end{array}\right\} +$

$$\boxed{(x-3)^2 + (y+2)^2 = 4^2 \cdot 1}$$ KRUG



$\underline{t=0}$: $x=7$
poč. tačka $y=-2$ $\quad A(7,-2)$

$t>0$ $x\searrow, y\nearrow$ kretanje ↺

zakon puta:

$$s(t) = \int_{t_0}^{t} ds \;, \quad ds = \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$$

$$\left.\begin{array}{l} \dot{x} = -4\sin t \\ \dot{y} = 4\cos t \end{array}\right\} ds = \sqrt{(-4\sin t)^2 + (4\cos t)^2}\, dt = 4dt$$

$$s(t) = \overset{\to 0}{s(0)} + \int_0^t ds = \int_0^t 4dt = \underline{4t}$$

2) brzina i ubrzanje: (u Dekartovom koord. sist.)

$$\vec{v} = \dot{x}\cdot\vec{i} + \dot{y}\cdot\vec{j}$$

$v_x = \dot{x} = -4\sin t$

$v_y = \dot{y} = 4\cos t$

$\Rightarrow$ $\vec{v} = -4\sin t \cdot \vec{i} + 4\cos t \cdot \vec{j}$

$v = |\vec{v}| = \sqrt{(-4\sin t)^2 + (4\cos t)^2} = 4 = \text{const}$

ubrzanje

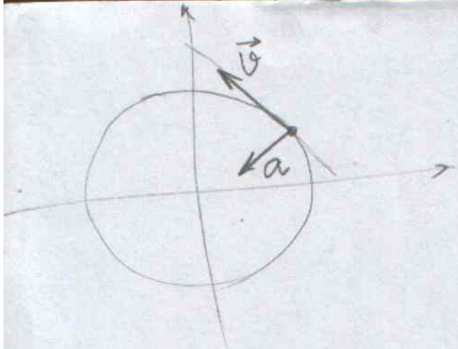
$\vec{a} = \ddot{x}\cdot\vec{i} + \ddot{y}\cdot\vec{j}$

$a_x = \ddot{x} = -4\cos t$

$a_y = \ddot{y} = -4\sin t$

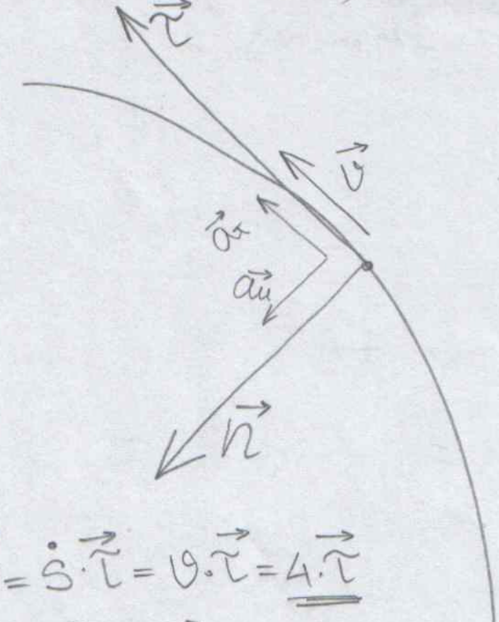
$\Rightarrow$ $\vec{a} = -4\cos t\,\vec{i} - 4\sin t \cdot \vec{j}$

$a = \sqrt{} = 4 = \text{const}$



ILI

(prirodni koord. sist.)



$$\vec{v} = \dot{s}\cdot\vec{\tau} = v\cdot\vec{\tau} = \underline{\underline{4\cdot\vec{\tau}}}$$

$$\vec{a} = \vec{a_\tau} + \vec{a_N}$$

$$\vec{a_\tau} = \dot{v}\cdot\vec{\tau} = 0$$

$$\vec{a_N} = \frac{v^2}{\rho}\cdot\vec{n} = \frac{4^2}{4}\cdot\vec{n} = 4\vec{n}$$

ρ-poluprečnik krivine

$$\underline{\underline{\vec{a} = 4\vec{n}}}$$

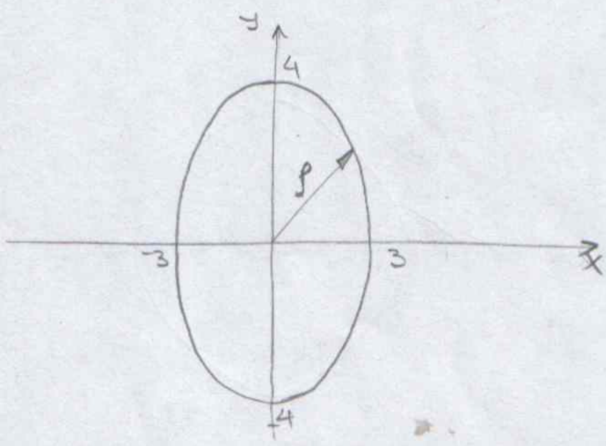
2. Tačka se kreće u ravni tako da je $x = 3\cos(2t)$, $y = 4\sin(2t)$, gde su x i y [m], a t [s]. Odredi brzinu, ubrzanje i poluprečnik krivine u trenutku $t = \frac{\pi}{8}$ [s].

$$x = 3\cos(2t)$$
$$y = 4\sin(2t)$$

$$\frac{x}{3} = \cos(2t) \;/^2$$
$$\frac{y}{4} = \sin(2t) \;/^2 \quad +$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

$$\boxed{\frac{x^2}{3^2} + \frac{y^2}{4^2} = 1} \text{ elipsa}$$



$$t = \frac{\pi}{8}$$

brzina:

$$v_x = \dot{x} = -6\sin(2t) \qquad v_x = -6\,\frac{\sqrt{2}}{2} = -4{,}243\,\frac{m}{s}$$

$$v_y = \dot{y} = 8\cos(2t) \qquad v_y = 8\cdot\frac{\sqrt{2}}{2} = 5{,}657\,\frac{m}{s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \underline{\underline{7{,}071\,\frac{m}{s}}}$$

ubrzanje: $t=\frac{\pi}{8}$

$a_x = \ddot{x} = -12\cos(2t) \qquad a_x = -12\cdot\frac{\sqrt{2}}{2} = -8{,}485\ \frac{m}{s^2}$

$a_y = \ddot{y} = -16\sin(2t) \qquad a_y = -16\cdot\frac{\sqrt{2}}{2} = -11{,}314\ \frac{m}{s^2}$

$$a = \sqrt{a_x^2 + a_y^2} = \underline{\underline{14{,}142\ \frac{m}{s^2}}}$$

poluprečnik krivine:

$\vec{a} = \vec{a_T} + \vec{a_N} = \dot{v}\cdot\vec{\tau} + \frac{v^2}{\rho}\cdot\vec{n}$

$|a|^2 = |\dot{v}|^2 + \left|\frac{v^2}{\rho}\right|^2$

$(14{,}142)^2 = (-3{,}96)^2 + \left(\frac{(7{,}071)^2}{\rho}\right)^2$

$\rho^2 = 13{,}56 \qquad \underline{\underline{\rho = 3{,}68m}}$

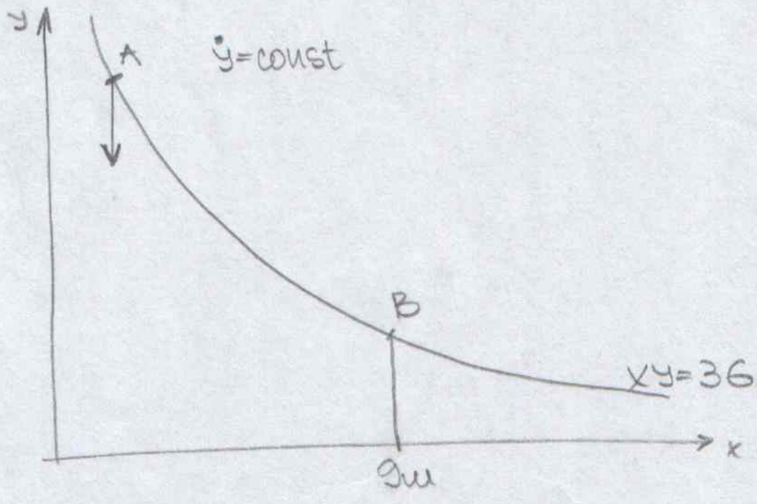
$\dot{v} = \vec{a}\cdot\vec{\tau}$

$\vec{\tau} = \frac{\vec{v}}{|\vec{v}|} = \frac{-6\sin(2t)\vec{i} + 8\cos(2t)\vec{j}}{7{,}071}$

$= \frac{-4{,}243\cdot\vec{i} + 5{,}657\vec{j}}{7{,}071} = -0{,}6\vec{i} + 0{,}8\vec{j}$

$\dot{v} = \underbrace{(-8{,}485; -11{,}314)}_{\vec{a}}\cdot\underbrace{(-0{,}6; 0{,}8)}_{\vec{\tau}} = -3{,}96$

3. Prsten A klizi bez trenja po žici koja ima oblik hiperbole $xy = 36m^2$. Ako je brzina prstena u pravcu y const i $\underbrace{v_y}_{\dot{y}} = -10\frac{m}{s}$, odredi ubrzanje prstena u $B(9m, ?)$.



$xy = 36m^2$

$v_y = -10\frac{m}{s} = const$

$a_B = ? \qquad B(9m, ?)$

$B(9,4)$

$\vec{a_B} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$

$\dot{y} = const \qquad \ddot{y} = 0$

$x = 36\cdot\frac{1}{y}$

$\dot{x} = 36\cdot\left(-\frac{1}{y^2}\right)\cdot\dot{y} = -36\left(\frac{\dot{y}}{y^2}\right)$

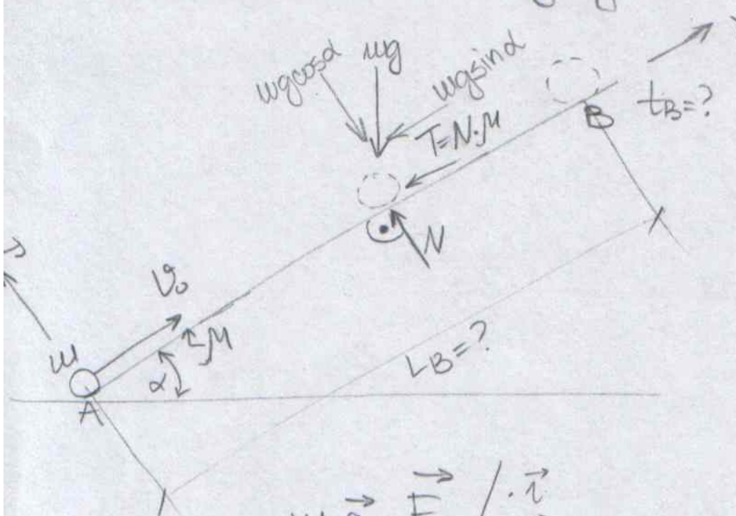
$\ddot{x} = -36\left(-\frac{2}{y^3}\cdot\dot{y}\cdot\dot{y} + \frac{1}{y^2}\cdot\ddot{y}\right)$

$\ddot{x} = -36\left(-\frac{2}{y^3}\cdot\dot{y}^2 + \frac{1}{y^2}\cdot\ddot{y}\right)$

$\ddot{x}_B = -36\left(-\frac{2}{4^3}(-10)^2 + \frac{1}{4^2}\cdot 0\right) = 112{,}5\frac{m}{s^2}$

DINAMIKA TAČKE

1. Mat. tačka mase m je u položaju A dobila početnu brzinu v_0 usled koje se kreće uz hrapavu strmu ravan. Nagib ravni u odnosu na horizontalu je α, a koef. trenja je μ. Odrediti vreme i mesto zaustavljanja tačke.



v_0, α, μ

$$\underline{m\vec{a}=\vec{F}_R} \Big/ \cdot \vec{i}, \ \cdot \vec{j}$$

$x: \quad m\cdot\ddot{x} = -mg\cdot\sin\alpha - \mu N$

$y: \quad m\cdot\ddot{y} = N - mg\cos\alpha \qquad N = mg\cos\alpha$

$\ddot{y} = 0$

$m\cdot\ddot{x} = -mg\sin\alpha - \mu\cdot mg\cdot mg\cos\alpha$

$$\ddot{x} = -\underbrace{\boxed{g(\sin\alpha + \mu\cos\alpha)}}_{A}$$

$\ddot{x} = -A \qquad \left(\frac{d\dot{x}}{dt} = -A\right)$

$\dot{x} = -At + C_1$

$x = -A\cdot\frac{t^2}{2} + C_1 t + C_2$

$t=0: \quad v_0 = -A\cdot 0 + C_1 \qquad v_0 = C_1$

$0 = C_2$

$$\boxed{\begin{aligned}\dot{x}(t) &= At + v_0\\ x(t) &= -\frac{1}{2}At^2 + v_0 t\end{aligned}}$$

uslov zaustavljanja tačke:

$\dot{x}(t_B) = 0 \qquad -At_B + v_0 = 0 \qquad \boxed{t_B = \frac{v_0}{A}}$

pređeni put:

$L_B = x(t_B) = -\frac{1}{2}A\cdot\left(\frac{v_0}{A}\right)^2 + v_0\cdot\frac{v_0}{A} = \frac{1}{2}\frac{v_0^2}{A}$

$$\boxed{L_B = \frac{1}{2}\frac{v_0^2}{A}}$$

II NAČIN

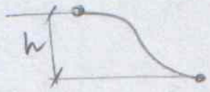
ZAKON O PROMENI KINETIČKE ENERGIJE

$$T_B - T_A = A_{A\to B}$$

$$\frac{1}{2} m \cdot \cancel{v_B^2}^{\to 0} - \frac{1}{2} m \cdot v_0^2 = -mg\sin\alpha \cdot L_B - \mu \cdot \overset{mg\cos\alpha}{N} L_B$$

$$\Sigma F_y = 0 \Rightarrow N = mg\cos\alpha$$

*SILA TEŽE



$$A = mg \cdot h \Rightarrow \frac{1}{2} m \cdot v_B^2 + \frac{1}{2} m\, v_0^2 = \overset{-mg \cdot L_B \sin\alpha}{-mg \cdot \sin\alpha L_B} - \mu \cdot N \cdot L_B$$

$$-\frac{1}{2} m\, v_0^2 = -mgL_B(\sin\alpha + \mu\cos\alpha)$$

$$\boxed{L_B = \frac{v_0^2}{2g(\sin\alpha + \mu\cos\alpha)}}$$

* ZAKON O PROMENI KOLIČINE KRETANJA*

$$\vec{K}_B - \vec{K}_A = \int_{t_A}^{t_B} \vec{F} \cdot dt$$

količina kretanja

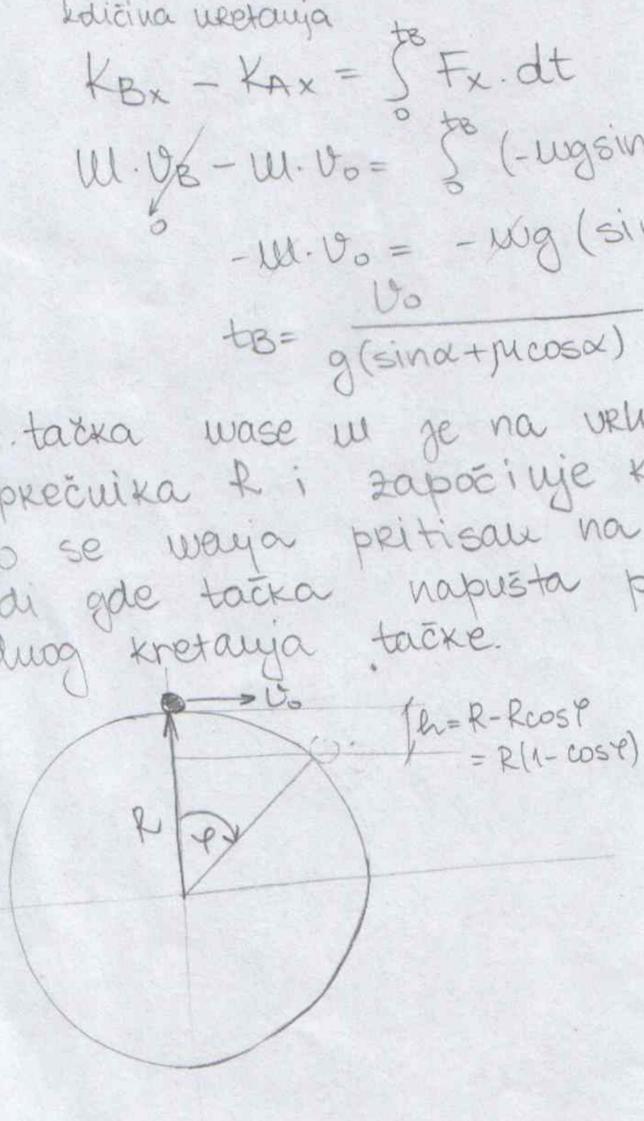
$$K_{Bx} - K_{Ax} = \int_0^{t_B} F_x \cdot dt$$

$$m \cdot \cancel{v_B}_{0} - m \cdot v_0 = \int_0^{t_B} (-mg\sin\alpha - \mu \cdot \overbrace{mg\cos\alpha}^{N})\, dt$$

$$-m \cdot v_0 = -mg(\sin\alpha + \mu \cdot \cos\alpha)\, t \Big|_0^{t_B}$$

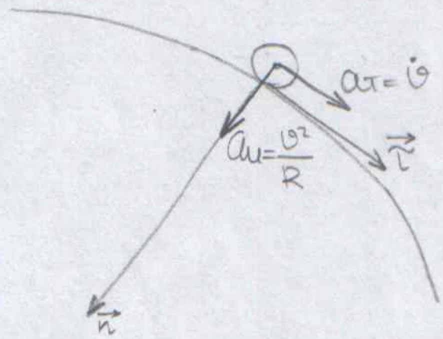
$$t_B = \frac{v_0}{g(\sin\alpha + \mu\cos\alpha)}$$

2. Mat. tačka mase m je na vrhu idealno glatkog cilindra poluprečnika R i započinje kretanje poč. brzinom v_0. 1) Napiši kako se menja pritisak na podlogu tokom kretanja, 2) odredi gde tačka napušta podlogu i 3) napiši konačne jedn. slobodnog kretanja tačke.



$$N(\varphi) = ?$$

$$m \cdot \vec{a} = \vec{F}_R$$

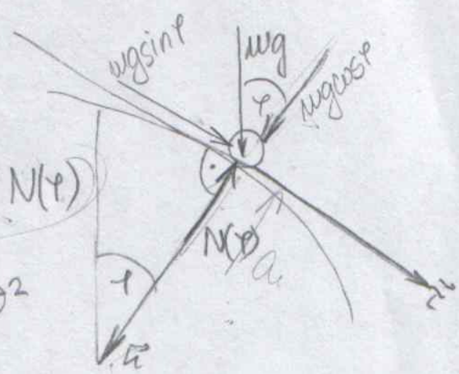


$$m \cdot \vec{a} = \vec{F}_R \quad / \cdot \vec{\tau}, \ \cdot \vec{n}$$

$$\vec{\tau}: m \cdot \dot{v} = mg\sin\varphi$$

$$\vec{n}: m \cdot \frac{v^2}{R} = mg\cos\varphi - N(\varphi)$$

$$N(\varphi) = mg\cos\varphi - \frac{m}{R} \cdot v^2$$



$$T_B - T_A = A_{A \to B}$$

$$\frac{1}{2} m v_B^2 - \frac{1}{2} m \cdot v_0^2 = mgR(1-\cos\varphi)$$

$$v_B^2 = v_0^2 + 2 \cdot g \cdot R \cdot (1-\cos\varphi) \quad (\star)$$

$$N(\varphi) = mg\cos\varphi - \frac{m}{R}\left(v_0^2 + 2gR - 2gR\cos\varphi\right)$$

$$\boxed{N(\varphi) = 3mg\cos\varphi - 2mg - \frac{m}{R} \cdot v_0^2}$$

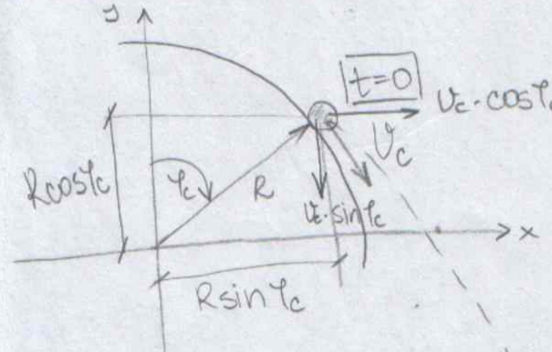
uslov napuštanja podloge:

$$N(\varphi_c) = 0$$

$$3mg\cos\varphi_c - 2mg - \frac{m}{R} v_0^2 = 0$$

$$\boxed{\cos\varphi_c = \frac{2}{3} + \frac{1}{3}\frac{v_0^2}{gR}}$$

slobodno kretanje tačke:



$$v_c^2 = v_0^2 + 2gR(1-\cos\varphi_c) \quad (\star)$$

?

$$v_c^2 = v_0^2 + 2gR\left(1 - \left(\frac{2}{3} + \frac{1}{3}\frac{v_0^2}{gR}\right)\right)$$

$$v_c^2 = \frac{1}{3} v_0^2 + \frac{2}{3} gR$$

diferencijalna jedn. kretanja:

$$m \cdot \vec{a} = \vec{F}_R \quad / \cdot \vec{i}, \ \cdot \vec{j}$$

$\downarrow mg$

$$x: \ m \cdot \ddot{x} = 0 \qquad \dot{x} = C_1 \qquad x = C_1 \cdot t + C_2$$

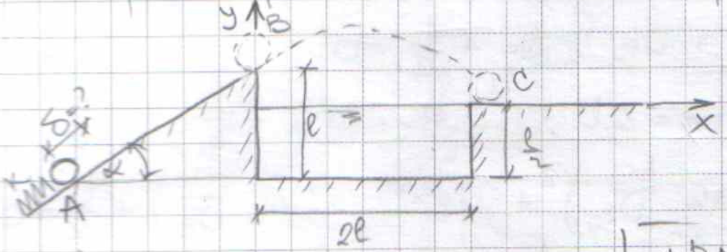
$$y: \ \cancel{m} \cdot \ddot{y} = -\cancel{m}g \qquad \dot{y} = -gt + C_3 \qquad y = -g\frac{t^2}{2} + C_3 \cdot t + C_4$$

$$t=0 \quad v_c\cos\varphi_c = C_1 \qquad -v_c \cdot \sin\varphi_c = C_3$$

$$R\sin\varphi_c = C_2 \qquad R\cos\varphi_c = C_4$$

$$\boxed{\begin{aligned} x(t) &= v_c \cdot t\cos\varphi_c + R\sin\varphi_c \\ y(t) &= \frac{1}{2}gt^2 - v_c \cdot t\sin\varphi_c + R\cos\varphi_c \end{aligned}}$$

1. Tačka mase m se nalazi u vertikalnoj ravni u položaju A na idealno glatkoj strmoj ravni nagiba α prema horizontali. Odredi potrebno sabijanje opruge krutosti k, da bi tačka posle napuštanja strme ravni dospela u položaj C.



$k = 10 \frac{mg}{l}$ $g = 10 \frac{m}{s^2}$

$\alpha = 45°$

- kretanje po strmoj ravni:

$$T_B - T_A = A_{A \to B}$$

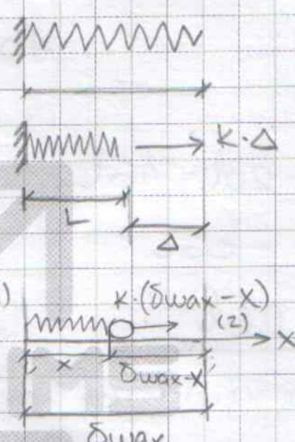
$$\frac{1}{2} m \cdot v_B^2 - 0 = -m \cdot g \cdot l + \frac{1}{2} k \delta^2$$

$$v_B^2 = \frac{k}{m}\delta^2 - 2gl =$$

$$= 10 \frac{\frac{m \cdot 10}{l}}{m} \cdot \delta^2 - 2 \cdot 10 \cdot l =$$

$$v_B^2 = 100 \cdot \frac{1}{l}\delta^2 - 20l$$

RAD SILE U OPRUZI



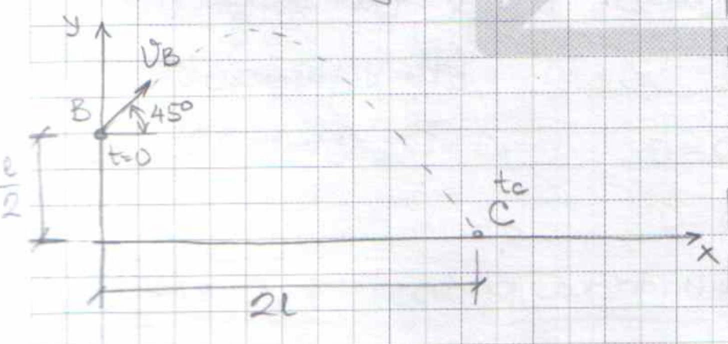
$$A = \int_{(1)}^{(2)} k(\delta_{max} - x)dx =$$

$$= k\delta_{max}\int_0^{\delta_{max}} dx - k\int_0^{\delta_{max}} x\,dx =$$

$$= k\delta_{max} \cdot \delta_{max} - k \cdot \frac{1}{2}\delta_{max}^2 =$$

$$\boxed{A = \frac{1}{2} k \delta_{max}^2}$$

- slobodno kretanje tačke:



$$m\vec{a} = \vec{F}_R \;/\cdot \vec{i}, \vec{j}$$

$x:\ m \cdot \ddot{x} = 0$ $\dot{x} = C_1$ $x = C_1 \cdot t + C_2$

$y:\ m \cdot \ddot{y} = -mg$ $\dot{y} = -g \cdot t + C_3$ $y = -\frac{1}{2}gt^2 + C_3 t + C_4$

početni uslovi

$x(0) = 0$ $\dot{x}(0) = v_B \cdot \frac{\sqrt{2}}{2}$ $t=0$ $v_B \cdot \frac{\sqrt{2}}{2} = C_1$ $0 = C_2$

$y(0) = \frac{l}{2}$ $\dot{y}(0) = v_B \cdot \frac{\sqrt{2}}{2}$ $v_B \cdot \frac{\sqrt{2}}{2} = C_3$ $\frac{l}{2} = C_4$

$$\boxed{x(t) = v_B \cdot \frac{\sqrt{2}}{2} \cdot t}$$

$$\boxed{y(t) = -\frac{1}{2} g t^2 + v_B \frac{\sqrt{2}}{2} \cdot t + \frac{l}{2}}$$

$$\dot{x}(t) = v_B \cdot \frac{\sqrt{2}}{2}$$

$$\dot{y}(t) = -gt + v_B \cdot \frac{\sqrt{2}}{2}$$

- uslov da mat. tačka dospe u položaj C:

$$y(t_C) = 0 \quad : \quad -\frac{1}{2} g \cdot t_C^2 + v_B \cdot \frac{\sqrt{2}}{2} t_C + \frac{l}{2} = 0$$

$$x(t_C) = 2l : \quad v_B \cdot \frac{\sqrt{2}}{2} t_C = 2l \qquad t_C = 2\sqrt{2} \frac{l}{v_B}$$

$$-\frac{1}{2} g \left(2\sqrt{2} \cdot \frac{l}{v_B}\right)^2 + v_B \cdot \frac{\sqrt{2}}{2} \cdot \left(2\sqrt{2} \cdot \frac{l}{v_B}\right) + \frac{l}{2} = 0$$

$$-5 \cdot 8 \cdot \frac{l^2}{v_B^2} + 2l + \frac{l}{2} = 0$$

$$40 \frac{l^2}{v_B^2} = \frac{5}{2} l$$

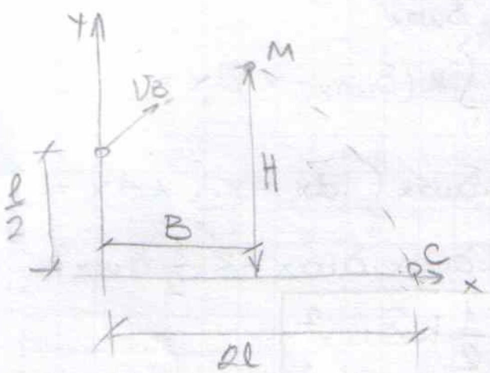
$$v_B^2 = 16 l$$

$$100 \cdot \frac{1}{l} \cdot \delta^2 - 20l = 16l$$

$$100 \frac{1}{l} \delta^2 = 36l$$

$$\underline{\delta = 0{,}6l}$$

+ odredi maximalnu visinu tokom kretanja:

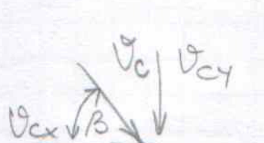


$$\dot{y}(t_M) = 0 \quad \ldots \quad t_M = \frac{\sqrt{2}}{2} \cdot \frac{v_B}{g}$$

$$H = y(t_M) = 0{,}9l$$

$$B = x(t_M) = 0{,}8l$$

+ odredi brzinu i ugao pod kojim tačka pada:



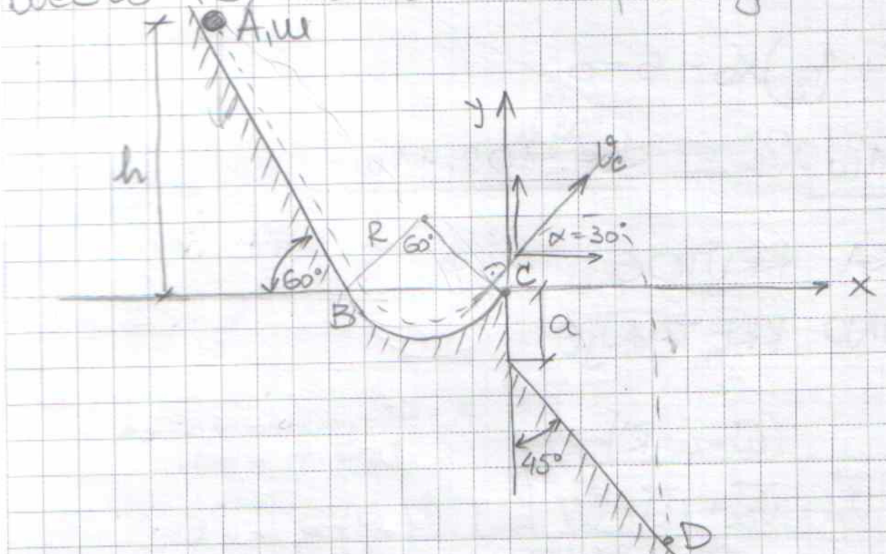
$$t_C = 2\sqrt{2} \frac{l}{v_B}$$

$$v_{CX} = \dot{x}(t_C) = \ldots = 2\sqrt{2} \cdot \sqrt{l}$$

$$v_{CY} = \dot{y}(t_C) = \ldots = -3\sqrt{2} \cdot \sqrt{l}$$

$$\text{tg}\,\beta = \frac{|v_{CY}|}{|v_{CX}|} = \frac{3}{2} \qquad \beta = 56{,}3^\circ$$

2. Skijaš koga treba tretirati kao mat. tačku mase m spušta se niz skakaonicu bez početne brzine. Odredi mesto (B) doskoka na podlogu.



$m = 80kg$

$h = 20m$

$a = 5m$

- kretanje po skakaonici

$$T_C - T_A = A_{A \to C}$$

$$\frac{1}{2} m v_C^2 - 0 = mg \cdot h \qquad v_C = \sqrt{2gh}$$

- slobodno kretanje tačke

$$m\vec{a} = F_R$$

$$m \cdot \ddot{x} = 0 \qquad \dot{x} = C_1 \qquad x = C_1 t + C_2$$

$$m \cdot \ddot{y} = -mg \qquad \dot{y} = -gt + C_3 \qquad y = -\frac{1}{2}gt^2 + C_3 t + C_4$$

$t = 0$

$$v_C \cdot \cos\alpha = C_1 \qquad 0 = C_2$$

$$v_C \cdot \sin\alpha = C_3 \qquad 0 = C_4$$

$$\boxed{x(t) = v_C \cdot t \cos\alpha}$$

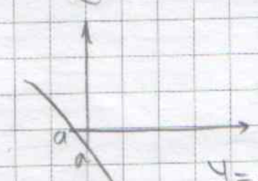
$$\boxed{y(t) = -\frac{1}{2}gt^2 + v_C \cdot t \cdot \sin\alpha}$$

$$t = \frac{x}{v_C \cdot \cos\alpha}$$

$$y = -\frac{1}{2} g \cdot \left(\frac{x}{v_C \cdot \cos\alpha}\right)^2 + v_C \cdot \frac{x}{v_C \cdot \cos\alpha} \cdot \sin\alpha$$

$$y = -\frac{g}{2v_C^2 \cdot \cos^2\alpha} \cdot x^2 + x \cdot tg\alpha$$ (jedn. trajektorije CD)

↑ jedn. skijača



$y = -x - a$

jedn. padine

$$X_D^{SK} = X_D^{PAD}$$
$$y_D^{SK} = y_D^{PAD}$$

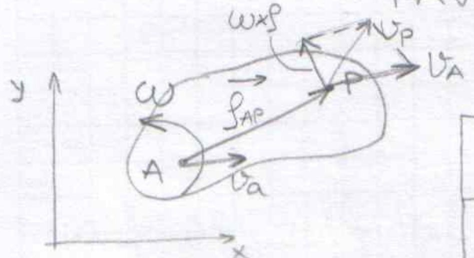
$$-\frac{g}{2gh\cos^2\alpha} x_D^2 + tg\alpha \cdot x_D = -x_D - a$$

$$\frac{1}{4\cdot 20\cdot \frac{3}{4}} x_D^2 - \left(1+\frac{1}{\sqrt{3}}\right)x_D - 5 = 0$$

$x_D = 97{,}71\,m$ $y_D = -102{,}71\,m$

KINEMATIKA KRUTOG TELA

RAVNO KRETANJE

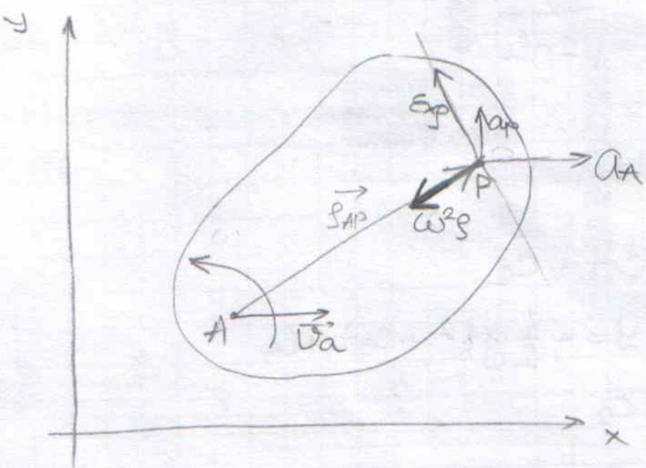


A - referentna tačka

$(\vec{\omega} = \omega\cdot\vec{k}) \leftarrow z$ osa

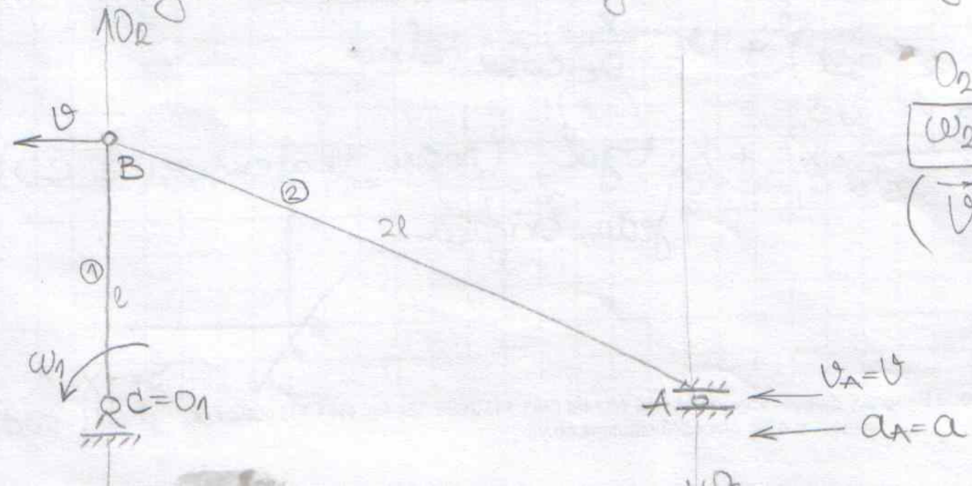
$$\boxed{\vec{v}_p = \vec{v}_a + \vec{\omega}\times\vec{\rho}_{AP}}$$

$$\boxed{\vec{a}_p = \vec{a}_A + \vec{\varepsilon}\times\vec{\rho}_{AP} - \omega^2\cdot\vec{\rho}_{AP}}$$



$\vec{\varepsilon} = \frac{d\vec{\omega}}{dt}$ $(\vec{\varepsilon} = \varepsilon\cdot\vec{k}) \leftarrow z$ osa

1. U prikazanom položaju mehanizma poznati su brzina i ubrzanje klipa A. Odrediti brzinu i ubrzanje tačke B kao i ugaone brzine i ugaona ubrzanja štapova AB i BC.



$O_2 \to \infty \to \omega_2 = 0$

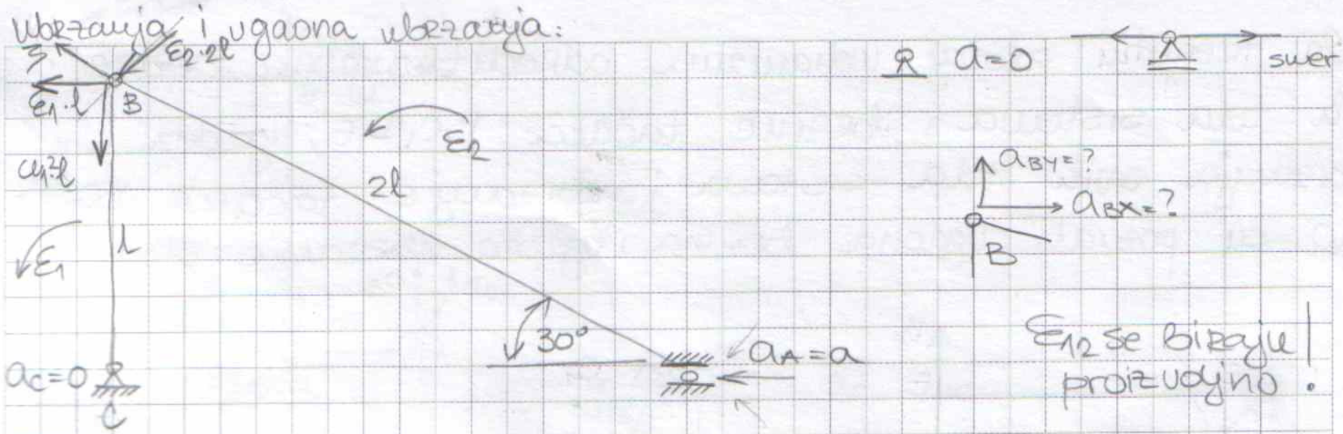
$\boxed{\omega_2 = 0} \Rightarrow v_B = v_A = v \to \omega_1$

$\left(\vec{v}_B = \cancelto{0}{\vec{v}_C} + \vec{\omega}\times\vec{\rho}_{CB}\right)$

$v_B = \omega_1\cdot l$

$$\boxed{\omega_1 = \frac{v}{l}}$$

Ubrzanja i ugaona ubrzanja:



$\varepsilon_{1,2}$ se biraju proizvoljno!

$$\vec a_B^C = \cancel{\vec a_C}^{\,0} + \underbrace{\vec\varepsilon_1\times\vec\rho_{CB}}_{a^C_{B,T}} - \underbrace{\omega_1^2\cdot\vec\rho_{CB}}_{a^C_{B,N}}$$

$$a^C_{B,T} = \varepsilon_1\cdot l$$

$$a^C_{B,N} = \left(\frac{v}{l}\right)^2\cdot l = \frac{v^2}{l}$$

$$\vec a_B^A = \vec a_A + \underbrace{\vec\varepsilon_2\times\vec\rho_{AB}}_{a^A_{B,T}} - \underbrace{\cancel{\omega_2^2\cdot\vec\rho_{AB}}}_{a^A_{B,N}\to 0}$$

$$\underline{\vec a_B^C = \vec a_B^A}$$

$$\cancel{\vec a_C}^{\,0} + \vec a^C_{B,T} + \vec a^C_{B,N} = \vec a_A + \cancel{\vec a^A_{B,N}}^{\,0} + \vec a^A_{B,T} \;\Big/\cdot\vec j$$

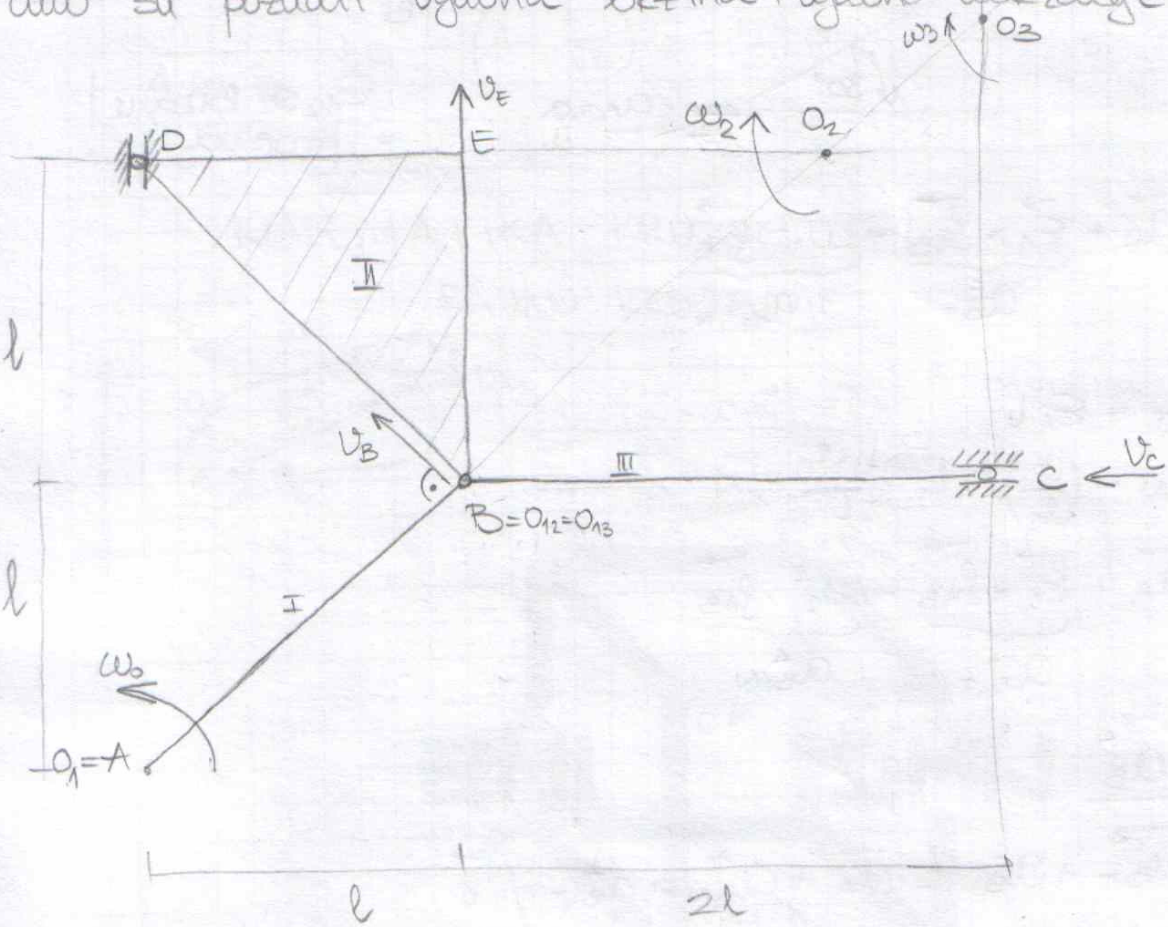
Y: $-\dfrac{v^2}{l} = -\varepsilon_2\cdot 2l\cdot\dfrac{\sqrt3}{2}$ $\qquad\boxed{\varepsilon_2 = \dfrac{1}{\sqrt3}\dfrac{v^2}{l^2}}$

ξ: $\varepsilon_1\cdot l\cdot\dfrac{\sqrt3}{2} - \dfrac{v^2}{l}\cdot\dfrac12 = a\dfrac{\sqrt3}{2}$

$$\boxed{\varepsilon_1 = \frac{a}{l} + \frac{1}{\sqrt3}\frac{v^2}{l^2}}$$

$a_{Bx} = -a - \dfrac{1}{\sqrt3}\dfrac{v^2}{l}$ $\qquad a_{By} = -\dfrac{v^2}{l}$

2. Za trenutni položaj mehanizma odrediti: ugaone brzine svih tela sistema i brzine tačaka B, C, D, E; ugaona ubrzanja svih tela sistema i ubrzanje tačaka B, C, D, E, ako su poznati ugaona brzina i ugaono ubrzanje štapa A.



$v_B = \omega_0 \cdot \overline{O_1B}$ $\boxed{v_B = \omega_0 \cdot l\sqrt{2}}$

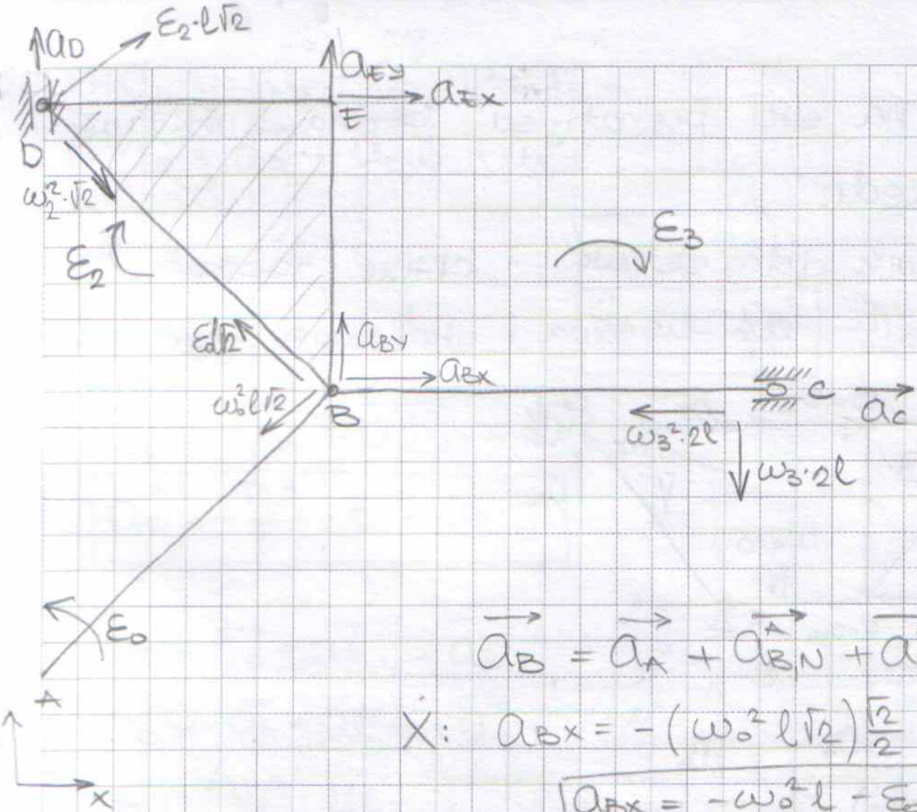
$v_B = \omega_2 \overline{O_2B}$ $\boxed{\omega_2 = \omega_0}$

$v_E = \omega_2 \cdot \overline{O_2E}$ $\boxed{v_E = \omega_0 \cdot l}$

$v_D = \omega_2 \cdot \overline{O_2D}$ $\boxed{v_D = 2\omega_0 \cdot l}$

$v_B = \omega_3 \cdot \overline{O_3B}$ $\boxed{\omega_3 = \frac{1}{2}\omega_0}$

$v_C = \omega_3 \cdot \overline{O_3C}$ $\boxed{v_C = \omega_0 \cdot l}$



$$\vec a_B = \vec a_A + \vec a^A_{B,N} + \vec a^A_{B,T} \;/\; \vec i,\ \vec j$$

X: $a_{Bx} = -(\omega_0^2 l\sqrt2)\frac{\sqrt2}{2} - (\varepsilon_0 l\sqrt2)\cdot\frac{\sqrt2}{2}$

$$\boxed{a_{Bx} = -\omega_0^2 l - \varepsilon_0 \cdot l}$$

Y: $a_{By} = -(\omega_0^2 l\sqrt2)\cdot\frac{\sqrt2}{2} + (\varepsilon_0 l\sqrt2)\cdot\frac{\sqrt2}{2}$

$$\boxed{a_{By} = -\omega_0^2 l + \varepsilon_0 \cdot l}$$

$$\vec a_B = a_{Bx}\cdot\vec i + a_{By}\cdot\vec j$$

$$\vec a_C = \vec a_B + \vec a^B_{C,N} + \vec a^B_{C,T} \;/\; \vec i,\ \vec j$$

X: $a_C = (-\omega_0^2 l - \varepsilon_0 l) - (\frac{1}{2}\omega_0)^2\cdot 2l$

$$\boxed{a_C = -\varepsilon_0 l - \frac{3}{2}\omega_0^2 l}$$

Y: $0 = (-\omega_0^2 l + \varepsilon_0 l) - \varepsilon_3\cdot 2l$ $\quad\boxed{\varepsilon_3 = \frac{1}{2}\varepsilon_0 - \frac{1}{2}\omega_0^2}$

$$\vec a_D = \vec a_B + \vec a^B_{D,N} + \vec a^B_{D,T} \;/\; \vec i,\ \vec j$$

X: $0 = (-\omega_0^2 l - \varepsilon_0\cdot l) + (\omega_0)^2 l\sqrt2\cdot\frac{\sqrt2}{2} + \varepsilon_2 l\sqrt2\cdot\frac{\sqrt2}{2}$ $\quad\boxed{\varepsilon_2 = \varepsilon_0}$

Y: $a_D = (-\omega_0^2 l + \varepsilon_0\cdot l) - (\omega_0)^2 l\sqrt2\cdot\frac{\sqrt2}{2} + \varepsilon_0 l\sqrt2\cdot\frac{\sqrt2}{2}$

$$\boxed{a_D = -2\omega_0^2 l + 2\varepsilon_0 l}$$

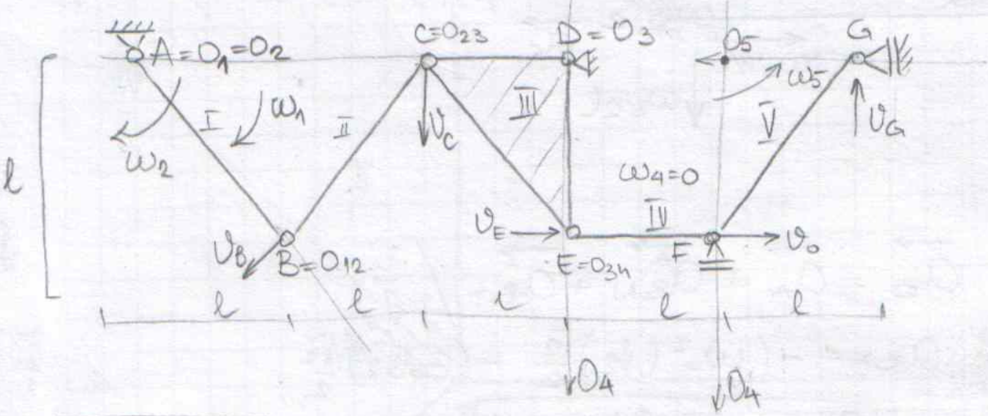
$a_{Ex} = -\omega_0^2 l \qquad a_{Ey} = -2\omega_0^2 l + \varepsilon_0 l$

Zadaci:

1. U prikazanom položaju na slici poznati su brzina i ubrzanje tačke F v_0 i a_0. Odredi:

a) ugaone brzine svih tela sistema i brzine tačaka B,C,E,G

b) Ugaone brzine svih tela sistema i ubrzanja tačaka C i E



$\boxed{v_E = v_F = v_0}$ (zbog translacije)

$\omega_5 \cdot l = v_0$ $\quad \boxed{\omega_5 = \frac{v_0}{l}}$

$v_G = \omega_5 \cdot l$ $\quad \boxed{v_G = v_0}$

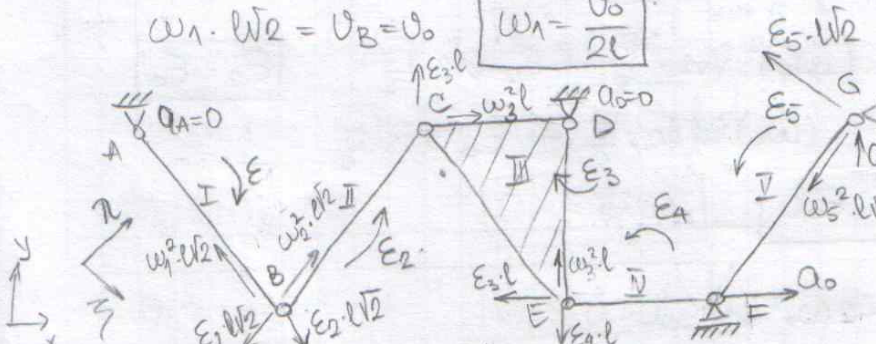
$\omega_3 \cdot l = v_E = v_0$ $\quad \boxed{\omega_3 = \frac{v_0}{l}}$

$v_C = \omega_3 \cdot l$ $\quad \boxed{v_C = v_0}$

$\omega_2 \cdot 2l = v_C = v_0$ $\quad \boxed{\omega_2 = \frac{v_0}{2l}}$

$v_B = \omega_2 \cdot l\sqrt{2}$ $\quad \boxed{v_B = \frac{\sqrt{2}}{2} v_0}$

$\omega_1 \cdot l\sqrt{2} = v_B = v_0$ $\quad \boxed{\omega_1 = \frac{v_0}{2l}}$



$$\vec{a}_G = \vec{a}_F + \vec{a}^F_{G,N} + \vec{a}^F_{G,T} \;/ : \vec{i}, \vec{j}$$

$$X: \quad 0 = a_0 - \left(\frac{v_0}{l}\right)^2 \cdot l\sqrt{2} \cdot \frac{\sqrt{2}}{2} - \varepsilon_5 \cdot l\sqrt{2} \cdot \frac{\sqrt{2}}{2} \qquad \boxed{\varepsilon_5 = \frac{a_0}{l} - \frac{v_0^2}{l^2}}$$

$$y: \quad a_G = 0 - \left(\frac{v_0}{l}\right)^2 \cdot l\sqrt{2} \cdot \frac{\sqrt{2}}{2} + \varepsilon_5 \cdot l\sqrt{2} \cdot \frac{\sqrt{2}}{2} \qquad \boxed{a_G = a_0 - 2\frac{v_0^2}{l}}$$

$$\vec{a}_E = \vec{a}_F + \vec{a}^F_{E,N} + \vec{a}^F_{E,T}$$

$$\vec{a}_E = \vec{a}_O + \vec{a}^O_{E,N} + \vec{a}^O_{E,T}$$

x: $a_O = -\varepsilon_3 \cdot l$ $\boxed{\varepsilon_3 = -\frac{a_0}{l}}$

y: $\left(\frac{v_0}{l}\right)^2 = -\varepsilon_4 \cdot l$ $\boxed{\varepsilon_4 = -\frac{v_0^2}{l^2}}$

$$\vec{a}_C = \vec{a}_O + \vec{a}^O_{C,N} + \vec{a}^O_{C,T}$$

$$\boxed{\vec{a}_C = \frac{v_0^2}{l} \cdot \vec{i} - a_0 \cdot \vec{j}}$$

$$\vec{a}_B = \vec{a}_C + \vec{a}^C_{B,N} + \vec{a}^C_{B,T} \qquad \vec{\eta}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}^A_{B,N} + \vec{a}^A_{B,T} \qquad \vec{\mu}$$

$$\xi: \left(\frac{v_0^2}{l} \cdot \frac{l\sqrt{2}}{2} - (-a_0) \cdot \frac{\sqrt{2}}{2}\right) + \varepsilon_2 \cdot l\sqrt{2} = \left(\frac{1}{2}\frac{v_0}{l}\right)^2 \cdot l\sqrt{2} \qquad \boxed{\varepsilon_2 = \frac{1}{2}\frac{a_0}{l} - \frac{3}{4}\frac{v_0^2}{l}}$$

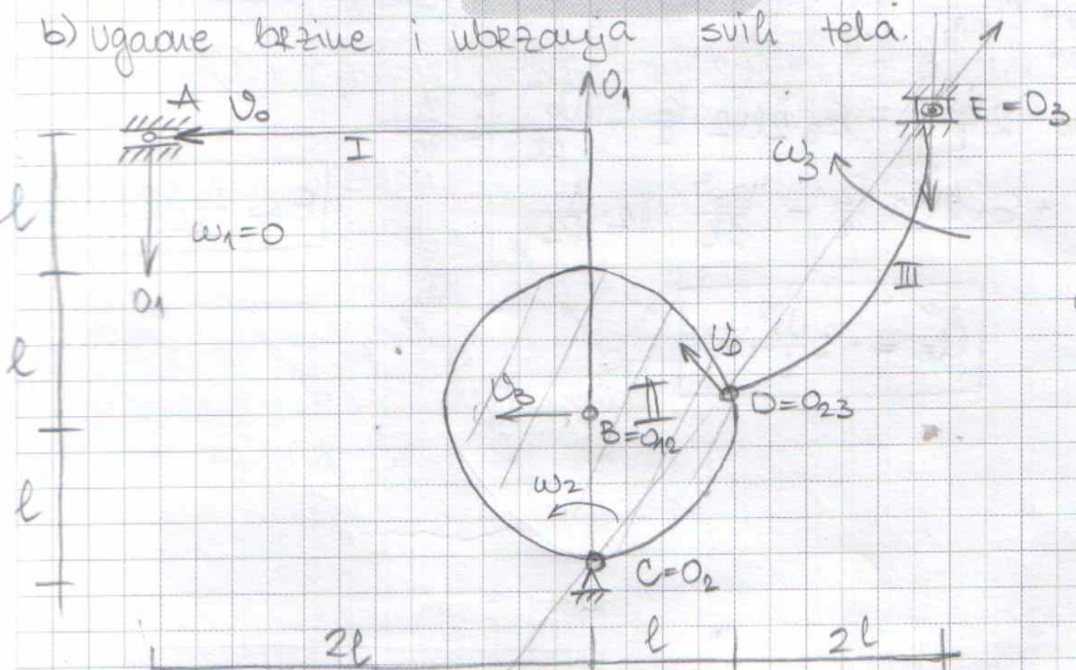
$$\eta: \left(\frac{v_0^2}{l} \cdot \frac{\sqrt{2}}{2} + (-a_0) \cdot \frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\frac{v_0}{l}\right)^2 \cdot l\sqrt{2} = -\varepsilon_1 l\sqrt{2}$$

$$\boxed{\varepsilon_1 = \frac{1}{2}\frac{a_0}{l} - \frac{3}{4}\frac{v_0^2}{l^2}}$$

2. Za mehanizam u zadatom položaju poznati su brzina i ubrzanje tačke A v_0 i a_0. Odredi:

a) brzine i ubrzanja tačaka B, D i E

b) ugaone brzine i ubrzanja svih tela.



$\boxed{v_B = v_A = v_0}$

$\omega_2 \cdot l = v_B = v_0$ $\boxed{\omega_2 = \frac{v_0}{l}}$

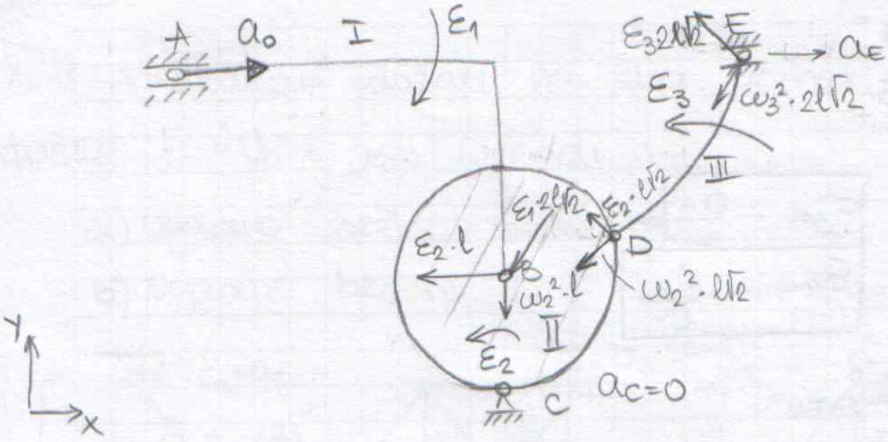
$v_D = \omega_2 \cdot l\sqrt{2}$

$\boxed{v_D = v_0 \cdot \sqrt{2}}$

$\omega_3 \cdot 2l\sqrt{2} = v_D = v_0\sqrt{2}$

$\boxed{\omega_3 = \frac{v_0}{2l}}$

$\boxed{v_E = 0}$



$$\vec{a_B} = \vec{a_A} + \vec{a^A_{B,N}} + \vec{a^A_{B,T}}$$
$$\vec{a_B} = \vec{a_C} + \vec{a^A_{C,N}} + \vec{a^A_{C,T}}$$
$\cdot \vec{i}$, $\cdot \vec{j}$

$$X:\ a_0 - \varepsilon_1 \cdot 2l\sqrt{2} \cdot \frac{\sqrt{2}}{2} = -\varepsilon_2 \cdot l \qquad \boxed{\varepsilon_2 = -\frac{a_0}{l} + \frac{v_0^2}{l^2}}$$
$$Y:\ -\varepsilon_1 2l\sqrt{2} \cdot \frac{\sqrt{2}}{2} = -\frac{v_0^2}{l} \qquad \boxed{\varepsilon_1 = \frac{v_0^2}{2l^2}}$$

$$\vec{a_B} = \vec{a_A} + \vec{a^A_{B,N}} + \vec{a^A_{B,T}}$$
$$\boxed{\vec{a_B} = \left(a_0 - \frac{v_0^2}{l}\right)\vec{i} - \frac{v_0^2}{l} \cdot \vec{j}}$$
$$\boxed{\vec{a_D} = \left(a_0 - 2\frac{v_0^2}{l}\right)\vec{i} - a_0 \cdot \vec{j}}$$

$$\vec{a_E} = \vec{a_D} + \vec{a^D_{E,N}} + \vec{a^D_{E,T}}$$
$\cdot \vec{i}$, $\cdot \vec{j}$

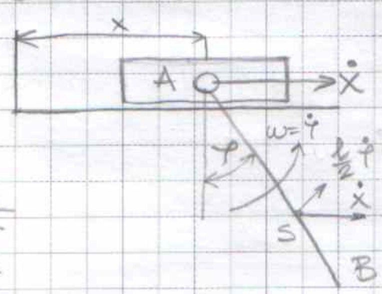
$$X:\ a_E = -\left(a_0 - 2\frac{v_0^2}{l}\right) - \varepsilon_3 \cdot 2l\sqrt{2} \cdot \frac{\sqrt{2}}{2} - \frac{v_0^2}{2l} \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}}$$
$$Y:\ 0 = -a_0 + \varepsilon_3 2l\sqrt{2} \cdot \frac{\sqrt{2}}{2} - \frac{v_0^2}{2l} \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} \qquad \boxed{\varepsilon_3 = \frac{1}{2l}\left(a_0 + \frac{v_0^2}{l}\right)}$$

$$\vec{a_E} = -3\frac{v_0^2}{l} \qquad \boxed{\vec{a_E} = -3\frac{v_0^2}{l} \cdot \vec{i}}$$

SLOŽENO KRETANJE KRUTOG TELA

1. Odrediti brzine središta masa krutih tela na slici.

$\overline{AB}=l$



$n=2$

$q_1=x$

$q_2=\varphi$

1° $x=x(t)$

$\varphi=const$

2° $x=const$

$\varphi=\varphi(t)$

- KOLICA: $\vec{v}_A=\dot{x}\,\vec{i}$

- ŠTAP AB:

rel. kretanje $v_S^{rel}=\dot{\varphi}\cdot\frac{l}{2}$

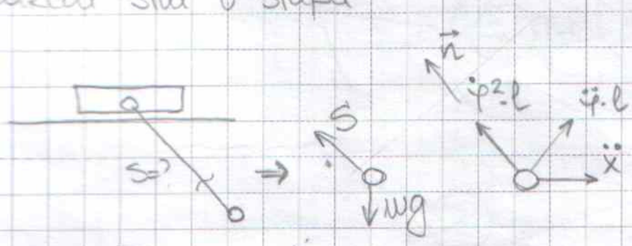
$\vec{v}_S^{\,rel}=\dot{\varphi}\cdot\frac{l}{2}$

pren. kretanje $v_S^{pr}=\dot{x}$

$$\vec{v}_S=\left(\dot{x}+\dot{\varphi}\cdot\frac{l}{2}\cos\varphi\right)\vec{i}+\dot{\varphi}\cdot\frac{l}{2}\sin\varphi\,\vec{j}$$

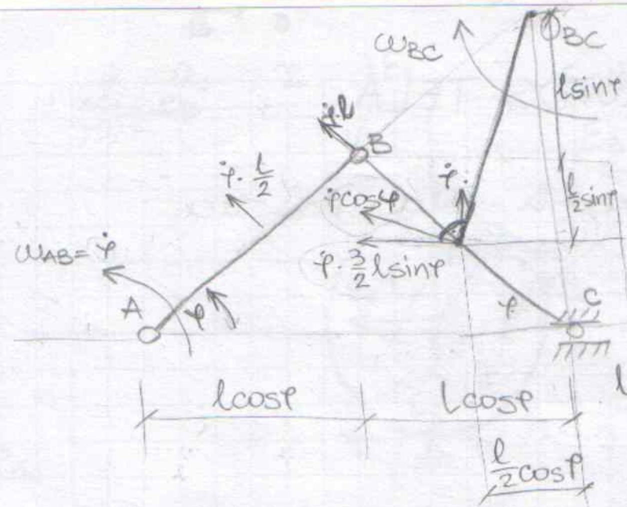
$$v_S^2=\left(\dot{x}+\dot{\varphi}\frac{l}{2}\cos\varphi\right)^2+\left(\dot{\varphi}\cdot\frac{l}{2}\sin\varphi\right)^2=\ldots=\dot{x}^2+\dot{x}\cdot\dot{\varphi}\,l\cos\varphi+\frac{1}{4}\dot{\varphi}^2\cdot l^2$$

* odredi silu u štapu



$\bar{F}=m\cdot\bar{a}\;/\vec{n}\;\rightarrow\;S=\ldots$

2.



$\overline{AB} = \overline{BC} = l$

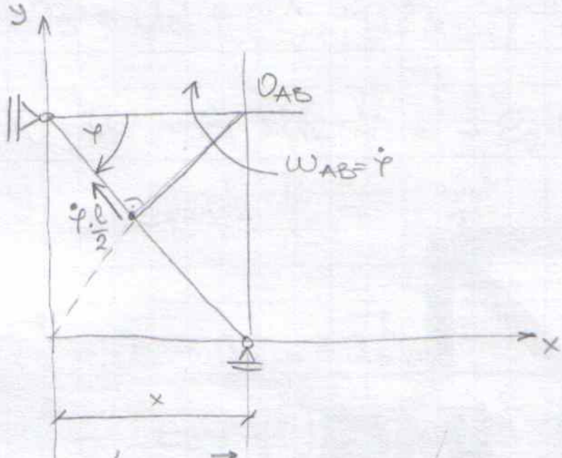
$\underline{n=1}$

$q_1 = \varphi$

$\omega_{AB} = \dot{\varphi}$

$$\vec{v}_S = -\dot{\varphi} \cdot \frac{3}{2} l \sin\varphi \cdot \vec{i} + \dot{\varphi} \cdot \frac{l}{2} \cos\varphi \cdot \vec{j}$$

3.

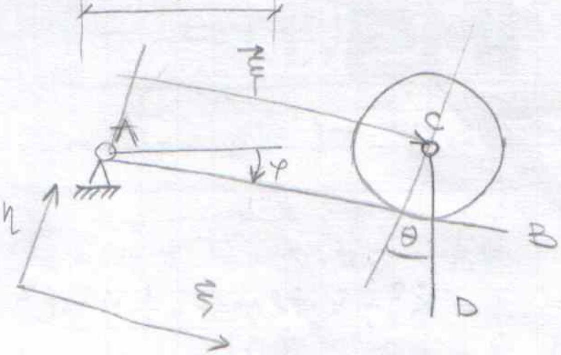


$\overline{AB} = l$

$\underline{n=1}$

$q_1 = \varphi$

4.



$\overline{AB} = l$

$\overline{CD} = l$

disk – polupreč. R

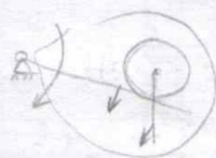
$\underline{n=}$

$q_1 = \varphi$

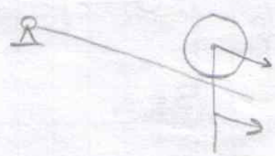
$q_2 = \xi$

$q_3 = \theta$

1° $\varphi = \varphi(t)$
$\xi = const$
$\theta = const$

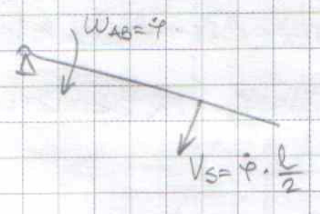


2° $\varphi = const$
$\xi = \xi(t)$
$\theta = const$

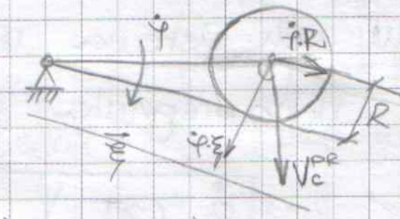


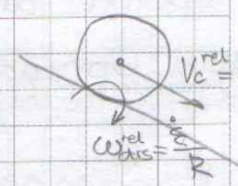
3° $\varphi = const$
$\xi = const$
$\theta = \theta(t)$

- ŠTAP AB:



- DISK:

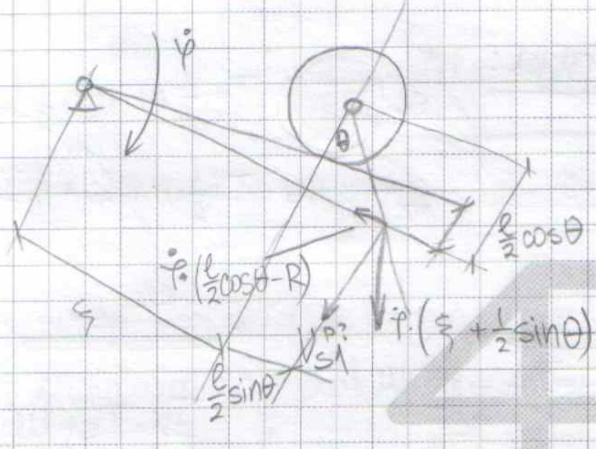


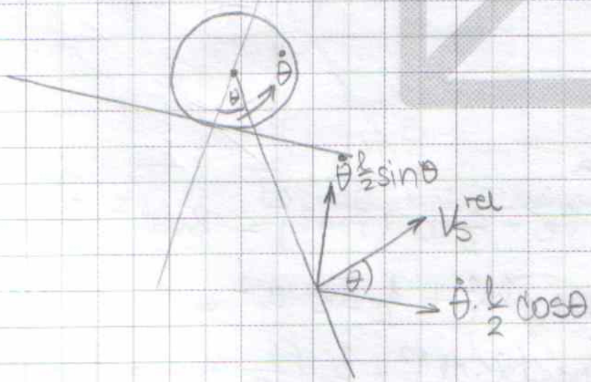


$$\vec{v}_C = (\dot\xi + \dot\varphi R)\vec{n} - \dot\varphi \xi \cdot \vec{\mu}$$

$$\omega_{DIS} = \dot\varphi + \frac{\dot\xi}{R}$$

-ŠTAP CD:



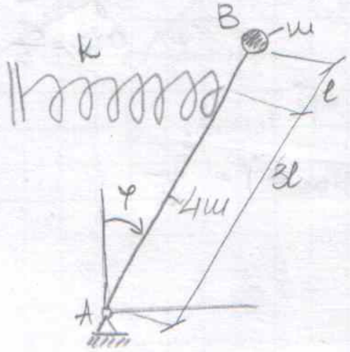


$$\vec{v}_S = \Big(\left(\dot\theta \tfrac{l}{2}\cos\theta + \dot\xi - \dot\varphi(\cos\theta - R)\right) + \left(\dot\theta \cdot \tfrac{l}{2}\sin\theta - \dot\varphi\left(\xi + \tfrac{l}{2}\sin\theta\right)\right)\Big) \cdot \vec{\mu}$$

$$\omega_{CD} = \dot\varphi - \dot\theta$$

DINAMIKA MEHANIČKIH SISTEMA

1. Napisati dif. jedn. meh. sistema koji se kreće u vertikalnoj ravni. Opruga je nenapregnuta kada je štap vertikalan.



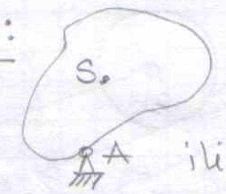
$$\boxed{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}=Q_i} \qquad i=1,2,\dots n$$

q - generalisana koordinata

$n=1$

$q_1=\varphi$

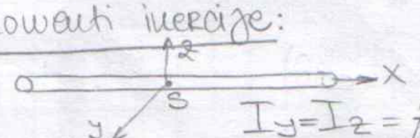
- kinetička energija:



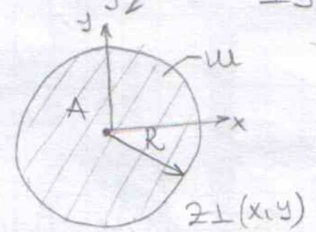
$T=\frac{1}{2}m\cdot v_S^2+\frac{1}{2}I_S\cdot\omega^2$

ili $T=\frac{1}{2}m\cdot \cancel{v_A^2}^{\,0}+\frac{1}{2}I_A\cdot\omega^2$

- momenti inercije:

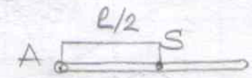


$I_y=I_z=\frac{1}{12}m\cdot l^2$



$z\perp(x,y)$

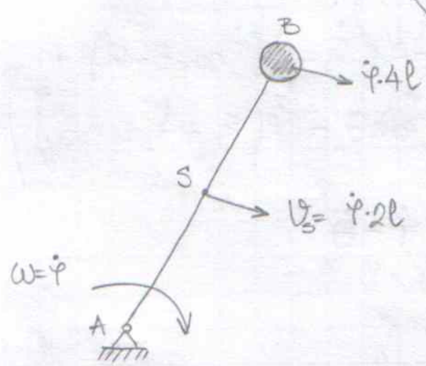
$I_z=\frac{1}{2}m\cdot R^2$



$I_A=I_S+m\cdot\overline{AS}^2$

$I_A=\frac{1}{12}ml^2+m\cdot\left(\frac{l}{2}\right)^2$

$I_A=\frac{1}{3}ml^2$



$T=T_B+T_{AB}$

$T_B=\frac{1}{2}m\cdot v_B^2=\frac{1}{2}m\cdot(\dot\varphi\cdot 4l)^2=8ml^2\cdot\dot\varphi^2$

$T_{AB}=\frac{1}{2}4m\cdot v_S^2+\frac{1}{2}I_S\cdot\omega^2=$

$=\frac{1}{2}4m\cdot(\dot\varphi\cdot 2l)^2+\frac{1}{2}\left(\frac{1}{12}4m\cdot(4l)^2\right)\cdot\dot\varphi^2$

$=\left(8+\frac{8}{3}\right)ml^2\cdot\dot\varphi^2=\frac{32}{3}m\cdot l^2\cdot\dot\varphi^2$

ILI $T_{AB}=\frac{1}{2}4m\cdot\cancel{v_A^2}^{\,0}+\frac{1}{2}I_A\cdot\omega^2=\frac{1}{2}\left(\frac{1}{3}4m\cdot(4l)^2\right)\cdot\dot\varphi^2=\frac{32}{3}ml^2\cdot\dot\varphi^2$

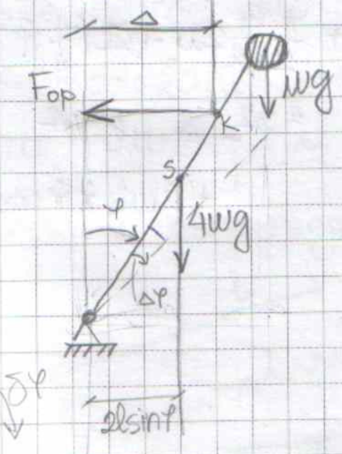
$T=\frac{56}{3}m\cdot l^2\cdot\dot\varphi^2$

$\frac{\partial T}{\partial\varphi}=0 \qquad \frac{\partial T}{\partial\dot\varphi}=\frac{112}{3}ml^2\cdot\dot\varphi$

$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot\varphi}\right)=\frac{112}{3}\cdot ml^2\cdot\ddot\varphi$

– GENERALISANA SILA:

$Q_\varphi \cdot \delta\varphi = \delta A$



sila u opruzi

$F_{op} = k \cdot \Delta$

$\Delta = 3l\sin\varphi$

za Δ je opruga sabijena/izdužena

$\delta A = 4mg \cdot \delta r_S^y + mg\,\delta r_3^x - F_{op}\,\delta r_K^x = Q_\varphi \cdot \delta\varphi$

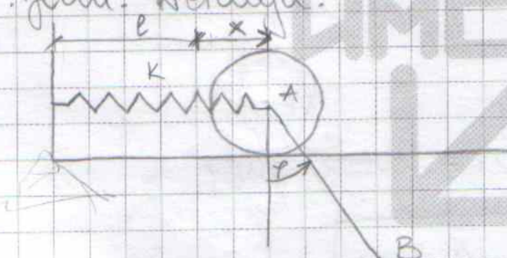
$4mg(\delta\varphi\, 2l\sin\varphi) + mg(\delta\varphi \cdot 4l\sin\varphi) - k \cdot 3l\sin(\delta\varphi\, 3l\cos\varphi)$

$= Q_\varphi\,\delta\varphi \quad / \cdot \frac{1}{\delta\varphi}$

$Q_\varphi = 12mgl\sin\varphi - 9kl^2\sin\varphi\cos\varphi$

$$\boxed{\frac{112}{3} ml^2 \cdot \ddot{\varphi} = 12mgl\sin\varphi - 9kl^2\sin\varphi\cos\varphi}$$ dif. jedn.!

2. Meh. sistem na slici se kreće u vert. ravni. Disk se kotrlja po podlozi bez klizanja, a dužina opruge u nenapregnutom stanju l. Odredi dif. jedn. kretanja.



disk: m, R = l/2

štap AB = 2m, 2l

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \quad (i = 1,2 \dots n)$$

$\underline{n = 2}$

$q_1 = x$

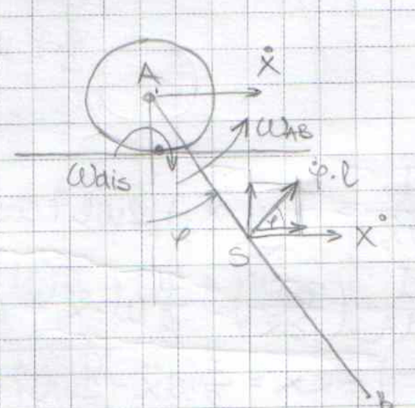
$q_2 = \varphi$

1° $x = x(t)$
$\varphi = const$

2° $x = const$
$\varphi = \varphi(t)$

1° + 2° →

brzine i ugaone brzine:



disk: $v_A = \dot{x}$

$\omega_d = \frac{\dot{x}}{\frac{l}{2}} = 2\frac{\dot{x}}{l}$

štap: $\omega_{AB} = \dot{\varphi}$

$\vec{V_S} = (\dot{x} + \dot{\varphi}(\cos\varphi) \cdot l)\vec{i} + \dot{\varphi} l\sin\varphi\,\vec{j}$

kinetička energija:

$$T_{disk} = \frac{1}{2}m\cdot(\dot{x})^2 + \frac{1}{2}\left(\frac{1}{2}m\cdot\left(\frac{l}{2}\right)^2\right)\cdot\left(2\cdot\frac{\dot{x}}{l}\right)^2 = \frac{3}{4}m\cdot\dot{x}^2$$

$$T_{AB} = \frac{1}{2}2m\cdot\left[(\dot{x}+\dot{\varphi}l\cos\varphi)^2 + (\dot{\varphi}l\sin\varphi)^2\right] + \frac{1}{2}\left(\frac{1}{12}2m\cdot(2l)^2\right)\cdot\dot{\varphi}^2$$

$$T_{AB} = m\left(\dot{x}^2 + 2\dot{x}(\dot{\varphi}l\cos\varphi) + \dot{\varphi}^2l^2\right) + \frac{1}{3}ml^2\dot{\varphi}^2 = \frac{4}{3}ml^2\dot{\varphi}^2 + m\dot{x}^2 + 2ml\dot{x}\dot{\varphi}\cos\varphi$$

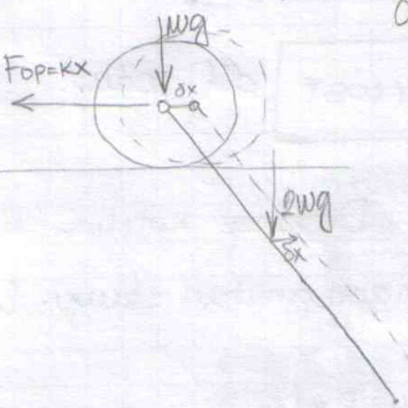
$$T = \frac{4}{3}ml^2\cdot\dot{\varphi}^2 + \frac{7}{4}m\dot{x}^2 + 2ml\dot{x}\dot{\varphi}\cos\varphi$$

GENERALISANE SILE:

$Q_x = ?$ $(\delta x \neq 0, \delta\varphi = 0)$

$$\delta A = -kx\delta x = Q_x\delta x \quad /\cdot\frac{1}{\delta x}$$

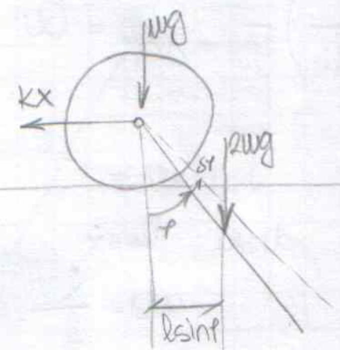
$$\boxed{Q_x = -kx}$$



$Q_\varphi = ?$ $(\delta x = 0, \delta\varphi \neq 0)$

$$\delta A = -2mgl\sin\varphi\,\delta\varphi = Q_\varphi\cdot\delta\varphi \quad /\cdot\frac{1}{\delta\varphi}$$

$$\boxed{Q_\varphi = -2mgl\sin\varphi}$$



$q_1 = x$: $\quad \frac{\partial T}{\partial x} = 0 \qquad \frac{\partial T}{\partial \dot{x}} = \frac{7}{2}m\cdot\dot{x} + 2ml\dot{\varphi}\cos\varphi$!!!

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = \frac{7}{2}m\cdot\ddot{x} + 2ml\ddot{\varphi}\cos\varphi + 2ml\dot{\varphi}(-\sin\varphi)\cdot\dot{\varphi}$$

$$\boxed{-kx = \frac{7}{2}m\ddot{x} + 2ml\ddot{\varphi}\cos\varphi - 2ml\dot{\varphi}^2\sin\varphi}$$

$q_2 = \varphi$: $\quad \frac{\partial T}{\partial \varphi} = 2ml\dot{x}\dot{\varphi}(-\sin\varphi)$

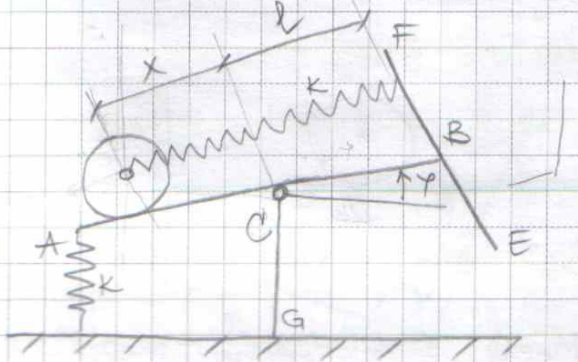
$$\frac{\partial T}{\partial \dot{\varphi}} = \frac{8}{3}ml^2\cdot\dot{\varphi} + 2ml\dot{x}\cos\varphi$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) = \frac{8}{3}ml^2\cdot\ddot{\varphi} + 2ml\ddot{x}\cos\varphi + 2ml\dot{x}(-\sin\varphi)\cdot\dot{\varphi}$$

$$\boxed{-2mgl\sin\varphi = \frac{8}{3}ml^2\cdot\ddot{\varphi} + 2ml\ddot{x}\cos\varphi}$$

Zadaci:

1. Štap AB kruto je spojen sa štapom EF u tački B i može da se obrće oko tačke C. Po njemu se bez klizanja kotrlja disk. U položaju statičke ravnoteže štap AB je horizontalan a opruga koja povezuje disk i štap EF nenapregnuta i njena dužina u nenapregnutom stanju je l. Napisati dif. jedn. sistema



AB: 3m, 3l (AC = 2l, CB = l)

EF: 2m, 2l (EB = FB = l)

disk: m, R = l/2

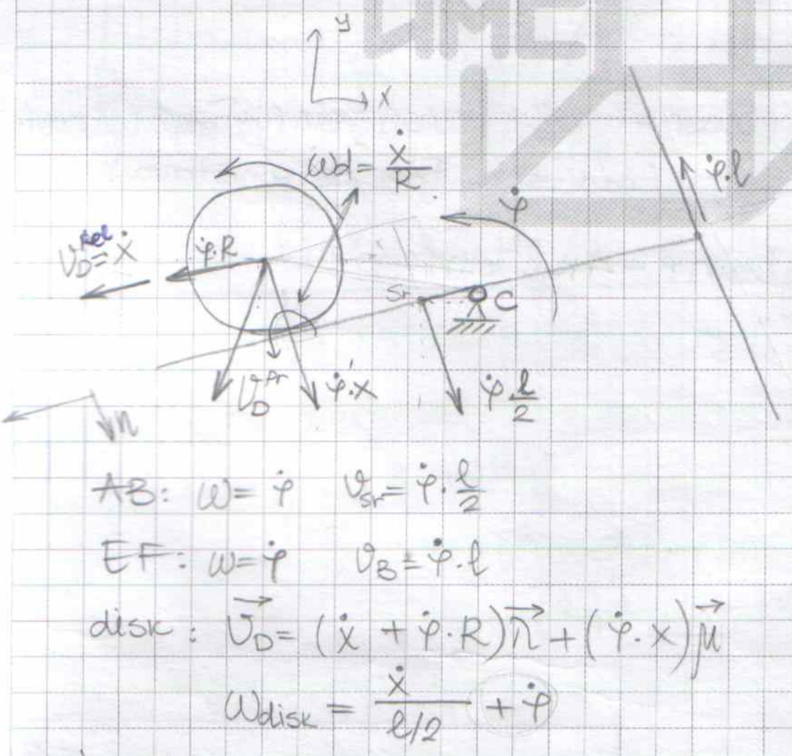
$n = 2$

$q_1 = \varphi$

$q_2 = x$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i$$

- brzine i ugaone brzine:



AB: $\omega = \dot{\varphi}$ $\quad v_{Sr} = \dot{\varphi}\cdot\frac{l}{2}$

EF: $\omega = \dot{\varphi}$ $\quad v_B = \dot{\varphi}\cdot l$

disk: $\vec{v}_D = (\dot{x} + \dot{\varphi}\cdot R)\vec{n} + (\dot{\varphi}\cdot x)\vec{\mu}$

$$\omega_{disk} = \frac{\dot{x}}{l/2} + \dot{\varphi}$$

- kinetička energija sistema:

$$T_{AB} = \frac{1}{2}3m\cdot\left(\dot{\varphi}\frac{l}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{12}3m\cdot(3l)^2\right)\cdot\dot{\varphi}^2 = \frac{3}{2}ml^2\cdot\dot{\varphi}^2$$

$\left(\text{ili} \quad T_{AB} = \frac{1}{2}3m\cdot \cancel{v_C^2} + \frac{1}{2}I_C\omega^2\right)$ ali C nije središte mase pa se teže radi I_C

$$T_{EF}=\frac{1}{2}2m\left(\dot{\varphi}\cdot l\right)^2+\frac{1}{2}\left(\frac{1}{12}2m\,(2l)^2\right)\cdot\dot{\varphi}^2=\frac{4}{3}ml^2\cdot\dot{\varphi}^2$$

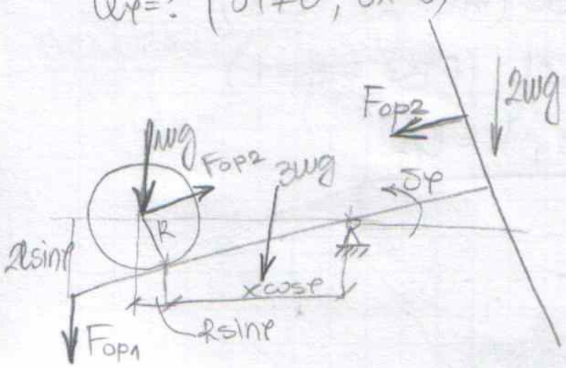
$$T_{disk}=\frac{1}{2}m\cdot\left[\left(\dot{x}+\dot{\varphi}\frac{l}{2}\right)^2+\left(\dot{\varphi}x\right)^2\right]+\frac{1}{2}\left(\frac{1}{2}m\left(\frac{l}{2}\right)^2\right)\cdot\left(\frac{2\dot{x}}{l}+\dot{\varphi}\right)^2=\ldots=$$

$$=\frac{3}{4}m\cdot\dot{x}+\frac{3}{4}ml\dot{x}\dot{\varphi}+\frac{31}{16}ml^2\dot{\varphi}^2+\frac{1}{2}mx^2\cdot\dot{\varphi}^2$$

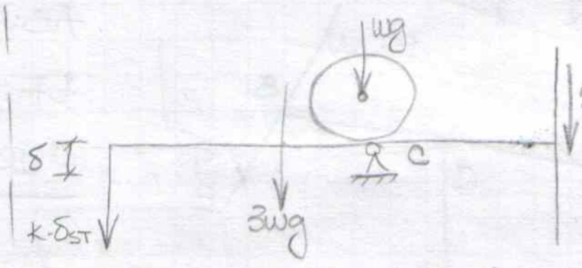
$$\boxed{T_{uk}=T_{AB}+T_{EF}+T_{disk}}$$

- GENERALISANE SILE:

$Q_\varphi=?$ $(\delta\varphi\neq0,\ \delta x=0)$



položaj statičke ravnoteže:



$$\Sigma M_C=0\qquad 2mg\cdot l-3mg\frac{l}{2}-k\delta_{ST}\cdot2l=0$$

$$\delta_{ST}=\frac{1}{4}\frac{mg}{k}$$

sile u opruzi: pretpostavimo da je $\delta_{ST}>2l\sin\varphi$

$$F_{op1}=k\left(\frac{1}{4}\frac{mg}{k}-2l\sin\varphi\right)$$

$$F_{op2}=k\cdot x$$

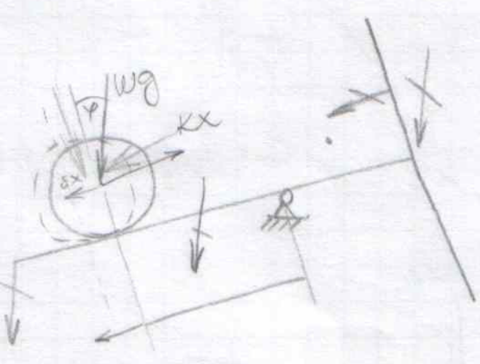
$$\delta A=3mg\cdot\delta\varphi\cdot\frac{l}{2}\cos\varphi-2mg\,\delta\varphi\, l\cos\varphi+mg\cdot\delta\varphi\left(x\cos\varphi+\frac{l}{2}\sin\varphi\right)+F_{op1}\,\delta\varphi\cdot2l\cos\varphi=Q_\varphi\delta\varphi$$

(rad sila F_{op2} se poništava)

...

$$Q_\varphi=mgx\cos\varphi+\frac{1}{2}mgl\sin\varphi-4kl^2\sin\varphi\cos\varphi$$

$Q_x=?$ $(\delta x\neq0\ \ \delta\varphi=0)$



$$\delta A=-kx\delta x+mg\sin\varphi\,\delta x=Q_x\cdot\delta x$$

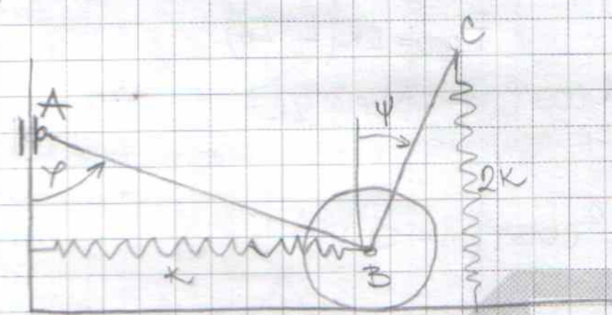
$$Q_x=mg\sin\varphi-kx$$

$$\boxed{\begin{aligned}&\frac{145}{24}ml^2\ddot{\varphi}+mx^2\ddot{\varphi}+\frac{3}{4}ml\ddot{x}+2mx\dot{x}\dot{\varphi}=\\&=mgx\cos\varphi-\frac{1}{2}mgl\sin\varphi-4kl\sin\varphi\cos\varphi\\&\frac{3}{2}m\cdot\ddot{x}+\frac{3}{4}ml\ddot{\varphi}-mx\dot{\varphi}^2=mg\sin\varphi-kx\end{aligned}}$$

dif. jedn.

2. Meh. sistem na slici može da se kreće u vertikalnoj ravni, disk se kotrlja bez klizanja po horizontalnoj podlozi dužine opruga u nenapregnutom stanju su $2l$. Odrediti:

- broj stepeni slobode i usvojiti generalisane koordinate
- brzine središta masa i ugaone brzine svih tela sistema
- kin. en. sistema i
- generalisane sile sistema



$\left.\begin{matrix} AB \\ BC \end{matrix}\right\} 2m, 2l$

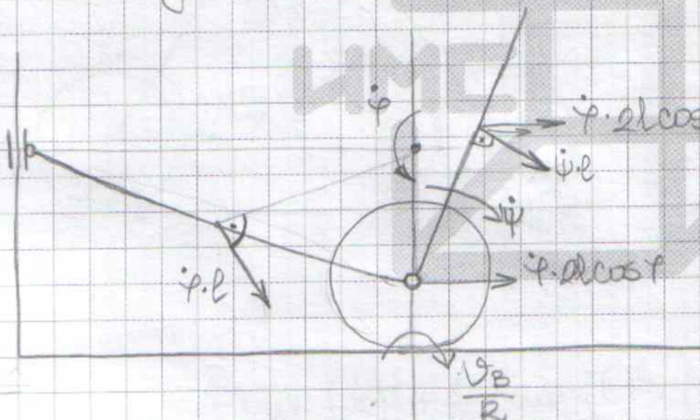
disk: m, $R = l/2$

$n = 2$

$q_1 = \varphi$ (ili $q_1 = x$)

$q_2 = \psi$

- brzine i ugaone brzine:



AB: $\omega = \dot\varphi$

$v_{S_{AB}} = \dot\varphi \cdot l$

disk: $v_B = 2\dot\varphi l\cos\varphi$

$\omega_{disk} = 4\dot\varphi\cos\varphi$

BC: $\omega = \dot\psi$

$v_{S_{BC}} = (\dot\varphi 2l\cos\varphi + \dot\psi l\cos\psi)\vec{i} - (\dot\psi l\sin\psi)\vec{j}$

- kinetička energija:

$T_{AB} = \ldots = \frac{4}{3} m l^2 \cdot \dot\varphi^2$

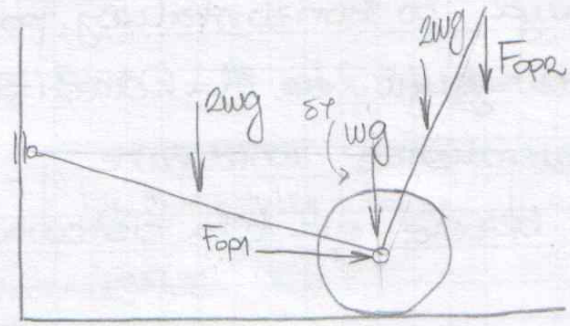
$T_{disk} = \ldots = 3ml^2\cos^2\varphi \cdot \dot\varphi^2$

$T_{BC} = \ldots = \frac{4}{3} ml^2\dot\psi^2 + 4ml^2\cos^2\varphi \cdot \dot\varphi^2 + 4ml^2\dot\varphi\dot\psi\cos\varphi\cos\psi$

$T = T_{AB} + T_{disk} + T_{BC} = \ldots$

-GENERALISANE SILE:

$Q_\varphi = ?$ $(\delta\varphi \neq 0, \delta\psi = 0)$



sile u opruzi:

* opruga 1 $L_0 = 2l$, $L = 2l\sin\varphi$ $\} L_0 > L$ ⇓ opruga je sabijena

$F_{op1} = k(2l - 2l\sin\varphi)$

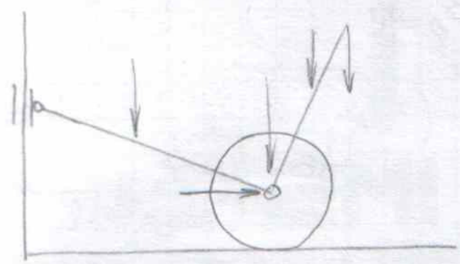
* opruga 2 $L_0 = 2l$, $L = \frac{l}{2} + 2l\cos\psi$ $\}$ pretpostavimo $L > L_0$ ⇓ zatezanje

$F_{op2} = 2k(L - L_0)$

$\delta A = 2mgl\delta\varphi\, l\sin\varphi + F_{op1}\cdot\cancel{\delta\varphi}\, 2l\cos\varphi = Q_\varphi\delta\varphi$

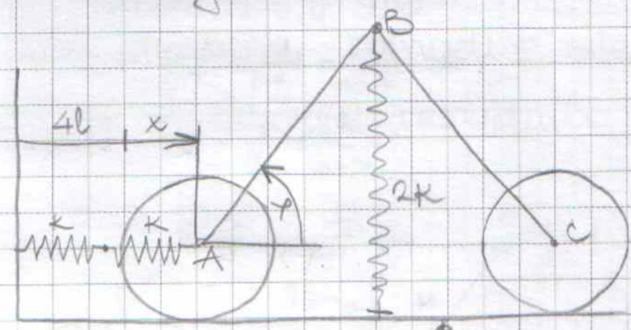
($\delta\psi = 0$ štap se translira i sila 2mg ne daje)

$Q_\varphi = 2mgl\sin\varphi + 4kl^2\cos\varphi(1 - \sin\varphi)$



$Q_\psi = 2mgl\sin\psi + kl^2(8\sin\psi\cos\psi - 6\sin\psi)$

1. Meh. sistem na slici kreće se u vertikalnoj ravni. Diskovi se kotrljaju po podlozi bez klizanja. Dužina redno vezanih opruga u nenapregnutom stanju je $2l$, a opruge krutosti $2k$ je $5l$. Napiši dif. jedn. kretanja sistema.



diskovi: $2m$, $R=l$

štap AB: $4m$, $4l$

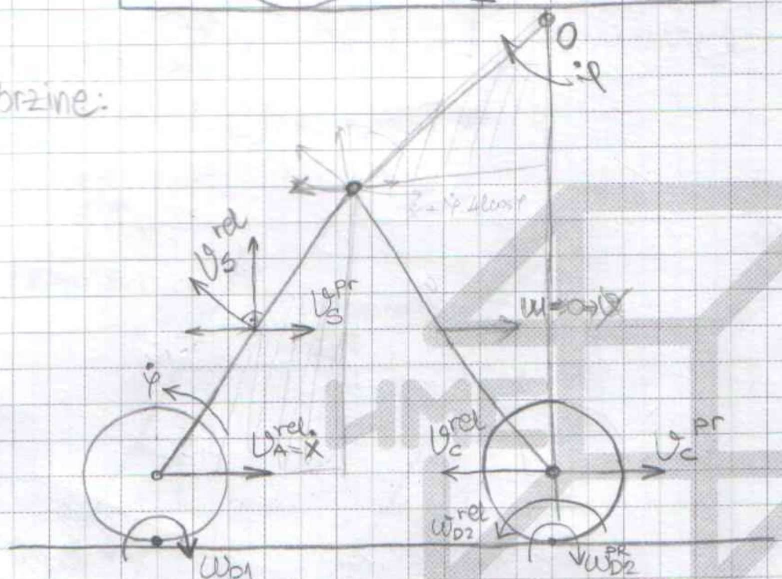
štap BC: $4l$, zanemarljiva masa

$n=2$

$q_1 = x$

$q_2 = \varphi$

- brzine:



$V_A^{rel} = \dot{x} \Rightarrow \omega_{D1} = \frac{\dot{x}}{l}$

$V_S^{pr} = \dot{x}$

$V_C^{pr} = \dot{x} \Rightarrow \omega_{D2}^{pr} = \frac{\dot{x}}{l}$

$\omega_{AB} = \dot{\varphi}$

$V_S^{rel} = \dot{\varphi} \cdot 2l \Rightarrow V_{S,x}^{rel} = \dot{\varphi} \cdot 2l \cdot \sin\varphi$, $V_{S,x}^{rel} = \dot{\varphi} \cdot 2l\cos\varphi$

$V_C^{rel} = \dot{\varphi} \cdot 8l\sin\varphi$

$\omega_{D2}^{rel} = 8\dot{\varphi}\sin\varphi$

$\omega_{D2} = \frac{\dot{x}}{l} - 8\dot{\varphi}\sin\varphi$

- kinetička energija:

$$T_{disk1} = \frac{1}{2} 2m \cdot (\dot{x})^2 + \frac{1}{2}\left(2m \cdot (l)^2 \cdot \left(\frac{\dot{x}}{l}\right)^2\right) = \frac{3}{2} m \cdot \dot{x}^2$$

$$T_{AB} = \frac{1}{2} 4m\left[(\dot{x} - \dot{\varphi} \cdot 2l\sin\varphi)^2 + (\dot{\varphi} \cdot 2l\cos\varphi)^2\right] + \frac{1}{2}\left(\frac{1}{12} 4m(4l)^2\right) \cdot \dot{\varphi}^2$$

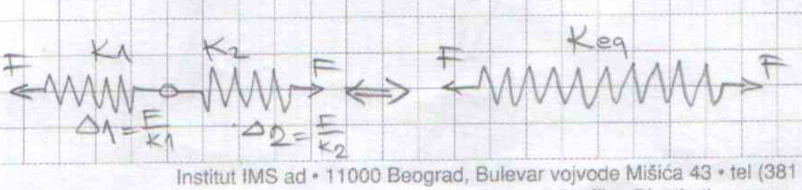
$$T_{AB} = 2m\dot{x}^2 + \frac{32}{3} ml \cdot \dot{\varphi}^2 - 8ml\,\dot{x}\,\dot{\varphi}\sin\varphi$$

$$T_{disk2} = \frac{1}{2} \cdot 2m \cdot (\dot{x} - \dot{\varphi} \cdot 8l \cdot \sin\varphi)^2 + \frac{1}{2}\left(\frac{1}{12} 2ml^2\right) \cdot \left(\frac{\dot{x}}{l} - 8\dot{\varphi}\sin\varphi\right)^2$$

$$T_{disk2} = \frac{3}{2} m \cdot \dot{x}^2 - 24ml\,\dot{x}\,\dot{\varphi}\sin\varphi + 96\, m\, l^2 \cdot \dot{\varphi}^2 \sin^2\varphi$$

$$T = \sum T_i$$

* REDNO VEZANE OPRUGE

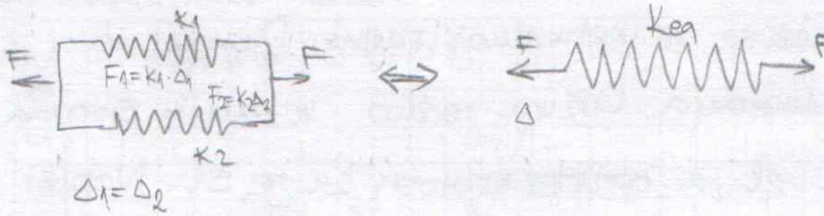


$\Delta_{eq} = \Delta_1 + \Delta_2$

$\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2}$

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$

PARALELNO VEZANE OPRUGE

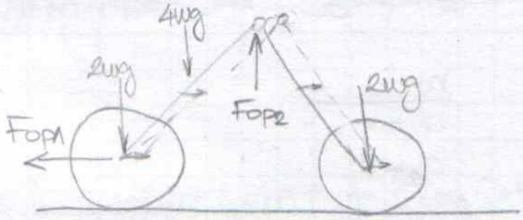


$F = k_1\Delta + k_2\Delta = k_{eq}\Delta$

$$\boxed{k_1 + k_2 = k_{eq}}$$

– GENERALISANE SILE:

$Q_x = ?\ (\delta x \neq 0, \delta\varphi = 0)$



– SILE U OPRUGAMA:

* $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$ –

$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{k} \quad k_{eq} = \frac{k}{2}$

$\Rightarrow F_{op1} = \frac{k}{2} \cdot x$

* $L_0 = 5l$

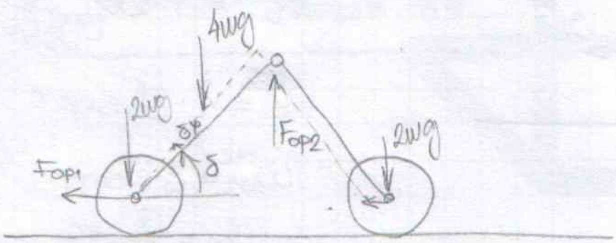
$L = l + 4l\sin\varphi$ } $L_0 > L$ (sabijena opruga)

$\Rightarrow F_{op2} = 2k \cdot (L_0 - L)$

$\delta A = -\left(\frac{k}{2}x\right)\delta x = Q_x \cdot \delta x$

$$\boxed{Q_x = -\frac{1}{2}kx}$$

$Q_\varphi = ?\ (\delta x = 0, \delta\varphi \neq 0)$



$\delta A = -4mg\,(\delta\varphi \cdot 2l\cos\varphi) + F_{op2}(\delta\varphi \cdot 4l\cos\varphi) = Q_\varphi \delta\varphi$

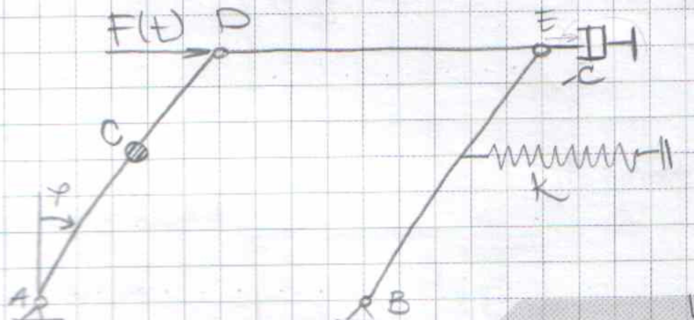
$$\boxed{Q_\varphi = -8mgl\cos\varphi + 32kl^2\cos\varphi\,(1 - \sin\varphi)}$$

dif. jedn. kretanja:

$10m\ddot{x} - 32ml\,\ddot{\varphi}\sin\varphi - 32kml\dot{\varphi}^2\cos\varphi = \frac{1}{2}kx$

$\frac{64}{3}ml^2\cdot\ddot{\varphi} - 32ml\ddot{x}\sin\varphi + 192\,ml^2\ddot{\varphi}\cdot\sin^2\varphi + 192ml^2\dot{\varphi}^2\sin\varphi\cos\varphi = Q_\varphi$

2. Meh. sistem na slici kreće se u vertikalnoj ravni i pri tome vrši male oscilacije. Opruga je nenapregnuta kada je štap BE vertikalan. Odrediti: a) dif. jedn. oscilovanja; b) kružnu frekvencu i period oscilovanja neprigušenog i prigušenog sistema; c) kružnu frekvencu prinudne sile pri kojoj dolazi do rezonance u slučaju prinudnih neprigušenih oscilacija.



AD, BE, DE : 2m, 2l ($\overline{AC}=\overline{CD}$)

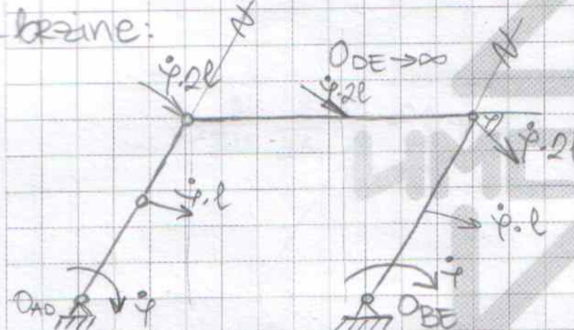
tačka C: m

$k = 15\frac{mg}{l}$ $\quad c = 2m\sqrt{\frac{g}{l}}$

$F(t) = F_0 \sin(\alpha t)$

$\frac{n=1}{q_1=\varphi}$

- brzine:



- kinetička energija:

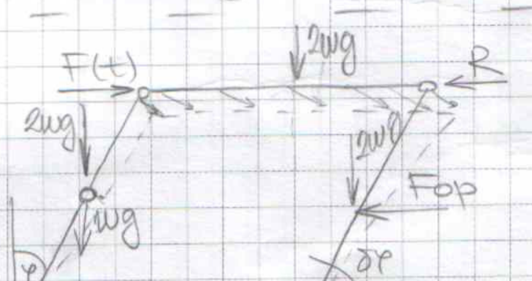
$T_{AD} = \frac{1}{2}2m(\dot\varphi \cdot l)^2 + \frac{1}{2}\left(\frac{1}{12}2m\cdot(2l)^2\right)\cdot\dot\varphi^2 = \frac{4}{3}m\cdot l^2\cdot\dot\varphi^2$

ili $T_{BE} = \frac{1}{2}2m\, v_B^2 \overset{=0}{} + \frac{1}{2}\left(\frac{1}{3}2m\cdot(2l)^2\right)\dot\varphi^2 = \frac{4}{3}m\, l^2\cdot\dot\varphi^2$

$T_{DE} = \frac{1}{2}2m\,(\dot\varphi\cdot 2l)^2 + \frac{1}{2}I_S\,\dot\phi = 4ml^2\cdot\dot\varphi^2$

$T_C = \frac{1}{2}m\,(\dot\varphi\cdot l)^2 = \frac{1}{2}m\cdot l^2\cdot\dot\varphi^2$

$\Rightarrow T = \frac{43}{6}m\cdot l^2\cdot\dot\varphi^2$



$Q_\varphi = ?\ (\delta\varphi \neq 0)$

sila u opruzi:

$F_{op} = k\cdot(l\sin\varphi)$

- sila u visk. prigušivaču:

$$\boxed{R=-c\cdot(\dot{\varphi}\cdot 2l\cos\varphi)}$$

$$\delta A = 3mg(\delta\varphi\cdot l\sin\varphi)+F(t)\cdot\delta\varphi\cdot 2l\cos\varphi+2mg(\delta\varphi 2l\sin\varphi)-R(\delta\varphi 2l\cos\varphi)+$$
$$+2mg(\delta\varphi\cdot l\sin\varphi)-F_{op}(\delta\varphi l\cos\varphi)=Q_\varphi\delta\varphi$$

$$\boxed{Q_\varphi = 9mgl\underset{\to\varphi}{\sin\varphi}+F(t)\cdot 2l\underset{\to 1}{\cos\varphi}-kl^2\underset{\to\varphi}{\sin\varphi}\underset{\to 1}{\cos\varphi}-4c\dot{\varphi}\cdot l^2\cos^2\varphi}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\varphi}}\right)-\frac{\partial T}{\partial\varphi}=Q_\varphi \qquad \frac{\partial T}{\partial\varphi}=0$$

$$\frac{\partial T}{\partial\dot{\varphi}}=\frac{43}{3}ml^2\cdot\dot{\varphi}$$

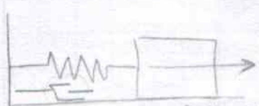
$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\varphi}}\right)=\frac{43}{3}ml^2\ddot{\varphi}$$

MALE OSCILACIJE: φ mali

$\sin\varphi\sim\varphi$

$\cos\varphi\sim 1$

$$\frac{43}{3}ml^2\cdot\ddot{\varphi}=9mgl\varphi+F(t)\cdot 2l-kl^2\varphi-4c\dot{\varphi}l^2 \quad \Big/\cdot\frac{3}{43}\frac{1}{ml^2}$$



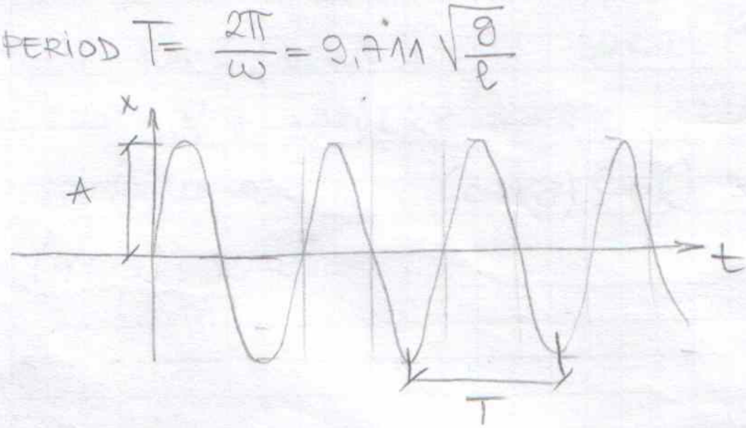
$$\ddot{x}+2\underset{\text{frekvenca}}{\omega}\zeta\cdot\dot{x}+\underset{\text{frekvenca}}{\omega^2}x=\dots$$

$$\boxed{\ddot{\varphi}+\frac{12}{43}\frac{c}{m}\dot{\varphi}+\left(\frac{3}{43}\frac{k}{m}-\frac{27}{43}\cdot\frac{g}{l}\right)\cdot\varphi=\frac{6}{43}\frac{F(t)}{m}}$$

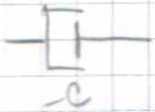
* NEPRIGUŠEN SISTEM: $c=0$

KRUŽNA FREKVENCA $\omega^2=\frac{3}{43}\frac{\overset{15mg/l}{k}}{m}-\frac{27}{43}\cdot\frac{g}{l}=\frac{18}{43}\frac{g}{l}$ $\quad\boxed{\omega=0{,}647\sqrt{\frac{g}{l}}}$

PERIOD $T=\frac{2\pi}{\omega}=9{,}711\sqrt{\frac{g}{l}}$



* PRIGUŠEN SISTEM * $c \neq 0$

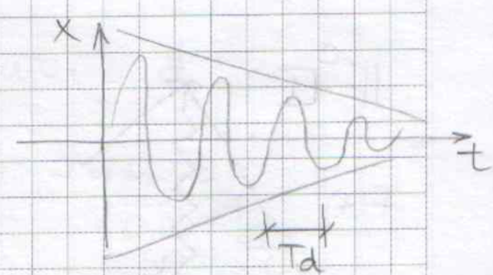


ξ – RELATIVNO PRIGUŠENJE

$$\boxed{R = c \cdot \theta}$$

$2\omega\sqrt{\frac{g}{\ell}}$

$$2\omega\,\xi = \frac{12}{43}\,\frac{c}{m}\cdot\xi \qquad \xi = 0{,}431$$

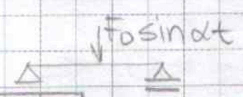


PRIGUŠENO

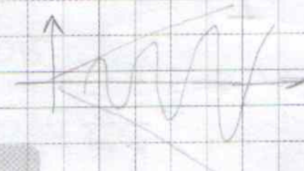
$$\omega_d = \omega\sqrt{1-\xi^2} \qquad \omega_d = 0{,}584\sqrt{\frac{g}{\ell}}$$

$$T_d = \frac{T}{\sqrt{1-\xi^2}} \qquad T_d = 10{,}764\sqrt{\frac{g}{\ell}}$$

* $F(t) = F_0 \sin\alpha t$

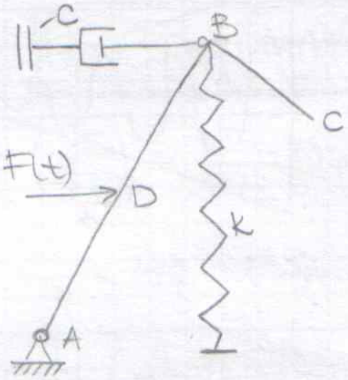


* REZONANCA *



$$\omega = \alpha \qquad \boxed{\alpha = 0{,}647\sqrt{\frac{g}{\ell}}}$$

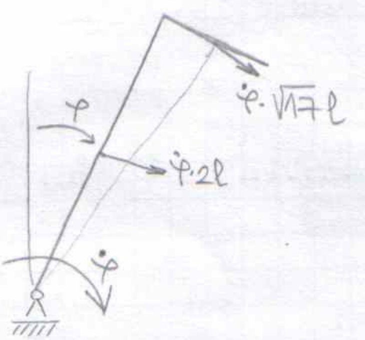
1. Meh. sistem kreće se u vert. ravni i pri tome vrši male oscilacije. Dužina opruge u nenapregnutom stanju je 5l. Odredi: 1) dif. jedn. oscilovanja, 2) kružnu frekvencu i period oscilovanja neprigušenog i prigušenog kretanja.



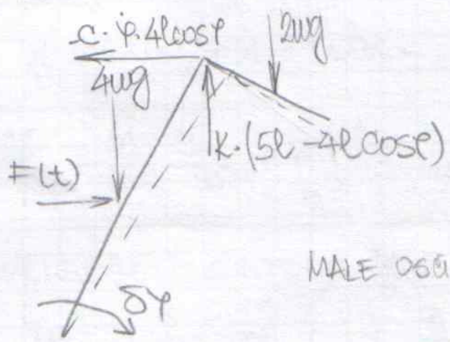
AB: $4l, 4m$ $(\overline{AD}=\overline{DB})$

BC: $2l, 2m$ $F(t)=F\sin\omega t$

$k=100\frac{mg}{l}$ $c=5m\sqrt{\frac{g}{l}}$



$T=28ml^2\dot\varphi^2$



$$Q_\varphi=16mgl\sin\varphi+2mgl\cos\varphi+2F(t)l\cos\varphi-16c\dot\varphi l^2\cos^2\varphi-20kl^2\sin\varphi+16kl^2\sin\varphi\cos\varphi$$

MALE oscilacije: $\sin\varphi\sim\varphi$

$\cos\varphi\sim 1$

$$\boxed{\ddot\varphi+\underbrace{\frac{2}{7}\cdot\frac{c}{m}}_{2\xi\omega}\cdot\dot\varphi+\underbrace{\left(\frac{1}{14}\frac{k}{m}-\frac{2}{7}\frac{g}{l}\right)}_{\omega^2}\varphi=\frac{1}{28}\frac{g}{l}+\frac{1}{28}\frac{F(t)}{ml}}$$

$\hookrightarrow \omega_d=2{,}52\sqrt{\frac{g}{l}}$

$\hookrightarrow \xi=0{,}273$

$\omega=2{,}619\sqrt{\frac{g}{l}}$

2. Za primer iz prethodnog zadatka odrediti konačne jedn. kretanja tačke C uz sled. pretpostavke:

* oscilacije su neprigušene $(c=0)$
* [illegible]

*usvojiti homogene početne uslove

$$\ddot{\varphi} + \frac{48}{7}\frac{g}{l}\varphi = \frac{1}{28}\frac{g}{l} + \frac{1}{28}\frac{F_0}{ml}\sin\alpha t$$

$\varphi = \varphi_H + \varphi_p$ $\qquad \ddot{\varphi} + \omega^2\varphi = 0$

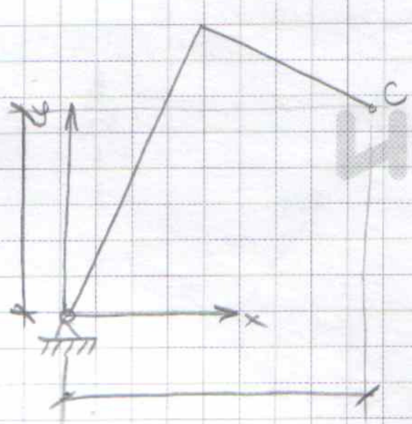
$$\varphi_H = C_1\cos\omega t + C_2\sin\omega t$$

$$\left.\begin{matrix}\varphi_p = A + B\sin\alpha t \\ \dot{\varphi}_p,\ \ddot{\varphi}_p\end{matrix}\right\} \begin{matrix} A = \dots = 0{,}005 \\ B = \dots = 0{,}007\dfrac{F_0}{mg}\end{matrix}$$

$$\underline{\varphi = C_1\cos\omega t + C_2\sin\omega t + 0{,}005 + 0{,}007\frac{F_0}{mg}\sin\alpha t}$$

početni uslovi: $\varphi(0) = 0 \Rightarrow C_1 = -0{,}005$

$\dot{\varphi}(0) = 0 \Rightarrow C_2 = -0{,}0035\dfrac{F_0}{mg}$ $\Rightarrow \varphi(t)$



$$x(t) = 4l\sin\varphi + 2l\cos\varphi$$

$$y(t) = 4l\cos\varphi - 2l\sin\varphi$$

3. Ako je amplituda prinudne harmonijske sile $F_0 = 10kN$, a krutost opruge $k = 1\frac{MN}{m}$, odredi masu tačke M tako da max amplituda prinudnih oscilacija bude manja od 2cm, ako se zna da je kružna frekvenca prinudne sile $\Omega = 100\frac{rad}{s}$

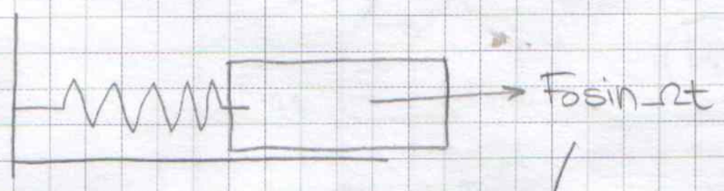
$F_0 = 10kN$

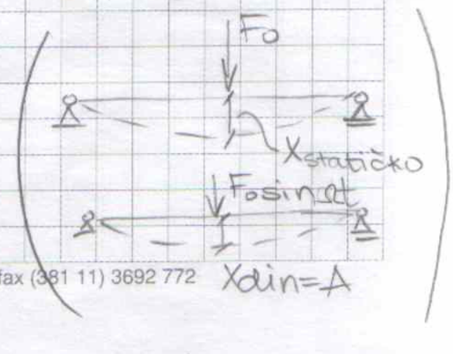
$k = 1\frac{MN}{m}$

$m = ?$

$x_m = A \leq 2cm$

$\Omega = 100\frac{rad}{s}$





$$D = \frac{A}{X_{stat}} \qquad X_{din} = \frac{F_0}{k} \cdot \frac{1}{1-\beta^2} \qquad \beta = \frac{\Omega}{\omega}$$

$$X_{stat} = \frac{F_0}{k}$$

$$D \leq \frac{2cm}{1cm} = 2 \qquad X_{st} = \frac{10}{1000} = 0{,}01\,m = 1cm$$

$$\boxed{D = \frac{1}{1-\beta^2}} \leq 2$$

$$1-\beta^2 \geq \frac{1}{2}$$

$$\beta^2 \leq \frac{1}{2}$$

$$\frac{\Omega^2}{\omega^2} \leq \frac{1}{2}$$

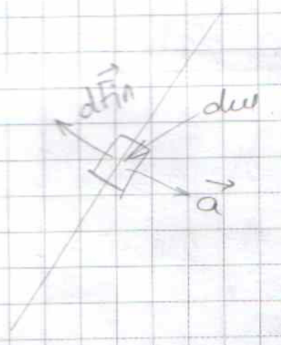
$$\omega^2 \geq 2\Omega^2 = 2 \cdot 100^2 = 20\,000 \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\frac{k}{m} \geq 20\,000$$

$$m \leq \frac{k}{20000} = \frac{1000}{20000} = 0{,}05\,t$$

$$\boxed{m \leq 50kg}$$

DALAMBEROV PRINCIP ZA ŠTAPASTE SISTEME

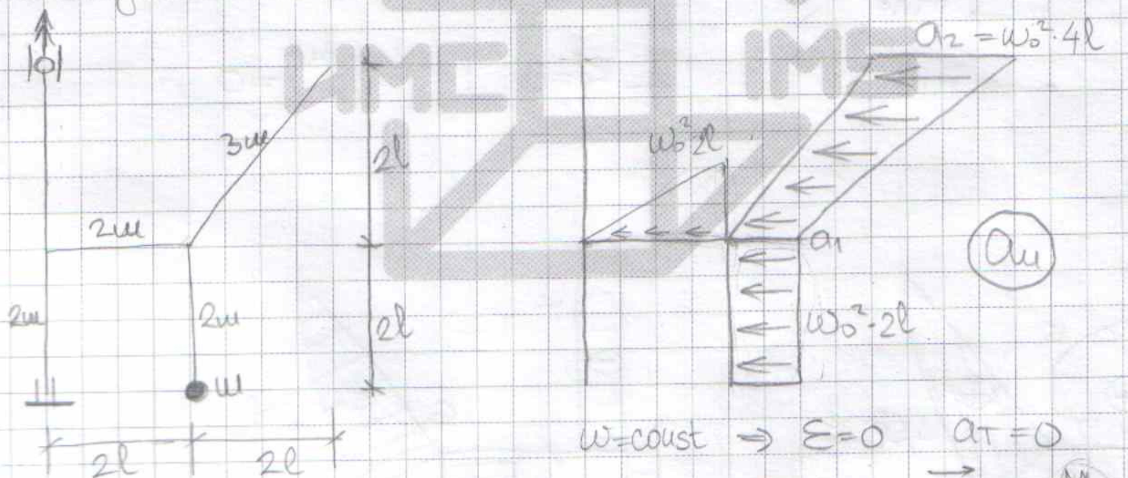


$$d\vec{F}_{in} = -dm \cdot \vec{a}$$

$$d\vec{F}_{in} = -\frac{m}{l} \cdot dx \cdot \vec{a}$$

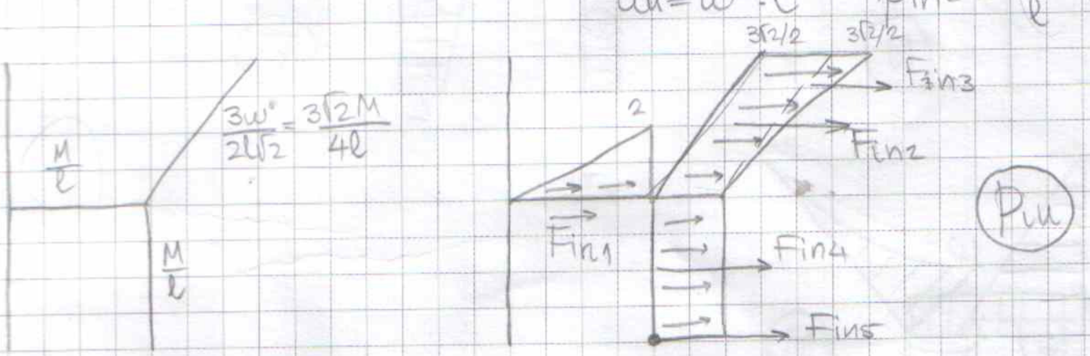
$$\frac{d\vec{F}_{in}}{dx} = -\frac{m}{l} \cdot \vec{a} \Rightarrow \boxed{\vec{p}_{in} = -\frac{M}{l} \cdot \vec{a}}$$

1. Sistem kruto vezanih štapova kao na slici obrće se konstantnom ugaonom brzinom oko vertikalne ose. Odredi dijagrame presečnih sila samo usled inercijalnog dinamičkog opterećenja. Vrednost ω_0 iz uslova da je reakcija veza u B usled inercijalnog opterećenja 2 puta veća od iste te reakcije usled statičkog opterećenja.



$\omega = const \Rightarrow \varepsilon = 0 \quad a_T = 0$

$a_n = \omega^2 \cdot l \quad \vec{p}_{in} = -\frac{M}{l} \cdot \vec{a}$



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$$p_{in1} = \frac{m}{l} \cdot \overset{a}{\left(\omega_0^2 \cdot 2l\right)} = 2m \cdot \omega_0^2 \qquad F_{in1} = \frac{1}{2} 2m \cdot \omega_0^2 \cdot 2l = 2m\,\omega_0^2 \cdot 2l$$

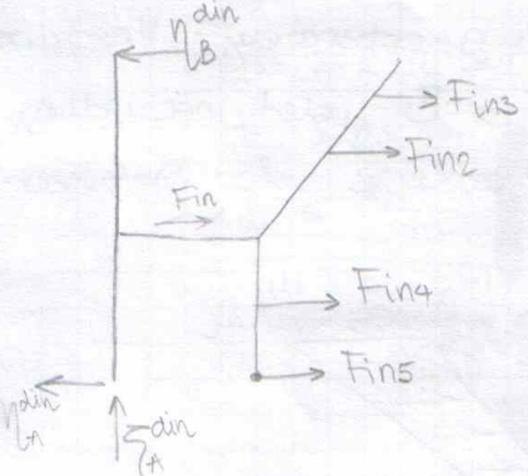
$$p_{in2} = \frac{m}{l} \cdot \omega_0^2 \cdot 2l = 2m \cdot \omega_0^2 \qquad F_{in2} = 1{,}5\sqrt{2}\, m \cdot \omega_0^2 \cdot 2\sqrt{2}l = 6m\,\omega_0^2 \cdot l$$

$$p_{in3} = \frac{3\sqrt{2}m}{4l} \cdot \omega_0^2 \cdot 2l = \frac{3}{2}\sqrt{2} M \omega_0^2 \qquad F_{in3} = \frac{1{,}5\sqrt{2}\, m \cdot \omega_0^2 \cdot 2\sqrt{2}\, l}{2} = 3m\,\omega_0^2 \cdot l$$

$$p_{in4} = \frac{3\sqrt{2}m}{4l} \cdot \omega_0^2 \cdot 4l = 3\sqrt{2}\, m \cdot \omega_0^2 \qquad F_{in4} = 2m \cdot \omega_0^2 \cdot 2l = 4m \cdot \omega_0^2 \cdot l$$

potiče od kuglice → $F_{in5} = \underset{\text{masa kuglice}}{M} \cdot \underset{a}{\omega_0^2 \cdot 2l} = 2m \cdot \omega_0^2 \cdot l$

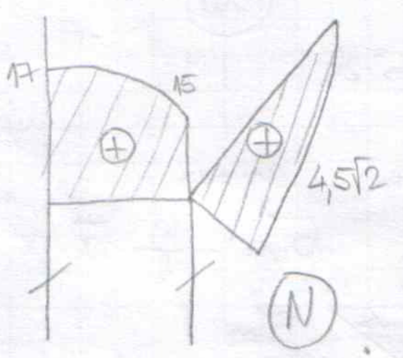
površine odgovarajućih figura ograničene opterećenjem Pin

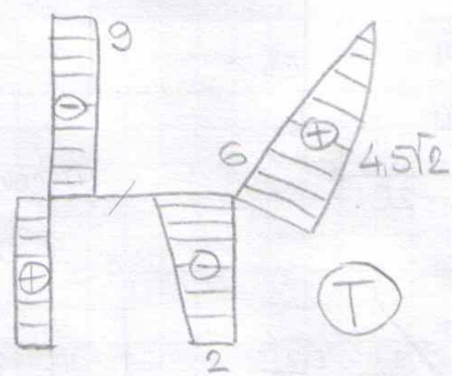


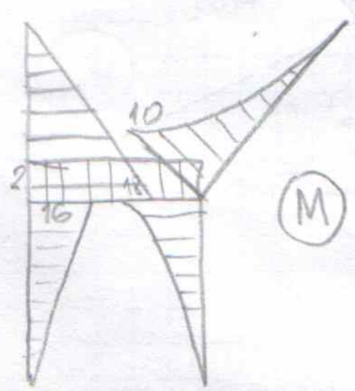
$$\Sigma M^A = 0: \quad \eta_B^{din} = 9m \cdot \omega_0^2 \cdot l$$

$$\Sigma \eta = 0: \quad \eta_A^{din} = 8m\,\omega_0^2 \cdot l$$

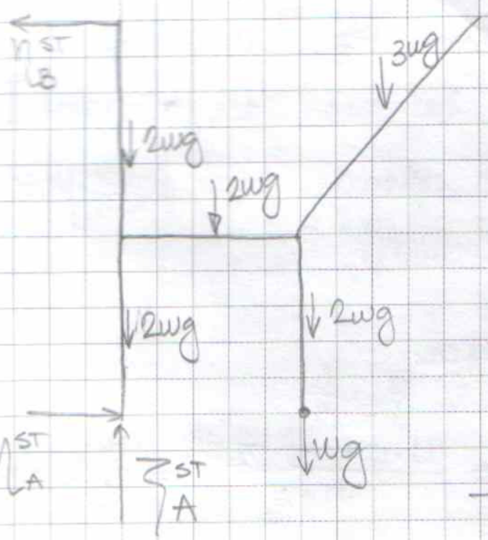
$$\Sigma \zeta = 0: \quad \zeta_A^{din} = 0$$







statičko opterećenje – sopstvene težine:



$\Sigma M = 0: \quad \eta_B^{ST} = 4{,}25mg$

$\Sigma \eta = 0: \quad \eta_A^{ST} = 4{,}25mg$

$\Sigma \zeta = 0: \quad \zeta_A^{ST} = 12mg$

uslov: $\eta_B^{din} = 2\eta_B^{ST}$

$$9\cancel{m} \cdot \omega_0^2 \cdot l = 2 \cdot 4{,}5 \cdot \cancel{m}g$$

$$\omega_0^2 = 0{,}944 \frac{g}{l}$$

$$\underline{\underline{\omega_0 = 0{,}972 \sqrt{\frac{g}{l}}}}$$

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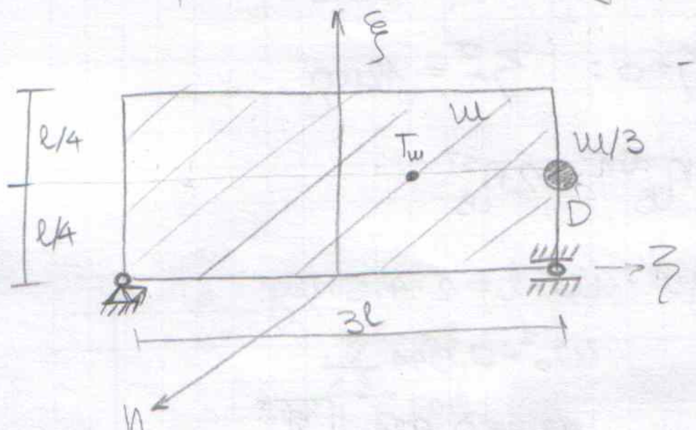
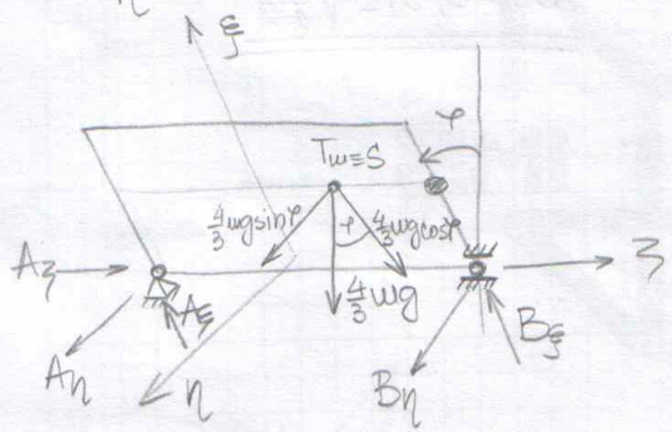
1. Pravougaona ploča mase m može da se okreće oko horizontalne osovine a u tački D se nalazi mat. tačka mase $\frac{m}{3}$. Ovako oformljen sistem počinje kretanje bez početne brzine iz prikazanog položaja. Napiši dif. jedn. kretanja i odredi reakcije veza kada ploča dodje u donji položaj ($\varphi=180°$)



- središte mase:

$$\eta_S=0 \quad , \quad \xi_S=\frac{l}{4}$$

$$\zeta_S=\frac{m\cdot 0+\frac{m}{3}\cdot\frac{1}{2}3l}{m+\frac{m}{3}}=\frac{3}{8}l$$



$\Sigma F_R = ma$

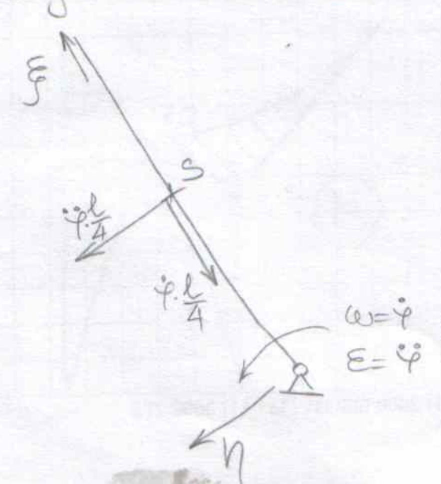
$\Sigma M_R = I\varepsilon$

- ZAKON O KRETANJU SRED. MASE:

$$\frac{d\vec{K}}{dt}=\vec{F}_R \quad : \quad \frac{4}{3}m\cdot\vec{a}_S=\vec{F}_R \quad \Big/ \cdot\vec{\lambda} \,,\; \cdot\vec{\mu} \,,\; \cdot\vec{\nu}$$

$\vec{K}$ - kol. kretanja

- ubrzanje središta mase:



$$1°\ \xi:\ \frac{4}{3}m\left(-\dot{\varphi}^2\cdot\frac{l}{4}\right)=A_\zeta+B_\zeta-\frac{4}{3}mg\cos\varphi$$

$$2°\ \eta:\ \frac{4}{3}m\left(\ddot{\varphi}\cdot\frac{l}{4}\right)=A_\eta+B_\eta+\frac{4}{3}mg\sin\varphi$$

$$3°\ \zeta:\ \frac{4}{3}m\cdot 0=A_\zeta$$

- ZAKON O PROMENI MOMENTA KOLIČINE KRETANJA:

$$\frac{d\vec{D}^{(0)}}{dt} = \vec{M}_R^{(0)} \quad \Big/ \; \vec{\pi}, \vec{\mu}, \vec{\nu}$$

$\vec{D}$ moment količine kretanja

$$\vec{D} = \begin{Bmatrix} D_\xi \\ D_\eta \\ D_\zeta \end{Bmatrix} = \begin{bmatrix} I_\xi & \cancel{I_{\xi\eta}}^{0} & I_{\xi\zeta} \\ \cancel{I_{\eta\xi}}^{0} & I_\eta & \cancel{I_{\eta\zeta}}^{0} \\ I_{\zeta\xi} & \cancel{I_{\zeta\eta}}^{0} & I_\zeta \end{bmatrix} \begin{Bmatrix} \cancel{\omega_\xi}^{0} \\ \cancel{\omega_\eta}^{0} \\ \omega_\zeta \end{Bmatrix} \Rightarrow$$

$$I_{\xi\zeta} = -\frac{4}{3}m\cdot\xi_S\cdot\zeta_S = -\frac{4}{3}\frac{l}{4}\cdot\frac{3}{8}l = -\frac{1}{8}ml^2$$

$$\Rightarrow \quad D_\xi = I_{\xi\zeta}\cdot\dot{\varphi}, \qquad D_\eta = 0, \qquad D_\zeta = I_\zeta\cdot\dot{\varphi}$$

$$I_\zeta = \frac{1}{12}m\cdot\left(\frac{l}{2}\right)^2 + m\cdot\left(\frac{l}{4}\right)^2 + 0 + \frac{m}{3}\cdot\left(\frac{l}{4}\right)^2 = \frac{5}{48}ml^2$$

$$\frac{d\vec{D}}{dt} = \overset{*}{\vec{D}} + \vec{\omega}\times\vec{D}$$

$$\overset{*}{\vec{D}} = \left(-\frac{1}{8}ml^2\cdot\ddot{\varphi}\right)\vec{\pi} + \left(\frac{5}{48}ml^2\cdot\ddot{\varphi}\right)\vec{\nu}$$

$$\vec{\omega}\times\vec{D} = \begin{vmatrix} \pi & \mu & \nu \\ 0 & 0 & \dot{\varphi} \\ D_\xi & 0 & D_\zeta \end{vmatrix} = -\frac{1}{8}ml^2\cdot\dot{\varphi}\cdot\dot{\varphi}\cdot\vec{\mu}$$

4° $\xi: \; -\frac{1}{8}ml^2\cdot\ddot{\varphi} = A_\eta\cdot\frac{3}{2}l - B_\eta\cdot\frac{3}{2}l - \frac{4}{3}mg\sin\varphi\cdot\frac{3}{8}l$

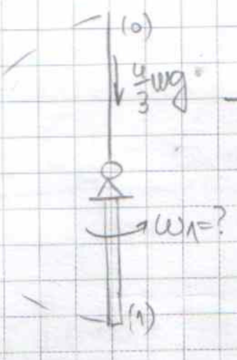
5° $\eta: \; -\frac{1}{8}ml^2\cdot\dot{\varphi}^2 = -A_\xi\cdot\frac{3}{2}l + B_\xi\cdot\frac{3}{2}l - \frac{4}{3}mg\cos\varphi\cdot\frac{3}{8}l$

6° $\boxed{\zeta: \; \frac{5}{48}ml^2\cdot\ddot{\varphi} = \frac{4}{3}\sin\varphi\cdot\frac{l}{4}}$ dif. jedn. kretanja

$\varphi = \pi \quad \Rightarrow \quad \boxed{\ddot{\varphi} = 0}$

$\dot{\varphi} = ?$ 1)

2) promena kinetičke energije:



$$T_1 - T_0 = A_{0-1}$$

$$\frac{1}{2}\left(\frac{5}{48}m\cdot l^2\right)\cdot\omega_1^2 - 0 = \frac{4}{3}mg\cdot\frac{l}{2}$$

$$\omega_1^2 = \dot{\varphi}^2(\varphi=\pi) = \frac{64}{5}\frac{g}{l}$$

1° $-\frac{1}{3}m\cdot\frac{64}{5}\cdot\frac{g}{l}\cdot l = A_\xi + B_\xi - \frac{4}{3}mg$

2° $0 = A_\eta + B_\eta$

3° $A_\eta = 0$

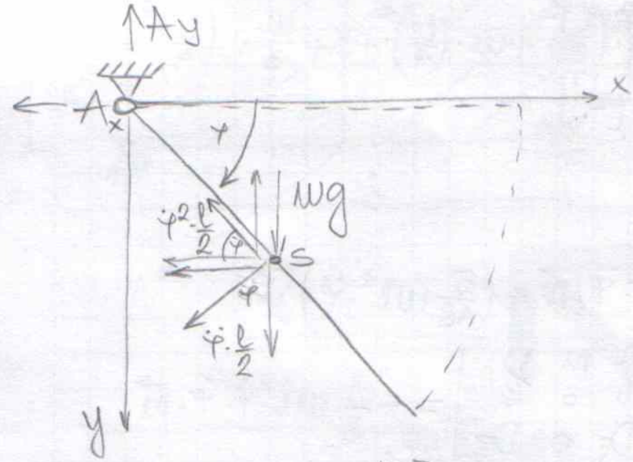
4° $0 = A_\eta\cdot\frac{3}{2}l - B_\eta\cdot\frac{3}{2}l$

5° $-\frac{1}{8}ml^2\cdot\frac{64}{5}\cdot\frac{g}{l} = -A_\xi\cdot\frac{3}{2}l + B_\xi\cdot\frac{3}{2}l + \frac{1}{2}mg$

$\boxed{A_\eta = 0}$ $\boxed{B_\eta = 0}$ $\boxed{A_\xi = -\frac{21}{10}mg}$ $\boxed{B_\xi = -\frac{35}{10}mg}$

2 Štap mase m i dužine l pušten je iz horizontalnog položaja bez poč. brzine da se kreće u vertik. ravni. Odredi reakcije veza u osloncu A u položaju $\varphi = 60°$.

štap: m, l



$* \; m\cdot\vec{a}_S = \vec{F}_R \; * \; / \vec{i}, \vec{j}$

1° x: $m\left(-\dot\varphi^2\cdot\frac{l}{2}\cos\varphi - \ddot\varphi\cdot\frac{l}{2}\sin\varphi\right) = -A_x$

2° y: $m\left(-\dot\varphi^2\cdot\frac{l}{2}\sin\varphi + \ddot\varphi\frac{l}{2}\cos\varphi\right) = mg - A_y$

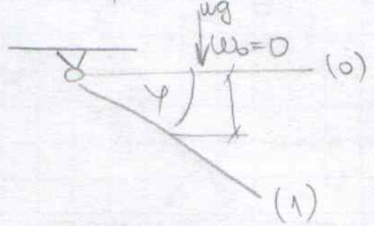
$* \; \frac{d\vec{D}^{(0)}}{dt} = \vec{M}_R^{(0)} \; * \; /\cdot\vec{k} \Rightarrow I\cdot\varepsilon = M_R$: $\frac{1}{3}ml^2\cdot\ddot\varphi = mg\frac{l}{2}\cos\varphi$ dif. jedn.

$\boxed{\ddot\varphi = \frac{3}{2}\frac{g}{l}\cos\varphi}$ dif. jedn. kretanja

$\varphi = 60°$ $\boxed{\ddot\varphi = \frac{3}{4}\frac{g}{l}}$

$\dot\varphi = ?$ 1) rešavanje dif. jedn.

2) promena kin. en.



$T_1 - T_0 = A_{0-1}$

$\frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega_1^2 - 0 = mg\frac{l}{2}\sin\varphi$

$\omega_1^2 = \dot\varphi^2 = \frac{3g}{l}\sin\varphi$

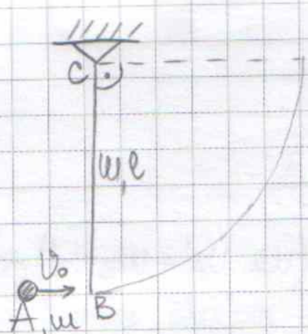
$\dot\varphi^2_{(\varphi=60°)} = \frac{3\sqrt{3}}{2}\cdot\frac{g}{l}$

$1^\circ\ A_x = \frac{1}{2}ml\left(\frac{3\sqrt{3}}{2}\frac{g}{l}\cdot\frac{1}{2} + \frac{3}{4}\frac{g}{l}\cdot\frac{\sqrt{3}}{2}\right) = 0{,}974\,mg$

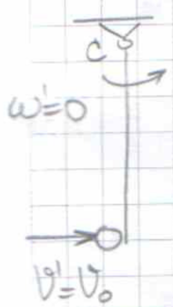
$2^\circ\ A_y = mg - \frac{1}{2}ml\left(-\frac{3\sqrt{3}}{2}\frac{g}{l}\cdot\frac{\sqrt{3}}{2} + \frac{3}{4}\frac{g}{l}\,\frac{1}{2}\right) = 1{,}938\,mg$

UDAR I SUDAR

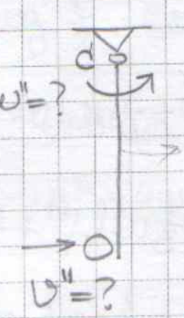
1. Mat. tačka A mase m krećući se duž horizontalnog pravca konstantnom brzinom v_0 naleti na donji kraj B vertikalnog štapa BC mase m i dužine l. Štap je pre udara bio u stanju mirovanja. Ako je udar idealno elastičan odredi potrebnu brzinu tačke A (v_0) tako da se štap posle udara zaustavi u horizontalnom položaju.

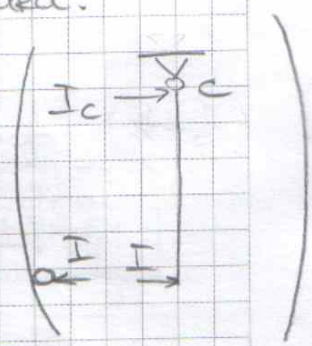


-brzine pre udara:



-brzine posle udara:



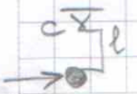


$1^\circ\ \boxed{\vec{D}''_C - \vec{D}'_C = \vec{M}_R = 0}$: $\boxed{[I_C\cdot\omega'' + m\cdot v''\cdot l] - [m\cdot v'\cdot l] = 0}$

$\frac{1}{3}m\cdot l^2\cdot\omega'' + mlv'' - mlv' = 0$

$v'' = v' - \frac{1}{3}l\omega''$ (★)

$\underline{D_C = I_C\cdot\omega}$



$K = m\cdot v \quad D = K\cdot l$

2° $0 \leq k \leq 1$ $k=0$ idealno plastičan udar

koef. udara $k=1$ idealno elastičan udar

idealno-~~plastičan~~ elastičan udar: nema promene kinetičke energije

$$\boxed{T'' - T' = 0} : \boxed{\left[\frac{1}{2} I_C \omega''^2 + \frac{1}{2} m v''^2\right] - \left[\frac{1}{2} m v'^2\right] = 0} \quad (☾)$$

$$T = \cancel{\frac{1}{2} m v_{\Delta}^2} + \frac{1}{2} I_{\Delta} \omega^2$$

* idealno plastičan udar

1° isti

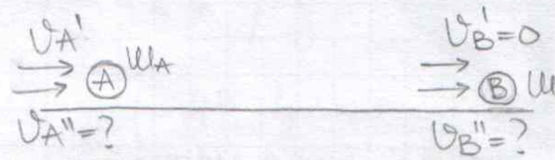
2° brzine dve tačke koje se sudare neposredno posle udara su jednake

$$v'' = \omega'' \cdot l$$

* koef udara: 1° isti uslov

$$2° \quad k = -\frac{\omega'' l - v''}{\omega' l - v'}$$

* sudar dve tačke:



1° $K'' - K' = I_R = 0$:

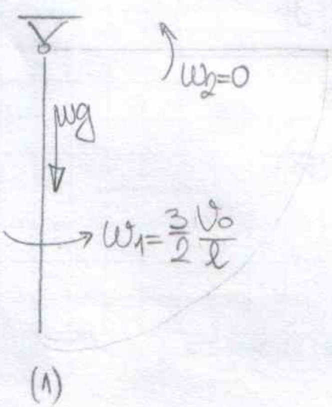
$$(m_A \cdot v_A'' + m_B v_B'') - (m_A \cdot v_A' + m_B \cdot v_B') = 0$$

2°

$$(☾ + \star) \quad \frac{1}{2} \cdot \frac{1}{3} m l^2 \omega''^2 + \frac{1}{2} m \left(v'^2 - 2 v' \frac{1}{3} l \omega'' + \frac{1}{9} l^2 \omega''^2\right) - \frac{1}{2} m v'^2 = 0$$

$$\omega'' \left(\frac{1}{6} m l^2 \omega'' - \frac{1}{3} m l v' + \frac{1}{18} m l^2 \omega''\right) = 0$$

$$\frac{2}{9} m l^2 \omega'' = \frac{1}{3} m l v' \qquad \boxed{\omega'' = \frac{3}{2} \frac{v_0}{l}}$$

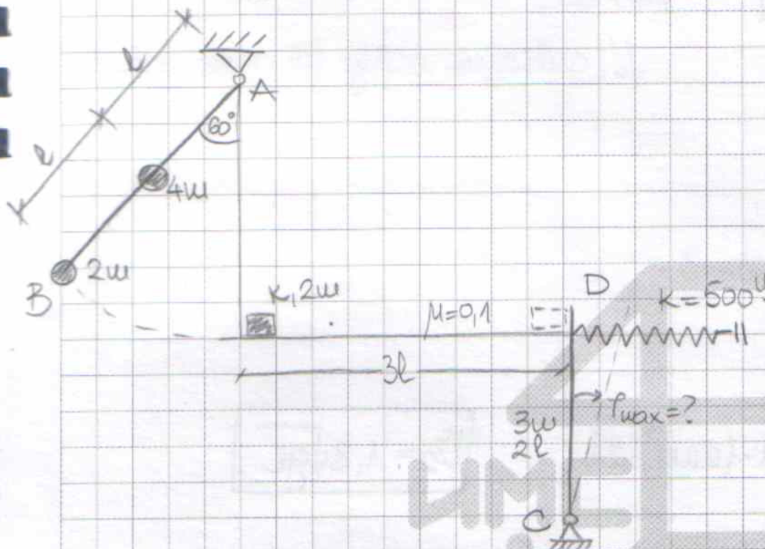


$$T_2 - T_1 = A_{1-2}$$

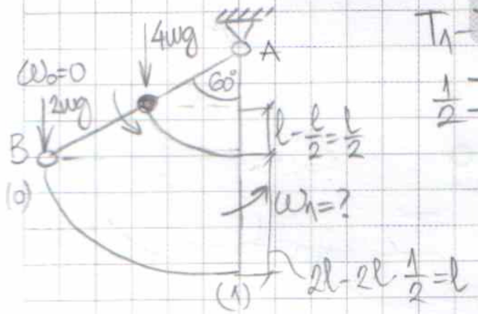
$$0 - \frac{1}{2}\left(\frac{1}{3} m \cdot l^2\right) \omega_1^2 = -mg \frac{l}{2}$$

$$\boxed{v_0 = 2 \cdot \sqrt{\frac{1}{3} g l}}$$

2. Štap AB počinje kretanje iz položaja sa skice. Prolazeći kroz vertikalan položaj, on udara u tačku K (idealno-elast. udar), koja se potom kreće po hrapavoj horizontalnoj ravni i konačno udara u vrh vertikalnog štapa CD (udar je idealno-plastičan). Odredi maximalno skretanje štapa CD posle udara tačke pretpostavljajući da su u pitanju male oscilacije.



- kretanje štapa AB



$$T_1 - T_0 = A_{0-1}$$

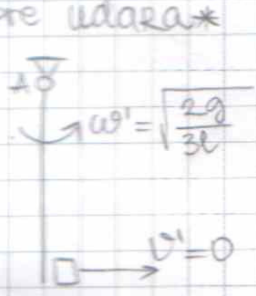
$$\frac{1}{2} I_A \cdot \omega_1^2 - 0 = 4mg\,\frac{l}{2} + 2mg\cdot l$$

$$I_A = 4m\,l^2 + 2m\cdot(2l)^2 = 12ml^2$$

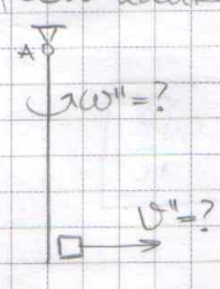
$$\frac{1}{2}\cdot 12ml^2\cdot\omega_1^2 = 4mgl \qquad \boxed{\omega_1 = \sqrt{\frac{2g}{3l}}}$$

- udar štapa u tačku:

pre udara



posle udara



$1^\circ \quad D_A'' - D_A' = 0:$

$$\left(I_A \cdot \omega'' + 2m \cdot v'' \cdot 2l\right) - \left(I_A \cdot \omega''\right) = 0$$

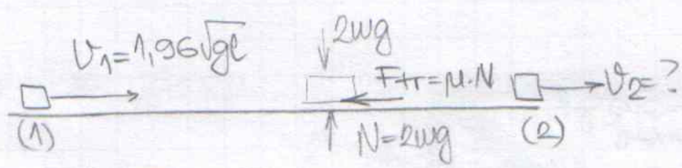
$2^\circ \quad T'' - T' = 0:$

$$\left(\frac{1}{2} I_A \cdot \omega''^2 + \frac{1}{2} 2m \cdot v''^2\right) - \left(\frac{1}{2} I_A \cdot \omega'^2\right) = 0$$

$$\omega'' = \omega' - \frac{4ml}{I_A} \cdot v'' \quad (\text{iz } 1^\circ)$$

$$\boxed{v'' = \frac{12}{5} l\omega' = 1{,}96\sqrt{gl}}$$

- kretanje tačke po podlozi:

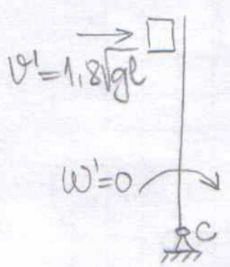


$$T_2 - T_1 = T_{1\to 2}$$

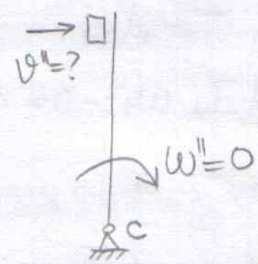
$$\frac{1}{2} 2m \cdot v_2^2 - \frac{1}{2} 2m \cdot v_1^2 = -\mu \cdot (2mg) \cdot 3l \qquad \boxed{v_2 = 1{,}8\sqrt{gl}}$$

- udar tačke u štap:

pre udara



posle udara



$$I_{C,\text{štapa}} = \frac{1}{3} \cdot 3m \cdot (2l)^2 = 4ml^2$$

$1^\circ \quad D'' - D' = 0:$

$$\left(I_C \omega'' + 2m \cdot v'' \cdot 2l\right) - \left(2m v' \cdot 2l\right) = 0$$

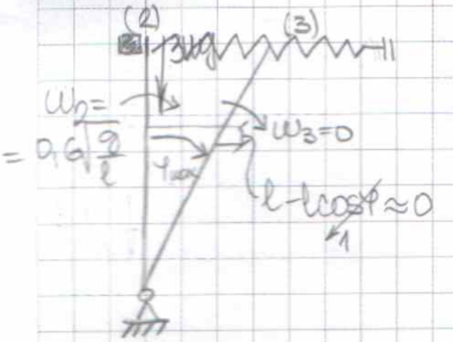
2° udar idealno-plastičan:

$$v'' = \omega'' \cdot 2l$$

$$\boxed{\omega'' = \frac{1}{3} \frac{v'}{l} = 0{,}6\sqrt{\frac{g}{l}}}$$

$$\boxed{v'' = 1{,}2\sqrt{gl}}$$

-kretanje štapa posle udara:



$$T_3 - T_2 = A_{2\to3}$$

$$0 - \left(\frac{1}{2} I_C \cdot \omega_2^2 + \frac{1}{2} 2m v_2^2\right) = -\frac{1}{2} k \delta^2$$

$$\delta = 2l \sin\varphi_{max}$$

$$\boxed{\varphi = 0{,}046\,\text{rad} = 2{,}66^\circ}$$

male oscilacije → φ mali → $\sin\varphi \approx \varphi$

$\cos\varphi \approx 1$

tada se | gleda zajedno!!!